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Geometric Electrodynamics: Source Self-Consistency, Foundations, and Unification of Electromagnetic and Gravitational Phenomena

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Abstract

Building on a previously proposed coupling between the Maxwell tensor and the Riemann-Christoffel curvature tensor [Ann. Phys. 465, 169661 (2024)], this manuscript develops the conceptual and foundational implications of that framework, with particular emphasis on source self-consistency and the unification of electromagnetic and gravitational source terms. The theory eliminates the need for externally introducing charge and mass and instead defines these quantities self-consistently in terms of the theory's fundamental fields. By construction, all solutions of the theory satisfy the equations of Classical Electrodynamics identically. This approach highlights a conceptual ambiguity in the merger of Classical Electrodynamics and General Relativity, where the standard merger can admit multiple, potentially inequivalent local definitions of mass density. Here, by enforcing source self-consistency, the geometric framework provides a unified and intrinsic foundation for both charge and mass, ensuring compatibility across all equations of motion. Notably, the global symmetries of the theory lead to the emergence of antimatter and dictate its behavior in electromagnetic and gravitational fields, in agreement with classical expectations. Despite the nontrivial integrability conditions imposed by the geometric coupling, exact particle-like and radiative solutions are presented which illustrate the unification of electromagnetic and gravitational phenomena and demonstrate the constraints imposed by the geometric structure. The results suggest new directions for foundational research, emphasizing the role of internal logical consistency in classical field theory and its potential implications for outstanding problems such as the equivalence principle and the nature of dark matter.

Keywords: geometric electrodynamics; source self-consistency; unification; gravitation

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1. Introduction

Classical Electrodynamics and General Relativity stand as two of the most successful pillars of modern physics, providing remarkably accurate descriptions of electromagnetic and gravitational phenomena across a vast range of scales. Maxwell's equations elegantly capture the behavior of electromagnetic fields, while Einstein's field equations form the foundation of our understanding of gravity as the curvature of spacetime. The empirical achievements of these theories are well established, from the prediction of electromagnetic waves to the recent detection of gravitational waves and precision tests of the equivalence principle.

Despite this success, the unification of electromagnetism and gravity at the classical level remains an open challenge. Historically, numerous attempts have been made to develop unified geometric frameworks, beginning with the pioneering work of Einstein and Weyl in the early twentieth century [2,3], and continuing through modern reviews and analyses [4]. A persistent issue in these efforts is the treatment of source terms—charge and mass—which are typically introduced as independent external entities in classical field theory. While this approach is effective for practical calculations, it can obscure the deeper geometric relationship between sources and the fields they generate.

Recent developments in geometric field theory indicate that a more coherent and logically consistent foundation may be achieved by defining source terms intrinsically, in terms of the underlying geometric fields. In particular, the energy-momentum tensor, which governs both electromagnetic and gravitational interactions, can admit multiple, potentially inequivalent definitions in the standard merger of Classical Electrodynamics and General Relativity [5,6]. This conceptual ambiguity is especially apparent in the definition of local mass density, where classical approaches yield distinct results depending on whether one adopts the electrodynamic or gravitational perspective.

The present work builds on these insights by developing a geometricized theory of electrodynamics in which Maxwell's equations and gravitational phenomena emerge from a single fundamental equation coupling the Maxwell tensor to the Riemann-Christoffel curvature tensor. Within this framework, charge and mass are not externally specified but arise self-consistently as geometric manifestations of spacetime curvature and the fundamental fields of the theory. This approach enforces source self-consistency, resolving ambiguities inherent in classical formulations and providing a unified foundation for both electromagnetic and gravitational phenomena.

In addition to reproducing all equations of Classical Electrodynamics, the theory naturally leads to gravitational effects and reveals new physical consequences arising from its global symmetries. Notably, the emergence of antimatter and its behavior in electromagnetic and gravitational fields are dictated by the geometric structure of the theory, offering a classical field-theoretic perspective on phenomena traditionally associated with quantum mechanics. Several exact solutions are presented, including particle-like and radiative cases, which illustrate the unification of electromagnetic and gravitational phenomena and suggest possible implications for outstanding problems such as the nature of dark matter and the quantization of physical quantities.

By focusing on logical consistency and the geometric foundations of classical field theory, this manuscript aims to advance the unification of electromagnetism and gravity, and to highlight the importance of source self-consistency in resolving longstanding conceptual issues. The results presented here may offer new directions for foundational research and shed light on unresolved problems in physics.

In the following sections, I review and develop the mathematical foundations of the geometricized theory, clarify its relationship to classical electrodynamics and general relativity, and present several exact solutions—including particle-like and radiative cases—that demonstrate the theory's unifying power and its implications for foundational questions in physics.

2. Relationship to Previous Work and What is New in This Manuscript

Building on a previously proposed coupling between the Maxwell tensor and the Riemann-Christoffel curvature tensor [Ann. Phys. 465, 169661 (2024)],[1] this manuscript develops the conceptual and foundational implications of that framework, with particular emphasis on source self-consistency and the unification of electromagnetic and gravitational source terms. To keep the presentation reasonably self-contained, I summarize key results from Ref. [1] but omit many details, derivations, and references already provided there. The main new contributions in this manuscript are:

- **Source Self-Consistency**: I introduce and analyze a criterion for internal logical consistency in classical field theory, requiring that all source terms—charge density, mass density, and four-velocity—be defined uniquely and intrinsically in terms of the fundamental fields and spacetime geometry.
- **Resolution of Ambiguities**: I show that the standard merger of Classical Electrodynamics and General Relativity can admit multiple, potentially inequivalent local definitions of mass density, highlighting a conceptual ambiguity that is resolved in the geometric framework.

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- **Antimatter and Symmetries**: I analyze the global symmetries of the theory, showing how the emergence and classical behavior of antimatter are dictated by the geometric structure.
- **Clarification of Coupling Field**: I clarify the role of the coupling vector field a^λ and its relationship to the classical electromagnetic vector potential.

These advances go beyond the technical derivations of Ref. [1], focusing on foundational, conceptual, and interpretive aspects of the geometric framework. For clarity in going forward, I refer to the geometricized theory of electrodynamics introduced in Ref. [1] as Geometric Electrodynamics to distinguish it from Classical Electrodynamics.

3. The Governing Equation and Fundamental Fields

The central postulate of Geometric Electrodynamics is that both electromagnetic and gravitational phenomena are governed by a single, unified geometric equation. This approach eliminates the need to introduce charge and mass as external sources, instead deriving these quantities intrinsically from the geometry of spacetime and the fundamental fields themselves. Such a formulation aims to resolve conceptual ambiguities in classical field theory, where sources and fields are traditionally treated as independent entities.

The foundational equation of the theory is

$$F_{\mu\nu;\kappa} = a^\lambda R_{\lambda\kappa\mu\nu}, \tag{1}$$

Here, $F_{\mu\nu}$ is the electromagnetic field tensor, $g_{\mu\nu}$ is the spacetime metric, $R_{\lambda\kappa\mu\nu}$ is the Riemann-Christoffel curvature tensor, and a^λ is a dynamical four-vector field that mediates the coupling between electromagnetic and geometric degrees of freedom. The inclusion of a^λ generalizes the role of the electromagnetic vector potential of Classical Electrodynamics and enables a direct geometric relationship between curvature and electromagnetic structure.

