

Review

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The Art of Quantum Computing for Finance: Brief Overview and Prospects

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Article

The Art of Quantum Computing for Finance: Brief Overview and Prospects

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Abstract

This review article discusses the application of quantum computing to financial problems while presenting current approaches and their future prospects. We also talk about quantum machine learning and deep learning in finance. In the banking industry, we look at the most recent developments and the state of the art in quantum computing. Following a quick introduction to financial derivatives, we go over the key models and techniques for estimating the effects of quantum computing. The most popular quantum financial algorithms and their quantum adversary are then described. Lastly, we discuss the main problems that must be solved in order for quantum algorithms to truly benefit the financial industry.

Keywords: survey; finance; quantum; future of finance

1. Introduction

Applications in asset management, investment banking, retail banking, and merchant banking provide a number of challenging computing issues in the financial services sector and quantum computing and its impact on actuarial modeling ¹. A completely new approach to computing is provided by quantum computing [1,2]; which solves intricate calculations by utilizing the inherent quantum mechanical features of materials. The use of quantum computing to financial issues [3–6] and the proof of quantum advantage in early applications are ongoing research subjects, as evidenced by the first noisy quantum devices that utilize the principles of quantum mechanics and are currently accessible to the general public. Quantum computers or quantum computing and communications [7] are expected to surpass the computing capabilities of classical computers this decade and transform many industrial sectors, including finance.

Compared to modern classical computers, quantum computing uses essentially different methods for processing and storing data. Because they are more capable than any conventional computer, quantum computers are now the most promising method for resolving certain issues. Deep insights may be gained from the vast amounts of data that are already available thanks to new computational models, especially in financial institutions that are dealing with less predictability and more complexity. In order to enhance contemporary financial models or systems, quantum computing provides a means of delivering new information processing paradigms in quantitative and computational finance [8]. For example, Scriba et al., in their article [4], present an autonomous algorithm that simulates in parallel an exponential number of asset trajectories without resorting to oracles. Method for obtaining a distribution of stock prices. Finance is strongly linked to uncertainty [9] over the future behavior of assets, their prices, and the gains (losses or profits) they may yield. The distribution of returns determines the risk measure. It measures volatility using the logarithm of the standard deviation of the rate of change of a set of stock prices over time. Analyzing an asset’s behavior by comparing it with market data is necessary to reduce risk. By carefully selecting investments in other complementary

¹ quantum in actuarial science

assets with inverse (volume) or irregular returns (diversification), one can reduce the risk [9] of asset ownership.

In this study, we are interested in the application of quantum computing (Figure 1) [10–13] in the financial sphere in a very broad way. While previous studies focused on certain applications or characteristics [3,9,14–36], [30,37–43], [18,44–47].

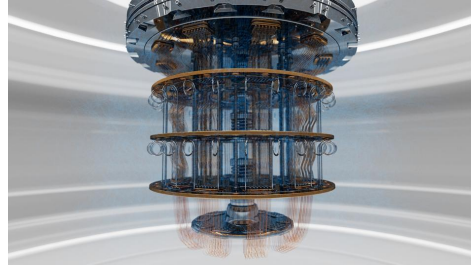


Figure 1. Quantum computing.

More specifically, we sought answers to three research questions (RQs):

- RQ1: What are the most commonly used methods in quantum finance?
- RQ2: How are the contributions of quantum approaches to finance evaluated?
- RQ3: What are the gaps, challenges, open questions, and future prospects of quantum computing?

To answer these questions, we searched different databases, namely, PubMed, MDPI, SCOPUS, Nature, Science Direct, IEEE Xplore, ACM, and Google Scholar, for the following keyword combinations: Finance * AND (“Quantum Finance” OR “Quantum Computing”). Articles were selected according to their publication dates. The search was designed to find research articles reporting the Quantum finance.

This paper is organized as follows: Section 1 presents related work. Section 2 discusses the fundamentals of financial issues, popular algorithms in quantitative finance, and quantum computing. Section 3 reviews financial applications of quantum computing. Section 4 introduces deep learning via quantum machine learning (QML) and quantum adversarial, and scenarios where it might be applicable to financial issues. Section 5 concludes and presents the viewpoints.

