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Article

Grand Unified Theories with Electro-Strong Interaction, New SU(5), and New SO(10) Model

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Abstract

Similar to electroweak interaction, strong force and electromagnetism can have similar Higgs mechanism mediated interaction. Thus, gluons can acquire mass. And, neural colored gluons have larger mass than colored gluons. Total, we can have eight gluons without red-anti-red gluon. The puzzle of proton or neutron mass can be solved. We can also derive a new SU(5) model to include all the above eight gluons, three W/Z bosons, photon, Higgs boson, three generations of leptons and quarks to make a new 5x5 SU(5) model. Wightman axioms can be fulfilled in this new SU(5) without causing proton decay crisis. We can also add the 4x4 four dimensional spacetime tensor integrating mass-energy density, light pressure, electric fields, and magnetic fields as well as four gradients to make a new contravariant SO(10) model. Weyl tensor and Ricci tensor related to the new SO(10) model are also given. Thus, grand unified theory or theory of everything can be obtained, that is compatible with four dimensional spacetime without extra-dimension needed in string theories.

Keywords: Electro-strong interaction; SU(5); SO(10)

Main Text

In previous studies, Professor Weinberg proposed the electroweak interaction to predict the masses of W and Z particles. His theory was very successful. However, it is actually the interaction of photons and W/Z bosons. Therefore, it is the weak light interaction. It is the interaction between the weak force and light. Here, I propose the interaction between the strong force and light. Therefore, it can solve the problem of the gluon mass.

Based on the Yang-Mills theory of the standard model, we know that the Yang-Mills equation is:

$$\mathrm{Fuv} = \partial \mathrm{uAv} - \partial \mathrm{vAu} - [\mathrm{Au}, \mathrm{Av}]$$

In addition, the QCD formula is:

$$\mathrm{U}\big(\mathrm{SU}(3)\big) = \exp\left[\mathrm{ig}\sum_{j=1}^{8} \mathrm{F_j}\, \mathrm{G_j}(\mathrm{x})\right]$$

Therefore, the covariant derivative is:

$$\partial^{\mu} = \partial^{\mu} + \mathrm{igF} * \mathrm{G}(\mathrm{x})$$

In addition, F = 1 / 2λ, λ is the Gell-Mann matrix:

$$\lambda_1 = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\lambda_2 = \begin{bmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

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$$\lambda_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\lambda_4 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

$$\lambda_5 = \begin{bmatrix} 0 & 0 & -\mathrm{i} \\ 0 & 0 & 0 \\ \mathrm{i} & 0 & 0 \end{bmatrix}$$

$$\lambda_6 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$\lambda_7 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -\mathrm{i} \\ 0 & \mathrm{i} & 0 \end{bmatrix}$$

$$\lambda_8 = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{bmatrix}$$

For photons, there is another matrix:

$$\lambda_9 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

We let R or $|\underline{R}>$ = (1,0,0), B or $|\underline{B}>$ = (0,1,0), and G or $|\underline{G}>$ = (0,0,1). Then, the entire matrix is:

$$\begin{bmatrix} r\bar{r} & b\bar{r} & g\bar{r} \\ r\bar{b} & b\bar{b} & g\bar{b} \\ r\bar{g} & b\bar{g} & g\bar{g} \end{bmatrix}$$

Furthermore, each matrix has its corresponding gluons and photons:

$$G_1 = \frac{1}{\sqrt{2}}\left(\mathrm{r}\bar{\mathrm{b}} + \mathrm{b}\bar{\mathrm{r}}\right)$$

$$G_2 = \frac{\mathrm{i}}{\sqrt{2}}\left(\mathrm{r}\bar{\mathrm{b}} - \mathrm{b}\bar{\mathrm{r}}\right)$$

$$G_3 = \frac{1}{\sqrt{2}}\left(\mathrm{r}\bar{\mathrm{r}} - \mathrm{b}\bar{\mathrm{b}}\right)$$

$$G_4 = \frac{1}{\sqrt{2}}\left(\mathrm{r}\bar{\mathrm{g}} + \mathrm{g}\bar{\mathrm{r}}\right)$$

$$G_5 = \frac{\mathrm{i}}{\sqrt{2}}\left(\mathrm{r}\bar{\mathrm{g}} - \mathrm{g}\bar{\mathrm{r}}\right)$$

$$G_6 = \frac{1}{\sqrt{2}}\left(\mathrm{g}\bar{\mathrm{b}} + \mathrm{b}\bar{\mathrm{g}}\right)$$

$$G_7 = \frac{\mathrm{i}}{\sqrt{2}}\left(\mathrm{b}\bar{\mathrm{g}} - \mathrm{g}\bar{\mathrm{b}}\right)$$

$$G_8 = \frac{1}{\sqrt{6}}\left(\mathrm{r}\bar{\mathrm{r}} + \mathrm{b}\bar{\mathrm{b}} - 2\mathrm{g}\bar{\mathrm{g}}\right)$$

Furthermore, the photon boson is:

$$B = G_9 = \frac{1}{\sqrt{3}}\left(r\bar{r} + b\bar{b} + g\bar{g}\right)$$

Therefore, there are a total of 9 bosons (8 gluons plus 1 photon) for the entire 3x3 matrix that interact with the Higgs boson. To maximize the total number of gluons, we need to use a complex scalar field that includes 6 Higgs bosons. We predict that six bosons will interact with the Higgs field, and the other three gluons are massless at this step. The Higgs field is:

$$\varphi(x) \equiv \frac{1}{\sqrt{2}}\begin{pmatrix}\varphi 1 + i\varphi 2\\ \varphi 3 + i\varphi 4\\ \varphi 5 + i\varphi 6\end{pmatrix}$$

Moreover, we let $\varphi 1 = \varphi 2 = \varphi 3 = \varphi 4 = \varphi 6 = 0$ and $\varphi 5 = v$

Therefore, the Higgs field should be $(0,0,V/\sqrt{2})$

The Lagrangian for a complex scalar field is:

$$L(\varphi) = (\partial_v\varphi)(\partial^v\varphi) - \mu^2\left(\varphi(x)\right)^2 - \lambda\left(\varphi(x)\right)^4$$

The complex scalar field mass term is m^2V^2

We then use QCD and Gell-Mann 's covariant derivatives

The Lagrangian becomes:

$$\frac{1}{4}\left|\left[\left(ig\lambda G(x)\right) * \varphi(x)\right]^{+}\left[\left(ig\lambda G(x)\right) * \varphi(x)\right]\right| =$$

$$\frac{1}{4}\left(\frac{1}{\sqrt{2}}gv(G_4 - iG_5), \frac{1}{\sqrt{2}}gv(G_6 - iG_7), v\left(\frac{1}{\sqrt{3}}kB - \frac{\sqrt{2}}{\sqrt{3}}gG_8\right)\right)$$

$$\times\left(\frac{1}{\sqrt{2}}gv(G_4 + iG_5), \frac{1}{\sqrt{2}}gv(G_6 + iG_7), v\left(\frac{1}{\sqrt{3}}kB - \frac{\sqrt{2}}{\sqrt{3}}gG_8\right)\right)$$

We set $G^4 = 1/\sqrt{2}\,(G_4 + iG_5)$, $G^5 = 1/\sqrt{2}\,(G_4 - iG_5)$ and G^6 and G^7 are obtained by analogy. We let $\sqrt{2}/\sqrt{3}g = g''$ and $1/\sqrt{3}k = g'$. The electromagnetic and strong coupling constant ratio g = k = 1. We calculate the above formula:

We set

$$G^{8u} = \left(g'B^u - g''G_8^u\right) \Big/ \sqrt{\left(g'^2 + g''^2\right)} \text{ and}$$

$$Au = \left(g'G_8^u + g''B^u\right) \Big/ \sqrt{\left(g'^2 + g''^2\right)}$$

Analogous to the electroweak theory, we obtain the mass of G^8

$$mG^8 = \frac{v\sqrt{g'^2 + g''^2}}{\sqrt{2}}$$

and the mass of the photon ^{Au} is still zero. Similar to the electroweak theory, we get the G^8 field and the photon:

$$G^8 = \frac{g'}{\sqrt{g''^2 + g'^2}}B - \frac{g''}{\sqrt{g''^2 + g'^2}}G_8 = B\sin\theta - G_8\cos\theta$$

$$A = \frac{g'}{\sqrt{g''^2 + g'^2}}G_8 + \frac{g''}{\sqrt{g''^2 + g'^2}}B = G_8\cos\theta + B\sin\theta$$

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In addition, the masses of the new gluons G^1,G^2 and G^3 are still zero. In addition, because the mass term is $M^2 V_u V^u$, the masses of the gluons G^4, G^5,G^6 and G^7 are 1/2 vg. Because the G^8 mass term after symmetry breaking is $1/2M^2G^{8u}G_{8u}$ after the Higgs mechanism, the G^8 gluon becomes $g\bar{g}$(mass v g/ $\sqrt{2}$). We know:

$$\frac{1}{\sqrt{2}}(G_1 - iG_2) = r\bar{b}, \text{ and} \frac{1}{\sqrt{2}}(G_1 + iG_2) = b\bar{r}$$

G_8 and the photon Higgs interaction is the right side and the bottommost position of the matrix, where we get a final $g\bar{g}$ gluon.

Therefore, we can get eight new gluons: $r\bar{b}$, $b\bar{r}$, $b\bar{b}$, $g\bar{b}$, $b\bar{g}$, $r\bar{g}$, $g\bar{r}$and $g\bar{g}$. $b\bar{b}$The form of is similar to that of neutral mesons:

$$\frac{1}{\sqrt{2}}(r\bar{r} - b\bar{b})$$

coupling constant ratio between photons (charges) and the strong force. However, if the α-ratio is 1, similar to the color force, we can get

$$\sin\theta = \frac{1}{\sqrt{3}}$$

$$\cos\theta = \frac{\sqrt{2}}{\sqrt{3}}$$

Therefore, we will get the result of G^8 (interaction)

$$G^8 = g\bar{g}$$

$$A = \frac{1}{\sqrt{2}}(r\bar{r} + b\bar{b})$$

The first step results in that the green-related gluons have mass, while the non- green gluons have no mass. This solves the Yang-Mills mass gap problem. This is why neutrons and protons are heavier than the quarks they contain. From the above we know that α decay is related to mesons and β decay is related to W bosons. Both are SU(2). From the strong light interaction, we can get five massive G^{4-8} gluons related to green: $g\bar{b}$, $b\bar{g}$, $r\bar{g}$, $g\bar{r}$and $g\bar{g}$. In addition, we have four massless bosons: $\lambda1$, $\lambda2$, $\lambda3$, & A. These four gluons can interact with the Higgs boson (0,V/$\sqrt{2}$) electrostrongly to obtain a sum of masses vg/2 for $r\bar{b}$ and $b\bar{r}$, $b\bar{b}$ mass vg/ $\sqrt{2}$, and a massless $r\bar{r}$, so there are a total of eight gluons of equal mass mediating the short-range strong force, which is a symmetry violation related to SU(3). In addition, the above A is the B0 boson in the electroweak theory. Interestingly, when the Higgs meson interacts, if we set Higgs (V/ $\sqrt{2}$, 0,0), we can use a neutral pion to fill in $g\bar{g}$the position to form a perfect 3 x3 matrix, so we finally have $r\bar{b}$, $b\bar{r}$, $b\bar{b}$, $g\bar{b}$, $b\bar{g}$, $r\bar{g}$, $g\bar{r}$ and $g\bar{g}$ a total of eight force gluons , plus a neutral gluon that is particularly closely related to πthe pion. With different coupling constants, these four bosons can also interact with the Higgs (0, V / $\sqrt{2}$) in the electroweak Pauli matrix to produce massive W+, W-, Z, and massless γ. From this, we can get a unified strong force, weak force, and light (electromagnetic) interaction.

