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The Quantum Measurement Problem and Two Famous Questions

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Abstract: By revisiting two famous questions concerning the Copenhagen interpretation of quantum mechanics, this article presents a concise analysis of the quantum measurement problems with measuring both microscopic and macroscopic objects. The method used here is mainly based on the concept "isolated point" in "point-set topology". The findings reported are as follows. (a) Einstein's argument has been misunderstood; he was opposed to the so-called "inherently probabilistic nature" attached by the Copenhagen interpretation to quantum mechanics rather than to the use of probability in quantum mechanics. (b) Probability used in Einstein's ensemble interpretation is identical to the quantum-mechanically calculated probability. (c) The wave-functions in Einstein's ensemble interpretation neither describe any single quantum object purportedly possessing mutually exclusive properties simultaneously when nobody looks nor collapse abruptly when an observer performs a measurement.

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1. Introduction

This article revisits two famous questions concerning the Copenhagen interpretation of quantum mechanics and presents a concise analysis of the quantum measurement problems with measuring both microscopic and macroscopic objects [1–3]. The method used here is mainly based on the concept "isolated point" in "point-set topology". Only familiar with real numbers and with the usual distance function defined on the set of real numbers is needed to understand the point-set topological analysis of the measurement problem.

The measurement problem concerns the measurement of a quantum-mechanically described *single* object, which can be a microscopic object or a macroscopic object. Unlike any microscopic object, a macroscopic object can be measured repeatedly. Both kinks of objects are expressed by quantum superpositions. The legitimacy of quantum-mechanical descriptions is the essence of the Einstein-Bohr debate [4,5].

2. Two Famous Questions

In the Einstein-Bohr debate, two questions concerning the Copenhagen interpretation of quantum mechanics are very famous. The questions are famous, because they bothered Einstein very much in his debate with Bohr.

- Question 1: Why are outcomes obtained by measuring a single quantum object inherently probabilistic rather than deterministic?
- Question 2: Can a single quantum object possess exclusive properties *simultaneously* when nobody looks?

The Copenhagen interpretation does not provide any reasonable answer to Question 1. For a single object described by a quantum superposition, the Copenhagen interpretation gives an *affirmative* answer to Question 2. Einstein disagreed with the Copenhagen interpretation and argued against quantum superpositions used to describe any object in the real world. Regrettably, Einstein's argument

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has been misunderstood, mainly because the experimental results of testing Bell inequalities are misinterpreted. Einstein was not opposed to the use of probability in quantum mechanics; he was opposed to the so-called "inherently probabilistic nature" attached by the Copenhagen interpretation to quantum mechanics. The present study concerns only Einstein's argument in the Einstein-Bohr debate. Bell inequalities and the corresponding experiments have been discussed intensively in the existing literature (for example, see [6–13]) and will not be considered here.

Questions mentioned above not only disturbed Einstein; they also disturb some people nowadays. For example, the answer to Question 1 given in the existing literature goes as follows [14]: "this is simply the way the world is. Where is it written that the laws of nature have to be deterministic?" Although this answer is standard and influential, it has not convinced everybody. Nevertheless, because the calculations in quantum mechanics are indeed unbelievably successful, people who used to worry the "inherently probabilistic nature of quantum mechanics" now just "shut up and calculate." They simply avoid considering any disturbing question and do not bother themselves anymore. However, the disturbing questions deserve reasonable answers. As shown below, a reasonable answer to Question 1 and a *negative* answer to Question 2 have been found.

3. Assumptions Underlying the Copenhagen Interpretation

The Copenhagen interpretation and its experimental verification rely on the following two assumptions.

- Assumption 1: Time modeled by $\mathbb{R}$, the set of real numbers (or its subsets) endowed with the usual point-set topology, could be measured perfectly precisely.
- Assumption 2: To characterize randomness involved in measuring a single quantum object, an observer needs to measure the object only once.