2. related work

Quantitative trading [48] is an integral part of financial markets with high calculation speed requirements, while no quantum algorithms^{2 3} have been introduced into this field yet. Zhuang et al., in [48] propose quantum algorithms for high-frequency statistical arbitrage trading by utilizing variable time condition number estimation and quantum linear regression. The algorithm complexity has been reduced from the classical benchmark $O(N^2d)$ to $O(\sqrt{dN}\kappa^2 \log(1/\epsilon)^2)$, where N is the length of trading data, and d is the number of stocks. In their article Arraut et al., in [49] analyze the patterns of effective symmetry breaking and the associated vacuum degeneracy for these particular circumstances. In the same scenario, they analyze the link between information flow and the multiplicity of martingale states, thus providing powerful tools for analyzing stock market dynamics.

Herman et al., in [45] present a comprehensive overview of quantum computing for financial applications, focusing on stochastic modeling, optimization, and machine learning. They explain how these methods, modified for a quantum computer, may be able to assist in resolving financial issues like fraud detection, risk modeling, portfolio optimization, derivatives pricing, and natural language understanding. They also show how these algorithms are applicable to a variety of financial use cases and talk about whether they can be implemented on quantum computers.

² Strategic Finance

³ Quantum Finance

Fintech is at the forefront of new technical applications. The rise of relatively new paradigms in a number of sciences, including physics (quantum), geometry (fractals), and database systems (distributed ledger—blockchain), appears to have the potential to further alter the framework of the financial sector while also posing issues (cyber threats) [50]. Mosteanu et al., in [51] studied of the reasonable potential impact of these new models (and their underlying technologies) is conducted, in [51], Mosteanu et al., confirms that the availability of information and the growing interconnection of cross-applications of each discovery in different scientific fields determine the rapid succession of revolutions, identified by significant evident changes in economic paradigms. Mosteanu et al., indicate that the quick succession of revolutions, marked by notable and obvious changes in economic paradigms, is determined by the availability of knowledge and the increasing connectivity of cross-applications of each discovery in other scientific domains. Pistoia et al., in [52] presents the state of the art of quantum algorithms for financial applications, focusing specifically on use cases that can be solved by machine learning.

Griffin et al., in [53] present an implementation of two quantum optimization algorithms applied to trade finance portfolios. The method used involves mapping the financial risk and returns of a trade finance portfolio to an optimization function of a quantum algorithm developed in a Qiskit tutorial [54]. The results show that, although no advantage is observed when using quantum algorithms, their performance does not suffer any statistically significant degradation. Therefore, it is promising that in the future, thanks to expected improvements in quantum hardware, the theoretically higher processing speeds and data volumes offered by quantum computing will also be applicable to trade finance. Albareti et al., in [14] provide a structured review of quantum computing platforms, algorithms, methodologies, and use cases for various financial applications.

Coyle et al., in [55], investigate and compare the capabilities of quantum and classical models for generative modeling in machine learning. They use a real financial dataset consisting of correlated currency pairs and compare two models for their ability to learn the resulting distribution: a restricted Boltzmann machine and a quantum circuit Born machine and demonstrates superior performance as the model evolves. They perform experiments on simulated and physical quantum chips using the Rigetti QCSTM platform.

Wilkens et al., in [32] analyses requirements and concrete approaches for the application to risk management in a financial institution. On the examples of Value-at-Risk for market risk and Potential Future Exposure for counterparty credit risk, the main contribution lies in going beyond textbook illustrations and instead exploring must-have model features and their quantum implementations. While conceptual solutions and small-scale circuits are feasible at this stage, the leap needed for real-life applications is still significant.

Miyamoto et al., in [56], are interested in derivative pricing based on solving the Black-Scholes partial differential equation by the finite difference method (FDM). This approach is suitable for certain types of derivatives, but it suffers from the problem of dimensionality, i.e., an exponential growth in complexity. They propose a quantum algorithm for pricing multi-asset derivatives by FDM, with an exponential acceleration of dimensionality compared to classical algorithms. This algorithm uses the quantum algorithm for solving differential equations, based on quantum linear systems algorithms.