The calculation at this time:

$$\frac{1}{4}\left(\frac{1}{\sqrt{2}}gv(G_1 - iG_2), v\left(\frac{1}{\sqrt{2}}kA - \frac{1}{\sqrt{2}}gG_3\right)\right) \times \left(\frac{1}{\sqrt{2}}gv(G_1 + iG_2), v\left(\frac{1}{\sqrt{2}}kA - \frac{1}{\sqrt{2}}gG_3\right)\right)$$

strong electric action (k=g=1), Isospin=Hypercharge=1. Under weak electric action, Isospin=1 and Hypercharge=1/2 are substituted into the result (at this time G1 =W1, G2=W2, G3=Z ʹ).

Then, why are there three generations of quarks and leptons? Electroweak interaction is the main source of three generations of leptons, such as electrons and neutrinos. After the Higgs interaction, W and Z bosons gain mass to produce electrons and neutrinos. We know that a large number of fermions do not show chiral symmetry. This is because the mass factor in the langragian mΨΨ breaks chiral symmetry. And we can use the principle of NXN Kappibo-Kobayashi-Maskawa matrix to determine the offspring of quarks. Its determining factor is (N-1)*(N-2)/2. If N=1, there is no quark mixing angle and CP violation. If N=2, there is one quark mixing angle and no CP violation. If N=3, there are 3 mixing angles and a CP violation. As we discussed above, weak light and strong light interactions cause spontaneous symmetry with CP violation after gauge bosons gain mass. This can help solve the strong CP problem. Therefore, there must be a 3x3 CKM matrix of quarks, which implies that there is a CP violation in strong interactions, such as parity violation in weak interactions. For leptons, there is a 3 x3 Maki-Nakagawa-Sakata matrix (MNS matrix). Similarly, the above algorithm N = 3 has 3 mixing angles and a CP violation, which means that there are three generations of leptons in weak interactions and the parity of weak interactions is not conserved. Quarks have three generations, but whether there is CP violation remains to be studied. However, the CKM matrix suggests that strong forces also have parity violations. Therefore, the three conditions of Sakharov are consistent: baryon number asymmetry, CP violation, and thermal equilibrium. This explains why there is more matter than antimatter in the universe. In addition, the three generations of elementary particles are related to the fact that space is three-dimensional.

Finally, I would like to discuss the relationship between charge, hypercharge, and isospin. By applying the Higgs mechanism described above, we can easily explain the phenomenon of this relationship. Let's use the example of left-handed quarks and leptons. In the strong interaction, we have the Gell-Mann formula:

$$Q = T_3 + 1/2Y \text{ (Q: charge, T: isospin, Y: hypercharge)}$$

First, let's look at the Higgs- gluon interaction. From above, we can see that a three- component Higgs (0,0,V) interacts with the gluon, giving the gluon mass. We know that the neutron isospin I_z and charge are -1 / 2 and 0, and the proton isospin I_z and charge are 1 / 2 and 1. From the Gell-Mann equation, we can get that both protons and neutrons have hypercharge Y = 1. In addition, we can get hypercharge Y = -1 for antiprotons or antineutrons. The strong force isospin g = 1 (gluons) & hypercharge g ' = 1 can be brought into the electric force to get the gluon mass.

Then we look at the up and down quarks, which makes the up quark isospin Iz and charge 1 / 2 and 2 / 3 in neutrons and protons. The down quark has isospin I_z and charge -1/2 and -1/3. Then, we can get two up quarks and one down quark with hypercharge Y = 1 / 3. Therefore, there must be three quarks which make a proton or neutron 1 (Y: 1 / 3 * 3 = 1). This can also explain strange quarks, charm quarks, top quarks and bottom quarks. Therefore, we know that the strong interaction is SU(3).

Then, we will look at the Higgs electroweak interaction. Here, we will use the weak hypercharge formula for the weak interaction:

$$Q = I_3 + Yw \text{ (}I_3\text{: weak isospin, Yw: weak hypercharge)}$$

We know that bosons decay into two parts: electrons and antineutrinos. > E^- + anti-ν. We know that the electron isospin Iz and charge are -1/2 and -1, the antineutrino isospin Iz and charge are -1/2 and 0, so we can get the electron's weak hypercharge to be -1/2 and the antineutrino 's hypercharge to be 1 / 2. We can also see this relationship in the decay of the Z boson. The Z boson decays into a neutrino and antineutrino pair. The neutrino isospin Iz and charge is 1 / 2 & The isospin Iz and charge of the 0 antineutrino are -1 / 2 and 0, so from the above weak supercharge formula, we can get the supercharge of the neutrino is -1 / 2 and the antineutrino supercharge is 1 / 2. Z boson decays into two parts (Yw = 0 = 1 / 2 - 1 / 2). Therefore, we know that electroweak force is SU (2). The weak force isospin g =1 (W/Z boson) & hypercharge g ' =1/2 can be brought into the electroweak interaction to obtain the W/Z mass. Since leptons include electrons and neutrinos, the above formula differs from the Gell-Mann formula by 1/2, but the two formulas are interchangeable. Neutrinos and antineutrinos are just left-handed Majorana fermions. Isospin is as follows: photon (0,1), gluon (0, spin=1), Higgs (+1), Z boson (0), W^+ boson (1), W- boson (-1), so bosons can be distinguished. In the following SU (5), it can be seen that electrons and neutrinos are separated, so the weak force hypercharge is -1/2 each, but quarks are in groups of three, and the strong force hypercharge is 1 / 3 * 3 = 1.

Since the Theory of Everything is a SO(10) model, the X charge is conserved:

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X+ 2Y w =5(BL) (BL is the number of baryon reducers, which is conserved in this model)

Q = T 3+1 / 2Y w (Q: charge, T: isospin, Y w: hypercharge)

Q = T 3L + T 3R +1 / 2 (BL) = T 3 +1 / 2 (BL) (T 3L or T 3R : left or right isospin)

It can be seen that the baryon reducer number corresponds to the hypercharge. Assuming that the chirality of the interaction between left-handed and right-handed fermions is not distinguished, the up quark X charge is -1, the down quark X charge is +3, and the electron X charge is -1. Then the proton X charge is +1, Ye is +2. The electron Ye is -2. The right-handed electron isospin T3 is 0, and the right-handed proton isospin T3 is 0. Then the proton Q = +1 and the electron Q = -1. This formula is valid.

In addition, let's take a look at the list of X charge values for all elementary particles:

Particle Name	Code	Left-handed X value	Right X value
Up quark	u	+1	-1
Charm Quark	c	+1	-1
Top quark	t	+1	-1
Down quark	d	+1	+3
Strange Quark	s	+1	+3
Bottom quarks	b	+1	+3
electronic	e^-	-3	-1
Mia	μ^-	-3	-1
Taozi	τ^-	-3	-1
Electron Neutrino	ν_e	-3	-5
Muon Neutrino	ν_μ	-3	-5
Taozi Neutrino	ν_τ	-3	-5
Anti-up quark	$\bar{u}$	+1	-1
Anti- charm quark	$\bar{c}$	+1	-1
Antitop quark	$\bar{t}$	+1	-1
Anti-down quark	$\bar{d}$	-3	-1
Anti-strange quark	$\bar{s}$	-3	-1
Anti-bottom quark	$\bar{b}$	-3	-1
Positron	e^+	+1	+3
Antimion	μ^+	+1	+3
Anti Tao Zi	τ^+	+1	+3
Anti-electron neutrino	$\bar{\nu}_e$	+5	+3
Antimion neutrino	$\bar{\nu}_\mu$	+5	+3
Anti- Tao neutrino	$\bar{\nu}_\tau$	+5	+3

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It can be seen that the X charge conserved in the SO(10) model has the same value for the three generations of particles in the same family. However, according to charge conjugation symmetry, the X charge of positive and antiparticles is interchanged and the positive and negative are opposite. For example, the left-handed X charge of the down quark is +1 and the right-handed X charge of the down quark is +3, then the left-handed X charge of the anti-down quark is -3 and the right-handed X charge of the anti-down quark is -1. Similarly, the three generations of electrons and neutrinos in the lepton family, but for the positive and negative up quark three generations of particles, this rule is not followed, which means that the up quark three generations do not follow the charge conjugation symmetry, that is, there is CP violation, which is evidence of CP violation in the strong interaction baryon family. We know that the bottom quark in the meson has a long life span, which can be explained by the CP violation of the up quark three generations. There have been experimental reports on the CP violation of charm quarks. However, the up quark itself has the smallest mass and is not easy to decay, and the down quark itself does not have CP violation, which can explain the stability of protons, neutrons and pions. In addition, there is the Jarlskog invariant in the CKM matrix: (CP violation disappears when J=0)

$$(m_t^2 - m_u^2)(m_t^2 - m_c^2)(m_c^2 - m_u^2)(m_b^2 - m_d^2)(m_b^2 - m_s^2)(m_s^2 - m_d^2)J \neq 0$$

$$J = c_{12}c_{13}^2c_{23}s_{12}s_{13}s_{23}\sin\delta = 3x10^{-5}$$

When Jarlskog invariant > 10^{-10}, the universe can produce CP violation and baryon generation conditions, thus meeting the three conditions of Sakharov. Since the PMNS lepton matrix has a similar Jarlskog invariant, it can be seen that both weak and strong interactions have the advantage of positive particles with CP violation, so it can explain the generation of positive matter baryons and leptons in our universe: (the maximum possible J value of strong interaction)

$$J_{max} = c_{12}c_{13}^2c_{23}s_{12}s_{13}s_{23} = 3\times 10^{-2}$$

Let's look at the role of quarks in this strong interaction model:

$$T_3 = F_3 = \frac{1}{2}\begin{bmatrix}1 & 0 & 0\\ 0 & -1 & 0\\ 0 & 0 & 0\end{bmatrix}$$

$$Y = \frac{2}{\sqrt{3}}F_8 = \frac{1}{\sqrt{3}}\lambda_8 = \frac{1}{3}\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & -2\end{bmatrix}$$

$$Q = T_3 + \frac{1}{2}Y = \begin{bmatrix}{}^2/_3 & 0 & 0\\ 0 & -{}^1/_3 & 0\\ 0 & 0 & -{}^1/_3\end{bmatrix}$$

We know the composition of the neutron (u,d,d), so the above 3x3 quark matrix plus the previous 3x3 gluon matrix constitutes the neutron. And R = (1,0,0), B = (0,1,0), and G = (0,0,1). This shows that the three quarks in this neutron must have different colors. If we set anti-R = (-1,0,0), anti-B = (0, -1,0), and anti-G = (0,0,-1). The antineutron has three anti-colors. The up quark Iz = 1/2, while the down quark Iz =-1/2, the anti-up quark Iz = -1/2, and the anti-down quark Iz =1/2.

