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Diquark Study in Quark Model

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Abstract

To find the diquark correlation in baryons, the baryon spectra with different light-heavy quark combinations are calculated with the help of Gaussian expansion method in naive quark model and chiral quark model. By calculating the diquark energies and separations between any two quarks in baryons, we analyze the diquark effect in the ud-q/Q, us-Q, ss-q/Q, QQ-q/Q($q = u,\ d$, or s; $Q = c,\ b$) systems. The results shown that there are diquark correlations in baryons, especially for qq-Q and QQ-q systems, the same diquark has almost the same energy and size in different baryons. For the orbital ground states of baryons, compared to the vector-isovector diquark, the scalar-isoscalar diquarks have lower energy and smaller size, making them good diquarks. For QQ-q systems, the larger the mass of Q, the smaller the diquark separation, and the more pronounced the diquark effect. For qq-Q systems, the separation between two light quarks is still larger than the separation between light and heavy quarks, so structure of these diquarks must be considered. By comparing the naive quark model and the chiral quark model, the introducing of meson exchange increases the size of diquark a little in most systems.

Keywords: diquark; baryon; quark model

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1. Introduction

In 1964, Gell-Mann and Zweig independently proposed the quark model, diquark was introduced during this period as an important component for explaining hadron structure [1,2]. In Quantum Chromodynamics (QCD) based on SU_3-color, diquark carries color charge. Due to the confinement of the strong interactions, diquark cannot observed experimentally and can only serve as internal components of hadrons. Understanding the structure of hadrons is a key issue in hadron physics. In early research, diquarks are considered as effective components of hadrons, baryons can be regarded as combinations of one quark and one diquark, diquarks were introduced to simplify the structural analysis of baryons, three-body problem is reduced to two-body one. In this case diquark was treated as a point-like particle. The important motivation for this treatment is to tackle the problem of missing states [3], the number of baryon states predicted by quark model is are much higher the states reported by experiments. The further theoretical studies indicated that diquarks possess spatial extension and cannot be simply regarded as point-like particles. So the modern studies of diquarks are focused on the quark+quark (diquark) correlations and emphasize the dynamical nature of diquarks [3–8]. Lattice QCD simulations supported the existence of diquark correlation [7,10]. Dyson-Schwinger equations and Bethe-Salpeter equations approach calculated the masses of mesons and diquarks, argued that two systems have similar behaviors. The comparative study of ground and excited states of light octet and decuplet baryons in three-body Faddeev framework and quark+diquark approximation showed that two approaches gave mutually consistent results [11]. Experiment also found signals for diquark correlations in the flavor-separation of the proton's electromagnetic form factors [9]. However, whether diquarks should be understood only as mathematical tools or as "physics" degrees of freedom in the hadrons is still in debate and under study. For more detailed information, good review articles [3,5], can be referred.

In the present work, a powerful method for few-body systems, Gaussian expansion method (GEM) [12], is employed to investigate the masses of the three-body systems, baryons in the framework of quark models. After obtaining the wavefunctions of the systems, the separations between any two quarks, and the masses of diquarks are calculated. By analyzing the separations and the masses of diquark, the diquark correlation are checked.

This paper is organized as follows. In Sec. 2 and Sec. 3, the model Hamiltonian, the wave functions and the calculation method are separately described. The results are given in Sec. 4 and a short summary is given in the last section.

2. Quark Model and Wave Functions

Two types of quark models are employed to do the calculations to check the model dependence of the diquark correlations. One is the naive quark model, where only gluon exchange are used. Another is the chiral quark model, in which the Goldstone bosons and corresponding scalar mesons exchange potentials are introduced.

2.1. The Naive Quark Model(NQM)

The constituent quark model has been successfully applied to describe hadron properties and baryon-baryon interactions. The naive quark model is a relatively simple model among the constituent quark models. In this model, the phenomenological Hamiltonian takes the form of kinetic energy term (T), confinement potential (V^{CON}), and one gluon exchange potential(V^{OGE}). The confinement potential reflects the long-range behavior of QCD, while the short-range behavior of QCD is asymptotically free, which is represented by one-gluon exchange (OGE) interaction potential [13,14].

