Article

Not peer-reviewed version

Geometric Electrodynamics – the Geometrization of Charge and Mass and Its Implications for General Relativity

Raymond Beach *

Posted Date: 21 July 2025

doi: 10.20944/preprints202507.1747.v1

Keywords: geometric electrodynamics; gravity; Maxwell's equations; general relativity

Article

Geometric Electrodynamics: The Geometrization of Charge and Mass and Its Implications for General Relativity

Raymond John Beach

Lawrence Livermore National Laboratory, L-465, 7000 East Avenue, Livermore, CA 94551, USA; beach2@llnl.gov

Abstract

This manuscript furthers a recently published theory, Geometric Electrodynamics, in which gravity emerges and mass and charge are defined as geometric quantities rather than as externally introduced entities as in classical electrodynamics [1]. The theory is founded on a single fundamental equation coupling the Maxwell tensor to the Riemann-Christoffel curvature tensor, and then developed through a logical progression based on Riemannian geometry as the underlying structure of spacetime. Geometric Electrodynamics presents a radically different perspective than Classical Electrodynamics: its solutions satisfy all the equations of Classical Electrodynamics, but gravitation also emerges in every solution. In this framework, the source terms—charge and mass—are defined geometrically in terms of the fundamental fields. Requiring that these definitions are self-consistent across the fundamental equations of a theory is taken as a criterion for the theory's logical consistency. Applying this check to Geometric Electrodynamics demonstrates its consistency, while a similar check of classical physics—defined as the merger of Classical Electrodynamics and General Relativity—reveals an inconsistency in its multiple definitions of mass. This discrepancy has significant implications for current problems in gravitational theory.

Keywords: geometric electrodynamics; gravity; Maxwell's equations; general relativity

---

I. Introduction

Maxwell's equations form the foundation of classical electromagnetism, traditionally derived using vector calculus and empirical assumptions about charge. This manuscript revisits a recently introduced alternative formulation, in which Maxwell's equations and gravitation emerge from a single fundamental equation and the geometric constraints imposed by Riemannian geometry as the underlying structure of spacetime. Building upon this prior effort to geometricize electrodynamics, the definitions that emerge for charge, mass, and their associated four-velocity, are used here to emphasize the differences in the physical picture that develops. Crucially, this framework reinterprets these quantities not as independent physical entities, but as intrinsic geometric manifestations of spacetime curvature, which dictates their precise definitions.

A major point in this manuscript is that these emerging definitions can be used as logical consistency checks across various physical theories. Specifically, for a continuous field theory such as the one considered here, the definitions for source terms like charge and mass—expressed in terms of the theory's fundamental fields—must be self-consistent. For a theory to be logically consistent, all its foundational equations must use the same derived definitions for the same source terms. This perspective will be expanded and exploited in the text to test the self-consistency of Geometric Electrodynamics, and then to compare it with classical physics defined as the merger of Classical Electrodynamics and General Relativity.

The manuscript begins with a brief review of the geometricized theory of electrodynamics presented in Ref. [1], including the geometric definitions of charge, mass, and their associated four-

 doi:10.20944/preprints202507.1747.v1

velocity. Because gravitation emerges naturally in this geometricized framework, the resulting picture differs fundamentally from that of Classical Electrodynamics. To keep this work reasonably self-contained, I summarize key results from Ref. [1] but omit many details, derivations, and references already provided there. Readers interested in a more comprehensive mathematical treatment, including full citations, are encouraged to consult Ref. [1], which is available as an open-access resource. For clarity, I refer to the geometricized theory as Geometric Electrodynamics throughout, to distinguish it from Classical Electrodynamics.