Throughout this manuscript I use the notational conventions of Weinberg [7]. Geometric units are used throughout ($c = G = 1$), and the metric tensor signature is (+,+,+,-) with spatial indices running from 1-3 and 4 the time index.

Because Equation (1) is a mixed system of first order-partial differential equations, any solution must satisfy the following integrability condition that arises from the geometric structure of the theory,

$$\left(a^\rho R_{\rho\kappa\mu\nu}\right)_{;\lambda} - \left(a^\rho R_{\rho\lambda\mu\nu}\right)_{;k} = -F_{\mu\sigma}R^{\sigma}{}_{\nu k\lambda} - F_{\sigma\nu}R^{\sigma}{}_{\mu\kappa\lambda}\,. \tag{2}$$

This integrability condition is derived in Ref [1] Section 2.6 and ensures that any solution of Equation (1) for the fundamental fields (a^λ, $g_{\mu\nu}$, $F_{\mu\nu}$) is self-consistently defined throughout spacetime by requiring that transport of $F_{\mu\nu}$ around closed loops be path independent. In Ref. [1] this integrability condition was mistakenly identified as being equivalent to the commutation of covariant derivatives of $F_{\mu\nu}$ which it is not, an error which is clarified here. Importantly, while the integrability conditions restrict the set of admissible solutions, they do not eliminate physically relevant configurations. Explicit solutions such as a spherically symmetric charged particle and radiative wave solutions already found demonstrate that Equation (1) admits nontrivial and physically meaningful field configurations within the geometric framework.

Equation (1) serves as the sole dynamical law for the three fundamental fields (a^λ, $g_{\mu\nu}$, $F_{\mu\nu}$). The system is closed by the algebraic and differential identities satisfied by the Riemann tensor, including its cyclicity and symmetry properties. Specifically, the cyclic identity,

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$$R_{\lambda\kappa\mu\nu}+R_{\lambda\mu\nu\kappa}+R_{\lambda\nu\kappa\mu}=0 \tag{3}$$

will be shown in the next section to be the basis of Maxwell's homogeneous equation, ensuring the gauge symmetry of $F_{\mu\nu}$.

The vector field a^{λ} plays a central role in the geometric framework, mediating the coupling between electromagnetic and gravitational degrees of freedom. In this theory, a^{λ} is treated as a fundamental field, on equal footing with the metric tensor $g_{\mu\nu}$ and the electromagnetic field tensor $F_{\mu\nu}$. Its dynamics are entirely encoded in Equation (1), and no separate Lagrangian or kinetic term is postulated. While a^{λ} is not identified with the classical electromagnetic vector potential A_{μ}, Equation (1) implies a nontrivial relationship between these quantities, as discussed in Ref. [1] Section 2.5. The geometric definitions of charge density and four-velocity arise directly from the interplay between a^{λ} and the spacetime curvature.

A key feature of this framework is that all physical source terms—charge density, mass density, and their associated four-velocities—are not introduced externally but are defined self-consistently in terms of the fundamental fields and spacetime geometry. This geometric origin of sources distinguishes the present theory from both Classical Electrodynamics and previous attempts at unification [3,8,9].

In summary, Equation (1) encapsulates the principle that the structure and dynamics of matter and fields are inseparable from the geometry of spacetime. The implications of this postulate—including the emergence of Maxwell's equations, conservation laws, and geometric definitions of charge and mass—are reviewed in the following sections.

4. Derivation of Maxwell's Equations and Geometric Charge Current

As shown in Ref. [1], Equation (1) implies Maxwell's equations. For completeness I summarize the key steps and resulting definitions in this and the following section.

Contracting the cyclic identity of the Riemann-Christoffel tensor Equation (3) with a^{λ} and using Equation (1) yields

$$F_{\mu\nu;\kappa}+F_{\nu\kappa;\mu}+F_{\kappa\mu;\nu}=0 \tag{4}$$

which is Maxwell's homogeneous equation in curved spacetime.

Contracting the μ and κ indices in Equation (1) gives

$$F^{\mu\nu}{}_{;\mu}=-a^{\lambda}R_{\lambda}{}^{\nu} \tag{5}$$

where $R_{\lambda}{}^{\nu}$ is the Ricci tensor. The electromagnetic charge current is then defined geometrically as

$$\rho_c u^{\nu}\equiv a^{\lambda}R_{\lambda}{}^{\nu}\ . \tag{6}$$

Substituting into Equation (5) recovers Maxwell's inhomogeneous equation in geometric form

$$F^{\mu\nu}{}_{;\mu}=-\rho_c u^{\nu}\ . \tag{7}$$

The four-velocity satisfies the normalization condition

$$u^{\lambda}u_{\lambda}=\pm 1 \tag{8}$$

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where the "$\pm$" sign distinguishes between subluminal (-1) and superluminal ($+1$) transport. At this stage, both possibilities are retained, as the governing equation does not restrict the causal character of $a^{\lambda}R_{\lambda}{}^{\nu}$.

The antisymmetry of $F^{\mu\nu}$ forces $F^{\mu\nu}{}_{;\mu;\nu}=0$. Combining this with Equations (5) and (7) then leads to

$$\left(a^{\lambda}R_{\lambda}{}^{\nu}\right)_{;\nu}=\left(\rho_c u^{\nu}\right)_{;\nu}=0, \tag{9}$$

which is the covariant conservation of charge.

In summary, both of Maxwell's equations and the conservation of charge—when expressed using the geometric definition of charge current given in Equation (6)—are derivative to Equation (1). Therefore, any solution of Equation (1) for the fundamental fields (a^{λ}, $g_{\mu\nu}$, $F_{\mu\nu}$) automatically satisfies both of Maxwell's equations and the conservation of charge identically, provided the classical source $\rho_c u^{\nu}$ is defined in terms of the fundamental fields as in Equation (6).

Further consequences of this geometric framework, including the conservation of mass, the Lorentz force, and geometric definitions for mass density, charge density, and their associated four-velocity are developed in the following section.

5. Intrinsic Geometric Definitions of Charge, Mass and Four-Velocity

Using the geometric definition for the charge current given in Equation (6), geometric definitions for the charge density and its associated four-velocity can be broken out separately

$$\rho_c \equiv \pm\sqrt{\left|a^{\gamma}R_{\gamma}{}^{\kappa}a^{\delta}R_{\delta\kappa}\right|} \tag{10}$$

and

$$u^{\nu} \equiv \pm\frac{a^{\lambda}R_{\lambda}{}^{\nu}}{\sqrt{\left|a^{\alpha}R_{\alpha}{}^{\sigma}a^{\beta}R_{\beta\sigma}\right|}}, \tag{11}$$

Here, "±" signs are linked and reflect the two possible orientations of the charge current for a given field configuration. The absolute value under the square root allows for both timelike and spacelike currents. Although the geometric definitions in Equations (10) and (11) are general enough to accommodate spacelike or even null current densities, in what follows I restrict attention to timelike charge and mass currents, since these are the cases with clear physical interpretation. Superluminal cases are briefly discussed in the conclusion as an open question.