And we got in the previous calculation:

$$\mathrm{A} = \frac{1}{\sqrt{2}}(\mathrm{r\bar{r}} + \mathrm{b\bar{b}})$$

This is equivalent to:

$$T_3 = F_3 = \frac{1}{2}\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 0\end{bmatrix}$$

Similar to the calculation above:

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$$Y=\frac{2}{\sqrt{3}}F_8=\frac{1}{\sqrt{3}}\lambda_8=\frac{1}{3}\begin{bmatrix}1&0&0\\0&1&0\\0&0&-2\end{bmatrix}$$

$$Q=T_3+\frac{1}{2}Y=\begin{bmatrix}{}^2/_3&0&0\\0&{}^2/_3&0\\0&0&-{}^1/_3\end{bmatrix}$$

We can get the composition of the proton (u,u,d). Therefore, the above 3x3 quark matrix plus the previous 3x3 gluon matrix constitutes the proton. And R = (1,0,0), B = (0,1,0), and G = (0,0,1). This shows that the three quarks in this proton must have different colors. If we set anti-R = (-1,0,0), anti-B = (0, -1,0), and anti-G = (0,0,-1). This can explain the color charge of the antiproton. The positive and negative quarks have opposite hypercharges.

We can apply the same logic to pions:

$$T_3=F_3=\frac{1}{2}\begin{bmatrix}1&0\\0&-1\end{bmatrix}$$

$$Y=\frac{1}{3}\begin{bmatrix}1&0\\0&-1\end{bmatrix}$$

Color Charge C>=(1,0) anti-color charge $\underline{C}$>=(0,-1)

In the $\pi0$ meson (U $\underline{U}$):

$$Q=T_3+\frac{1}{2}Y=\begin{bmatrix}{}^2/_3&0\\0&-{}^2/_3\end{bmatrix}$$

In $\pi+$ meson (U $\underline{D}$):

$$T_3=F_3=\frac{1}{2}\begin{bmatrix}1&0\\0&1\end{bmatrix}$$

$$Y=\frac{1}{3}\begin{bmatrix}1&0\\0&-1\end{bmatrix}$$

$$Q=T_3+\frac{1}{2}Y=\begin{bmatrix}{}^2/_3&0\\0&{}^1/_3\end{bmatrix}$$

In π -meson ($\underline{UD}$): color charge C>=(0,1) anti-color charge $\underline{C}$>=(-1,0)

$$T_3=F_3=\frac{1}{2}\begin{bmatrix}-1&0\\0&-1\end{bmatrix}$$

$$Y=\frac{1}{3}\begin{bmatrix}-1&0\\0&1\end{bmatrix}$$

$$Q=T_3+\frac{1}{2}Y=\begin{bmatrix}-{}^2/_3&0\\0&-{}^1/_3\end{bmatrix}$$

This also shows that neutral gluons are coupled to up and down quarks in pions. So in practice we can write the strong hypercharge as:

$$Y=\frac{1}{3}(n_u-n_{\bar{u}})+\frac{1}{3}(n_d-n_{\bar{d}})-\frac{2}{3}(n_s-n_{\bar{s}})$$

In addition, in the previous chapter, we discussed the T3 matrix of the pion:

$$\mathrm{T_z}=\begin{bmatrix}1&0&0\\0&0&0\\0&0&-1\end{bmatrix}$$

In addition, I would like to discuss how quarks and leptons obtain mass. If there is a matrix (composed of up - type quarks and down -type quarks) such as:

$$\begin{bmatrix} u\bar{u} & \bar{u}d \\ u\bar{d} & d\bar{d} \end{bmatrix}$$

The Pauli matrix of the electroweak interaction is:

$$\sigma_x = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$\sigma_y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$$

$$\sigma_z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

So W or Z boson and Higgs can form a matrix:

$$\begin{bmatrix} Z & W_- \\ W_+ & Z \end{bmatrix}$$

In addition, we know the pion matrix:

$$\mathrm{T_x} = \frac{1}{\sqrt{2}}\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$\mathrm{T_y} = \frac{1}{\sqrt{2}}\begin{bmatrix} 0 & -\mathrm{i} & 0 \\ \mathrm{i} & 0 & -\mathrm{i} \\ 0 & \mathrm{i} & 0 \end{bmatrix}$$

Look at the four numbers in the lower right corner. From the above, we can see that the W + boson corresponds to the π + meson and also corresponds to $u\bar{d}$. The W - boson corresponds to the π - meson and also corresponds to $\bar{u}d$. And the Z boson corresponds $u\bar{u}$ & $d\bar{d}$:

$$\mathrm{T_z} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

The Z boson does not completely correspond to the π0 meson, unless the complete π0 meson Tz is seen:

$$\pi_0 = \frac{u\bar{u} - d\bar{d}}{\sqrt{2}}$$

Therefore, the π0 meson decays without passing through the Z boson, while the π+ or π- meson decays through the W boson. That is:

$$\begin{bmatrix} \pi_0 & \pi_- \\ \pi_+ & \pi_0 \end{bmatrix}$$

Leptons can also be written as matrices (composed of electrons $e$and neutrinos υ):

$$\begin{bmatrix} \upsilon\bar{\upsilon} & e\bar{\upsilon} \\ \upsilon\bar{e} & e\bar{e} \end{bmatrix}$$

σ_x corresponds to Tx、σ_y corresponds to Ty、σ_z corresponds to Tz.

From the above, we can see that the W + boson corresponds to $\upsilon\bar{e}$. The W- boson corresponds to $e\bar{\upsilon}$. And the Z boson corresponds to $e\bar{e}$ and $\upsilon\bar{\upsilon}$.

The W- boson will decay to $\bar{u}d$or $e\bar{\upsilon}$. The W+ boson will decay to$u\bar{d}$or $\upsilon\bar{e}$. And the Z boson corresponds to and decays to$\frac{u\bar{u}}{d\bar{d}}$ and $\frac{e\bar{e}}{\upsilon\bar{\upsilon}}$.

Therefore, when a proton emits a π+ meson (including neutral gluons), it can annihilate into a massive W+ boson, and the W+ boson decays $u\bar{d}$into a quark-antiquark pair, which is the source of quark mass. The W+ boson can also decay into a positron and a neutrino, because both of them are derived from the decay of the massive W+ boson, so it can also explain the source of lepton mass. When a neutron emits a π- meson (including neutral gluons), it can annihilate into a massive W- boson, and the W- boson decays into $\bar{u}d$a quark-antiquark pair, which is the source of quark mass. The W- boson can also decay into an electron and an antineutrino, because both of them are derived from the decay of the massive W- boson, so it can also explain the source of lepton mass by the same logic. The Z- boson corresponds to $u\bar{u}$ or $d\bar{d}$, The Z- boson decaying into a quark-antiquark pair, and the massive Z- boson is also the source of quark mass. The Z boson does not completely correspond to the π0 meson, so protons or neutrons will not decay by emitting π0 mesons. Therefore, there is no spontaneous decay of protons. In the nucleus, the force between protons and neutrons is constantly maintained by the π0 meson, and the π0 meson will not decay through the Z boson. Neutrons with a larger mass than protons must emit π-/W- particles when decaying. From the above, we can know that during β decay:

$$d = u + e$$

There is no mention of antineutrinos here. Antineutrinos have a small mass because they are antimatter and decay into energy, while neutrinos are Majorana particles. Both neutrinos and antineutrinos have positive mass and opposite parity, and neutrinos are electrically neutral elementary particles and should not have magnetic moments, otherwise they would participate in electromagnetic interactions in addition to weak interactions. Protons can also emit π+/W+ particles and transform into neutrons, but this requires energy absorption rather than spontaneity. Then I will use matrix operations to explain the phenomenon of QCD. If we have a gluon matrix:

$$\begin{bmatrix} \gamma & b\bar{r} & g\bar{r} \\ r\bar{b} & b\bar{b} & g\bar{b} \\ r\bar{g} & b\bar{g} & g\bar{g} \end{bmatrix}$$

Let’s assume that the red quark is R (1,0,0), the blue quark is B (0,1,0), and the green quark is (0,0,1)

When a blue quark emits a blue anti-blue gluon:

$$[0 \quad 1 \quad 0] \times \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} = [0 \quad 1 \quad 0] = B$$

When a blue quark from an adjacent proton or neutron receives a blue anti-blue gluon:

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \times \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = B$$

the colors of the two red quarks that emit or receive red anti-red gluons remain unchanged, but if we use blue quarks B (0,1,0) or green quarks G (0,0,1) instead, we can only get zero matrices, proving that blue quarks or green quarks have no interaction with red anti-red gluons in terms of emission and absorption. Therefore, they can only act on adjacent proton or neutron red quarks.