II. The Governing Equation of Geometric Electrodynamics

I begin by briefly reviewing the development of Geometric Electrodynamics as presented in Ref. [**1**], starting with the postulated fundamental equation

$$F_{\mu\nu;\kappa} = a^{\lambda} R_{\lambda\kappa\mu\nu} \quad . \tag{1}$$

Here $F_{\mu\nu}$ is the electromagnetic Maxwell field tensor encoding electric and magnetic fields; $R_{\lambda\kappa\mu\nu}$ is the Riemann-Christoffel curvature tensor encapsulating spacetime geometry and gravitation [2]. As in Ref. [**1**], I use the notational style and definitions used by Weinberg. The definition of the Riemann-Christoffel curvature tensor is $R^{\lambda}{}_{\mu\nu\kappa} \equiv \partial_{\kappa}\Gamma^{\lambda}{}_{\mu\nu} - \partial_{\nu}\Gamma^{\lambda}{}_{\mu\kappa} + \Gamma^{\eta}{}_{\mu\nu}\Gamma^{\lambda}{}_{\kappa\eta} - \Gamma^{\eta}{}_{\mu\kappa}\Gamma^{\lambda}{}_{\nu\eta}$ and the definition of the Ricci tensor is $R_{\mu\kappa} \equiv R^{\lambda}{}_{\mu\lambda\kappa}$; and a^{λ} is a coupling field governing interactions between electromagnetism and curvature. A complete physical solution is described by 20 independent field components: 10 metric tensor components $g_{\mu\nu}$ defining spacetime curvature, 6 Maxwell tensor components $F_{\mu\nu}$ representing electromagnetic degrees of freedom, and 4 coupling components a^{λ} that link the Maxwell tensor to spacetime curvature.

These field components are determined by an equal number of independent governing equations. At first glance, the fundamental equation appears to involve 24 independent component equations. However, the four algebraic cyclicity conditions satisfied by the Riemann tensor

$$R_{\lambda\kappa\mu\nu} + R_{\lambda\mu\nu\kappa} + R_{\lambda\nu\kappa\mu} = 0 \tag{2}$$

impose four constraints, reducing the system to 20 independent equations—precisely matching the degrees of freedom in the component fields.

III. Emergence of Maxwell's Equations

Maxwell's homogeneous equation follows directly from Equation (1) by contracting the algebraic cyclicity condition Equation (2) with a^{λ}

$$a^{\lambda}\left(R_{\lambda\kappa\mu\nu} + R_{\lambda\mu\nu\kappa} + R_{\lambda\nu\kappa\mu} = 0\right) \;\rightarrow\; F_{\mu\nu;\kappa} + F_{\nu\kappa;\mu} + F_{\kappa\mu;\nu} = 0 \quad . \tag{3}$$

Maxwell's inhomogeneous equation is obtained by contracting the μ and κ indices in Equation (1)

$$F^{\mu\nu}{}_{;\mu} = -a^{\lambda} R_{\lambda}{}^{\nu} \;\rightarrow\; F^{\mu\nu}{}_{;\mu} = -\rho_c u^{\nu} \tag{4}$$

Preprints.org (www.preprints.org) | NOT PEER-REVIEWED | Posted: 21 July 2025 doi:10.20944/preprints202507.1747.v1

where the four-vector $a^{\lambda}R_{\lambda}{}^{\nu}$ is identified as the charge current density

$$\rho_c u^{\nu} \equiv a^{\lambda}R_{\lambda}{}^{\nu} = \pm\sqrt{\left|a^{\gamma}R_{\gamma}{}^{\kappa}a^{\delta}R_{\delta\kappa}\right|}\left(\pm\frac{a^{\lambda}R_{\lambda}{}^{\nu}}{\sqrt{\left|a^{\alpha}R_{\alpha}{}^{\sigma}a^{\beta}R_{\beta\sigma}\right|}}\right). \tag{5}$$

Unlike in classical electromagnetism, where the charge density and its associated four-velocity are introduced as external source terms, here they are defined geometrically in terms of the fundamental field components, a^{λ} and $g_{\mu\nu}$,

$$\rho_c \equiv \pm\sqrt{\left|a^{\gamma}R_{\gamma}{}^{\kappa}a^{\delta}R_{\delta\kappa}\right|} \ , \tag{6}$$

and

$$u^{\nu} \equiv \pm\frac{a^{\lambda}R_{\lambda}{}^{\nu}}{\sqrt{\left|a^{\alpha}R_{\alpha}{}^{\sigma}a^{\beta}R_{\beta\sigma}\right|}} \ . \tag{7}$$

Note, "$\pm$" signs introduced in Equation (5) are linked, they are either both "+" or both "-" and this convention is maintained in Equations (6) and (7). The reason for this will become clear when considering charge conjugation and matter-antimatter symmetries (See Ref. [1]).