To ensure that Geometric Electrodynamics encompasses the full scope of Classical Electrodynamics—including the Lorentz force law—it is necessary to formulate a definition of inertial mass directly in terms of the theory's fundamental fields. In pursuit of consistency with classical results, I adopt the same form of the conserved energy-momentum tensor used in classical theory

$$T^{\mu\nu}{}_{;\nu}=\left(\rho_m u^{\mu}u^{\nu}+F^{\mu}{}_{\lambda}F^{\nu\lambda}-\frac{1}{4}g^{\mu\nu}F_{\rho\sigma}F^{\rho\sigma}\right)_{;\nu}=0. \tag{12}$$

From the conservation of energy-momentum in Equation (12) and the previously derived Maxwell equations, two essential results follow, the conservation of mass

$$\left(\rho_m u^{\nu}\right)_{;\nu}=0, \tag{13}$$

and the Lorentz force law

$$\rho_m \frac{Du^\mu}{D\tau} = \rho_m u^\mu{}_{;\nu}\, u^\nu = \rho_c F^\mu{}_\lambda u^\lambda \,. \tag{14}$$

(For detailed derivations, see Ref. [1], Section 2.4).

To develop a geometric definition for mass density ρ_m that is self-consistent with the assumed energy-momentum tensor Equation (12), I use the Lorentz force law Equation (14). Contracting both sides of Equation (14) with itself

$$\rho_m{}^2 u^\mu{}_{;\nu} u^\nu u_{\mu;\gamma} u^\gamma = \rho_c{}^2 F^\kappa{}_\lambda u^\lambda F_{\kappa\eta} u^\eta \tag{15}$$

gives $\rho_m{}^2$ as,

$$\rho_m{}^2 \equiv \rho_c{}^2 \frac{F^\kappa{}_\lambda u^\lambda F_{\kappa\eta} u^\eta}{u^\mu{}_{;\nu} u^\nu u_{\mu;\gamma} u^\gamma} \;. \tag{16}$$

Substituting the geometric definitions of ρ_c and u^ν given by Equations (10) and (11) , respectively, into the right-hand side of Equation (16) then gives the desired definition of ρ_m in terms of the fundamental fields (a_μ , $g_{\mu\nu}$, $F_{\mu\nu}$). Another equivalent definition of ρ_m , but one that is often simpler to calculate than that given in Equation (16) follows directly from any nonzero component of the Lorentz Force Equation (14). Taking the μ' component of Equation (14) to be nonzero, ρ_m is given by

$$\rho_m \equiv \frac{F^{\mu'}{}_\lambda a^\sigma R_\sigma{}^\lambda}{\left(\dfrac{a^\rho R_\rho{}^{\mu'}}{\sqrt{\left|a^\gamma R_\gamma{}^\kappa a^\delta R_{\delta\kappa}\right|}}\right)_{;\nu}\left(\dfrac{a^\alpha R_\alpha{}^\nu}{\sqrt{\left|a^\beta R_\beta{}^\eta a^\zeta R_{\zeta\eta}\right|}}\right)} \quad (\text{no sum on } \mu') \tag{17}$$

Both forms above—whether using the contraction or a specific component—are mathematically equivalent and serve as the geometric definition of mass density within this framework. Importantly, this definition is constructed to satisfy the conservation of mass and conservation of energy-momentum, as required by the structure of the theory.

It is essential to note that introducing mass via the specific energy-momentum tensor given in Equation (12) is itself an axiomatic choice within Geometric Electrodynamics. Should an alternative form of the energy-momentum tensor be adopted, the resulting geometric definition of mass density would likewise change. This underscores the foundational role of the energy-momentum tensor in shaping the theory's physical predictions.

6. Foundations of Geometric Electrodynamics

Table 1 summarizes the geometric definitions for charge density ρ_c , mass density ρ_m , and their associated four- velocity u^λ

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Table 1. Geometric definitions for source terms.

Quantity	Geometric Definition	Classical Role
Charge density ρ_c	$\pm\sqrt{\left\|a^{\gamma}R_{\gamma}{}^{\kappa}a^{\delta}R_{\delta\kappa}\right\|}$ Eq. (10)	External source in Maxwell's equations
Mass density ρ_m	$\dfrac{F^{\mu'}{}_{\lambda}a^{\sigma}R_{\sigma}{}^{\lambda}}{\left(\dfrac{a^{\rho}R_{\rho}{}^{\mu'}}{\sqrt{\left\|a^{\gamma}R_{\gamma}{}^{\kappa}a^{\delta}R_{\delta\kappa}\right\|}}\right)_{;\nu}\left(\dfrac{a^{\alpha}R_{\alpha}{}^{\nu}}{\sqrt{\left\|a^{\beta}R_{\beta}{}^{\eta}a^{\zeta}R_{\zeta\eta}\right\|}}\right)}$ (no sum on μ') Equation (17)	External source in conserved energy-momentum tensor
Four-velocity u^{λ}	$\pm\dfrac{a^{\lambda}R_{\lambda}{}^{\nu}}{\sqrt{\left\|a^{\alpha}R_{\alpha}{}^{\sigma}a^{\beta}R_{\beta\sigma}\right\|}}$ Eq. (11)	Prescribed for charge/mass transport

Any solution to Equation (1) for the fields (a_{μ}, $g_{\mu\nu}$, $F_{\mu\nu}$) can be mapped onto a corresponding classical physics solution using these geometric definitions and automatically satisfies the following equations of Classical Electrodynamics identically:

- Maxwell's homogeneous eq. $F_{\mu\nu;\kappa}+F_{\nu\kappa;\mu}+F_{\kappa\mu;\nu}=0$ (4)
- Maxwell's inhomogeneous eq. $F^{\mu\nu}{}_{;\mu}=-\rho_c u^{\nu}$ (7)
- Conservation of charge: $\left(\rho_c u^{\nu}\right)_{;\nu}=0$ (9)
- Conservation of energy-momentum: $\left(\rho_m u^{\mu}u^{\nu}+F^{\mu}{}_{\lambda}F^{\nu\lambda}-\frac{1}{4}g^{\mu\nu}F^{\rho\sigma}F_{\rho\sigma}\right)_{;\nu}=0$ (12)
- Conservation of mass: $\left(\rho_m u^{\nu}\right)_{;\nu}=0$ (13)
- The Lorentz force law: $\rho_m\dfrac{Du^{\mu}}{D\tau}=\rho_m u^{\mu}{}_{;\nu}\,u^{\nu}=\rho_c F^{\mu}{}_{\lambda}u^{\lambda}$ (14)

This framework reinterprets Classical Electrodynamics as an effective theory derived from Equation (1) under the specific geometric identifications given in Table 1 which represent genuine physical restrictions and not just a relabeling of sources. Mathematically, this is a significant shift from the traditional paradigm, where sources are prescribed and fields are subsequently determined. Here, the fundamental fields (a_{μ}, $g_{\mu\nu}$, $F_{\mu\nu}$) are primary, and all physical sources are consequences of the geometry. Once a solution for the fundamental fields is obtained, the source terms are fully determined, and the classical equations automatically fulfilled. The hierarchal structure of Geometric electrodynamics is summarized in Figure 1.