Similarly, when a green quark emits a green-antiblue gluon, it turns blue:

$$[0 \quad 0 \quad 1] \times \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} = [0 \quad 1 \quad 0] = B$$

When a blue quark receives a green-antiblue gluon it turns green:

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \times \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = G$$

If we use red quarks (1,0,0) and green anti-blue gluons to do calculations, we can only get a zero matrix, indicating that there is no interaction between them. This explains the color change of colored gluons in the nucleus when quarks are exchanged and the overall color neutrality is maintained, which is the basic spirit of Q CD. When gluons or quarks are zero, the matrix multiplication must be

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zero, so gluons and quarks must be coupled. The same can be deduced for antineutrons and antiprotons. At this time, the antigluon matrix is:

$$\begin{bmatrix} \gamma & r\bar{b} & r\bar{g} \\ b\bar{r} & b\bar{b} & b\bar{g} \\ g\bar{r} & g\bar{b} & g\bar{g} \end{bmatrix}$$

And suppose the anti-red quark is R (-1,0,0), the anti-blue quark is B (0,-1,0), and the anti-green quark is G (0, 0, -1)

When an anti-blue quark emits a blue anti-blue gluon:

$$\begin{bmatrix} 0 & -1 & 0 \end{bmatrix} \times \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 \end{bmatrix} = -B$$

When an adjacent antiproton or antineutron antired quark receives a red antired gluon:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \times \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix} = -R$$

Similarly, when an anti-green quark emits a blue anti-green gluon, it turns anti- blue:

$$\begin{bmatrix} 0 & 0 & -1 \end{bmatrix} \times \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 \end{bmatrix} = -B$$

When an anti-blue quark receives a blue anti-green gluon, it becomes anti- green:

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \times \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix} = -G$$

I would like to talk about the basic phenomenon of natural radiation decay. We know that the basic radiation decay includes alpha decay, beta decay and gamma decay. Here, I suggest that alpha decay is the release of alpha particles, which are related to the strong nuclear force such as gluons (SU (3)) or pions. β decay is the release of W particles. And, gamma decay is the release of photons. And, we all know that beta decay is related to W^+ Emission or absorption or W^- Emission or absorption of bosons It is more common for a W boson to decay into an electron and an antineutrino. Also, a nuclear neutron will become a proton. Therefore, I say that beta decay is the release of a W (SU(2)) particle. Gamma decay, gamma rays can be released from an excited nucleus. However, the charge-mass is not affected. And gamma decay is the release of a photon (U (1)) from the nucleus. This will help explain the characteristics of these three basic nuclear decays.

Here we should also add Yang Mi field theory of the standard model

$$\text{Fuv} = \partial_u A_v - \partial_v A_u - [A_u, A_v]$$

The torsion tensor is

$$\text{T}(X,Y) = D_x Y - D_y X - [X,Y]$$

[X, Y] are Lie brackets.

$$[X,Y](f) = X\big(Y(f)\big) - Y\big(X(f)\big)$$
$$[X,Y] = \text{XY} - \text{YX}$$

It is worth noting that Yangmi field theory, which describes strong and weak forces, has Lie brackets of non - commutative terms, which means that strong and weak forces are non -holonomic non-conservative forces, which are different from gravity or electromagnetic force, which are holonomic conservative forces, although they are still field forces. This explains why strong and weak forces need path integrals. I think that formulas containing Dirac spinor (without geometric meaning) are wrong, and QM, QCD, QED containing Dirac spinor need to be re-examined, but formulas based on Yang-Mills field theory such as gluon field strength tensor are correct. Lagrangian can be derived from Klein-Gordon equation. Since Yang-Mills field theory describes the interaction between strong

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and weak forces, it can be seen that Yang-Mills field theory is also a torsion tensor like the electromagnetic field tensor (but the electromagnetic field tensor does not have Lie brackets), so the electromagnetic and strong and weak forces can be unified by geometric methods. The gravitational field is a curvature tensor, and the grand unified field theory is to unify curvature and torsion by geometric methods. It can be seen that strong and weak forces also have attractive and repulsive forces. In the strong force, like colors repel each other while opposite colors attract each other. The three colors correspond to the three axes XYZ of the space coordinate system. Positive values represent positive time while negative values represent negative time. For example, the +X axis is red while the -X axis is anti - red, the + Y axis is blue while the -Y axis is anti-blue, and the + Z axis is green while the -Z axis is anti-green. The red anti-blue gluon moves from the + X axis to the -Y axis, and they attract each other according to the torsion tensor. Because space is only three-dimensional, there is no SU(4) and no the fourth generation particles. The reason for quark confinement is that each gluon has two colors, just like a magnet with magnetic moment has north and south poles and cannot have magnetic monopoles. Similarly, we cannot separate single-color quarks or gluons to cause quark confinement. Anti-color represents that negative time is not easy to exist, so even neutral gluons cannot be separated.

To add to the beta decay in the weak force, why do free neutrons decay but neutrons in the nucleus bound to protons are not easy to decay? Because the mass of neutrons is large, they will emit π^- mesons to become protons and then form W^- bosons, and finally decay into electrons and antineutrinos. Since protons are composed of the smallest possible quark unit charge and gluons, they will not decay or have a very long half-life and are very stable. Let's talk about the important concept of QCD. There are a total of eight plus one pion in the composition matrix of protons or neutrons, that is, six colored gluons and three neutral gluons, one of which is closely related to the meson. There are three different colors of red, blue and green quarks in protons or neutrons, which can exchange colored gluons with each other $r\bar{b}$, $b\bar{r}$, $g\bar{b}$, $b\bar{g}$, $r\bar{g}$, $g\bar{r}$ and change colors to maintain the strong force in protons or neutrons. However, the quarks inside the proton or neutron, for example, a blue quark can emit a $b\bar{b}$ neutral gluon, but the other two quarks in the proton or neutron are blue and red and cannot accept this $b\bar{b}$ gluon. Therefore, this $b\bar{b}$ gluon must exchange the mediating force with the blue quark of the adjacent proton or neutron to perform internuclear bonding. This explains why the nuclear force πis mediated by mesons as a residual strong force. So why when a proton and a neutron are connected, the neutron will not emit πa meson (containing a neutral gluon) and decay. Neutral gluons πmediate the internuclear force through mesons. The nuclear meson itself can change πcolor from blue-anti-blue to green-anti-green and other three neutral colors (including neutral gluons) by exchanging blue-anti-green gluons. Neutral gluons mediate the nuclear force through neutral mesons due to color charge πconfinement (no gluon waves).

The author would like to talk about the existence of Yang-Mills field theory. It can be proved later that Yang-Mills field theory is a quantum field theory. After careful deduction, I found that quantum mechanics and quantum electrodynamics are not correct. The massless Yang-Mills field theory gives the mediator mass through the Higgs field. These mediators spin-1 have mass bosons (gluons, π mesons) and can be described by Yukawa interaction. This is also the spin-1 solution of the Klein-Gordon equation. The latter is the scope of quantum field theory. It can be seen that Einstein's field equation is a curvature tensor and Yang-Mills field theory is a torsion tensor. It is wrong to describe the so-called proton field or electron field with quantum electrodynamics or Dirac field. The so-called positron field is not a field excited state because it has negative energy, but antimatter moves toward the negative time axis, so matter moving toward the positive time axis will not all become antimatter, which solves Dirac's original doubts. And Yang-Mills field theory:

$$\text{Fuv} = \partial_u A_v - \partial_v A_u - [A_u, A_v]$$

If it is a left invariant vector, then all particles in space, such as neutrinos, all spin to the left. At this time:

$$[A_u, A_v] = 2\partial_u A_v$$

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Then the above formula becomes:

$$\mathrm{Fuv} = \partial_u(-A_v) - \partial_v A_u$$

It becomes the vortex line equation of the standard electromagnetic force, so the clockwise or counterclockwise vortex field can be used to explain the attraction or repulsion of strong or weak forces. If it is a right invariant vector, such as particles all spin to the right:

$$[A_u, A_v] = -[A_u, A_v]$$

Then the above formula becomes:

$$\mathrm{Fuv} = \partial_u(3A_v) - \partial_v A_u$$

It will not become the standard vortex line equation and cannot explain the attraction or repulsion of strong or weak forces. Therefore, the strong or weak forces of the standard model can only act on left-handed particles, which explains why parity is not conserved. The left-hand restriction can make the gauge field theory have Lorentz invariance, so that the strong and weak forces have time and space symmetry, that is, the laws of the strong and weak force equations in any space and time in the universe are unchanged, only the physical system itself, such as left-handed or mass characteristics, has spontaneous symmetry breaking. Since U (1) electromagnetic and Higgs interaction can produce electrons and protons, electrons and protons can be left-handed or right-handed. However, neutrinos can only be produced by weak interactions, so neutrinos are all left-handed. Neutrons are all produced by strong interactions, so it can be inferred that neutrons are all left-handed. This inference is reasonable, because beta decay only acts on left-handed neutrons to decay into protons, so right-handed neutrons will not decay. However, the mass of neutrons is greater than that of protons, and it is not correct that right-handed neutrons will not decay. Therefore, the author believes that the above contradiction can be resolved by not having right-handed neutrons at all.

Here, we use the classification of group theory to discuss the four major interactions: gravity , electromagnetic force, strong force, and weak force . These have been discussed in my works, but the orthogonal group is added to these interactions. I also proposed a model to modify the original Georgi-Glashow SU(5) model into the Hu SU(5) model to unify all discovered elementary particles. SU (5) is the smallest and simplest Lie group to unify the U (1)xSU(2)xSU(3) of the standard model . The model is as follows. The second matrix is the antiparticle matrix (L represents left-handed particles and R represents right-handed particles, and right-handed particles have zero isospin). The first matrix is used in the following discussion. The first matrix is most consistent with the axioms of quantum field theory , and uses positive and negative charges and supercharges to arrange them:

$$\begin{bmatrix} \gamma & b\bar{r} & g\bar{r} & d & e \\ r\bar{b} & b\bar{b} & g\bar{b} & s & \mu \\ r\bar{g} & b\bar{g} & g\bar{g} & b & \tau \\ u & c & t & Z & W^- \\ \nu_{eL} & \nu_{\mu L} & \nu_{\tau L} & W^+ & H \end{bmatrix}$$

$$\begin{bmatrix} \gamma & r\bar{b} & r\bar{g} & \bar{d} & \bar{e} \\ b\bar{r} & b\bar{b} & b\bar{g} & \bar{s} & \bar{\mu} \\ g\bar{r} & g\bar{b} & g\bar{g} & \bar{b} & \bar{\tau} \\ \bar{u} & \bar{c} & \bar{t} & Z & W^+ \\ \bar{\nu}_{eR} & \bar{\nu}_{\mu R} & \bar{\nu}_{\tau R} & W^- & H \end{bmatrix}$$

$$U(1) \times SU(2) \times SU(3) \to S\left(U(2) \times U(3)\right) \subset SU(5)$$

$$U(1)y \times SU(2)L \times SU(3)c \to SU(5)$$

Kernal :$\left\{\left(\alpha, \alpha^{-3} Id_2, \alpha^2 Id_3 \middle| \alpha \in c, \alpha^6 = 1\right)\right\} \cong Z6$

When zero power: $\Lambda^0\ C^5$

This fits the left-handed neutrino and fills in the upper right corner of the matrix:$C_0 \otimes C \otimes C$

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When first exterior power: $\wedge^1 C^5 \cong C^5$

$$C^5 \cong C^2 \oplus C^3$$

When $\alpha^{-3} = \alpha^{6Y}$

At this time, the SU(2) weak interaction hypercharge Y =-1/2, which means that the 2 x2 matrix is filled to the lower right corner .