Finally, because $F^{\mu\nu}$ is antisymmetric, it enforces $F^{\mu\nu}{}_{;\mu;\nu} = 0$, which, together with Equation (4), ensures charge conservation

$$\left(F^{\mu\nu}{}_{;\mu} = -a^{\lambda}R_{\lambda}{}^{\nu}\right)_{;\nu} \ \rightarrow \ 0 = \left(a^{\lambda}R_{\lambda}{}^{\nu}\right)_{;\nu} \ \rightarrow \ 0 = \left(\rho_c u^{\nu}\right)_{;\nu} \ . \tag{8}$$

In summary, the geometric derivation of Maxwell's homogenous and inhomogeneous equations follows from Equation (1) with Riemannian geometry as the underlying structure of spacetime. Although the vector field a^{λ} is newly introduced and unfamiliar to classical physics, as shown in Ref. [**1**] it is related to the classical electromagnetic vector potential A_{λ} by

$$-a_{\kappa;\mu;\nu} + a_{\kappa;\nu;\mu} = A_{\nu;\mu;\kappa} - A_{\mu;\nu;\kappa} \ . \tag{9}$$

IV. Constraints on Physically Meaningful Solutions

For a solution to fundamental Equation (1) to be physically meaningful, transporting that solution around any closed curve must leave it unchanged—in other words, the fields must be single-valued at all points in spacetime. To establish a condition that ensures this requirement, I analyze the transport of a solution for $F_{\mu\nu}$ around an infinitesimal closed path, using the fundamental Equation (1) to evaluate its accumulated change. A detailed analysis reveals that the requirement for this accumulated change to vanish is

$$\left(a^{\rho}R_{\rho\kappa\mu\nu}\right)_{;\lambda} - \left(a^{\rho}R_{\rho\lambda\mu\nu}\right)_{;k} = -F_{\mu\sigma}R^{\sigma}{}_{\nu k\lambda} - F_{\sigma\nu}R^{\sigma}{}_{\mu\kappa\lambda} \ . \tag{10}$$

This condition serves as a consistency requirement, guaranteeing that solutions remain single-valued. This same equation (10) appears in Ref. [**1**] as Equation (31) where it is followed by the statement, "which is equivalent to requiring the covariant derivatives of $F_{\mu\nu}$ commute, *i.e.*, $F_{\mu\nu;\kappa;\lambda} - F_{\mu\nu;\lambda;\kappa} = 0$". This follow-on statement is erroneous and not what Equation (10) says, it should be deleted. It appears correctly here in the body of the manuscript. The emergence of this constraint further underscores the geometric nature of electrodynamics, imposing additional structural restrictions on the permissible field configurations.

V. Comparison of the Geometric and Classical Physics Derivations of Maxwell's Equations

Before I introduce mass into the Geometrical framework, it is insightful to compare the foregoing geometric derivation of Maxwell's equations to their conventional classical formulation. In the classical approach, Maxwell's homogeneous equation is taken as an axiom, an assumption required because the assumed Lagrangian

$$L = -\rho_m \sqrt{\left|u^\mu u_\mu\right|} + \rho_c A_\lambda u^\lambda - \frac{1}{4} F^{\rho\sigma} F_{\rho\sigma} \tag{11}$$

necessitates a vector potential A_λ for the matter-field interaction term. This is also the stage where charge density ρ_c and mass density ρ_m are introduced as external entities in the classical formulation.