$$H = T + V_{ij}^{CON} + V_{ij}^{OGE}, \tag{1}$$

$$T = \sum_{i=1}^{3}\left(m_i + \frac{p_i^2}{2m_i}\right) - T_{CM} \tag{2}$$

$$V_{ij}^{CON} = -a_c \boldsymbol{\lambda}_i^c \cdot \boldsymbol{\lambda}_j^c (r_{ij}^2 - V_0) \tag{3}$$

$$V_{ij}^{OGE} = \frac{1}{4}\alpha_s \boldsymbol{\lambda}_i^c \cdot \boldsymbol{\lambda}_j^c \{\frac{1}{r_{ij}} - \frac{\pi}{2}\delta(\mathbf{r}_{ij})(\frac{1}{m_i^2} + \frac{1}{m_j^2} + \frac{4\boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j}{3m_i m_j})\} \tag{4}$$

where λ_c, σ are the SU_3 color and SU_2 spin matrices; T_{CM} is the center-of-mass kinetic energy; α_s is the quark-gluon coupling constant. However, in a non-relativistic quark model, the wide energy range covered to describe the systems with light, strange and heavy quarks requires an effective scale-dependent strong coupling constant α_s that cannot be obtained from the usual one-loop expression of the running coupling constant because it diverges when $Q \to \Lambda_{QCD}$. So we use an effective scale-dependent strong coupling constant explained by Ref. [15].

$$\alpha_s = \frac{\alpha_0}{\ln\left(\frac{\mu^2}{\Lambda_0^2} + \frac{\mu_0^2}{\Lambda_0^2}\right)}, \tag{5}$$

where μ is the reduced mass of two interacting quarks and α_0, μ_0 and Λ_0 are model parameters. For the confinement potential V_{ij}^{CON}, quadratic form is used in our calculations. The δ function, arising as a consequence of the non-relativistic reduction of the one-gluon exchange diagram between point-like particles, has to be regularized in order to perform exact calculations. It reads [16,17]

$$\delta(\mathbf{r}_{ij}) = \frac{1}{\beta^3 \pi^{3/2}} e^{-r_{ij}^2/\beta^2}, \tag{6}$$

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where β is a parameter.

2.2. The Chiral Quark Model (ChQM)

The Salamanca version of ChQM is chosen as a representative of chiral quark models [18,19]. It has been successfully applied to describe both hadron spectroscopy and hadron-hadron interactions. The model details can be found in Refs. [18,19]. Here only the Hamiltonian in the baryon-baryon sector is given below.

$$H = T + V_{ij}^{\rm CON} + V_{ij}^{\rm OGE} + V_{ij}^{\rm OBE} \quad (7)$$

$$T = \sum_{i=1}^{3}\left(m_i + \frac{p_i^2}{2m_i}\right) - T_{CM} \quad (8)$$

The kinetic energy term (T) is same as the naive quark model.

Compared to the confinement potential in the NQM, the ChQM employs a screened confinement, introducing an additional parameter μ_c.

$$V_{ij}^{\rm CON} = \left[-a_c(1-e^{-\mu_c r_{ij}}) + V_0\right](\lambda_i^c \cdot \lambda_j^c), \quad (9)$$

$$V_{ij}^{\rm OGE} = \frac{1}{4}\alpha_s(\lambda_i^c\cdot\lambda_j^c)\left[\frac{1}{r_{ij}} - \frac{(\sigma_i\cdot\sigma_j)}{6m_i m_j}\frac{e^{-r_{ij}/r_0(\mu)}}{r_{ij}r_0^2(\mu)}\right], \quad (10)$$

where the contact term has been regularized as

$$\delta(\mathbf{r}_{ij}) \sim \frac{1}{4\pi r_0^2}\frac{e^{-r_{ij}/r_0}}{r_{ij}}. \quad (11)$$

The ChQM is based on the fact that a nearly massless current light quark acquires a dynamical, momentum-dependent mass, namely, the constituent quark mass due to its interaction with the gluon medium. To preserve chiral invariance of the QCD Lagrangian new interaction terms, given by Goldstone-boson exchanges, should appear between constituent quarks. The partner of Goldstone boson, scalar mesons also appear. Therefore, the chiral part of the quark-quark interaction can be expressed as follows:

$$\begin{aligned}
V_{ij}^{\rm OBE} &= (v_{ij}^{\pi} + v_{ij}^{a_0})\sum_{a=1}^{3}\lambda_i^{f,a}\lambda_j^{f,a} + (v_{ij}^{K} + v_{ij}^{\kappa})\sum_{a=4}^{7}\lambda_i^{f,a}\lambda_j^{f,a} \\
&\quad + (v_{ij}^{\eta}\cos\theta_P + v_{ij}^{f_0})\lambda_i^{f,8}\lambda_j^{f,8} + (-v_{ij}^{\eta}\sin\theta_P + v_{ij}^{\sigma})\lambda_i^{f,0}\lambda_j^{f,0}, \quad (12)\\
v^{\chi}(\mathbf{r}_{ij}) &= \frac{g_{ch}^2}{4\pi}\frac{m_\chi^2}{12m_i m_j}\frac{\Lambda_\chi^2}{\Lambda_\chi^2 - m_\chi^2}m_\chi\left[Y(m_\chi r_{ij}) - \frac{\Lambda_\chi^3}{m_\chi^3}Y(\Lambda_\chi r_{ij})\right](\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j), \quad \chi = \pi, K, \eta, \\
v^{s}(\mathbf{r}_{ij}) &= -\frac{g_{ch}^2}{4\pi}\frac{\Lambda_s^2}{\Lambda_s^2 - m_s^2}m_s\left[Y(m_s r_{ij}) - \frac{\Lambda_s}{m_s}Y(\Lambda_s r_{ij})\right], \quad s = \sigma, a_0, \kappa, f_0.
\end{aligned}$$