When $\alpha^{2} = \alpha^{6Y}$

At this time, the SU(3) strong interaction hypercharge Y = 1/3, which represents a 3x3 matrix that we fill to the upper left corner .

The superloads obtained by both are completely consistent with the author's inference.

When second power: $\wedge^2 C^5$

Obtained by the following formula:

$$\wedge^2 (V \oplus W) = \wedge^2 V^2 \oplus (V \otimes W) \oplus \wedge^2 V^2$$

Hodge duality (upper three power) :

$$\wedge^P C^5 \cong \left(\wedge^{5-P} C^5 \right)^*$$

When P = 2, then 5-P = 3 , which means there are two three-generation fermion elementary particles. Fill in the remaining columns with Y = 1/3 of the up quark and down quark generations , and Y = -1/2 of the electron and neutrino generations according to the duality principle . The upper right (1-i) is the positive charge (Q) or positive color charge (Q^c) and the lower left (1+i) is the negative charge or negative color charge. The charge of the elementary fermion can be calculated by the NNG formula. The positive and negative charges are determined by the opposite rotation direction of the complex plane. Both bosons and fermions are filled in according to this rule:

Supplementary quantum chromodynamics supercharge and isospin:

	Y	I^c		Y	I^c
r	1 /3	1 /2	$\bar{r}$	- 1/3	- 1/2
b	1 /3	- 1/2	$\bar{b}$	- 1/3	1 /2
g	- 2/3	0	$\bar{g}$	twenty three	0

We can get :

	Y	I^c	$Q^c = I^c + 1/2 Y^c$
$r\bar{g}$	1	1/2	1
$g\bar{r}$	- 1	-1/2	- 1
$r\bar{b}$	0	1	1
$b\bar{r}$	0	- 1	- 1
$b\bar{g}$	1	- 1 /2	0
$g\bar{b}$	- 1	1/2	0
$b\bar{b}$	0	0	0
$g\bar{g}$	0	0	0

In addition, W^+ charge + 1 and W^- charge - 1 (W particles and neutrinos are only left-handed) .

If we look at the electroweak interaction W particle:

$$W^{\pm} = \frac{1}{\sqrt{2}} \left(W_1 \pm i W_2 \right)$$

The matrix can be interpreted by defining a -bi as positively charged particles and a +bi as negatively charged particles in the conjugate complex numbers .

Since SU (2) is isomorphic to the three-dimensional spin Spin(3) group, the inclusion matrix of SU (2) is:

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

You can get:

$$\sigma^{\pm} = \frac{1}{2}\left(\sigma^1 \pm i\sigma^2\right)$$

And the corresponding charged current j:

$$j_u^{\pm} = j_u^1 \pm i j_u^2$$

You can get:

$$W^{\pm} = \frac{1}{\sqrt{2}}\left(W_1 \pm iW_2\right)$$

and:

$$\bar{5}_F = (r, b, g, e, -\nu)$$

Add the neutral bosons on the diagonal: $(\gamma, b\bar{b}, g\bar{g}, Z, H)$

$$5H = \left(T_1, T_2, T_3, H', H\right)$$

It can be used to indicate the $A_\mu^a T^a$3x3 gluon G matrix of SU(3) on the upper left and the 2x2 weak force W matrix of SU(2) on the lower right:

$$\begin{pmatrix} G_\mu^a T^a & 0 \\ 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 \\ 0 & \frac{\sigma^a W_\mu^a}{2} \end{pmatrix}$$

3x3 matrix λ^1at the upper left is λ^8equivalent to the various group representations of gluons, while the 2x2 matrix at the lower right is equivalent $\lambda^{21}$to $\lambda^{23}$the Pauli matrix of weak interactions, representing the various group representations, λ^9 to λ^{20} representing the three generations of fermions.

In addition, there is U (1) overload:

$$NB_\mu^0 diag(1/3, 1/3, 1/3, -1/2, -1/2)$$

If we list the representation of the 24th group:

$$\lambda^{24} = \frac{1}{\sqrt{15}} diag\left(2,2,2,-3,-3\right)$$

$$= \frac{2\sqrt{3}}{\sqrt{5}} diag\left(1/3, 1/3, 1/3, -1/2, -1/2\right)$$

And SU(5) generator:

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$$Y_w = \frac{1\sqrt{5}}{2\sqrt{3}}\lambda^{24} = diag(1/3, 1/3, 1/3, -1/2, -1/2)$$

The SU(5) symmetry breaking occurs in the Higgs field, which corresponds to the hypercharge and obtains the vacuum expectation value, which can explain the origin of the mass of elementary particles:

$$\langle 24H \rangle = v_{24} diag(1/3, 1/3, 1/3, -1/2, -1/2)$$

Because of the supercharge Y, SU(5) is symmetric broken into a subgroup, and SU(5) is the same as SU(3)xSU(2)xU(1), and the commutator relation is obtained:

$$[\langle 24H \rangle, G_\mu] = [\langle 24H \rangle, W_\mu] = [\langle 24H \rangle, B_\mu] = 0$$

Let's examine the group theory 5 x5 matrix again:

$$[\bar{5}] \times [5] = (8,1) + (\bar{3}, 2) + (3, \bar{2}) + (1,3) + 2(1,1)$$

25th position (1,1) in the matrix is filled with the Higgs particle, and the remaining 24 cells are:

$$[24] = (8,1,0) + \left(\bar{3}, 2, -\frac{5}{6}\right) + \left(3, \bar{2}, \frac{5}{6}\right) + (1,3,0) + (1,1,0)$$

The first term (8,1,0) represents the SU(3) matrix of eight gluons with zero hypercharge, which we fill in the upper left block. The fourth term (1,3,0) represents the SU(2) matrix of three weak bosons with zero hypercharge, which we fill in the lower right block. The fifth term (1,1,0) represents the photon boson with zero supercharge and zero mass, which we fill in the upper left block. The remaining two terms have non-zero supercharges. These two 3x2 and 2x3 matrices are not filled with bosons but with basic fermions, and their conjugate hypercharges are: -1/3-1/2=-5/6 or 1 /3+1/2=5/6, which come from quarks and leptons. Again: ($[5] + [10] + 1$)

$$[5] \to \left(3,1,\frac{1}{3}\right) + \left(1,2,\frac{-1}{2}\right)$$

$$\left(3,1,\frac{1}{3}\right) \otimes \left(1,2,\frac{-1}{2}\right) \cong \left(3,2,\frac{-1}{6}\right)$$

$$[10] \to \left(3,2,\frac{1}{6}\right) + \left(\bar{3}, 1, -\frac{2}{3}\right) + (1,1,1)$$

The first equation can represent $\left(T_1, T_2, T_3, H', H\right)$ the above, while the second equation means that the interaction between the two can generate a 3 x2 matrix and the hypercharge added is 1 /3 $-1/2$ = $-1/6$ or 1 / 2 $-1/3$ =1/6 to fill in the fermions. By $[10]$, the same token, the six basic fermions with positive charge, three colored gluons with positive color charge, and W^+ particles can be placed in the upper triangular matrix. For negative charge or negative color charge, we can also place them in the lower triangular matrix. In this way, we can unify the six colored gluons, two neutral gluons, photons, W^{+-}/Z particles, three generations of up quarks, three generations of down quarks, three generations of electrons, three generations of neutrinos, and Higgs particles, a total of 25 basic particles, into the 5 x5 SU(5) model. The arrangement of this model is lighter as it moves to the upper left corner and heavier as it moves to the lower right corner. The diagonal from the upper left to the lower right corner is all neutral particles, so the trace is zero. The upper right corner is positively charged or has positive color charge, and the lower left corner is negatively charged or has negative color charge. The difference between the two tables above is that matter or antimatter is filled in, and the boson axis is swapped. The upper left 3x3 matrix is the gluon SU (3) matrix, because there are no $r\bar{r}$ bosons, their positions are replaced by photons. The lower right is the weak interaction particle 2x2 SU(2) matrix, with the Higgs boson placed in the heaviest position. The upper left 3x4 or 4x3 matrix can be analogous to the C KM matrix, explaining the interaction between gluons and quarks to explain why quarks can get color. The rightmost row represents the W^+ particle that decays into neutrinos and anti-electron leptons (antiparticles are not shown in this table), and the bottom column represents the W $^-$ particle that decays into electron leptons and anti-neutrinos (antiparticles are not shown in this table). The second row from the left and the second row from the bottom explain why Z particles tend to decay into quark and antiquark pairs. It is worth noting that the top quark (t) in the second row on the left and the bottom quark (b) in the second row below must be formed by the combination of Z and W particles (especially the right-handed top quark in the second matrix is most consistent with this prediction). The mass of the Higgs boson plus the W $^-$ particle divided by the electric coupling constant 1.37 or the weak coupling constant 1836 can be estimated to be 1.5 GeV for the tau and 1.10 MeV for the muon. This model also indicates that neutrinos are all left-handed and can also gain mass through the Higgs mechanism. This modified model does not propose a new mediating force XY boson, so there will be no proton decay and violation of baryon and lepton number conservation, or the two-three fission problem or the magnetic monopole problem predicted by the Georgi -Glashow SU(5) model. Moreover, the dimension (rank) of the SU(5) model is 5-1=4, which just matches our four-dimensional Minkowski spacetime with three-dimensional space and one-dimensional time. There will be no problems of super-high dimensions and superpartners in string theory. This model unifies all elementary particles. The Euclidean space of SO(4) is equivalent to the Minkowski spacetime of the Lorentz group S O $^+$ (1,3), which can be expressed by SU(2)xSU(2). Therefore, the six-dimensional (six types) elementary particles can be obtained by the interaction between particles W, Z, H, etc. in the 2 x2 SU(2) matrix at the bottom right.