Assumption 1 is not practically meaningful as indicated by the point-set topological analysis (see below). Assumption 2 violates a necessary condition needed to characterize the randomness. The necessary condition is a banal fact: a large number of repetitions (i.e., runs) of an experiment must be performed to characterize the randomness based on the measurement outcomes obtained in *different* repetitions of the experiment. In one repetition, only one measurement outcome can be obtained, which makes no sense statistically. The above assumptions make the Copenhagen interpretation and its experimental verification questionable.

4. The Point-Set Topological Analysis of the Measurement Problem

Consider first the affirmative answer to Question 2 (see Section 2) given by the Copenhagen interpretation.

4.1. The Measurement Problem with Measuring a Single Microscopic Object

When describing a single microscopic object in general, a quantum superposition, namely, a wave-function, consists of orthonormal vectors spanning an n-dimensional Hilbert space. The orthogonality of superposed vectors is associated with properties possessed by the object. Representing alternative outcomes obtained by measuring the object, the properties are exclusive. There is no limit to the number of superposed orthonormal vectors. For the purpose of the present study, it is sufficient to consider a two-dimensional Hilbert space $\mathcal{H}$.

The Copenhagen interpretation of quantum mechanics, which is defended by Bohr in the Einstein-Bohr debate, may be summarized briefly as follows. Consider a wave-function ψ consisting of orthonormal vectors ψ_1 and ψ_2 spanning $\mathcal{H}$.

$$\psi = c_1\psi_1 + c_2\psi_2$$

where c_1 and c_2 are complex numbers. According to the Copenhagen interpretation, a single microscopic object possesses exclusive properties represented by ψ_1 *and* ψ_2 *simultaneously* when nobody

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measures (or observes) it. For example, if the object is a particle, two of its different energy levels are such mutually exclusive properties. Once an observer performs a measurement on the object, ψ collapses abruptly onto ψ_1 *or* ψ_2 according to the measurement outcome. The outcomes obtained by measurements are inherently probabilistic. The probability of finding the measurement outcome represented by ψ_i is $|c_i|^2, i = 1, 2$.

Denote by $E = \{q_n, n \geq 1\}$ an ensemble of single microscopic objects. The objects are all described by the wave-function ψ. In other words, E is a pure ensemble consisting of microscopic objects characterized by the same pure state. Einstein's argument can be better appreciated in this simple, ideal situation. According to the answer given by the Copenhagen interpretation, each object in E possesses exclusive properties represented by ψ_1 *and* ψ_2 at any time before an observer measures the object. However, this answer relies on Assumption 1 (see Section 3), which does not hold in practice. Thus the affirmative answer and its experimental verification are not practically meaningful.

To justify the above claim, denote by $H(\psi_1, \tau)$ and $H(\psi_2, \tau)$ two propositions, where $H(\psi_1, \tau)$ means "ψ_1 represents the outcome obtained by measuring an element of E at time τ." The meaning of $H(\psi_2, \tau)$ is similar. In $H(\psi_1, \tau)$ and $H(\psi_2, \tau)$, the time τ is fixed. If time could be measured perfectly precisely, $H(\psi_1, \tau)$ and $H(\psi_2, \tau)$ would hold *simultaneously* in the corresponding experiment, which requires τ to be an isolated point of $\mathbb{R}$. By definition, if τ is an isolated point of $\mathbb{R}$ equipped with its usual point-set topology, then there exists a number $r > 0$, such that the distance between τ and any other element of $\mathbb{R}$ is at least r. As can be readily verified, τ is not an isolated point of $\mathbb{R}$. Actually, $\mathbb{R}$ does not have any isolated point.