where $\lambda^{f,a}$ is a-th Gell-Mann matrix of flavor SU_3^f. $\lambda^{f,0}$ is just the 3×3 identity matrix multiplied by a factor of $\sqrt{2/3}$ which is according to the normalization property of Gell-Mann matrices. In fact, The different terms of the OBE potential contain central, tensor and spin-orbit contributions; only the central ones will be considered attending the goal of the present manuscript and for clarity in our discussion.

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Table 1. Model parameters.

Model		NQM	ChQM
Quark mass	m_u (MeV)	313	313
	m_d (MeV)	313	313
	m_s(MeV)	589	555
	m_c(MeV)	1860	1620
	m_b(MeV)	5209	5030
Confinement	a_c (MeV)	60.845	202.1
	μ_c (fm^{-1})	-	0.677
	V_0 (MeV)	21.38	64.57
OGE	α_0	5.02	0.852
	Λ_0 (fm^{-1})	0.1874	1.8445
	μ_0 (MeV)	109.298	659.93
	β	0.485	-
	r_0 (MeV fm)	-	40.73
Goldstone boson	m_π (fm^{-1})	-	0.70
	m_K (fm^{-1})	-	2.51
	m_η (fm^{-1})	-	2.77
	$\Lambda_\pi = \Lambda_\sigma$ (fm^{-1})	-	4.20
	Λ_η (fm^{-1})	-	5.20
	Λ_K (fm^{-1})	-	5.20
	$\theta_P(^\circ)$	-	-15
	$g_{ch}^2/(4\pi)$	-	0.54
SU(3)	m_σ (fm^{-1})	-	3.42
Scalar nonet	Λ_s (fm^{-1})	-	5.20
$s = \sigma, a_0, \kappa, f_0$	m_s (fm^{-1})	-	4.97

3. Wave Functions

As for the baryon's wave function, each quark has color, spin, flavor and spatial degrees-of-freedom. According to the empirical fact that color sources have never seen as isolated particles, the color wave function of a baryon must be color singlet, which can be easily written as

$$\chi^c = \frac{1}{\sqrt{6}}(rgb - rbg + gbr - grb + brg - bgr). \tag{13}$$

The spin wave functions $\chi^\sigma_{SM_S}$ of a 3-quark system taking into account all possible quantum number combination are as below.

$$\chi^\sigma_{\frac{3}{2},\frac{3}{2}}(3) = \alpha\alpha\alpha, \tag{14}$$

$$\chi^{\sigma_1}_{\frac{1}{2},\frac{1}{2}}(3) = \frac{1}{\sqrt{6}}(2\alpha\alpha\beta - \alpha\beta\alpha - \beta\alpha\alpha), \tag{15}$$

$$\chi^{\sigma_2}_{\frac{1}{2},\frac{1}{2}}(3) = \frac{1}{\sqrt{2}}(\alpha\beta\alpha - \beta\alpha), \tag{16}$$

The charm and bottom quarks are much heavier than the light ones: u, d and s quark. Therefore, we investigate the baryon with quark content u, d, s and c or b in $SU(3)$-flavor case and the corresponding flavor wave functions $\chi^f_{IM_I}$ are given by

$$\chi^{N_1}_{\frac{1}{2}\frac{1}{2}} = \frac{1}{\sqrt{6}}(2uud - udu - duu), \tag{17}$$

$$\chi^{N_2}_{\frac{1}{2}\frac{1}{2}} = \frac{1}{\sqrt{2}}(ud - du)u, \quad \chi^\Delta = uuu, \tag{18}$$

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$$\chi_{00}^{\Lambda_1} = \frac{1}{2}(usd - dsu + sud - sdu), \tag{19}$$

$$\chi_{00}^{\Lambda_2} = \frac{1}{\sqrt{12}}(2uds - 2dus + usd - dsu - sud + sdu), \tag{20}$$

$$\chi_{10}^{\Sigma_1} = \frac{1}{\sqrt{12}}(2uds + 2dus - usd - dsu - sud - sdu), \tag{21}$$

$$\chi_{10}^{\Sigma_2} = \frac{1}{2}(usd - sud + dsu - sdu), \tag{22}$$

$$\chi_{10}^{\Sigma^*} = \frac{1}{\sqrt{6}}(uds + usd + dus + dsu + sud + sdu), \tag{23}$$