$$10 \rightarrow \begin{bmatrix} 0 & b\bar{r} & g\bar{r} & d & e \\ r\bar{b} & 0 & g\bar{b} & s & \mu \\ r\bar{g} & b\bar{g} & 0 & b & \tau \\ u & c & t & 0 & W^- \\ \nu_{eL} & \nu_{\mu L} & \nu_{\tau L} & W^+ & 0 \end{bmatrix}$$

composed of the group theory of U(1)xSU(2)xSU(3), the numbers in the brackets 1 +2+3=6 or 1 x2x3=6 represent six types of baryons, leptons or bosons, and the sum of the three dimensions 1 +(2^2-1)+(3^2-1)=12 represents a total of 12 types of fermions (including quarks and leptons) or a total of twelve mediating bosons. The division into three generations requires the introduction of the concept of the Lorentz group SO$^+$ (1,3). The Lorentz group was first introduced by relativity. Almost all physical quantities can be included in the Lorentz group as long as they conform to the Lorentz transformation. Its four units are one-dimensional time and three-dimensional space. The Lorentz group represents a global symmetry of time and space, that is, the physical quantity or physical law is invariant in every time and space in the universe. The structure of the Lorentz group is a six-dimensional non-compact, non-connected, non-Abelian real Lie group. Non-compact means that the state space of a single particle is an infinite-dimensional Hilbert space that can connect the infinite potential wells of elementary particles. Non-connected means that the three-generation 3x2 matrix of up quarks and neutrinos is not connected to the three-generation 2x3 matrix of down quarks and electrons, and they are not inverses of each other and therefore not connected. Non-Abelian means the non-commutativity of Lorentz transformations and the Standard Model is a non-Abelian theory. Lie group means that the universe, time and space are smooth manifolds, including the smallest level of quantum field theory symmetry such as symplectic group or unitary group. The Lorentz group has two important components: Lorentz rotation and Lorentz boost. The former is the meaning of rotation, which corresponds to isospin and chirality in elementary particles. The quark family can be divided into up quark and down quark groups, which correspond to each other as a Lorentz group. The lepton family can be divided into electron and neutrino groups, which correspond to each other as another Lorentz group. They are divided into two major groups because of the opposite rotation of isospin signs. Isospin also has a linear relationship with charge. It can be said that the up quark isospin +1/2 turns to the down quark isospin -1/2 with a difference of one unit, and the up quark charge +2/3e turns to the down quark charge -1/3e with a difference of 1e unit. The neutrino isospin +1/2 turns to the electron isospin -1/2 with a difference of one unit, and the neutrino charge 0e turns to the electron charge -1e with a difference of 1e unit. The latter has the meaning of motion, which is a linear transformation. It can be connected to the motion of particles in the three axes of space, X, Y, and Z. Therefore, there will be three generations of quarks and three generations of leptons. Due to the Higgs mechanism, the mass of particles near the Z axis is larger, and the mass of particles near the X axis is smaller. This can correspond to why the Lorentz group is six-dimensional. The Lorentz group has four connected units (t, x, y, z), which is one-dimensional time plus three-dimensional space. These four connected units are not simply connected but path connected. This can explain the characteristics of our four-dimensional space-time universe, such as the universe is FLRW metric is uniform, isotropic, and connected with expansion, but not necessarily simply connected. Here we can compare that the six vectors of the charge tensor (Ex, Ey, Ez, Bx, By, Bz) can also form a Lorentz group, which is a six-dimensional quantity. Non-compact means that the electric and magnetic fields are not closed sets, non-connected means that the electric and magnetic fields are not inverse elements of each other, and non-commutative means that vector multiplication is non-commutative ExB ≠BxE. The rotation of the electric field causes the magnetic field, and the electric field or magnetic field each has three axes XYZ, so there are also the concepts of Lorentz rotation and Lorentz boost, and the electromagnetic fields can be transformed by Lorentz transformation, so it is reasonable to form a Lorentz group.

Here, the author uses the Hu SU(5) model to prove that the standard model Yang-Mills field theory conforms to the Wightman axioms, which makes the standard model conform to the mathematical construction axioms of quantum field theory.

$$\begin{bmatrix} \gamma & b\bar{r} & g\bar{r} & d & e \\ r\bar{b} & b\bar{b} & g\bar{b} & s & \mu \\ r\bar{g} & b\bar{g} & g\bar{g} & b & \tau \\ u & c & t & Z & W^- \\ \nu_{eL} & \nu_{\mu L} & \nu_{\tau L} & W^+ & H \end{bmatrix}$$

Axiom 1. There exists a Hilbert space H on which the Poincare spinor P acts, and the unitary U-representation group:

Since the matrix is SU(5), it is itself a special unitary U-representation group, and the rank of the SU(5) group is 5-1=4, which can be expressed by a three-dimensional plus one-dimensional Minkowski spacetime, which is isomorphic to the Poincare group. This proves that there exists a unitary U-representation group of the Hilbert space H.

Axiom 2. The spectrum of energy-momentum operators P' is enclosed in the upper light cone:

Since the Einstein-Dirac energy-momentum relation can also be used for elementary particles, we can prove:

$$E^2 = (pc)^2 + (mc^2)^2$$

$$P_0^2 - p^2 = (mc)^2 \geq 0$$

$$P_0 = \frac{E}{c} \geq 0$$

In this SU (5) matrix, all the energy and mass are positive. The first matrix does not contain antiparticles with negative time, so it is in the upper light cone of the future, and thus it can conform to this axiom.

Axiom 3. There exists a unit vector | 0> in the Hilbert space H. This is the definition of a vacuum that is invariant to space-time transformations.

In the author's SU (5) model, if all fermions and bosons are removed, the zero vector represents the vacuum state. This vector | 0> does not change in time and space conversion. And SU(5):

$$U(1)y \times SU(2)L \times \frac{SU(3)c}{Z6} \rightarrow SU(5)$$

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Z 6 represents a cyclic group and is related to the definition of vacuum.

Axiom 4. A quantum field $\varphi$is a linear distribution in Schwarz space whose domain is defined as D and D contains the unit vector | 0>.

Since SU(5) is a matrix representing linear operations, and elementary particles can be regarded as having high energy, high charge or high mass in a certain small fixed space, the elementary particles can be regarded as a field that conforms to the Schwarz distribution as a convex function, and the convex function value of the space adjacent to the elementary particle is zero. We can define the field domain of these elementary particle fields as D, and the empty set, namely the vacuum | 0>, is included in it.

Axiom 5. The quantum field is covariant under the action of the Poincare group. Under the Lorentz transformation, the representation group S is the Lorentz group or SL (2,C):

$$U(a,L)^{\dagger}\varphi(x)U(a,L) = S(L)\varphi\big(L^{-1}(x-a)\big)$$

Taking S U (2) as an example, it can be proved as follows: Lorentz transformation is defined as:

$$\Lambda \in SO^{+}(1,3)$$

for Λ^{μ}_{ν}, spacetime indexes$\mu,\nu = 0,1,2,3$

The four vectors can be represented by the Pauli matrix:

$$\sigma^{\mu} = (I,\vec{\sigma})$$

$$\overline{\sigma^{\mu}} = (I,-\vec{\sigma})$$

and:

$$\overline{\sigma_{\mu}} = \sigma^{\mu}$$

And the Lorentz transformation:

$$x^{\mu} \to x'^{\mu} = \Lambda^{\mu}_{\nu}x^{\nu}$$

$$S \in SL(2,C)$$

$$x'^{\mu}\overline{\sigma_{\mu}} = \overline{\sigma_{\mu}}\Lambda^{\mu}_{\nu}x^{\nu} = Sx^{\nu}\overline{\sigma_{\nu}}S^{\dagger}$$

$$\overline{\sigma_{\mu}}\Lambda^{\mu}_{\nu} = S\overline{\sigma_{\nu}}S^{\dagger}$$

And we know SU(5):

$$U^{-1} = U^{\dagger}$$

Substituting it back gives:

$$U(a,L)^{\dagger}\varphi(x)U(a,L) = S(L)\varphi\big(L^{-1}(x-a)\big)$$

Another proof is from physics.stackexchange.com:

$$G \ni g \to U\,g$$

$$Lg: g \to g$$

$$ge^{ts}g^{-1} = e^{tLgS}$$

$$G \ni g \to L\,g$$

$$L\_g\ s\_k = \sum A\ \hat{}r\_k(g)\ s\_r$$

$$U\ g{=}exp\ ({-}i\ \sum a\hat{}kS\_k)$$

$$s_k \to -iS_k$$

$$U_gS_kU_g^{-1} = \sum A\hat{}r\_k(g)Sr$$

When g represents the Lorentz transformation and S represents the quantum field, it can be proved. We know that in the SU (5) model, there are six third-generation fermions with positive and negative charges in the upper right and lower left respectively, which belong to the Lorentz group and conform to the Lorentz transformation. In the standard model SU (2) and SU (3), there are six colored gluons and six colorless bosons, which also belong to the Lorentz group and conform to the Lorentz transformation. In fact, this SU (5) model can be described by the Minkowski four-dimensional spacetime isomorphic to the Poincare group. The Lorentz group is included in the Poincare group and can naturally be described. Similar to the covariant Faraday tensor, the upper right corner of the matrix is positively charged, so this SU(5) is also covariant. This standard model is a covariant equation.

Axiom 6. The two components of a quantum field are either $\varphi(x)$connected (commute) or $\varphi(y)$not connected (non -commute) for the spatially separated x and y.

Since this SU (5) is decomposable and contains the strong interaction SU (3) and the weak interaction SU (2), which are both on the diagonal, SU (3) is a 3x3 Cayley table and SU (2) is a 2x2 Cayley table, which means that these spin 1 bosons have connectivity. For example , $r\bar{g}$a -bi is a +bi, which are conjugate complex numbers and $g\bar{r}$are therefore inverses of each other and conform to the Cayley table, even though neither is Abelian. Spin 1/2 fermions are subgroups in the 3x2 and 2x3 matrices on the upper right and lower left, and have different multiplication dimensions and therefore have no connectivity (AB BA). Unlike the two SU(2) and SU(3) Cayley tables mentioned above, which have inverses of each other on the diagonal, $\neq$the up quark generation 3 corresponds to the down quark generation 3 and the neutrino generation 3 corresponds to the electron generation 3 in the fermion matrix, but they are not inverses of each other and have no connectivity.

Axiom 7. The linear combination of the previous vectors D is dense in the Hilbert space H.

every position in the SU(5) model, its complement is the empty set, which meets the definition of dense.

Here is a supplementary explanation of the difference between bosons and fermions:

Symmetric state S of wave function in space:

$$\Psi_s\left(r_1,r_2\right)=\frac{1}{\sqrt{2}}\left(\varphi_a\left(r_1\right)\varphi_b\left(r_2\right)+\varphi_b\left(r_1\right)\varphi_a\left(r_2\right)\right)$$

Antisymmetric state of wave function in space A:

$$\Psi_A\left(r_1,r_2\right)=\frac{1}{\sqrt{2}}\left(\varphi_a\left(r_1\right)\varphi_b\left(r_2\right)-\varphi_b\left(r_1\right)\varphi_a\left(r_2\right)\right)$$

Symmetric states correspond to bosons and antisymmetric states correspond to fermions, which can be explained by the author's SU (5) model. Bosons are located on the diagonal and the Carré table, so they are symmetrical, while fermions are located in the antisymmetric part of the matrix, which is completely consistent . Since the antisymmetric state has two identical particles, the wave function will be zero, which is not allowed, indicating that the antisymmetric state can be explained by the Pauli exclusion theorem as Pauli repulsion. Boson exchange has an attractive effect, indicating that bosons mediate the force and are related to Bose-Einstein condensation. This can also be linked to the statistical properties of spin 1 or 1/2.