Maxwell's inhomogeneous equation and the Lorentz force law are then derived using a stationary-action principle, with the action defined in terms of the Lagrangian

$$I_M \equiv \int d^4x \sqrt{g(x)} \left(-\rho_m \sqrt{\left|u^\mu u_\mu\right|} + \rho_c A_\lambda u^\lambda - \frac{1}{4} F^{\rho\sigma} F_{\rho\sigma} \right) . \tag{12}$$

In this formulation, A_λ and the spatial positions of masses and charges serve as dynamic variables to be varied to derive Maxwell's inhomogeneous equation and the Lorentz force law, respectively.

Finally, invoking general covariance, a conserved energy-momentum tensor $T^{\mu\nu}$ is defined as the functional derivative of the scalar action I_M with respect to the metric tensor $g_{\mu\nu}$,

$$T^{\mu\nu} \equiv \rho_m u^\mu u^\nu + F^\mu{}_\lambda F^{\nu\lambda} - \frac{1}{4} g^{\mu\nu} F^{\rho\sigma} F_{\rho\sigma} , \tag{13}$$

where

$$T^{\mu\nu}{}_{;\nu} = 0 . \tag{14}$$

VI. The Introduction of Mass

For the geometric development pursued here, mass is introduced using the same conserved energy-momentum tensor used in the classical formulation of Maxwell's equations. A direct consequence of this energy-momentum tensor and the previously derived Maxwell equations (3) and (4), are the conservation of mass equation

 doi:10.20944/preprints202507.1747.v1

$$\left(\rho_m u^\nu\right)_{;\nu} = 0 \quad , \tag{15}$$

and the Lorentz force law

$$\rho_m \frac{Du^\mu}{D\tau} = \rho_c F^\mu{}_\lambda u^\lambda \quad , \tag{16}$$

where

$$\frac{Du^\mu}{D\tau} \equiv u^\mu{}_{;\nu}\, u^\nu \tag{17}$$

(See Ref. [**1**] for the derivation). Rather than adopting the classical perspective in which mass density ρ_m is treated as an externally introduced entity, here it is defined geometrically. To see this explicitly, consider the Lorentz force law (16) and assume the right-hand side is nonzero for some μ-component, yielding

$$\rho_m\, u^\mu{}_{;\nu} u^\nu = \rho_c F^\mu{}_\lambda u^\lambda \,. \tag{18}$$

Substituting the geometric definitions for ρ_c and u^λ from Equations (6) and (7), respectively, gives

$$\rho_m \equiv \frac{F^\mu{}_\lambda a^\sigma R_\sigma{}^\lambda}{\left(\dfrac{a^\rho R_\rho{}^\mu}{\sqrt{\left|a^\gamma R_\gamma{}^\kappa a^\delta R_{\delta\kappa}\right|}}\right)_{;\nu}\left(\dfrac{a^\alpha R_\alpha{}^\nu}{\sqrt{\left|a^\beta R_\beta{}^\eta a^\zeta R_{\zeta\eta}\right|}}\right)} \quad \text{(no sum over μ).} \tag{19}$$

The value of ρ_m defined here using the Lorentz force law is the inertial mass density, a distinction that will become important later when considering the Weak Equivalence Principle. The same definition of ρ_m given in Equation (19) also satisfies the conservation of mass Equation (15), and the conservation of energy-momentum

$$T^{\mu\nu}{}_{;\nu} = \left(\rho_m u^\mu u^\nu + F^\mu{}_\lambda F^{\nu\lambda} - \frac{1}{4} g^{\mu\nu} F^{\rho\sigma} F_{\rho\sigma}\right)_{;\nu} = 0\,. \tag{20}$$

from which the Lorentz Force Law (16) was derived.