$$\chi_{\frac{1}{2}\frac{1}{2}}^{\Xi_1} = \frac{1}{\sqrt{6}}(uss + sus - 2ssu), \tag{24}$$

$$\chi_{\frac{1}{2}\frac{1}{2}}^{\Xi_2} = \frac{1}{\sqrt{2}}(us - su)s, \tag{25}$$

$$\chi_{\frac{1}{2}\frac{1}{2}}^{\Xi^*} = \frac{1}{\sqrt{3}}(uss + sus + ssu), \tag{26}$$

$$\chi_{00}^{\Omega} = sss. \tag{27}$$

For the light-heavy and full heavy baryons where Q represents either c- or b-quark, the flavor wave functions are given by

$$\chi_{00}^{\Lambda_Q} = \frac{1}{2}(ud - du)Q, \tag{28}$$

$$\chi_{10}^{\Sigma_Q} = \frac{1}{2}(ud + du)Q, \tag{29}$$

$$\chi_{\frac{1}{2}\frac{1}{2}}^{\Xi_Q} = \frac{1}{2}(us - su)Q, \tag{30}$$

$$\chi_{\frac{1}{2}\frac{1}{2}}^{\Xi'_Q} = \frac{1}{2}(us + su)Q, \tag{31}$$

$$\chi_{00}^{\Omega_Q} = ssQ, \tag{32}$$

$$\chi_{\frac{1}{2}\frac{1}{2}}^{\Xi_{QQ}} = uQQ, \tag{33}$$

$$\chi_{00}^{\Omega_{QQ}} = sQQ, \tag{34}$$

$$\chi_{00}^{\Omega_{QQQ}} = QQQ, \tag{35}$$

The total wave functions of baryons are

$$\Psi_{IM_IJM_J} = \mathcal{A}\Big[[\psi_L(\boldsymbol{\rho},\boldsymbol{\lambda})\chi_S^{\sigma}]_{JM_J}\chi_{00}^{c}\chi_{IM_I}^{f}\Big], \tag{36}$$

where $\psi_{LM_L}(\boldsymbol{\rho},\boldsymbol{\lambda})$ is the spatial wavefunction, $\boldsymbol{\rho},\boldsymbol{\lambda}$ are Jacobi coordinates which are defined as,

$$\boldsymbol{\rho} = \mathbf{r}_1 - \mathbf{r}_2, \quad \boldsymbol{\lambda} = \mathbf{r}_3 - \frac{m_1\mathbf{r}_1 + m_2\mathbf{r}_2}{m_1 + m_2}. \tag{37}$$

$\mathcal{A}$ is the antisymmetrization operators, $\mathcal{A} = 1 - (13) - (23)$ for three identical particles, $\mathcal{A} = 1$ for other cases. Because the permutation symmetry of the first two-particle has been considered by choosing the appropriate wave functions of color, spin, flavor and spatial degrees of freedom.

Among the different methods to solve the three-body Schrödinger equation we use the Rayleigh–Ritz variational principle, which is one of the most extended tools to solve eigenvalue problems due to its simplicity and flexibility. However, it is of great importance how to choose the basis on which to expand the wave function. In this work, we choose a set of gaussians to expand the radial part of the spatial wave function. So the spatial wave function of a 3-quark system is written as follows:

$$\psi_{LM_L} = \left[\phi_{l_1}(\boldsymbol{\rho})\phi_{l_2}(\boldsymbol{\lambda})\right]_{LM_L}. \tag{38}$$

$$\phi_{l_1 m_1}(\boldsymbol{\rho}) = \sum_{n_1=1}^{n_{max}} c_{n_1 l_1} N_{n_1 l_1} \rho^{l_1} e^{-\nu_{n_1}\rho^2} Y_{l_1 m_1}(\hat{\rho}), \tag{39}$$

$$N_{n_1 l_1} = \left[\frac{2^{l_1+2}(2\nu_{n_1})^{l_1+\frac{3}{2}}}{\sqrt{\pi}(2l_1+1)!!}\right]^{\frac{1}{2}}. \tag{40}$$

This choice is convenient because, for a nonrelativistic system, the center-of-mass kinetic term T_{CM} can be completely eliminated. To deal with the complicate case, orbital angular momentum is not zero, the infinitesimally-shifted Gaussians (ISG) can be employed [**?**],

$$\phi_{lm}(\boldsymbol{\rho}) = \sum_{n=1}^{n_{max}} c_{nl} N_{nl} \lim_{\varepsilon \to 0} \frac{1}{(\nu_n \varepsilon)^l} \sum_{k=1}^{k_{max}} C_{lm,k} e^{-\nu_n (r - \varepsilon D_{lm,k})^2}, \tag{41}$$

where the limit $\varepsilon \to 0$ must be carried out after the matrix elements have been calculated analytically. This new set of basis functions makes the calculation of three- and, in general, few-body matrix elements very easy without the laborious Racah algebra. Moreover, all the advantages of using Gaussians remain with the new basis functions. In order to make the calculation tractable, the sizes of gaussians are arranged in a geometric progression,

$$\nu_n = \frac{1}{r_n^2}, \quad r_n = r_1 a^{n-1}, \quad a = \left(\frac{r_{n_{max}}}{r_1}\right)^{\frac{1}{n_{max}-1}}. \tag{42}$$