4.2. The Measurement Problem with Measuring a Macroscopic Object

The measurement problem with measuring a macroscopic object has an analogy in popular science: Schrödinger's cat in a box [1]. Consider a macroscopic object denoted by Q, which has two macroscopically distinguishable states [2]. Let Q^+ and Q^- represent the states. The macroscopic object Q, with its states Q^+ and Q^-, is a popular analogy to the quantum-mechanical description of Schrödinger's cat. According to the affirmative answer given by the Copenhagen interpretation, the cat is both alive and dead simultaneously if nobody lifts the lid of the box and looks inside. As shown below, concerning the measurement problem with measuring this macroscopic object, the answer given by the Copenhagen interpretation and the corresponding experimental verification are not practically meaningful either.

A macroscopic object can be measured repeatedly. Thus, when measurements are performed on Q, the measurements are all performed on the same object. Let $H(Q^+, \tau)$ and $H(Q^-, \tau)$ be two propositions. The proposition $H(Q^+, \tau)$ states: "Q^+ is observed by measuring Q at time τ." The other proposition has a similar meaning. In $H(Q^+, \tau)$ and $H(Q^-, \tau)$, the time τ is fixed in a "time ensemble" needed to measure Q [2]. Analyzing the quantum measurement problem with measuring this macroscopic object is exactly the same as the analysis presented in Subsection 4.1. If time could be measured perfectly precisely, both $H(Q^+, \tau)$ and $H(Q^-, \tau)$ would hold *simultaneously* in the corresponding experiment, which requires τ to be an isolated point of $\mathbb{R}$. However, $\mathbb{R}$ does not have any isolated point as shown by the point-set topological analysis.

Therefore, for the measurement problems with measuring both microscopic and macroscopic quantum objects, the affirmative answer to Question 2 given by the Copenhagen interpretation and the experimental verifications are indeed not practically meaningful, simply because time cannot be measured perfectly precisely in practice.

5. The Origin of Randomness

Now consider the standard answer to Question 1 (see Section 2) given in the literature [14]. This answer and its experimental verification rely on both Assumption 1 and Assumption 2 (see Section 3). The answer given in [14] cannot be considered convincing, because it is actually not a reasonable answer. The point-set topological analysis together with the necessary condition needed to characterize randomness involved in measuring a single quantum object can provide a reasonable

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answer. According to the necessary condition, the randomness can only be characterized based on the measurement outcomes obtained in *different* repetitions of the experiment in question. Needless to say, any single quantum object can possess mutually exclusive properties; it just cannot have such properties *simultaneously*. By no means can mutually exclusive properties of a single quantum object be observed in only one repetition by measuring the object only once. Because time cannot be measured perfectly precisely in practice, mutually exclusive properties of a single quantum object are actually observed at *different* times in *different* repetitions of the corresponding experiment.

For a single microscopic object, its mutually exclusive properties correspond to *different* outcomes obtained by measuring *different* objects of the same kind at *different* times in *different* repetitions. For a macroscopic object, its mutually exclusive properties correspond to *different* outcomes obtained by measuring the same object at *different* times in *different* repetitions. The origin of randomness involved in measuring a quantum-mechanically described single quantum object is concealed by attaching mutually exclusive properties to an *imaginary* object, which does not exist in the real world. Consequently, the "inherently probabilistic nature" is attached by the Copenhagen interpretation to quantum mechanics because of the incorrectly interpreted experimental results. The "inherently probabilistic nature" attached to quantum mechanics can be detached by revealing the origin of the randomness, which is helpful to understand why Einstein argued against using quantum superpositions to describe any object in the real world.

6. Einstein's Argument Grounded on Ensemble Interpretation

In the spirit of Einstein's argument grounded on his ensemble interpretation of wave-functions [15], single microscopic objects of the same kind are measured at *different* times in *different* repetitions of the corresponding experiment; the objects form an ensemble described by the wave-function in question. Each element of the ensemble possesses mutually exclusive properties; however, none of them possesses such properties *simultaneously*. As can be readily seen, Einstein's ensemble interpretation is also applicable to macroscopic objects, and the wave-functions in Einstein's ensemble interpretation neither describe any single object purportedly possessing mutually exclusive properties *simultaneously* when nobody looks nor collapse abruptly when an observer performs a measurement. Einstein was not opposed to the use of probability in quantum mechanics; he was only opposed to the Copenhagen interpretation of quantum mechanics. In Einstein's ensemble interpretation, the use of probability is still needed to characterize the randomness, and probability used in Einstein's ensemble interpretation is identical to the quantum-mechanically calculated probability.