It is important to recognize that introducing mass via the specific energy-momentum tensor given by Equation (13) constitutes an axiom in the development of Geometric Electrodynamics as presented here. Choosing a different conserved energy-momentum tensor could yield a different definition of mass density, reflecting the non-uniqueness of mass concepts in relativistic theories. In what follows, I will refer to mass defined operationally via the response to the Lorentz force law as

 doi:10.20944/preprints202507.1747.v1

"inertial mass," and to mass defined via the source term in the General Relativity field equations as "gravitational mass." While the distinction is not always clear, particularly considering the Weak Equivalence Principle, this terminology will provide a useful working framework for the subsequent discussion [3].

VII. Summary: The Geometrization of Electrodynamics

The geometrically dictated definitions of mass, charge, and their associated four-velocity, in Geometric Electrodynamics offer a fundamentally different approach to describing physical systems compared to Classical Electrodynamics. To illustrate this, consider any solution of Equation (1) for the fundamental fields (a^{λ} , $g_{\mu\nu}$, $F_{\mu\nu}$). Next express (ρ_m , ρ_c , u^{λ}) in terms of these fundamental fields using their geometric definitions:

- $$\rho_m \equiv \frac{F^{\mu}{}_{\lambda} a^{\sigma} R_{\sigma}{}^{\lambda}}{\left(\dfrac{a^{\rho} R_{\rho}{}^{\mu}}{\sqrt{\left|a^{\gamma} R_{\gamma}{}^{\kappa} a^{\delta} R_{\delta\kappa}\right|}}\right)_{;\nu}\left(\dfrac{a^{\alpha} R_{\alpha}{}^{\nu}}{\sqrt{\left|a^{\beta} R_{\beta}{}^{\eta} a^{\zeta} R_{\zeta\eta}\right|}}\right)} \quad \text{(no sum on } \mu) \qquad (19)$$
- $$\rho_c \equiv \pm\sqrt{\left|a^{\gamma} R_{\gamma}{}^{\kappa} a^{\delta} R_{\delta\kappa}\right|} \qquad (6)$$
- $$u^{\nu} \equiv \pm\frac{a^{\lambda} R_{\lambda}{}^{\nu}}{\sqrt{\left|a^{\alpha} R_{\alpha}{}^{\sigma} a^{\beta} R_{\beta\sigma}\right|}} \qquad (7)$$

This collection of six fields (a^{λ} , $g_{\mu\nu}$, $F_{\mu\nu}$, ρ_m , ρ_c , u^{ν}), where the last three are expressed in terms of the first three, constitutes a solution to the following equations of Classical Electrodynamics:

- Maxwell's homogeneous eq. $F_{\mu\nu;\kappa} + F_{\nu\kappa;\mu} + F_{\kappa\mu;\nu} = 0$ (3)
- Maxwell's inhomogeneous eq. $F^{\mu\nu}{}_{;\mu} = -\rho_c u^{\nu}$ (4)
- The conservation of charge: $\left(\rho_c u^{\nu}\right)_{;\nu} = 0$ (8)
- Conservation of energy-momentum: $\left(\rho_m u^{\mu} u^{\nu} + F^{\mu}{}_{\lambda} F^{\nu\lambda} - \frac{1}{4} g^{\mu\nu} F^{\rho\sigma} F_{\rho\sigma}\right)_{;\nu} = 0$ (20)
- The conservation of mass: $\left(\rho_m u^{\nu}\right)_{;\nu} = 0$ (15)
- The Lorentz force law: $\rho_m \dfrac{Du^{\mu}}{D\tau} = \rho_c F^{\mu}{}_{\lambda} u^{\lambda}$ (16)

This approach differs fundamentally from Classical Electrodynamics, where charge and mass are introduced as external entities, and their trajectories and accompanying electromagnetic fields must be found from self-consistent solutions to Equations (3), (4), and (20).