By using Rayleigh–Ritz variational principle, the three-body Schrödinger equation can be reduced to the following generalized eigen-equation,

$$\sum_{n=1}^{n_{max}} (H_{n'n} - E N_{n'n}) C_n = 0, \quad n = (n_1, n_2). \tag{43}$$

$$\begin{aligned} H_{n'n} = & \Big\langle N_{n_1' l_1'} N_{n_2' l_2'} \rho^{l_1'} \lambda^{l_2'} e^{-\nu_{n_1'}\rho^2} e^{-\nu_{n_2'}\lambda^2} \left[[Y_{l_1'}(\hat{\rho}) Y_{l_2'}(\hat{\lambda})]_{L'} \chi_S^{\sigma} \right]_{JM_J} \chi^c \chi^f_{IM_I} \\ & |H| N_{n_1 l_1} N_{n_2 l_2} \rho^{l_1} \lambda^{l_2} e^{-\nu_{n_1}\rho^2} e^{-\nu_{n_2}\lambda^2} \left[[Y_{l_1}(\hat{\rho}) Y_{l_2}(\hat{\lambda})]_{L} \chi_S^{\sigma} \right]_{JM_J} \chi^c \chi^f_{IM_I} \Big\rangle, \end{aligned} \tag{44}$$

$$\begin{aligned} N_{n'n} = & \Big\langle N_{n_1' l_1'} N_{n_2' l_2'} \rho^{l_1'} \lambda^{l_2'} e^{-\nu_{n_1'}\rho^2} e^{-\nu_{n_2'}\lambda^2} \left[[Y_{l_1'}(\hat{\rho}) Y_{l_2'}(\hat{\lambda})]_{L'} \chi_S^{\sigma} \right]_{JM_J} \chi^c \chi^f_{IM_I} \\ & \Big| N_{n_1 l_1} N_{n_2 l_2} \rho^{l_1} \lambda^{l_2} e^{-\nu_{n_1}\rho^2} e^{-\nu_{n_2}\lambda^2} \left[[Y_{l_1}(\hat{\rho}) Y_{l_2}(\hat{\lambda})]_{L} \chi_S^{\sigma} \right]_{JM_J} \chi^c \chi^f_{IM_I} \Big\rangle. \end{aligned} \tag{45}$$

After obtaining the eigen-energie E and eigen-function Ψ^E of a baryon, the energy and the size of diquark can be calculated as,

$$E_{12} = \langle \Psi^E | H_{12} | \Psi^E \rangle, \tag{46}$$

$$\sqrt{r_{12}^2} = \sqrt{\langle \Psi^E | r_{12}^2 | \Psi^E \rangle}. \tag{47}$$

4. The Results and Discussions

Before the numerical calculation, we discuss the properties of diquark in a baryon analytically. To simplify the discussion, the orbital angular momentum between two quarks is set to 0, the ground state diquark. Because of the requirement of color singlet, only symmetric flavor-spin diquarks are allowed in a baryon. There are two types of diquark, one is spin scalar with flavor antisymmetric, another

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is spin vector with flavor symmetric. In the constituent quark model, the confinement potential is responsible for confining the quarks in a baryon, it is proportional to operator $-\boldsymbol{\lambda}_i \cdot \boldsymbol{\lambda}_j$. Applying to color-antisymmetric quark pair, the operator gives $\frac{8}{3}$. The contribution of confinement potential to the energy of the diquark increases with the increasing separation between two quarks. It has the effect of confinement. For the one-gluon-exchange potential, the first term is color-Coulomb with the color operator $\boldsymbol{\lambda}_i \cdot \boldsymbol{\lambda}_j$, the factor $-\frac{8}{3}$ makes the attraction of color-Coulomb term. The second term is color magnetic interaction (CMI), it has color-spin operator $-\boldsymbol{\lambda}_i \cdot \boldsymbol{\lambda}_j \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j$, it gives $-(-\frac{8}{3}) \times (-3) = -8$ for scalar diquark, and $-(-\frac{8}{3}) \times 1 = \frac{8}{3}$ for vector diquark. So CMI lowers the energies of scalar diquarks and lifts the energies of vector diquarks.