3. Quantum Quantitative Finance

3.1. Problems in Financial Services

The forward-thinking financial services [51,57–59] sector has always been on the lookout for ways to use emerging technologies to boost earnings [15,43,60].

3.2. Black-Scholes PDE for option pricing

Let $r \in (0, \infty)$ be the risk-free interest rate, let $T \in (0, \infty)$ be a finite time horizon determining the maturity, and let $d \in \mathbb{N}$ be the number of assets. The multiple-asset Black-Scholes PDE is,

$$\frac{\partial u}{\partial t} + \frac{1}{2} \sum_{i,j=1}^d C_{ij} x_i x_j \frac{\partial^2 u}{\partial x_i \partial x_j} + \sum_{i=1}^d r x_i \frac{\partial u}{\partial x_i} - ru = 0, \quad (1)$$

in $[0, T] \times \mathbb{R}_+^d$. Let to a terminal condition $u(T, \cdot) = h(\cdot)$. Here, $h : \mathbb{R}_+^d \rightarrow \mathbb{R}$ is the payoff function, $u(t, x)$ is the option price at time t with price x .

3.2.1. Geometric Brownian motion process

Let $(\Omega, \mathcal{F}, \mathbb{P})$ be a probability space $W = (W^1, \dots, W^d) : [0, T] \times \Omega \rightarrow \mathbb{R}^d$ be a standard d -dimensional Brownian motion. let $\sigma_i := \|\sigma_i\|_{\ell^2(\mathbb{R}^d)}$. Let $S = (S^1, \dots, S^d) : [0, T] \times \Omega \rightarrow \mathbb{R}_+^d$ be the stock price process governed by,

$$dS_t^i = S_t^i \left(r dt + \sum_{j=1}^d \sigma_{ij} dW_t^j \right), \quad \text{for } i = 1, \dots, d, \quad (2)$$

With for initial price $S_0 \in \mathbb{R}_+^d$. Here $S_t = (S_t^1, \dots, S_t^d)$ are the values of each stock $i = 1, \dots, d$ at time $0 \leq t \leq T$. Let $R = (R^1, \dots, R^d) : [0, T] \times \Omega \rightarrow \mathbb{R}^d$ be the log-return process defined component-wise by $R_t^i = \ln(S_t^i / S_0^i)$ for $i = 1, \dots, d$. It follows from Itô's formula for all $t \in [0, T]$ that,

$$dR_t^i = (r - \frac{1}{2} \sigma_i^2) dt + \sum_{j=1}^d \sigma_{ij} dW_t^j, \quad \text{for } i = 1, \dots, d, \quad (3)$$

3.2.2. Quantum Black-Scholes Equation

let $V_t := F(t, \zeta_t(X))$, $F : [0, T] \times \mathcal{B}(\mathcal{H} \otimes \Gamma) \rightarrow \mathcal{B}(\mathcal{H} \otimes \Gamma)$ is the extension [61]

to self-adjoint operators $x = \zeta_t(X)$ of the analytic function $F(t, x) = \sum_{n,k=0}^{+\infty} a_{n,k}(t_0, x_0) (t - t_0)^n (x - x_0)^k$, where x and $a_{n,k}(t_0, x_0)$ are in $\mathbb{C}$, and for $\Lambda, \mu \in \{0, 1, \dots\}$,

$$\begin{aligned} F_{\Lambda\mu}(t, x) &:= \frac{\partial^{\Lambda+\mu} F}{\partial t^\Lambda \partial x^\mu}(t, x) \\ &= \sum_{n=\Lambda, k=\mu}^{+\infty} \frac{n!}{(n-\Lambda)!} \frac{k!}{(k-\mu)!} a_{n,k}(t_0, x_0) (t - t_0)^{n-\Lambda} (x - x_0)^{k-\mu} \end{aligned}$$

if 1 denotes the identity operator then,

$$a_{n,k}(t_0, x_0) = a_{n,k}(t_0, x_0) 1 = \frac{1}{n!k!} F_{nk}(t_0, x_0)$$

For $(t_0, x_0) = (0, 0)$ we have,

$$V_t = \sum_{n,k=0}^{+\infty} a_{n,k}(0, 0) t^n \zeta_t(X)^k = \sum_{n,k=0}^{+\infty} a_{n,k}(0, 0) t^n \zeta_t(X^k)$$

$(a_t, b_t), t \in [0, T]$ is a self -financing trading strategy then the value of the portfolio at time t is given by $V_t = a_t X_t + b_t \beta_t$.