This intrinsic approach offers several advantages. First, it enforces logical self-consistency: all source terms are uniquely defined in terms of the fundamental fields, eliminating possible ambiguities. Second, it provides a unified description of electromagnetic and gravitational phenomena, with both phenomena emerging from a single geometric foundation. Finally, it offers a new perspective on the relationship between fields and sources, suggesting that the traditional distinction may be less fundamental than previously assumed.

It is important to note that not every classical solution with externally prescribed sources corresponds to a solution in Geometric Electrodynamics. The geometric framework imposes additional constraints on the allowed source distributions, which may restrict the class of physically realizable solutions. This distinction highlights the potential for Geometric Electrodynamics to resolve longstanding conceptual issues in classical field theory and suggests new directions for foundational research.

Taking the approach adopted here for the development of Geometric Electrodynamics one step further, theories postulated on multiple fundamental equations can be analyzed for source term self-consistency. For such theories to be logically self-consistent all the fundamental equations of the theory must lead to the same definition for its sources, i.e., differing definitions for the same source would indicate a lack of internal self-constancy in the theoretical formulation. This point will be discussed further in the following section.

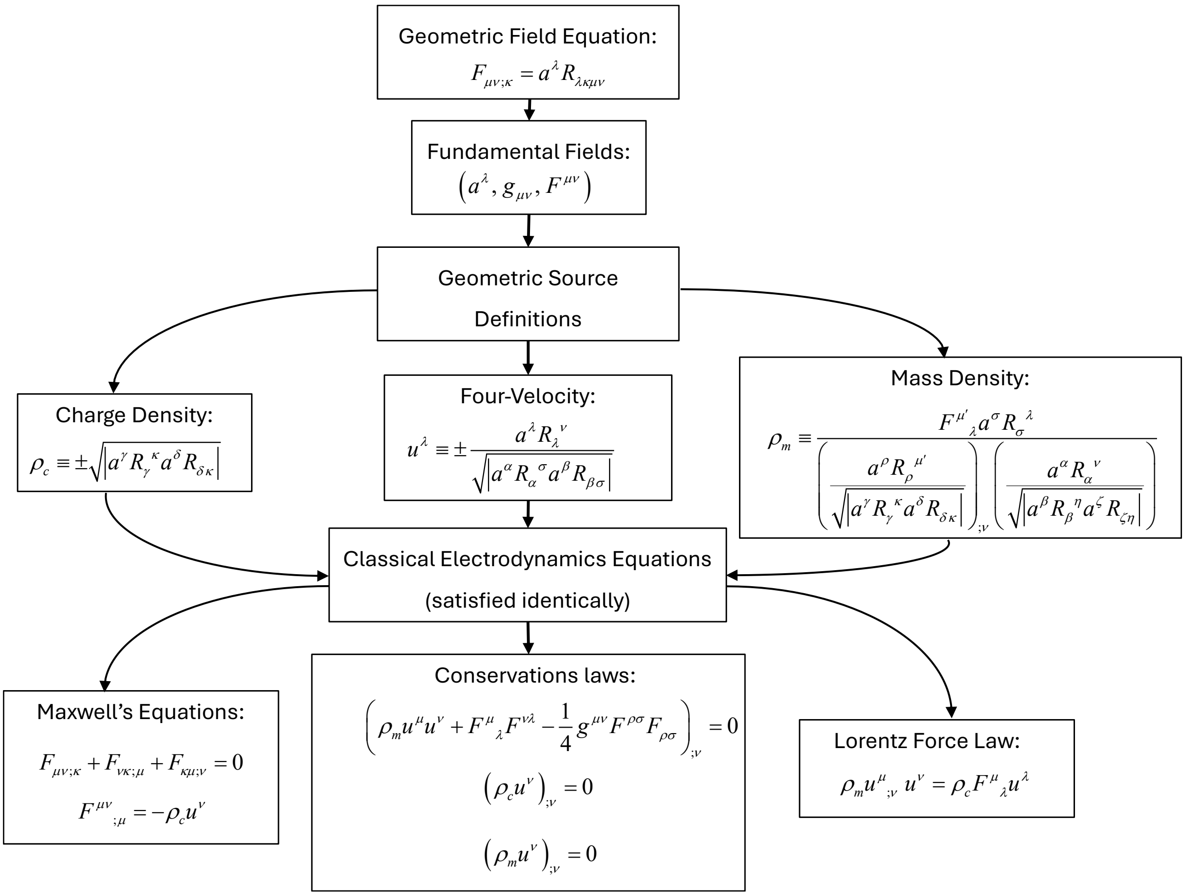

Figure 1. Hierarchal structure of Geometric Electrodynamics: The geometric field equation is solved for the fundamental fields (a^{λ}, $g_{\mu\nu}$, $F_{\mu\nu}$) which then determine the sources (ρ_c, u^{λ}, ρ_m) from their geometric definitions. All such solutions satisfy the Classical Electrodynamics equations identically because these equations are themselves derivative to the geometric field equation and the geometric source definitions.

7. Comparison of Geometric Electrodynamics and General Relativity

A central premise of this manuscript is that, for any continuous field theory, source terms—when expressed in terms of the theory's fundamental fields—should be defined in a logically self-consistent manner. Specifically, all foundational equations should yield compatible definitions for each source term. Geometric Electrodynamics satisfies this criterion: it is built upon a single governing equation, Equation (1), and the source terms (ρ_c, u^{λ}, ρ_m) each have unique geometric definitions, given by Equations (10), (11), and (17), respectively.

In contrast, the situation in standard Classical Physics is more nuanced. To clarify the foundational differences between Classical Physics and Geometric Electrodynamics, I apply the same geometric methodology to the classical merger of Electrodynamics and General Relativity. In this context, one can derive distinct definitions for local mass density depending on whether one starts from the electromagnetic or gravitational sector.

For example, beginning with Maxwell's inhomogeneous equation

$$F^{\mu\nu}{}_{;\mu} = -\rho_{c\,CP}\, u_{CP}{}^{\nu} \quad , \tag{7}$$

where the subscript "CP" indicates definitions of charge density and four-velocity that are specific to Classical Physics. The charge density can be represented as

$$\rho_{c\,CP} \equiv \pm\sqrt{\left|F^{\mu\nu}{}_{;\mu} F^{\sigma}{}_{\nu;\sigma}\right|} \quad , \tag{18}$$

and its associated four-velocity as

$$u_{CP}{}^{\lambda} \equiv \mp \frac{F^{\mu\lambda}{}_{;\mu}}{\sqrt{\left|F^{\mu\nu}{}_{;\mu} F^{\sigma}{}_{\nu;\sigma}\right|}} \quad . \tag{19}$$

These definitions are closely related to their analogues in Geometric Electrodynamics and would be identical under the substitution $F_{\mu\nu;\kappa} \to a^{\lambda} R_{\lambda\kappa\mu\nu}$. The absolute values inside the square roots are retained for consistency with the geometric definitions of Equations (10) and (11).