For the one-boson-exchange potential, the situation is complicate. The spatial part of the Goldstone-boson exchange interaction is $Y(m_\chi r) - \left(\frac{\Lambda_\chi}{m_\chi}\right)^3 Y(\Lambda_\chi r)$ with $\Lambda_\chi > m_\chi$, it is negative for the small separation ($r < r_0 = \frac{2(\ln \Lambda_\chi - \ln m_\chi)}{\Lambda_\chi - m_\chi}$, for π, $r_0 = 1.02$ fm, for K, $r_0 = 0.54$ fm) and positive for the large separation. The matrix elements of flavor operators on light diquarks are shown in Table 2. Combining four degrees-of-freedom, one can see that the Goldstone-boson exchange potentials are negative for the small separation between two quarks, and are positive for large separation between two quarks. The contributions of Goldstone-bosons are attractive or repulsive depending on the wave function of diquarks. For the scalar meson exchange, the spatial part is positive. so the contributions of scalar nonet are universally attractive.

From the above analyse, one can see that the “best” diquark is the one with antisymmetric color, spin and flavor, $(ud - du)/\sqrt{2}$, in which all the potentials are attractive.

Table 2. Matrix elements of flavor operators on light diquarks.

diquarks	flavor operators $\sum_{a=1}^{3} \lambda_i^{f,a} \lambda_j^{f,a}$	$\sum_{a=4}^{7} \lambda_i^{f,a} \lambda_j^{f,a}$	$\lambda_i^{f,8} \lambda_j^{f,8}$	$\lambda_i^{f,0} \lambda_j^{f,0}$
uu	1	0	$\frac{1}{3}$	$\frac{2}{3}$
$\sqrt{\frac{1}{2}}(ud + du)$	1	0	$\frac{1}{3}$	$\frac{2}{3}$
dd	1	0	$\frac{1}{3}$	$\frac{2}{3}$
$\sqrt{\frac{1}{2}}(ud - du)$	-3	0	$\frac{1}{3}$	$\frac{2}{3}$
$\sqrt{\frac{1}{2}}(us + su)$	0	2	$-\frac{2}{3}$	$\frac{2}{3}$
$\sqrt{\frac{1}{2}}(us - su)$	0	-2	$-\frac{2}{3}$	$\frac{2}{3}$
ss	0	0	$\frac{4}{3}$	$\frac{2}{3}$

In the following, two quark models, NQM [17] and ChQM [20], are used to do the numerical calculations. The model parameters are fixed by fitting orbital ground state baryons and are listed in Table 1. The GEM parameters are determined by requiring the convergence of the results, $r_1 = 0.1$ fm, $r_{max} = 3$ fm and $n_{max} = 12$. The calculated results are shown in Tables 3–6. In the following, we discuss the results in detail.

Table 3. Mass spectra (unit: MeV) and the distances (unit: fm) between two quarks of ud-diquark in baryons.

Baryon $I(J^P)$	Exp.	Theo.		E_{12}		r_{12}		r_{13}	
		NQM	ChQM	NQM	ChQM	NQM	ChQM	NQM	ChQM
$N(uud)\ \frac{1}{2}(\frac{1}{2}^+)$	939	939	939	765	824	0.729	0.674	0.729	0.674
$\Delta(uuu)\ \frac{3}{2}(\frac{3}{2}^+)$	1232	1232	1232	826	812	0.939	1.203	0.939	1.203
$\Lambda(uds)\ \frac{1}{2}(\frac{1}{2}^+)$	1116	1150	1206	769	813	0.708	0.694	0.610	0.622
$\Sigma(uds)\ \frac{1}{2}(\frac{1}{2}^+)$	1193	1172	1302	778	831	0.719	0.779	0.609	0.672
$\Lambda_c(udc)\ 0(\frac{1}{2}^+)$	2286	2288	2246	675	674	0.592	0.598	0.482	0.594
$\Sigma_c(udc)\ 1(\frac{1}{2}^+)$	2455	2471	2416	846	823	0.812	1.034	0.545	0.736
$\Lambda_b(udb)\ 0(\frac{1}{2}^+)$	5620	5608	5616	678	677	0.585	0.577	0.442	0.515
$\Sigma_b(udb)\ 1(\frac{1}{2}^+)$	5811	5816	5811	845	821	0.817	0.976	0.521	0.634
$\Sigma^*(uds)\ \frac{3}{2}(\frac{3}{2}^+)$	1383	1363	1397	835	825	0.881	1.095	0.737	0.936
$\Sigma_c^*(udc)\ 1(\frac{3}{2}^+)$	2520	2523	2456	839	811	0.845	1.133	0.581	0.835
$\Sigma_b^*(udb)\ 1(\frac{3}{2}^+)$	5830	5835	5826	842	817	0.829	1.075	0.533	0.722