3.3. Black-Scholes pricing formulae

$$dS = \mu S dt + \sigma S dW, \quad (4)$$

Here μ represents drift, σ variance, and W is a standard Wiener process.

$$dB = rB dt, \quad (5)$$

$B(t) = B(0)e^{rt}$, where B_0 . Let an option on a stock with strike price K and time to maturity $T = T_p - t$, where T_p is the fixed duration between the issuance of the option and its maturity. The stochastic differential equation for $V(S, t)$ is, from Itô's lemma,

$$dV = \left(\mu S \frac{\partial V}{\partial S} + \frac{\partial V}{\partial t} + \frac{\sigma^2 S^2}{2} \frac{\partial^2 V}{\partial S^2} \right) dt + \sigma S \frac{\partial V}{\partial S} dW. \tag{6}$$

Let a portfolio [62,63] that contains the option, which has been sold, and Δ shares of the underlying asset. The value Π of this portfolio is,

$$\Pi = \Delta S - V(S(t), t). \tag{7}$$

According to Itô's lemma, the stochastic differential equation for Π is,

$$d\Pi = - \left(\frac{\partial V}{\partial t} + \frac{\sigma^2 S^2}{2} \frac{\partial^2 V}{\partial S^2} \right) dt + \left(\Delta - \frac{\partial V}{\partial S} \right) dS. \tag{8}$$

$$\frac{\partial V}{\partial t} + \frac{\sigma^2 S^2}{2} \frac{\partial^2 V}{\partial S^2} + rS \frac{\partial V}{\partial S} - rV = 0. \tag{9}$$

For European call options $C(S, T)$,

$$C(S, 0) = \max(S - K, 0), \tag{10}$$

$$C(0, T) = 0, \tag{11}$$

$$\lim_{S \rightarrow \infty} C(S, T) = S, \tag{12}$$

For $T_p \geq T \geq 0$. the solution to the Black-Scholes equation is,

$$C(S, T) = SN(d_1) - Ke^{-rT}N(d_2), \tag{13}$$

$N(x)$ is the cumulative normal distribution function,

$$\begin{aligned} d_1 &= \frac{\ln(S/K) + (r + \frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}}, \\ d_2 &= \frac{\ln(S/K) + (r - \frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}}. \end{aligned} \tag{14}$$

where, K is the strike price, S is the current stock price, T is the time to expiration, r is the risk-free interest rate, σ the volatility.

The Greeks' formula for a European vanilla call and put option on a single asset is then given as follows:

	Calls	Puts
Delta, $\frac{\partial C}{\partial S}$	$N(d_1)$	$N(d_1) - 1$
Gamma, $\frac{\partial^2 C}{\partial S^2}$	$\frac{N'(d_1)}{S\sigma\sqrt{T-t}}$	
Vega, $\frac{\partial C}{\partial \sigma}$	$SN'(d_1)\sqrt{T-t}$	
Theta, $\frac{\partial C}{\partial t}$	$-\frac{SN'(d_1)\sigma}{2\sqrt{T-t}} - rKe^{-r(T-t)}N(d_2)$	$-\frac{SN'(d_1)\sigma}{2\sqrt{T-t}} + rKe^{-r(T-t)}N(-d_2)$
Rho, $\frac{\partial C}{\partial r}$	$K(T-t)e^{-r(T-t)}N(d_2)$	$-K(T-t)e^{-r(T-t)}N(-d_2)$