Similarly, using the Lorentz force law Equation (14) and assuming the right-hand side is nonzero for some μ-component, call it μ', the mass density can be solved for

$$\rho_{m\,CP} = \frac{\rho_{c\,CP}\, F^{\mu'}{}_{\lambda}\, u_{CP}{}^{\lambda}}{u_{CP}{}^{\mu'}{}_{;\nu}\, u_{CP}{}^{\nu}} = \frac{-F^{\mu'}{}_{\lambda} F^{\sigma\lambda}{}_{;\sigma}}{\left(\dfrac{F^{\rho\mu'}{}_{;\rho}}{\sqrt{\left|F^{\gamma\alpha}{}_{;\gamma} F^{\delta}{}_{\alpha;\delta}\right|}}\right)_{;\nu} \left(\dfrac{F^{\alpha\nu}{}_{;\alpha}}{\sqrt{\left|F^{\beta\eta}{}_{;\beta} F^{\zeta}{}_{\eta;\zeta}\right|}}\right)} \quad (\text{no sum on } \mu') \tag{20}$$

where $\rho_{m\,CP}$ denotes the Classical Physics definition for mass density as dictated by Classical Electrodynamics.

Next, consider the General Relativity field equation, the gravitational wing of Classical Physics, which employs the conserved energy-momentum tensor Equation (12) on its right-hand side

$$R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R = -8\pi T^{\mu\nu} = -8\pi\left(\rho_{m\,CP}\, u_{CP}{}^{\mu}\, u_{CP}{}^{\nu} + F^{\mu}{}_{\sigma} F^{\nu\sigma} - \frac{1}{4} g^{\mu\nu} F^{\rho\sigma} F_{\rho\sigma}\right) \quad . \tag{21}$$

Contracting both sides of (21) gives

$$\rho_{m\,CP} = -\frac{1}{8\pi} R \tag{22}$$

where R is the Ricci scalar and $\rho_{m\,CP}$ is now the gravitational mass density implied by the field equations.

These two definitions are not generally equivalent, as can be seen by comparing the right-hand sides of Equations (20) and (22). The electromagnetic definition depends on the local field configuration and its derivatives, while the gravitational definition depends on the spacetime curvature. Unless additional constraints are imposed, there is no guarantee that the mass density defined by the electromagnetic sector matches that defined by the gravitational sector.

This ambiguity does not affect the empirical success of General Relativity or Maxwell's equations, but it does indicate a lack of internal self-consistency at the level of theoretical formulation. It raises questions about the interpretation of the weak equivalence principle—now experimentally verified to an accuracy of 10^{-15} in the Eötvös ratio [10]—and the nature of dark matter [11,12], which may reflect deeper issues in the way sources are represented in classical field theories.

By contrast, Geometric Electrodynamics provides a unique, geometric definition of mass density Equation (17), and both the "electromagnetic" and the "gravitational" roles of mass refer back to the same object. This internal logical consistency strengthens the theoretical foundation of Geometric

Electrodynamics and, in this respect, distinguishes it from the classical physics merger of electrodynamics and General Relativity.

It should be noted that in standard General Relativity, mass is often defined quasi-locally or asymptotically, rather than via a local scalar constructed purely from the Ricci scalar. If one insists on defining a local scalar mass density using the energy-momentum tensor in the electrodynamic sector and a separate local scalar using the curvature in the gravitational sector, then one obtains in general distinct expressions, Equations (20) and (22). This reflects a conceptual ambiguity in the local attribution of mass density in the classical merger of Electrodynamics and General Relativity, not a contradiction in standard General Relativity. The comparison here is intended to highlight how different sectors of classical theory can yield different local geometric definitions for mass density, rather than to claim that there is a unique or preferred definition in all contexts.

8. Physical Content of Geometric Electrodynamics: Antimatter, Gravity, and Symmetries

8.1. The Emergence of Antimatter

Having reviewed the mathematical foundation and formulation of Geometrical Electrodynamics, I next turn to the physical predictions of the theory. To begin, I review the emergence of antimatter which is dictated by the global symmetries of the fundamental fields and their geometric sources. Consider any solution of Equation (1) for fundamental fields (a^λ, $g_{\mu\nu}$, $F_{\mu\nu}$). The global symmetries of Equation (1) and the geometric definitions of the source terms (ρ_m, ρ_c, u^λ) immediately give a total of three additional related solutions.

Starting with the solution (a^λ, $g_{\mu\nu}$, $F_{\mu\nu}$), there are two solutions for the source terms due to the linked "$\pm$" in the geometric definitions of ρ_c and u^λ given by Equations (10) and (11), respectively. If one of these solutions is (ρ_m, ρ_c, u^λ) then the other is (ρ_m, $-\rho_c$, $-u^\lambda$).

Next, because (a^λ, $g_{\mu\nu}$, $F_{\mu\nu}$) is a solution for the fields, then so is ($-a^\lambda$, $g_{\mu\nu}$, $-F_{\mu\nu}$) which follows from Equation (1). This solution also supports two source terms (ρ_m, ρ_c, $-u^\lambda$) and (ρ_m, $-\rho_c$, u^λ), which again follows from the linked "$\pm$" in the geometric definitions of ρ_c and u^λ, and additionally the change in sign of u^λ on transforming $a^\lambda \to -a^\lambda$. Table 2 summarizes the four related solutions of Equation (1).

Table 2. Every solution to Equation (1) leads to three additional related solutions.

Solution	Fields	Sources	Relation relative to solution (1)
(1)	$(a^\lambda, g_{\mu\nu}, F_{\mu\nu})$	$(\rho_m, \rho_c, u^\lambda)$	Assumed solution to $F_{\mu\nu;\kappa} = a^\lambda R_{\lambda\kappa\mu\nu}$
(2)	$(a^\lambda, g_{\mu\nu}, F_{\mu\nu})$	$(\rho_m, -\rho_c, -u^\lambda)$	Charge and matter-antimatter conjugated
(3)	$(-a^\lambda, g_{\mu\nu}, -F_{\mu\nu})$	$(\rho_m, \rho_c, -u^\lambda)$	Matter-antimatter conjugated

(4)	$(-a^{\lambda}, g_{\mu\nu}, -F_{\mu\nu})$	$(\rho_m, -\rho_c, u^{\lambda})$	Charge conjugated

One of the remarkable features of these four solutions is that in addition to covering charge conjugation, $\rho_c \to -\rho_c$, as would be expected in a theory of electrodynamics, also covered is matter-antimatter conjugation. Because antimatter traditionally emerges in quantum theories and so is unexpected in Geometric Electrodynamics which is a classical field theory, I briefly review its emergence and related phenomenology to show how these conform with expected behavior.