When two particles are exchanged with each other, there is an exchange operator P:

$$P(|\psi>|\phi>)\equiv|\phi>|\psi>$$

P is a Hermitian operator and a unitary operator and $P^2=1$, so its value can only be + 1 or - 1, and the corresponding eigenstates are the symmetric state and the antisymmetric state:

$$P|n1,n2;S\geq+|n1,n2;S>$$

$$P|n1,n2;A\geq-|n1,n2;A>$$

After particle exchange, the symmetric state and the antisymmetric state do not change basically, they are just multiplied by + 1 or - 1 instead of being rotated in the Hilbert space. This is the concept of identical particles. The other matrix is the 4x4 matrix of Riemann tensor with 20 components. General relativity says that the original 4x4x4x4=256 components of Riemann tensor can be contracted into a 20 components tensor. In this 20 component tensor, 10 is for Weyl tensor and the other 10 is for

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Ricci tensor. In fact, if we consider the Lorentz symmetry, the final independent invariants should be 20-6=14 components. The matrix is given below:

Since the Georgi- Gl ashow SU(5) model and 4x4 unified field theory are both included in a larger group of n (n-1)/2=45 particles in dimension SO(10) or Spin(10), SO(10) may be more integrated. The Spin group is an arrangement related to charge. Since the rank of SO(10) is 5, it needs to be symmetry broken to be reduced to rank 4 of SU(5) or 4x4 unified field theory. The basic subgroup concept is as follows:

$$SSO(2n) \supset SU(n) \;\; \therefore SO(10) \supset SU(5)$$

$$SO(6) \cong SU(4)$$

$$SO(4) \cong SU(2)xSU(2)$$

$$SO(1,3) \cong \frac{SU(2)xSU(2)}{Z2}$$

$$SO(6)xSO(4) \to SO(10)$$

The S O group has branching rules:

$$SO(n-2)xSO(n) \subset SO(n)$$

S O(10) branching rules: Decomposing S O(10) 4 5 field branches:

$$45 \to (15,1,1)\oplus(6,2,2)\oplus(1,3,1)\oplus(1,1,3)$$

Considering elementary particles, time and space as fields, particles are the excitation of fields or particles are mass, charge, or energy confined in a small space:

From this we can write a contravariant SO(10) matrix: (four gradient is fundementally contravariant)

$$\begin{bmatrix} -\rho & -Ex & -Ey & -Ez & \gamma & b\bar{r} & g\bar{r} & d & e \\ Ex & Px & -Bz & By & r\bar{b} & b\bar{b} & g\bar{b} & s & \mu \\ Ey & Bz & Py & -Bx & r\bar{g} & b\bar{g} & g\bar{g} & b & \tau \\ Ez & -By & Bx & Pz & u & c & t & Z & W^- \\ \partial t & -\partial x & -\partial y & -\partial z & \nu_{eL} & \nu_{\mu L} & \nu_{\tau L} & W^+ & H \end{bmatrix}$$

$$\begin{bmatrix} -\rho & -Ex & -Ey & -Ez \\ Ex & Px & -Bz & By \\ Ey & Bz & Py & -Bx \\ Ez & -By & Bz & Pz \\ \partial t & -\partial x & -\partial y & -\partial z \end{bmatrix}$$

This second marix is the 20 components Riemann matrix in four dimmensions (n=4). The actual independent invariants are 14 because E and B fields are not totally independent.

$$N = \frac{\left[n^2\left(n^2-1\right)\right]}{12} = 20$$

The Weyl tensor (not invloving volume change or divergence) is below. It is a pure magnetic because pure electric means there is a static universe or electrovac spacetime which is only with electromagnetism without gravity. Thus, pure electric is not the case in our universe. On the other hand, magnetic part is related to Weyl tensor.

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$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & -Bz & By \\ 0 & Bz & 0 & -Bx \\ 0 & -By & Bz & 0 \\ \partial t & -\partial x & -\partial y & -\partial z \end{bmatrix}$$

$$N = \frac{[n(n+1)(n+2)]}{12} = 10$$

The Ricci tensor (involving volume change or divengence) is:

$$\begin{bmatrix} -\rho & -Ex & -Ey & -Ez \\ Ex & Px & 0 & 0 \\ Ey & 0 & Py & 0 \\ Ez & 0 & 0 & Pz \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$N = \frac{[n(n+1)]}{2} = 10$$

The 20 component Riemann tensor can be divided into three main parts: electric, magnetic, and mixed time-space components. First of all, the electric part has six components including R0101(E11), R0102(E12), R0103(E13), R0202(E22), R0203(E23), and R0303(E33): The index number 1,2,3 can be related to x,y,z, and R stands for Riemann tensor.

$$\begin{bmatrix} 0 & -Ex & -Ey & -Ez \\ Ex & 0 & 0 & 0 \\ Ey & 0 & 0 & 0 \\ Ez & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Then, the magnetic part also has six components including R1212(B33), R1213(B32), R1223(B31), R1313(B22), R1323(B21), and R2323(B11).

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & -Bz & By \\ 0 & Bz & 0 & -Bx \\ 0 & -By & Bz & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Finally, the mixed time-space part has eight components including R0112, R0113, R0123, R0212, R0213, R0223, R0312, and R0313.

$$\begin{bmatrix} -\rho & 0 & 0 & 0 \\ 0 & Px & 0 & 0 \\ 0 & 0 & Py & 0 \\ 0 & 0 & 0 & Pz \\ \partial t & -\partial x & -\partial y & -\partial z \end{bmatrix}$$

The covariant SO(10) matrix is a combination of SU(5) and the unified field theory matrix SO(1,3) ≅SO(4). (15,1,1) represents the unified field theory matrix except -ϱ. (6,2,2) represents the six third-generation quarks, six third-generation leptons, and twelve bosons (three W/Z particles, eight gluons, and one Higgs boson) in SU(5). (1, 1, 3) and (1, 3 , 1) represent the time-related γ, ∂t, and -ϱ , respectively , and represent the three axes of space $-\partial x.\ -\partial y,\ -\partial z$, this dt, dx, dy, dz also shows that space-time can be quantized with the smallest unit and has the concept of differentiable field and is connected to the gravitational field or electromagnetic field above. The lower part is the elementary particle and the upper part is the field. Because according to quantum field theory, the field and the particle are inseparable (Particle-field interaction) , they are placed in the same matrix. The particle field can be regarded as a quantized unit containing a fixed amount of mass or charge in an extremely small space. The first four on the far right are filled with the gradient four-vector of the most basic four-vector. The lower part of this SO(10) matrix is the weak force and strong force of SU(2) and SU(3), the upper side is the gravitational field, electromagnetic field, and light pressure, and time and space are the field of field-particle interaction. In the SU(5) model, the SU(3) matrix represents a strong interaction 3x3 matrix or a similar 3x4 CKM or 4x3 CKM matrix, which may not include the right-handed up quark third generation and the neutrino third generation, which may represent CP violation. However, if the electromagnetic 4x9 matrix is included in the SO(10) matrix, all up quark third generations can be included, so the up quark third generation and the electron third generation can also be right-handed , but the neutrino third generation is not included so it can only be left-handed. Since the vacuum zero-point energy predicted by quantum electrodynamics is 10^{114} erg, but the vacuum zero-point energy measured by the Voyager exploration satellite is 10^{-8} erg, this exaggerated prediction of quantum electrodynamics is considered to be a major flaw of QED, which is called the vacuum catastrophe. If we look at the rightmost row of the SO(10) matrix , we can find the relationship between time, space and neutrinos, which also shows that the Higgs boson is everywhere in time and space. The geometric unit r=2m (derived from the Schwarzschild radius) implies that the Planck length of space quantization is related to the mass of neutrinos. The smallest unit of space, Planck length Lh=r=1.6x10^-35 meter, and the mass of electron neutrinos is 1ev/c2 , then 2x2x1.6x10^-19 joule / ≈(3x10^8)^ 2*10^-35, which is not much different. Left-handed neutrinos are very likely the main dark matter. The energy levels of the three generations of neutrinos are 10^-12 erg, 10^-7 erg, and 10^-5 erg, respectively. Therefore, the sum of the three flavors of continuous neutrino oscillation is closer to the observed vacuum zero-point energy of 10^-8 erg. The medium of strong and weak forces is particles (gluons or W/Z particles), and the medium of electromagnetism and gravity is field (geometry of space-time torsion or curvature). When the symmetry is broken, the four forces separate. When the initial Planck energy of the universe was 10^{19} GeV, the four forces were united (Planck time of Planck epoch). When the energy dropped below 10^{16} GeV, the four forces separated. The SO(10) matrix unified the four force fields and became the Theory of Everything, which is also the Greatest Unified Field Theory (GUTSiest).

Removing the negative signs of the ideal fluid and the four gradients gives this matrix (set to be covariant).

Decompose SO(10) (45 field branches) for SU(3)cx SU(2)L x U(1)y/Z6 :

$$[45] = (8,1,0)\oplus(1,3,0)\oplus(1,1,0)\oplus\left(\bar{3},2,-\tfrac{5}{6}\right)\oplus\left(3,\bar{2},\tfrac{5}{6}\right)\oplus\left(3,1,\tfrac{2}{3}\right)$$

$$\oplus\left(\bar{3},1,-\tfrac{2}{3}\right)\oplus(1,1,1)\oplus(1,1,0)\oplus(1,1,-1)\oplus\left(\bar{3},2,\tfrac{-1}{6}\right)\oplus\left(3,\bar{2},\tfrac{1}{6}\right)$$

Among them, (8,1,0) represents the eight strong gluon boson matrices in SU(5), (1,3,0) represents the three W/Z weak boson matrices in SU(2), (1,1,0) is the Higgs boson in the lower right corner, ($\bar{3}$,2,-5/6) and (3, $\bar{2}$,5/6) are filled with three third-generation up quarks and three third-generation electrons, and three third-generation down quarks and three third-generation neutrinos, respectively, and (3,1,2/3) is filled with the three light pressure terms Px, Py, Pz of the upper matrix, ($\bar{3}$,1,-2/3) is filled in with the three quadratic gradient differential terms on the right $-\partial x, -\partial y, -\partial z$. This is

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because the two are almost of the same dimension. 1/length^2 can be obtained as magnitude from the inner product of the four gradients. In this way, we can put the wave equation (including mechanical waves, heat conduction, sound waves, fluid mechanics, etc.) into the framework of this universal theory. Originally, the SO(10) matrix was filled in with the position four-vector, but the ideal fluid (- $\rho c^2, \rho c v$)can be reduced to four velocities or four positions or other four-vectors by multiplying and dividing certain constants. Therefore, there is no need to fill in the position four-vector to represent the special theory of relativity. On the contrary, the four gradients cannot be obtained by such operations, and the four gradients can obtain other four-vectors such as the four heat flows. If (1,1,1) is filled in, ∂t the wave equation will be fully presented (the inner product becomes the D'Alembert operator) and the bottom column still retains the relationship between the left-handed neutrino and time and space. (1,1,0) is filled in with the photon boson, and (1,1,-1) is filled in with - ρ, ($\overline{3}$,2,-1/6) and (3, $\overline{2}$,1/6) is to fill in six electric and magnetic field terms, a total of twelve (quark 1/3 and lepton -1/2 are positive and negative additions and subtractions). Supplementary explanation: U(1) hypercharge is 1, SU(2) hypercharge is -1/2, and SU(3) hypercharge is 1/3. Adding and subtracting two by two can give the above group theory numbers such as 2/3, 1/6, and 5/6. Pay attention to the relativity of positive and negative signs when filling in.