3.4. Quantum Finance: Quantum Black-Scholes Model and Pricing

$\varphi((a + ib)c_t) = \varphi(f(t))$, where φ is a mapping from vectors in a complex Hilbert space [56,64–66] $\mathcal{H}$ to Hermitian operators in the quantum field Hilbert space $\mathcal{K}$ and c_t is the characteristic function of the interval $[0, t]$. For any element f of a Hilbert space $\mathcal{H}$, $e^{i\varphi(f)}$ is the corresponding Weyl operator, whose definition is restricted to the interval $[0, t]$,

$$dS = \mu S \, dt + \sigma S \, dW + bS \, dX \tag{15}$$

$$= \mu S \, dt + S \, d\varphi(f(t)) \, . \tag{16}$$

For arbitrary f, g in a real Hilbert space, the usual canonical commutation (Weyl) relations in a complex Hilbert space $\mathcal{H}$.

$$e^{i\varphi(f)}e^{i\varphi(g)} = e^{i\varphi(f+g)}e^{\frac{1}{2}i\text{Im}(\langle f,g \rangle)} \tag{17}$$

The operators $\varphi(f)$ mutually commute if $f \in \mathbb{R}$ in $\mathcal{H}$, and likewise for $\varphi(if)$;

$$[\varphi(f), \varphi(ig)] = i\langle f, g \rangle \, , \tag{18}$$

For $f, g \in \mathbb{R}$, where the right-hand side is an inner product defined on $\mathcal{H}$.

$$[\varphi(f), \varphi(ig)] = -i\text{Im}(\langle f, ig \rangle) \, , \tag{19}$$

3.5. Quantum Hardware

The quantum⁴ computers [9,67,68] are based on quantum circuits and gates. Google holds the record for the most qubits in a gate architecture with 72 quantum computing qubits. There are several physical approaches to induce qubits. Furthermore, the leading manufacturers of consumer (military) quantum computers are Microsoft (using topological qubits), Xanadu (developing photonic quantum computing), IonQ (customizing solid ion qubits), Google, IBM, Alibaba, and Rigetti (using superconducting qubits).

3.6. Financial Applications of Quantum Computing

In finance [69] (potential advantages of quantum mechanics in the financial sector), risk refers to the uncertainty surrounding the future behavior of an asset, as well as its future prices and returns. It measures the likelihood that the asset’s actual return will deviate from the expected return, which was initially projected by the investor. The distribution of returns in this instance determines the risk measure. This is the definition of volatility, which is the standard deviation of logarithmic returns used to quantify the degree of variation of a series of stock prices over time. By connecting the asset to market data, an analysis of its behavior is conducted in order to lower this risk. In order to mitigate the risk of holding the asset, either with anticorrelated returns (*hedging*) or with uncorrelated returns (*diversification*) [6,70].

Table 1. Quantum Finance survey.

Quantum Finance	References
Transaction Settlement	[71]
Quantum Accounting	[72]
Predicting Financial Crashes	[73]
Quantum (Norm-Sampling)	[74–78], [79,80]
Quantum Money	[81–90]
Blockchain	[91–93]
Risk Management	[94–98]
Fraud Detection	[99–101]
Asset Pricing	[27,35,56,102–104]
Portfolio Optimization	[75,105–114]

⁴ Quantum computing: Tools

3.7. Optimal Trading

Let's look at the dynamic portfolio optimization problem. Finding the best course in the portfolio sector while accounting for transaction costs and market effect is our goal [9,42,53,70,109,115–124].

$$\varphi = \sum_{t=1}^T \left(\mu_t^T \varphi_t - \frac{\gamma}{2} \varphi_t^T \Sigma_t \varphi_t - \Delta \varphi_t^T \Lambda_t \Delta \varphi_t + \Delta \varphi_t^T \Lambda_t' \varphi_t \right), \quad (20)$$

with μ representing the expected returns, φ the holdings, Σ the expected covariance tensor and γ the risk aversion. The remaining terms represent the different contributions to transaction costs.