Another important distinction is that, in Geometric Electrodynamics, gravity is an integral part of every solution because the metric field $g_{\mu\nu}$ enters the fundamental Equation (1) via the Riemann-Christoffel curvature tensor. In Classical Electrodynamics, gravitational effects require the separate introduction of the Einstein field equations to solve for the metric tensor.

 doi:10.20944/preprints202507.1747.v1

VIII. The Emergence of Gravity in Geometric Electrodynamics and Comparison to General Relativity

As discussed more fully in Ref. [**1**], gravity emerges naturally in the solutions of fundamental Equation (1). For example, an exact solution was developed in Ref. [**1**] for a spherically symmetric, non-rotating, charged particle, employing the Reissner-Nordström metric. This solution demonstrates that the emergent gravitational fields for this case are identical to those predicted by General Relativity, namely, the Reissner-Nordström metric. However, the charge and mass densities derived from the solution to Equation (1) do not correspond to point-like distributions. Instead, they exhibit spatial extents that decay as $1/r^4$. This leads to the intriguing situation in which the solution to Equation (1) predicts gravitational fields that coincide with those of General Relativity for a point charge, while Geometric Electrodynamics yields spatially extended charge and mass distributions.

This discrepancy highlights an inconsistency between the geometric definition of inertial mass in Geometric Electrodynamics and the definition of gravitational mass in General Relativity. To illustrate this, consider the General Relativity field equation, which employs the conserved energy-momentum tensor (13) on its right-hand side

$$R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R = -8\pi T^{\mu\nu} = -8\pi \left(\rho_{m\,GR} u^\mu u^\nu + F^\mu{}_\sigma F^{\nu o} - \frac{1}{4} g^{\mu\nu} F^{\rho\sigma} F_{\rho\sigma} \right) . \tag{21}$$

Contracting both sides of Equation (21) leads to the following mass density definition

$$\rho_{m\,GR} = -\frac{1}{8\pi} R \ , \tag{22}$$

where R is the Ricci scalar, and $\rho_{m\,GR}$ denotes the General Relativity definition for gravitational mass density, to distinguish it from ρ_m, the Geometric Electrodynamic definition for mass density given by Equation (19) and repeated here

$$\rho_m = \frac{F^\mu{}_\lambda a^\sigma R_\sigma{}^\lambda}{\left(\dfrac{a^\rho R_\rho{}^\mu}{\sqrt{\left| a^\gamma R_\gamma{}^\kappa a^\delta R_{\delta\kappa} \right|}} \right)_{;\nu} \left(\dfrac{a^\alpha R_\alpha{}^\nu}{\sqrt{\left| a^\beta R_\beta{}^\eta a^\zeta R_{\zeta\eta} \right|}} \right)} \quad \text{(no sum on } \mu\text{)}. \tag{19}$$

The General Relativity definition depends soley on the metric tensor $g_{\mu\nu}$ and its derivatives, while the electrodynamic definition depends on (a^λ, $g_{\mu\nu}$, $F_{\mu\nu}$), making it impossible for both definitions to be correct simultaneously.

Despite both definitions arising from logical developments based on Riemannian geometry as the structure of spacetime, and employing the same energy-momentum tensor, the differences in their foundational axioms lead to incompatible definitions. As such, Geometric Electrodynamics and General Relativity cannot be merged into a single, logically consistent theory satisfying the Weak Equivalence Principle. This incompatibility is also evident in the solution from Ref. [**1**], where the same gravitational field requires different mass distributions depending on whether it is calculated using Geometric Electrodynamics or General Relativity.

Nevertheless, this logical inconsistency does not undermine the internal consistency of Geometric Electrodynamics as a standalone theory with emergent gravity; there is only one definition for mass density in Geometric Electrodynamics, and it serves both the inertial and gravitational parts of the theory. In the next section, I examine the self-consistency of mass definitions used in Classical Electrodynamics and General Relativity, which together form the accepted foundation of classical physics.