The Table 3 shows the mass spectra and the distances between two quarks of ud-q/Q system. When the ud orbital is in the ground state, the scalar diquark with color, spin, and flavor wave functions being all antisymmetric is the "best" diquark, resulting in a lower energy E_{12} for these systems such as $N(uud)$, $\Lambda(uds)$, $\Lambda_c(udc)$, $\Lambda_b(udb)$. For $\Lambda_c(udc)$ and $\Lambda_b(udb)$, the energies of diquark are almost same, 675 MeV and 678 MeV in NQM, 674 MeV and 677 MeV in ChQM, and the separations have the same behavior, 0.592 fm and 0.585 fm in NQM, 0.598 fm 0.577 fm in ChQM. However, the separations between two light quarks are still larger than the separations between light and heavy quarks. So the point-like approximation of diquark is not a good one, even for the "best" diquark. For baryons $N(uud)$ and $\Lambda(uds)$, the masses of diquarks are a little larger, due to the using of SU_3^f symmetry, in which all three particles are identical. For the uu or ud diquark (vector diquark) in baryons $\Delta(uuu)$, $\Sigma(uds)$, $\Sigma_c(udc)$, $\Sigma_b(udb)$, $\Sigma^*(uds)$, $\Sigma_c^*(udc)$, $\Sigma_b^*(udb)$, the masses of diquark are in the range, 826$\sim$ 846 MeV, about 170 MeV higher than the masses of "best" diquark. The separations between two quarks in vector diquarks are also larger compared to scalar diquark. The differences can be explained by CMI and Goldstone-boson-exchange, which have larger contribution to the energy in the vector diquark than that in the scalar diquark. Our results are also show that the heavier the Q, the smaller the diquark, and the more pronounced the diquark effect. Generally the size of diquark in ChQM is a little larger than that in NQM, this effect may come from the different model parameters.

Table 4. Mass spectra (unit: MeV) and the distances (unit: fm) between two quarks of us-diquark in baryons.

Baryon $I(J^P)$	Exp.	Theo.		E_{12}		r_{12}		r_{13}	
		NQM	ChQM	NQM	ChQM	NQM	ChQM	NQM	ChQM
$\Xi_c(usc)\ \frac{1}{2}(\frac{1}{2}^+)$	2470	2504	2511	953	966	0.516	0.597	0.574	0.474
$\Xi_c'(usc)\ \frac{1}{2}(\frac{1}{2}^+)$	2578	2601	2571	1052	1028	0.617	0.794	0.634	0.524
$\Xi_c^*(usc)\ \frac{1}{2}(\frac{3}{2}^+)$	2645	2644	2613	1044	1015	0.640	0.874	0.558	0.794
$\Xi_b(usb)\ \frac{1}{2}(\frac{1}{2}^+)$	5797	5817	5880	956	970	0.506	0.572	0.440	0.534
$\Xi_b'(usb)\ \frac{1}{2}(\frac{1}{2}^+)$	5935	5935	5963	1052	1026	0.615	0.796	0.496	0.649
$\Xi_b^*(usb)\ \frac{1}{2}(\frac{3}{2}^+)$	-	5951	5979	1050	1031	0.624	0.826	0.506	0.682

In Table 4 the mass spectra and the distances between two light quarks of us-q/Q system are listed. Similar to the above discussion, the scalar us diquarks have lower energy, 953$\sim$ 956 MeV in NQM, than that of vector diquark, 1044$\sim$ 1052 MeV. The energy difference $\sim$ 100 MeV between vector and scalar us diquark is smaller than that of ud diquark. The separation has the similar behavior.

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Table 5. Mass spectra (unit: MeV) and the distances (unit: fm) between two quarks of ss-diquark in baryons.

Baryon $I(J^P)$	Exp.	Theo.		E_{12}		r_{12}		r_{13}	
		NQM	ChQM	NQM	ChQM	NQM	ChQM	NQM	ChQM
$\Xi(ssu)$ $\frac{1}{2}(\frac{1}{2}^+)$	1315	1341	1438	1202	1229	0.486	0.528	0.577	0.608
$\Xi^*(ssu)$ $\frac{3}{2}(\frac{3}{2}^+)$	1530	1502	1559	1222	1200	0.580	0.722	0.673	0.826
$\Omega(sss)$ $\frac{3}{2}(\frac{3}{2}^+)$	1672	1613	1684	1238	1214	0.492	0.605	0.492	0.605
$\Omega_c(ssc)$ $0(\frac{1}{2}^+)$	2695	2729	2713	1248	1218	0.438	0.568	0.330	0.450
$\Omega_c^*(ssc)$ $0(\frac{3}{2}^+)$	2766	2763	2757	1244	1204	0.450	0.625	0.344	0.519
$\Omega_b(ssb)$ $0(\frac{1}{2}^+)$	6045	6052	6103	1252	1217	0.429	0.526	0.292	0.359
$\Omega_b^*(ssb)$ $0(\frac{3}{2}^+)$	-	6064	6121	1250	1201	0.568	0.591	0.400	0.423

Table 5 shows the mass spectra and the distances between two quarks of ss-q/Q system. In this case, only one type of orbital ground state diquark allowed, so the energies E_{12} and the separation between two s quarks r_{12} are all similar for different baryons. The separation between two s quarks is still larger than the separation between s and heavy quarks, so the point-like particle approximation is still a rough one.