To begin, consider a region of normal matter described by solution (1) of Table 1. Making the global transformation (a^{λ} , $g_{\mu\nu}$, $F_{\mu\nu}$) ⟶ ($-a^{\lambda}$, $g_{\mu\nu}$, $-F_{\mu\nu}$) takes solution (1) to solution (3), transforming the source terms (ρ_m, ρ_c, u^{λ}) ⟶ (ρ_m, ρ_c, $-u^{\lambda}$). This is an example of matter-antimatter conjugation which is realized by the transformation

$$u^{\lambda} \to -u^{\lambda} . \tag{23}$$

All charge and matter are associated with specific four-velocities describing their spacetime trajectory, and it is this four-velocity that imparts either normal matter or antimatter properties. To see this, consider u^{λ} in a specific coordinate system (*x*, *y*, *z*, *t*)

$$u^{\lambda} \equiv \frac{dx^{\lambda}}{d\tau} \tag{24}$$

where $d\tau$ is defined by

$$d\tau = \pm\sqrt{\left|g_{\mu\nu}dx^{\mu}dx^{\nu}\right|} \ . \tag{25}$$

By convention I take the "+" sign in the Equation (25) as corresponding to matter and the "−" sign to antimatter. The absolute value taken inside the square root in Equation (25) is included for the same reason it is included in the geometric definitions of ρ_c and u^{λ} in Equations (10) and (11), respectively. Equation (24) makes clear that the matter-antimatter conjugation transformation $u^{\lambda} \to -u^{\lambda}$ is equivalent to $\tau \to -\tau$

$$u^{\lambda} \to -u^{\lambda} \quad \equiv \quad \tau \to -\tau \tag{26}$$

These transformations are physically equivalent to the view that a particle's antiparticle is the particle moving backwards through time [13]. To illustrate the emerging phenomenology of antimatter in Geometric Electrodynamics, I start by considering the response of a particle and its antiparticle to an external electromagnetic field. The response of an ordinary matter test particle is given by Lorentz force law

$$\rho_m u^{\mu}{}_{;\nu} u^{\nu} = \rho_c F^{\mu}{}_{\lambda} u^{\lambda} \ . \tag{14}$$

Applying the matter-antimatter conjugation transformation given by Equation (26) and then simplifying gives

$$\begin{gathered} \rho_m\left(-u^{\mu}{}_{;\nu}\right)\left(-u^{\nu}\right) = \rho_c F^{\mu}{}_{\lambda}\left(-u^{\lambda}\right) \\ \downarrow \\ \rho_m u^{\mu}{}_{;\nu} u_{;\nu} = -\rho_c F^{\mu}{}_{\lambda} u^{\lambda} \end{gathered} \tag{27}$$

which is equivalent to equation (14) representing an ordinary matter test particle but with its charge conjugated. In this geometric framework, the scalar charge density ρ_c is invariant under matter–

antimatter conjugation; however, the combination $\rho_c u^\lambda$, which is the physically relevant current, changes sign when u^λ does. Because all electromagnetic interactions depend on the charge current, this reproduces the usual phenomenology of opposite charge for matter and antimatter in external fields. This also explains how particle–antiparticle pairs which have the same sign mass and the same sign charge can annihilate. Particle-antiparticle annihilation zeroes both the total final mass and the total final charge, which can only be effected if the 4th component of both the charge current $\rho_c u^\lambda$ and mass current $\rho_m u^\lambda$ have opposite signs for the particle and antiparticle.

Finally, the matter-antimatter conjugation defined by Equation (26) when applied to the behavior of antimatter in a gravitational field predicts that it will be indistinguishable from normal matter. This follows from the Lorentz Force law Equation (27) on zeroing the Maxwell tensor, leaving only a gravitational interaction described by the geodesic equation

$$u^\mu{}_{;\nu} u^\nu = 0 \tag{28}$$

which transforms into itself under the matter-antimatter conjugation transformation given in Equation (26). This reinforces experimental expectations that antimatter falls like normal matter in gravity [14].

As already mentioned, the emergence of antimatter is an unexpected development in a classical field theory such as Geometric Electrodynamics. This suggests that some features usually attributed to quantum field theory (charge-conjugated partners, time reversal interpretations) might have classical geometric analogues. Finally, I note that the sign of ρ_m is invariant under all symmetry transformation, an observation that has relevance to the discussion of antigravity that will come up Section 9.3.

8.2. The Emergence of Gravity

Fundamental Equation (1) leads to a theory of Geometric Electrodynamics that agrees with Classcial Electrodynamic, but then goes further with every solution to Equation (1) including gravitation due to the coupling of the Riemann-Christoffel tensor to the Maxwell tensor. In addition to the gravity fields being part of every solution, the mass density distributions that generate the gravity fields are also a part of every solution as given by the geometrically defined mass density in Equation (17).

To illustrate these and other aspects of the gravitation that emerges in Geometric Electrodynamics, I review two previously developed solutions to fundamental Equation (1). The first of these represents a spherically symmetric nonrotating charged mass, and the second, free propagating electromagnetic and gravitational waves. In both cases, the emergent gravitational fields are consistent with those found using General Relativity.

9. Exact Solutions

9.1. Particle-like Solution

Here I will use a specific solution to investigate the emergence of gravity in Geometric Electrodynamics. The following is a previously published solution to Equation (1) for the fundamental fields (a^λ, $g_{\mu\nu}$, $F_{\mu\nu}$) representing a spherically symmetric nonrotating charged mass (See Ref [1] Section 3.1)

$$a^\lambda = \left(0,0,0,\frac{q}{m}\right) , \tag{29}$$

$$g_{\mu\nu} = \begin{pmatrix} \dfrac{1}{\left(1-\dfrac{2m}{r}+\dfrac{q^2}{r^2}\right)} & 0 & 0 & 0 \\ 0 & r^2 & 0 & 0 \\ 0 & 0 & r^2\mathrm{Sin}^2(\theta) & 0 \\ 0 & 0 & 0 & -\left(1-\dfrac{2m}{r}+\dfrac{q^2}{r^2}\right) \end{pmatrix}, \tag{30}$$

and

$$F_{\mu\nu} = \begin{pmatrix} 0 & B_\phi & -B_\theta & E_r \\ -B_\phi & 0 & B_r & E_\theta \\ B_\theta & -B_r & 0 & E_\phi \\ -E_r & -E_\theta & -E_\phi & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & \dfrac{q}{r^2}\left(1-\dfrac{q^2/m}{r}\right) \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -\dfrac{q}{r^2}\left(1-\dfrac{q^2/m}{r}\right) & 0 & 0 & 0 \end{pmatrix}. \tag{31}$$

The solution for the fundamental fields found above uses the Reisner-Nordström metric and leads to a static particle-like solution with electric fields that agree with Coulomb's law to leading order in 1/r. If the charge q, for example, represents an electron, then the coefficient of correction term to the Coulomb field is recognized as the classical radius of an electron q^2/m. Using this solution for the fundamental fields, I can now write down expressions for the sources (ρ_c, u^λ, ρ_m) using the geometric definitions for them given in Equations (10), (11), and (17), respectively,

$$\rho_c = \frac{q^3}{m}\,\frac{\sqrt{1-\dfrac{2m}{r}+\dfrac{q^2}{r^2}}}{r^4}, \tag{32}$$

$$u^\lambda = \left(0, 0, 0, \frac{1}{\sqrt{1-\dfrac{2m}{r}+\dfrac{q^2}{r^2}}}\right), \tag{33}$$

and

$$\rho_m = \frac{q^4}{m^2}\,\frac{1-\dfrac{2m}{r}+\dfrac{q^2}{r^2}}{r^4}. \tag{34}$$

Requiring $q^2 \geq m^2$ is the only restriction on the q and m parameters in the above solution and ensures that both ρ_c is real valued and $\rho_m \geq 0$ for all r. It is straight forward to check by direct substitution that the fundament fields (a^λ, $g_{\mu\nu}$, $F_{\mu\nu}$) given by Equations (29)-(31) satisfy fundamental Equation (1), and that the sources (ρ_c, u^λ, ρ_m) given by Equations (32)-(34) satisfy their geometric definitions given by Equations (10), (11), and (17), respectively [15]. Finally, the fundamental fields and source terms found here also identically solve all the equations of Classical Electrodynamics as detailed in Section 6, which is also easily verified by direct substitution.