IX. Mass Definition Discrepancy – The Classical Physics View

In this section, I compare the mass definitions dictated by Classical Electrodynamics and General Relativity. The approach parallels the development in Geometric Electrodynamics, but here the starting point is Maxwell's inhomogeneous equation (4) rather than Equation (1). In Classical Electrodynamics, the definitions of charge density and the four-velocity describing its motion are given in terms of the Maxwell tensor and its derivatives. Starting from Maxwell's inhomogeneous equation

$$F^{\mu\nu}{}_{;\mu} = -\rho_c u^\nu \quad , \tag{4}$$

the charge density can be represented as

$$\rho_c \equiv \pm\sqrt{\left|F^{\mu\nu}{}_{;\mu} F^{\sigma}{}_{\nu;\sigma}\right|} \quad , \tag{23}$$

and its associated four-velocity as

$$u^\lambda \equiv \mp \frac{F^{\mu\lambda}{}_{;\mu}}{\sqrt{\left|F^{\mu\nu}{}_{;\mu} F^{\sigma}{}_{\nu;\sigma}\right|}} \quad . \tag{24}$$

These definitions are closely related to their analogues in Geometric Electrodynamics and would be identical under the substitution $F_{\mu\nu;\kappa} \rightarrow a^\lambda R_{\lambda\kappa\mu\nu}$.

Using the Lorentz force law (16) and assuming the right-hand side is nonzero for some μ-component

$$\rho_{m\ CE} = \frac{\rho_c F^{\mu}{}_{\lambda} u^\lambda}{u^\mu{}_{;\nu} u^\nu} = \frac{-F^{\mu}{}_{\lambda} F^{\sigma\lambda}{}_{;\sigma}}{\left(\frac{F^{\rho\mu}{}_{;\rho}}{\sqrt{\left|F^{\gamma\alpha}{}_{;\gamma} F^{\delta}{}_{\alpha;\delta}\right|}}\right)_{;\nu}\left(\frac{F^{\alpha\nu}{}_{;\alpha}}{\sqrt{\left|F^{\beta\eta}{}_{;\beta} F^{\zeta}{}_{\eta;\zeta}\right|}}\right)} \quad \text{(no sum on } \mu\text{)} \tag{25}$$

where $\rho_{m\ CE}$ is the Classical Electrodynamic definition for inertial mass density.

Comparing the expression in Equation (25) for $\rho_{m\,CE}$ to the expression in Equation (22) for $\rho_{m\,GR}$, we see that these definitions are inconsistent—analogous to the situation found with Geometric Electrodynamics and General Relativity in the previous section, except here General Relativity provides the gravitational foundation for classical physics.

X. Discussion and Conclusion

This work adopts the perspective that nature can be described by a continuous field theory, in which the fundamental fields are governed by one or more equations describing their evolution in the presence of source terms. Traditionally, these source terms have been introduced as external entities—defined independently of the fields they generate and interact with. In contrast, the approach taken here defines source terms themselves as continuous fields, determined self-consistently by the governing equations of the theory.

A key implication of this approach is that a logically consistent theory should yield a unique definition for each source term. If a theory produces multiple, incompatible definitions for the same source, it lacks internal self-consistency. This issue arises in classical physics—here defined as the union of Classical Electrodynamics and General Relativity—where two distinct, non-equivalent definitions of mass coexist. Resolving this inconsistency and thereby aligning classical theory with the Weak Equivalence Principle—now experimentally verified to an accuracy of 10^{-15} in the Eötvös ratio [4]—would require modifying the foundational equations so that their definitions of mass coincide. This realization of an inconsistency in the foundational equations of classical physics may offer a new perspective on outstanding fundamental problems, such as the nature of dark matter [5,6].

Geometric Electrodynamics, with its emergent gravity, occupies a distinctive position: it provides a single, self-consistent definition of mass. While only a few exact solutions to date have been found for Equation (1) (see Ref. [**1**]), no fundamental obstacles have emerged, and the theory exhibits several attractive features as a candidate for a unified classical field theory.

XI. Summary

In this manuscript, I have outlined the development of Geometric Electrodynamics, beginning with Equation (1) as the fundamental postulate and Riemannian geometry as the underlying structure of spacetime. This framework yields Maxwell's equations and a geometrically dictated definition of charge current density, from which both charge density and its associated four-velocity are derived. Mass is introduced via a conserved energy-momentum tensor, leading to a geometric definition of mass density that is consistent with the charge density and automatically satisfies the equivalence of inertial and gravitational mass.