Table 6. Mass spectra (unit: MeV) and the distances (unit: fm) between two quarks of cc-diquark and bb-diquark in baryons.

Baryon J^P	Exp.	Theo.		E_{12}		r_{12}		r_{13}	
		NQM	ChQM	NQM	ChQM	NQM	ChQM	NQM	ChQM
$\Xi_{cc}(ccu)$ $\frac{1}{2}^+$	3622	3698	3574	3607	3235	0.196	0.321	0.448	0.644
$\Xi_{cc}^*(ccu)$ $\frac{3}{2}^+$	-	3761	3624	3604	3258	0.205	0.353	0.490	0.757
$\Omega_{cc}(ccs)$ $\frac{1}{2}^+$	-	3842	3728	3614	3243	0.179	0.295	0.293	0.432
$\Omega_{cc}^*(ccs)$ $\frac{3}{2}^+$	-	3879	3777	3611	3261	0.184	0.328	0.308	0.511
$\Omega_{ccb}(ccb)$ $\frac{1}{2}^+$	-	8243	8107	3647	3275	0.139	0.270	0.108	0.206
$\Omega_{ccb}^*(ccb)$ $\frac{3}{2}^+$	-	8247	8118	3646	3272	0.140	0.280	0.109	0.216
$\Xi_{bb}(bbu)$ $\frac{1}{2}^+$	-	10268	10313	10183	10013	0.075	0.143	0.396	0.563
$\Xi_{bb}^*(bbu)$ $\frac{3}{2}^+$	-	10292	10233	10183	9961	0.076	0.120	0.411	0.531
$\Omega_{bb}(bbs)$ $\frac{1}{2}^+$	-	10383	10463	10187	10015	0.068	0.132	0.229	0.354
$\Omega_{bb}^*(bbs)$ $\frac{3}{2}^+$	-	10397	10364	10186	9964	0.069	0.112	0.234	0.326
$\Omega_{bbc}(bbc)$ $\frac{1}{2}^+$	-	11458	11457	10201	10019	0.056	0.123	0.093	0.189
$\Omega_{bbc}^*(bbc)$ $\frac{3}{2}^+$	-	11463	11470	10201	10017	0.056	0.128	0.093	0.201
$\Omega_{ccc}(ccc)$ $\frac{3}{2}^+$	-	4980	4751	3636	3269	0.160	0.302	0.160	0.302
$\Omega_{bbb}(bbb)$ $\frac{3}{2}^+$	-	14640	14819	10229	10023	0.047	0.119	0.047	0.119

Table 6 gives the mass spectra and the distances between two quarks of QQ-q/Q system. Same as the system ss-q/Q, there is only one type of orbital ground state diquark, vector diquark. The values of energy E_{12} of cc (bb), $\sim$ 3610 (10200) MeV and the separation r_{12}, $\sim$ 0.2 (0.1) fm are similar for different baryons. diquark correlation is clear. Only for the heavy diquark, the separations between two heavy quarks are smaller than that of heavy-light quarks.

5. Summary

We use the Gaussian Expansion Method to dynamically calculate the baryon spectra and the distances between quarks in various heavy-light quark combinations. By analyzing the energy and separation of two-particle systems, we study the diquark effect in systems such as ud-q/Q, us-Q, ss-q/Q, and QQ–q/Q (where $q = u$, d, or s; $Q = c$ or b).

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The results show that the same diquark almost has the same energy and the same size, which means that diquark correlation really exists in baryons. When the *ud*, *us* orbital is in the ground state, the color, spin, and flavor wave functions are all antisymmetric, leading to lower energy and smaller quark separations, making these systems good diquarks. For the diquark with the same flavor, *uu*, *ss*, and *QQ*, there is only one type of orbital ground state diquark, which the spin is 1. They have higher energy and larger separation that that of scalar diquark, which spin is 0. The diquark effect is more pronounced with larger *Q* values. However, the hierarchy of the separations between two quarks is the same as the hierarchy of quark mass, the smaller the separation, the heavier the quark mass. In most case, the separations of diquark are not small enough to take the diquark as a point-like particle.

In baryon models, the structure of these diquarks must be considered. Comparing the Naive Quark Model (NQM) and the Chiral Quark Model (ChQM), we find that introducing meson exchange in ChQM generally increases the distance between quarks in most systems.

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