3.8. Optimal Arbitrage

The concept of arbitrage [32,48,125–127] is to take advantage of price differences of the same asset in different markets. In general, the conversion rates are not symmetric, i.e.: $c_{ij} \neq c_{ji}$, i represent the assets and transaction costs are assumed to be included in the variable. The optimization problem can be solved by,

$$\begin{aligned} w = & \sum_{(i,j) \in E} x_{ij} \log c_{ij} \\ & - \chi_1 \sum_{i \in \xi} \left(\sum_{j, (i,j) \in E} x_{ij} - \sum_{j, (j,i) \in E} x_{ji} \right)^2 \\ & - \chi_2 \sum_{i \in \xi} \sum_{j, (i,j) \in E} x_{ij} \left(\sum_{j, (i,j) \in E} x_{ij} - 1 \right). \end{aligned} \quad (21)$$

E represents the edges, ξ the vertices of the graph and the third term constrains x_{ij} to be equal to 0 or 1, so that cycles can only pass through a given asset once, where χ_1 and χ_2 are adjustable penalty parameters [6,42,116,128,128].

3.9. Risk Analysis

The VaR function, which establishes the distribution of losses throughout the portfolio, is one quantitative method for risk assessment [9,97,129]. Conditional Value at Risk (CVaR) is another useful risk assessment technique for a certain probability distribution. When a portfolio exceeds the VaR, it calculates the expected loss. In quantitative finance, VaR and CVaR are commonly calculated from related probability distributions using Monte Carlo sampling [94,130,131].

4. Quantum Machine Learning

The field of *machine learning* [36,132,133], [134,135], [136,137], [138–147] broadly amounts to the design and implementation of algorithms that can be trained to perform a variety of tasks. These include pattern recognition, data classification, and many others. The field of classical machine learning⁵, has grown enormously, mainly due to hardware and algorithmic developments (allowing, for instance, to train deep learning networks). The basic principles of machine learning are at the root of a number of vastly successful fields, the most prominent of which is probably neural networks, which includes deep learning, recurrent networks, generative neural networks and generative adversarial network.

4.1. Generative Neural Networks and Generative Adversarial Network

Deep neural networks [45], [140,141,149–153] have proven extremely effective in predicting markets and analyzing credit risk. The key to this success lies in their ability to learn from the training

⁵ Quantum machine learning, emmanoulopoulos2022quantum

data provided to them in order to tackle tasks requiring intuitive judgment and to draw conclusions even from incomplete data sets. While machine learning algorithms are generally extremely efficient, their training can be computationally expensive, but neural networks also have weaknesses such as generative adversarial networks ⁶ [154,155,155–170] and quantum poisoning [171–173] which can create vulnerabilities at the very heart of deep learning models or machine learning applied to financial quantum computing or general quantum computing.

5. Quantum Economics and Finance in Stock Markets

An option in the financial industry (Table 1) is a contract between a buyer and a seller that, depending on the underlying financial securities, like stocks or indexes, guarantees the buyer a future return after expiration. In recent years, numerical methods have rapidly evolved to solve quantitative finance problems using quantum computers. Quantum economics and finance [9,45, 56,64,174–178] have thus emerged to interpret the erratic behavior of stock markets using quantum mechanical concepts [179,180]. The financial market is an intricate dynamic system that is not linear. The introduction of derivative instruments aims to lower the risks involved in its operations. These financial instruments, like futures and options, are traded similarly to stocks, bonds, and other assets. To reduce financial risk, financial options are the most frequently utilized of them. These financial trading tactics entail figuring out how much financial instruments like bonds, options, and interest-rate derivatives are worth. Usually, stochastic differential equations derived from a Black-Scholes model [181,182] control these computations. The Monte Carlo method [102,183–188], , IIF, [35,63,189,190,190,191] is a technique used to estimate the properties of a system stochastically by statistically sampling.

$$\Pr(|\tilde{\mu} - \mu| \geq \epsilon) \leq \frac{\sigma^2}{k\epsilon^2},$$

(22)

where ϵ is the error and $\tilde{\mu}$ is the approximation to μ obtained from k samples.