For the physical situation considered here—a spherically symmetric nonrotating charged mass—the emergent gravitational fields agree with those found using General Relativity, i.e., the Reisner-Nordström metric. However, the mass distribution which is also a part of the Equation (1) solution does not agree with that used classically. The geometrically defined mass and charge distributions both roll off as $1/r^4$. The discrepancy in mass distributions hints at differences in the underlying local definition of mass in Geometric Electrodynamics versus that used classically in General Relativity.

9.2. Self-Consistency and Quantization

Because the foregoing solution to Equation (1) includes the geometrically defined charge and mass distributions, self-consistency boundary conditions requiring the total integrated charge and the total integrated mass be consistent with the asymptotic electric and gravitational fields, respectively, can be developed. For the solution just considered, this condition for charge takes the form

$$\lim_{r\to\infty}\left(r^2 F_{14}\right)=\int \rho_c u^4 \sqrt{\gamma_{sp}}\, d^3x \tag{35}$$

where γ_{sp} is the determinant of the spatial metric defined by [16]

$$\gamma_{sp\ i\,j}=g_{i\,j}-\frac{g_{i4}\,g_{j4}}{g_{44}} \tag{36}$$

and i and j run over the spatial dimensions 1, 2 and 3. For the mass, the analogous self-consistency boundary condition is

$$\lim_{r\to\infty}\left(r\,\frac{1+g_{44}}{2}\right)=\int \rho_m \left|u^4\right| \sqrt{\gamma_{sp}}\, d^3x \tag{37}$$

The use of the absolute value in the mass self-consistency condition Equation (37), but not in the charge condition Equation (35), reflects the symmetries of Equation (1) and the geometric source definitions in Equations (10), (11), and (17), and ensures the boundary conditions respect the same underlying symmetries.

I conjecture that only solutions satisfying both self-consistency conditions (35) and (37) represent physically realizable solutions. While this approach is not pursued further here, it is worth noting that similar boundary conditions could be applied to metrics with nonzero angular momentum, such as those describing particles with intrinsic magnetic moments. In such cases, the quantization of angular momentum could arise in parallel with mass and charge quantization. Traditionally, quantization of these quantities is introduced via quantum mechanics; here, however, it is conjectured within a classical continuous field-theoretic framework, illustrating a key departure from conventional physics.

For the spherically symmetric solution examined above, both the charge and mass self-consistency conditions (35) and (37) diverge on their right-hand sides, and are therefore not satisfied. This highlights that the externally inserted q and m parameters used in the Reisner-Nordström metric Equation (30) do not agree with the geometric definitions given in Equations (10) and (17). Therefore, while this solution formally describes the asymptotic gravitational and electric fields of a particle, it does not correspond to a physically admissible configuration. Whether static particle-like solutions exist that satisfy such self-consistency boundary conditions is not yet known. However, alternative metrics—such as the modified Reissner-Nordström and Kerr-Newman solutions developed by S.M. Blinder [17]—can yield finite values for the right-hand sides of Equations (35) and (37), suggesting promising directions for future work.

9.3. On the Sign of Mass Density and Antigravity

Equipped with the previous solution I can now establish that the mass density must be nonnegative always. To see this, I set $q = 0$ in the previous solution which leaves a Schwarzschild metric. Consider two test particles, one of mass m and the other of mass -m, initially at rest with respect to each other and separated by some distance $r_0 >> m$. Assuming the test particle with positive mass is at the origin of the coordinate system, the acceleration seen by the negative test mass particle is

$$\frac{\partial u^r}{\partial t} = -\frac{m}{r^2}\left(1-\frac{2m}{r}\right) \tag{38}$$

where u^r is the radial component of the four-velocity of the negative test mass particle. In this case the acceleration is negative and the two particles are attracted, the normal gravity case. However, if the negative mass test particle is at the origin of the coordinate system, then Equation (38) gives a positive acceleration for the positive mass test particle and the two particles are repelled, the antigravity case. Since the situation in these two scenarios is symmetric and arises from masses of differing sign, there is a logical inconsistency if both positive and negative masses are permitted by the theory. The only resolution consistent with normal gravity is to require $\rho_m \geq 0$ always which then additionally eliminates the possibility of antigravity.

This condition on ρ_m is directly related to its invariance under the symmetry transformations discussed in Section 8.1 and identifies it as a consistency requirement of Geometric Electrodynamics as developed within. Allowing negative mass density leads to contradictory predictions for the motion of test particles in gravitational fields. This requirement is not intended to rule out all exotic energy conditions in general, but rather to maintain logical consistency within the present framework.

9.4. Radiative Gravitational and Electromagnetic Waves

Here I review a solution to Equation (1) describing radiating electromagnetic and gravitational waves (See Ref [1] Section 3.2). The solution is derived in the weak field limit defined using

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} e^{i\omega(t-z)} \tag{39}$$

where $h_{\mu\nu}$ are complex constants satisfying $|h_{\mu\nu}| \ll 1$, and $\eta_{\mu\nu} = diag[1,1,1,-1]$. In deriving the solutions here, I only retain terms to first order in the fields $h_{\mu\nu}$ and $F_{\mu\nu}$, both of which are assumed to be small and of the same order. I take the electromagnetic plane wave to have frequency ω, propagate in the +*z*-direction, and be polarized in the *x*-direction. In this case, the solution for the fundamental fields (a^λ, $g_{\mu\nu}$, $F_{\mu\nu}$) is

$$a^\lambda = \left(a^1, a^1 \frac{h_{12}}{h_{11}}, a^4, a^4\right) \tag{40}$$

where a^1 and a^4 are constants,

$$g_{\mu\nu} = \eta_{\mu\nu} + \begin{pmatrix} h_{11} & h_{12} & 0 & 0 \\ h_{12} & -h_{11} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} e^{i\omega(t-z)} , \tag{41}$$

and

$$F_{\mu\nu} = \begin{pmatrix} 0 & 0 & -B_y & E_x \\ 0 & 0 & 0 & 0 \\ B_y & 0 & 0 & 0 \\ -E_x & 0 & 0 & 0 \end{pmatrix} e^{i\omega(t-z)}, \text{ where } E_x = B_y = i\,\omega \frac{\left(h_{11}{}^2 + h_{12}{}^2\right)}{2\,h_{11}} a^1 \quad . \tag{42}$$

The only restriction on this solution is $h_{11} \neq 0$.