7. Conclusions

Based on the point-set topological analysis together with the necessary condition needed to characterize randomness involved in measuring a single quantum object, this article revisited two famous questions concerning the Copenhagen interpretation of quantum mechanics and analyzed the quantum measurement problems with measuring both microscopic and macroscopic objects. The findings reported are as follows. (a) Einstein's argument has been misunderstood; he was opposed to the so-called "inherently probabilistic nature" attached by the Copenhagen interpretation to quantum mechanics rather than to the use of probability in quantum mechanics. (b) Probability used in Einstein's ensemble interpretation is identical to the quantum-mechanically calculated probability. (c) The wave-functions in Einstein's ensemble interpretation neither describe any single quantum object purportedly possessing mutually exclusive properties simultaneously when nobody looks nor collapse abruptly when an observer performs a measurement.

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References

1. Legget, A. The Quantum Measurement Problem. *Science* **2005**, *307*, 871–872. https://doi.org/10.1126/science.1109541.
2. Leggett, A. The EPR-Bell Experiments: The Role of Counterfactuality and Probability in the Context of Actually Conducted Experiments. *Philosophies* **2024**, *9*, 133.
3. Hance, J.R.; Hossenfelder, S. What does it take to solve the measurement problem? *Journal of Physics Communicacctions* **2022**, *6*. https://doi.org/10.1088/2399-6528/ac96cf.
4. Einstein, A.; Podolsky, B.; Rosen, N. Can quantum-mechanical description of physical reality be considered completed? *Physical Review* **1935**, *47*, 777–780. https://doi.org/10.1103/PhysRev.47.777.
5. Bohr, N. Can quantum-mechanical description of physical reality be considered complete? *Physical Review* **1935**, *48*, 696–702. https://doi.org/10.1103/PhysRev.48.696.
6. Bell, J.S. On the Einstein-Podolsky-Rosen paradox. *Physics* **1964**, *1*, 195–200. https://doi.org/10.1103/PhysicsPhysiqueFizika.1.195.
7. Maudlin, T. What Bell did. *Journal of Physics A: Mathematics and Theory* **2014**, *47*, 424010.
8. Hess, K. A Critical Review of Works Pertinent to the Einstein-Bohr Debate and Bell's Theorem. *Symmetry* **2022**, *14*, 163.
9. Aspect, A. Bell's inequality test: more ideal than ever. *Nature* **1999**, *398*, 189–190. https://doi.org/10.1038/18296.
10. Aspect, A. Closing the door on Einstein and Bohr's quantum debate. *Physics* **2015**, *8*, 123. https://doi.org/10.1103/Physics.8.123.
11. Aspect, A. Bell's theorem: the naive view of an experimentalist. In Proceedings of the Quantum [Un]speakables-From Bell to Quantum information; Bertlmann, R.; Zeilinger, A., Eds., Berlin, 2002.
12. Clauser, J.F.; Shimony, A. Bell's theorem: experimental tests and implications. *Reporting Progress Physics* **1978**, *41*, 1881–1927. https://doi.org/10.1088/0034-4885/41/12/002.
13. Clauser, J.F.; Horne, M.A.; Shimony, A.; Holt, R.A. Proposed experiment to test local hidden variable theories. *Physical Review Letters* **1969**, *23*, 880–84. https://doi.org/10.1103/PhysRevLett.23.880.
14. Weinberg, S. The greatest physics discovery of the 20th century. *J. Phys.: Conf. Ser* **2024**, *2877*, 012114.
15. Howard, D. Einstein on Locality and Separability. *Stud. Hist. Phil. Sci* **1985**, *16*, 171–201.