The Geometric Electrodynamics perspective asserts that solutions to Equation (1) for the fundamental fields (a^{λ}, $g_{\mu\nu}$, $F_{\mu\nu}$) provide a complete description of a physical system. Such solutions, along with the geometric definitions of mass density, charge density, and their associated four-velocity in terms of the fundamental fields (a^{λ}, $g_{\mu\nu}$, $F_{\mu\nu}$), satisfy all the equations of Classical Electrodynamics and incorporate gravitational effects. This unified treatment of electromagnetism and gravity that emerges from Equation (1) strongly supports this approach, as does its logical consistency.

To date, only a handful of exact solutions to Equation (1) have been developed and cross-checked against analogous results from Classical Electrodynamics and General Relativity. For example, the particle-like solution described in Ref. [**1**] yields gravitational fields in agreement with General Relativity, but with a distinct mass distribution. Another solution from Ref. [**1**] derives expressions for radiating electromagnetic and gravitational waves in the weak-field limit, again enabling direct comparison with classical theories. These comparisons highlight the unification of gravitational and electromagnetic phenomena achieved by Equation (1).

Several compelling aspects of the theory, not explicitly covered in this manuscript but discussed in Ref. [**1**], deserve mention. Notably, the properties of antimatter and its behavior in gravitational and electromagnetic fields emerge naturally from the symmetries of Equation (1)—an unexpected result for a non-quantum continuous field theory. Furthermore, the particle-like solution to Equation (1) suggests the possibility of quantized values for mass, charge, and angular momentum, again, a surprising feature for a classical field theory.

Finally, a more speculative aspect of the theory concerns the possibility of superluminal transport. Notably, there appears to be no a priori restriction requiring $a^{\lambda}R_{\lambda}^{\ \nu}$ in solutions to Equation (1) to be exclusively timelike; the existence of spacelike solutions would, via Equation (5), correspond to a superluminal charge current density. While such solutions challenge conventional notions of causality, they may have relevance to quantum mechanical nonlocal correlations.

 doi:10.20944/preprints202507.1747.v1

Whether the superluminal features suggested by the present theory can be reconciled with, or are fundamentally distinct from, quantum nonlocality remains an open question. Further investigation is warranted to clarify the physical admissibility and potential implications of these solutions, should they be found to exist [7].

This work was performed under the auspices of the U.S. Department of Energy by Lawrence Livermore National Laboratory under Contract DE-AC52-07NA27344.

References

1. Beach, R. J. (2024). The geometrization of Maxwell's equations and the emergence of gravity and antimatter. *Annals of Physics*, *465*, 169661.
2. Weinberg, S. (1972). *Gravitation and Cosmology*, John Wiley & Sons, New York, NY 1972.
3. C.M. Will, Theory and Experiment in Gravitational Physics, Cambridge University Press, Cambridge, 1993.
4. Touboul, P., Métris, G., Rodrigues, M., Bergé, J., Robert, A., Baghi, Q., ... & (MICROSCOPE Collaboration). (2022). MICROSCOPE mission: final results of the test of the equivalence principle. *Physical review letters*, *129*(12), 121102
5. Milgrom, M. (1994). Dynamics with a nonstandard inertia-acceleration relation: An alternative to dark matter in galactic systems. *Annals of Physics*, *229*(2), 384-415
6. Bertone, G., Hooper, D., & Silk, J. (2005). Particle dark matter: Evidence, candidates and constraints. *Physics reports*, *405*(5-6), 279-390
7. Handsteiner, J., Friedman, A. S., Rauch, D., Gallicchio, J., Liu, B., Hosp, H., ... & Zeilinger, A. (2017). Cosmic Bell test: measurement settings from milky way stars. *Physical review letters*, *118*(6), 060401.