5.1. Financial Quantum Approach in Option Pricing

With the assumption of transaction costs, a variable risk-free interest rate, or stochastic asset price volatility, Black-Scholes models [49,192–194] are frequently employed in the literature (Table 1). These models are regarded more accurate for option pricing since these assumptions are more likely to reflect actual market conditions. However, the risk of model data poisoning (DP) has emerged with the use of artificial intelligence (AI) models in the financial industry [195]. The financial quantum approach is starting to emerge as a substitute strategy for the stock market as a result. This section will examine option pricing models [27,66,94] that are based on the Black-Scholes equation in a quantum setting using Reinforcement Learning [134,149,196–199].

5.2. Reinforcement Learning

A typical reinforcement learning [198–201] setting is based on Markov decision processes. A Markov decision process is defined as follows: $\{S, A_{(i)}, p_{ij}(a), r_{(i,a)}, V, i, j \in S, a \in A_{(i)}\}$. S denotes the set of the states, $A_{(i)}$ denotes the set of actions corresponding to state i , $p_{ij}(a)$ denotes the probability of transitioning from state i to j when action a is executed, $r_{(i,a)}$ denotes the reward of executing action a in state i , V is the value function that the agent tries to maximize. Reward function is defined from Γ to $(-\infty, +\infty)$, where $\Gamma = \{(i, a) : i \in S, a \in A_{(i)}\}$. π denotes the policy that the agent tries to learn and it is defined from $S \times \bigcup_{i \in S} A_{(i)}$ to $[0, 1]$. The value function is defined as the following:

⁶ Quantum adversarial computing

$$\begin{aligned}
 V_s^\pi &= \mathbb{E}[r_{t+1} + \gamma r_{t+2} + \dots \mid s_t = s, \pi] = \\
 &= \mathbb{E}\left[r_{t+1} + \gamma V_{s_{t+1}}^\pi \mid s_t = s, \pi\right] = \\
 &= \sum_{a \in A_s} \pi(s, a) \left[r_s^a + \gamma \sum_{s'} p_{ss'}^a V_{s'}^\pi \right]
 \end{aligned}$$

where t denotes a timestep and γ is the discount factor in the range $[0, 1]$. $p_{ss'}^a = P[s_{t+1} = s' \mid s_t = s, a_t = a]$, $r_s^a = \mathbb{E}[r_{t+1} \mid s_t = s, a_t = a]$.

$$V(s) \leftarrow V(s) + \alpha (r + \gamma V(s') - V(s))$$

Optimal value: $V_s^* = \max_{a \in A_s} [r_s^a + \gamma \sum_{s'} p_{ss'}^a V_{s'}^*]$, Optimal policy: $\pi^* = \operatorname{argmax}_\pi V_s^\pi$.

5.3. Challenges for quantum computing

One of the most important problems in quantum computing [67,202,202–207] is decoherence, i.e., uncontrolled interactions between the system and its environment. This leads to a loss of quantum behavior in the quantum processor. The decoherence time therefore imposes a strict limit on the number of operations. Designing higher-fidelity qubits is a major hardware challenge. Nevertheless, decoherence can be fixed via error-correction techniques. The fact that a single qubit may need thousands of physical qubits is a significant barrier. Numerous researchers have resorted to algorithms for so-called Noisy Intermediate-Scale Quantum (NISQ) quantum processors in response to these challenges [208–211]. Despite decoherence, these are made to function well on malfunctioning quantum computers.

6. Conclusions

This article looked at how quantum computing might be used in the financial industry. Partly because of conceptual advancements that promise large speedups for broadly applicable algorithms [212], and in part because of experimental breakthroughs in quantum hardware that surpass all expectations, this field is expanding quickly. However, it will take more experimental work before a universal quantum processor that can outperform existing supercomputers can be constructed. Using quantum parallelism, the solution is roughly calculated:

$$V(S, t) \approx \sum_{i=1}^n c_i \phi_i(S) e^{-r(T-t)}.$$

Nelson-Siegel-Svensson model [213]:

$$r(t) = \beta_0 + \beta_1 \frac{1 - e^{-t/\tau_1}}{t/\tau_1} + \beta_2 \frac{1 - e^{-t/\tau_2}}{t/\tau_2}.$$

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