This covariant matrix arrangement is to stipulate that the lower left has negative charge and the upper right has positive charge . Since the charge is related to the direction of rotation, the charge is linear with the isospin or hypercharge. SO(10) is double covered by the spinor group Spin(10). The spinor group means that it is reasonable to arrange this SO(10) matrix by charge or color charge. If the charge or color charge is zero, then the positive or negative hypercharge is considered to make the arrangement . The diagonal of the 4x4 unified field theory matrix and the SU(5) matrix is electrically neutral. This arrangement also preserves the relationship between neutrinos and time and space. In SU(5), the third generation of neutrinos and the third generation of up quarks in the left also have CP violation. The mass source of the third generation of up quarks is similar to the addition of the Young tableau, the vertical (u,c,t) plus the horizontal (Z, W^+). Therefore, the mass of the up quark comes from (Z, W^+) and decays in sequence. This Young tableau addition is also applicable to the mass source of the third generation of down quarks . This model can help explain the Georgi-Jarlskog down quark and electron mass relationship and the Barr & Babu up quark and neutrino mass relationship. The covariant SO (10) model is more representative of the theory of everything , because the Einstein field equations of general relativity, the four gradients of special relativity (the denominator is distance), the Faraday tensor of charge relativity, and the Yang-Mills field theory of the standard model are basically covariant equations . In this arrangement, the upper right is positive charge or positive color charge and the lower left is negative charge or negative color charge. It is worth noting that the standard model SU(5) tends to be left invariant vector, that is, the left triangular matrix in the lower left corner is less likely to have CP violation problems, while the right triangular matrix in the upper right corner is more likely to have CP violation problems. SO(10) has no perturbative local anomalies and no non-perturbative global anomalies , and is consistent with the correct theory .

Finally, let me add Hilbert's sixth problem about the axiomatization of physics. The above universal theory matrix contains the Lorentz group or the Lorentz vector or the Lorentz tensor, which means that it has the Lorentz invariance, which means that it has the time symmetry, space symmetry, and rotational symmetry of Noether's theorem, that is, the conservation of energy, conservation of momentum, and conservation of angular momentum. Among Newton's three laws of motion, the law of inertia and the law of acceleration come from the curvature of space-time, which is the equivalence principle of general relativity. It can be explained by the curvature of the unified field theory general relativity term (ideal fluid) to include the law of universal gravitation, and the action-reaction principle comes from the conservation of momentum, which is included in the above Lorentz invariance. The charge relativistic term (Faraday tensor) includes the Maxwell equations, and the general relativity term (ideal fluid) also includes the Maxwell equations of gravity and spin force, because the light pressure term and the spin force field are interchangeable. Maxwell's equations can

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be used to derive the conservation of charge in the continuity equation, and the Maxwell's equations for the gravity and rotation force can be used to derive the conservation of the continuity equation. Maxwell's equations can also be used to derive the light waves with a constant speed of light. In addition, the mass continuity equation and the conservation of momentum can help derive the fluid mechanics equations. Among the three laws of thermodynamics, the first law is the conservation of energy, which is included in the above-mentioned Lorentz invariance. The principle of least action can also be derived from the conservation of energy. The second law can be seen from the light pressure term of the ideal fluid, that is, the thermal radiation term. As the universe evolves and expands, the entropy continues to increase (entropy is the number of Planck spaces). Similarly, the maximum entropy principle of statistical mechanics is applied. The universe must reach thermal equilibrium, which also explains the zeroth law of thermodynamics. The third law says that absolute zero cannot be reached. The ratio of -ϱ to P in the ideal fluid term can be fixed at 1:1. The conservation of mass and energy in the universe will not disappear, so the radiation temperature of the light pressure term will not drop to zero. The Hu SU(5) elementary particle field completes the quantum field theory of the standard model, representing the structure of U(1)xSU(2)xSU(3). The U(1) gauge transformation includes the conservation of charge, lepton number and supercharge, the SU(2) gauge transformation includes the conservation of isospin and weak isospin, and the SU(3) gauge transformation includes the conservation of quark flavor and baryon number. In addition, quantum field theory also means that the CPT theorem is true. The separation of fermions and bosons can be linked to Fermi statistics and Bose statistics in statistical mechanics. In addition, the general relativity terms also satisfy Gauss's wonderful theorem, so they also have intrinsic symmetry. Among the four gradient vectors, ∂t>0 represents a time-like vector, and the positive time term means that the universe time is only evolving in the direction of the future. The four vectors also imply the principle of relativity and the principle of the constancy of the speed of light, which are the basic axioms of special relativity. The four-gradient theory contains the basic spirit of special relativity and contains many conservation laws, such as the derivation of the four-dimensional space-time, charge conservation, continuity equation, particle number conservation, conservation equations of re-rotation waves and electromagnetic waves, energy conservation, momentum conservation, angular momentum conservation and intrinsic time invariant. Therefore, this universal theory (unified field theory) completely covers general relativity, special relativity, Newtonian mechanics, electromagnetism, fluid mechanics, acoustic optics, thermodynamics, time and space, particles and waves, and the standard model, thus solving Hilbert's sixth problem. Since this SO (10) contains the general relativity framework and time is positive, there is no antimatter matrix version.

In addition, the mass derivation of other non-elementary particles (mainly baryons and mesons) is supplemented. Based on the above demonstration of the three generations of quarks, the mass of other baryons and mesons can be calculated by adding the mass of the quarks they carry. First, the baryon with J=1/2, taking $\Lambda^0$the particle as an example, its quark composition is uds, and its mass is 1115Mev in experimental observation. We know that the proton uud or neutron udd carries six colored gluons, two neutral gluons, and a neutral Pion (containing a neutral green anti-green gluon), and its total mass is 940Mev. The mass of the strange quark s is 101Mev. Add 940Mev and subtract the mass of an up quark u, 1-2Mev, to get 1040Mev, which is close to the experimentally observed mass. Then there is the baryon with J=3/2. Taking Δ^{++} the particle as an example, its quark composition is uuu, and its mass is experimentally observed to be 1232Mev. Since J=3/2 means that three quarks with three positive charges spin in the same direction and produce additional repulsion, additional gluons are needed to increase the attraction. Since neutral gluons cannot mediate the exchange of force between two quarks in a single baryon, we have to use the smallest possible combination, namely three colored gluons (red against blue, blue against green, and green against red) to maintain the total color of the baryon as white. The colored gluons are 92Mev x 3 plus 940Mev to get 1216Mev, which is close to the experimentally observed mass. Next is $J^p = 0$the meson. For $\eta_c$example, its quark composition is c $\bar{c}$, and its mass is 2983Mev. The mass of the charm quark is 1280Mev, and the most basic pion meson (containing a neutral gluon) is 130Mev. Antimatter has

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positive mass, so 1280x2+130=2690Mev. Or if there is an extra neutral gluon and two colored gluons to maintain the attraction, then: 1280x2 + 130x 2+ 92x2 = 3004 Mev, close to the experimentally observed mass. Next is $J^p = 1$the meson. Taking $\rho$the particle as an example, its quark composition is $u\bar{d}$, and its experimentally observed mass is 775 Mev. The Pion meson is 130 Mev. At this time, because the quark spin increases the repulsion, we assume that there are two neutral gluons and six colored gluons to mediate $J^p = 1$the meson, then 130x 2+92x6 = 812 Mev, close to the experimentally observed mass. The antiquark with the anti-color needs to be combined with the gluon with the positive color to maintain the white color, which is also related to the quark confinement.

Since the Theory of Everything is a SO(10) model and contains the SU(5) group, it has the Georgi-Jarlskog relation for the lepton mass:

$$m_e \approx \frac{1}{3(m_{dGUT})}$$

$$m_\mu \approx 3(m_{sGUT})$$

$$m_\tau \approx m_{bGUT}$$

$$m_{dGUT} \approx \frac{1}{3(m_d)}$$

$$m_{sGUT} \approx \frac{1}{3(m_s)}$$

$$m_{bGUT} \approx \frac{1}{3(m_b)}$$

$$\left(\frac{m_\mu}{m_\tau}\right) \approx 3\left(\frac{m_s}{m_b}\right)$$

$$\left(\frac{m_e}{m_\mu}\right) \approx \frac{\left(\frac{m_d}{m_s}\right)}{3^2}$$

The factor 3 is because the three quarks have a red, blue, and green color relationship. In the SO(10) matrix, we can see that the three generations of down quarks and electrons are adjacent to each other. We can find that the actual measurement error is within 20% . If we look at the previous Barr & Babu research report, there is a similar mass relationship between neutrinos and up quarks:

$$\left(\frac{m_{\upsilon_\mu}}{m_{\upsilon_\tau}}\right) \approx 16\left(\frac{m_c}{m_t}\right)$$

$$\left(\frac{m_{\upsilon_e}}{m_{\upsilon_\mu}}\right) \approx \frac{\left(\frac{m_u}{m_c}\right)}{16^2}$$

The upper left of the three generations of neutrinos is the 4x4 unified field theory four-dimensional space-time matrix. Neutrinos are closely related to space-time, so there is a factor of 16. In the SO(10) matrix, the basic charge of the 4x4 unified field theory four-dimensional electromagnetic matrix is 1, and in the matrix, we can see that the upper quark and the three generations of neutrinos are adjacent to each other. If we use the current experimentally measured mass of the electron neutrino 1ev/c^2, the mass of the muon neutrino 0.17Mev/c^2, and the mass of the tau neutrino 18Mev/c^2, we will find that it is consistent with the theory. To supplement the CP violation of the charm quark decay mentioned above, it is actually the parity violation of the two decays of the D0 particle (c):$\bar{u}$

$$D^0 \to K^+ + K^- \to K^- + \pi^0 + \pi^+$$

$$D^0 \to \pi^+ + \pi^- \to \pi^- + \nu_e + e^+$$

The first equation later had a parity of -1 and the second equation later had a parity of +1. Since mesons contain gluons and quarks, they involve SU(2) and SU(3). Therefore, the charm quark decay of the upper quark series causes CP violation, which helps to explain that strong interactions can also cause CP violation.

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