The form of the plane wave solution for the free propagating electromagnetic field given in Equation (42) is described by a Maxwell tensor that is in agreement with that found using the Classical Maxwell equations. What is unusual about this solution is that it requires the presence of a simultaneous free propagating gravitational wave described by Equation (41) which matches the General Relativity plane wave solution in a transverse-traceless gauge [18,19].

The previous solution for an electromagnetic plane wave Equation (42) illustrates the requirement that electromagnetic waves must always have an accompanying gravitational wave Equation (41). However, it is possible to have a standalone gravitational wave with no electromagnetic wave present using a different solution for a^λ

$$a^\lambda = \left(a^1, -a^1 \frac{h_{11}}{h_{12}}, a^4, a^4 \right) \quad , \tag{43}$$

which then leads to

$$g_{\mu\nu} = \eta_{\mu\nu} + \begin{pmatrix} h_{11} & h_{12} & 0 & 0 \\ h_{12} & \dfrac{h_{12}{}^2}{h_{11}} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} e^{i\omega(t-z)} \quad , \tag{44}$$

and

$$F_{\mu\nu} = 0 \quad . \tag{45}$$

Here the Maxwell tensor vanishes, leaving a standalone gravitational wave.

Geometric Electrodynamics provides a unified geometric description of electromagnetic and gravitational waves, and in the weak-field regime reproduces standard General Relativity predictions while linking electromagnetism and gravity more tightly than in the classical split-framework. Additionally, gravitational and electromagnetic waves are predicted to have identical propagation speeds, a result that has been confirmed experimentally to an uncertainty of $\lesssim 10^{-15}$ by the binary neutron star merger GW170817 [20].

10. Discussion: Source Self-Consistency, Uniqueness and Open Questions

Geometric Electrodynamics represents a significant departure from the traditional development of Classical Electrodynamics. At its core, the theory unifies electromagnetic and gravitational phenomena through a single foundational equation, Equation (1), in which the coupling vector field a^λ links the derivatives of the Maxwell tensor to the Riemann-Christoffel curvature tensor. This construction ensures that spacetime geometry and gravitational effects are intrinsically present in every solution, rather than being added as separate ingredients.

A key conceptual shift in Geometric Electrodynamics is the treatment of source terms. In Classical Physics, charge and mass densities are introduced as external parameters—once specified,

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the field equations are solved to determine the resulting electromagnetic and gravitational fields. In contrast, Geometric Electrodynamics reverses this logic: the fundamental fields (a^{λ}, $g_{\mu\nu}$, $F_{\mu\nu}$) are solved for directly, and the source terms (ρ_c, u^{λ}, ρ_m) are then defined self-consistently in terms of these fields and their derivatives, as given by Equations (10), (11), and (17), respectively.

This approach blurs the traditional distinction between fields and sources, describing physical systems entirely in terms of their fundamental fields. Any solution to Equation (1) thus represents a physically allowed configuration, with the associated source terms emerging as derived quantities. Importantly, all solutions to Equation (1) can be mapped onto the framework of Classical Electrodynamics, using the source terms (ρ_c, u^{λ}, ρ_m) determined by their geometric definitions. As detailed in Section 6, the equations of Classical Electrodynamics are satisfied identically by all solutions to Equation (1). However, the converse is not true; not every classical solution with externally prescribed sources corresponds to an Equation (1) solution. This is due to the freedom in classical theory to insert arbitrary sources, which may not be compatible with the geometric constraints imposed by Equation (1). For example, in the case of the spherically symmetric nonrotating charged mass, the Geometric Electrodynamics solution yields gravitational fields consistent with General Relativity but imposes stricter constraints on the charge and mass distributions than the classical theory. This distinction highlights a key advantage of the geometric framework: by deriving source terms from the fields themselves, the theory enforces internal logical consistency and restricts the class of physically realizable solutions. This opens new avenues for foundational research, including the possibility of finding stable, particle-like solutions that represent elementary particles in terms of the fundamental fields (a^{λ}, $g_{\mu\nu}$, $F_{\mu\nu}$). While the search for such solutions has a long history, Geometric Electrodynamics introduces a novel coupling between gravity and electromagnetism, potentially enabling new types of unified classical field models.

In summary, Geometric Electrodynamics adopts the perspective that nature is governed by continuous fields, with all physical quantities determined self-consistently by the governing equations. This contrasts with the traditional approach, where sources are defined independently of the fields they generate. Although only a limited number of exact solutions to Equation (1) have been published to date, the theory appears to admit a wide class of physically interesting configurations and exhibits promising features as a candidate for a unified classical field theory encompassing both gravitation and electromagnetism.

Several open questions remain, including the physical admissibility of superluminal solutions, the possible phenomenological signatures of the geometric framework, and the compatibility of Geometric Electrodynamics with experimental tests of the equivalence principle. Further investigation of these issues may provide new insight into the foundations of classical field theory.

11. Conclusions

This manuscript has presented Geometric Electrodynamics as a unified classical field theory, founded on a single geometric postulate, Equation (1), with Riemannian geometry as the underlying structure of spacetime. Within this framework, Maxwell's equations and the gravitational field emerge naturally, and charge and mass are defined intrinsically in terms of the fundamental fields rather than introduced externally. This approach resolves longstanding ambiguities in the classical merger of electrodynamics and gravitation, providing unique and self-consistent definitions for charge density, four-velocity, and local mass density. Importantly, all solutions to Equation (1) yield physical systems that satisfy the full set of equations of Classical Electrodynamics while simultaneously incorporating gravitational effects. Several exact solutions have been constructed and compared with their classical counterparts, demonstrating consistency with General Relativity and Maxwell's theory, but also revealing new constraints on mass and charge distributions. The unified treatment of electromagnetic and gravitational phenomena underscores the internal logical consistency and conceptual strength of the geometric approach.

A striking feature of the theory is the emergence of antimatter and its classical behavior in electromagnetic and gravitational fields, dictated by the symmetries of Equation (1). This result suggests that certain phenomena typically associated with quantum theory—such as charge conjugation and time reversal—may have geometric analogues in a classical context.

Looking forward, Geometric Electrodynamics opens new directions for foundational research. Notably, the theory permits the possibility of superluminal charge transport, as there is no inherent restriction to timelike solutions for the charge current. While at present no explicit superluminal solutions have been constructed. Whether such solutions exist and, if so, whether they can be reconciled with causal structure and experimental constraints remains an open question. Although such solutions challenge conventional causality, they may offer insight into quantum nonlocality or other unexplored physical regimes. Clarifying the physical admissibility and implications of these solutions remains an important avenue for future investigation [21].

In summary, although Geometric Electrodynamic as developed within has not yet systematically confronted data beyond the specific cases where it has reproduced Classical Electrodynamic and Geneal Relativity results, it has achieved conceptual unification, internal consistency of source definitions, and the existence of nontrivial solutions. The primary contribution of this work is to propose and analyze a logically self-consistent geometric framework in which the classical notions of source, field, and spacetime are unified, suggesting that several outstanding conceptual questions in classical physics may be addressed within a single continuous field theory.

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