Concept Paper

Non-Statistical Analytic Entropy-Multiplicative Entropy Formula Answers Planck's Question

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Non-Statistical Analytic Entropy-Multiplicative Entropy Formula Answers Planck's Question

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Abstract

This paper introduces a novel entropy formulation — multiplicative entropy — defined as the product of energy values across all units in a **quantized homogeneous invariant network**. Unlike traditional statistical entropy, this approach explicitly tracks irreversible energy redistribution pathways, offering an analytic and path-dependent description of entropy growth. Logarithmic transformation recovers classical entropy forms, while preserving temporal directionality and quantum-scale resolution. The model enables precise simulation of thermodynamic processes, supporting the development of Analytic Quantum Thermodynamics as a new framework for understanding entropy-driven dynamics. Crucially, the analytical multiplicative entropy formula proposed here responds to Planck's long-sought vision of an entropy expression in analytic form, enabling the resolution of entropy to match the high resolution of quantum processes.

Keywords: multiplicative entropy; energy product entropy; analytic entropy; computable entropy; entropy coordinate; analytic quantum thermodynamics; non-statistical entropy

Introduction

In our previous study, Time-Entropy Mapping; Mass-Gravity Duality; Metric-Frequency Mirroring—A Two-Layer Fiber Bundle Model with Topologically Invariant Space Configuration (Preprints 2025, https://doi.org/10.20944/preprints202505.0270.v10), we proposed a topologically invariant two-layer base space model aiming to embed the thermodynamic arrow of time into the frameworks of General Relativity (GR) and Quantum Field Theory (QFT). This model consists of a macroscopic geometric layer and a microscopic quantum layer—the Space Elementary Quanta (SEQ) layer—whose dynamical behavior is characterized by resonant frequencies that encode the spatial distribution of kinetic and potential energy. Building upon this structural foundation, entropy can be reduced from its traditional statistical formulation to a fully analytic, path-dependent function that evolves deterministically with each discrete energy redistribution event.

Entropy, as traditionally defined in statistical mechanics, is a measure of disorder based on the logarithm of the number of accessible microstates. While this formulation has been foundational in thermodynamics, it often lacks the resolution and intuitiveness needed to describe the step-by-step evolution of entropy in physical systems. The classical approach focuses primarily on macroscopic end states and provides limited insight into the irreversible dynamics of spontaneous entropy increase.

In contrast, the proposed energy product entropy — or multiplicative entropy — offers a fundamentally new perspective. By defining entropy as the product of energy values across all system units, this approach explicitly tracks the microscopic redistribution of energy at each stage of evolution. It provides a high-resolution, path-dependent description of entropy growth, naturally encoding time's arrow through local energy flows. Furthermore, this formulation supports the development of analytic quantum thermodynamics, enabling precise simulation and mapping of entropy changes even at the Planck time scale.

This entropy definition realizes the analytical form of Planck's entropy concept within a discrete spacetime network and naturally embeds the thermodynamic arrow of time into the ADM formalism of General Relativity. In this framework, temporal evolution emerges as a statistical ordering of energy redistributions in SEQ network, unifying quantum thermodynamics and spacetime geometry under a single discrete dynamical structure.

This paper demonstrates how multiplicative entropy overcomes the limitations of traditional entropy measures and opens new avenues for understanding thermodynamic processes from a dynamic, computable perspective at the quantum scale.

1. The Concept of Entropy: From Statistical Description to Analytic Dynamics

Traditionally, entropy is understood as a measure of "disorder" or "randomness" in a system. However, "disorder" is inherently abstract and difficult to quantify directly—it relies on statistical counting of microstates (e.g., Boltzmann's $S = k \ln W$), rather than dynamic tracking of actual energy flows. Although Planck sought an analytic expression for entropy at the birth of quantum theory, entropy has remained largely confined within probabilistic and statistical frameworks, failing to emerge as a real-time evolving dynamical variable.

This statistical formulation leads to a fundamental mismatch in resolution: quantum processes occur on timescales as fine as the Planck time, governed precisely by the Schrödinger equation; spacetime geometry in general relativity evolves continuously and deterministically. Yet, thermodynamic entropy—and its irreversibility—has no place in the core equations of either theory. As a result, the two pillars of modern physics—quantum field theory and general relativity—fail to naturally incorporate the laws of thermodynamics, especially the second law and its arrow of time.

The deeper issue lies in the fact that statistical entropy is essentially a "retrospective summary"—a macroscopic description that cannot capture the path of entropy change during individual quantum transitions or local geometric changes. It cannot answer: "How does entropy change during a single energy exchange event? " Consequently, it fails to embed thermodynamic dynamics into quantum evolution or spacetime geometry.

The multiplicative analytic entropy proposed here aims to overcome this limitation. By defining entropy as the product of energy values across all Space Elementary Quanta (SEQs), $S = \prod m_i$, this form abandons statistical assumptions and instead tracks every discrete energy redistribution event, enabling high-resolution dynamic modeling of entropy. Each energy transfer satisfying $m_i > m_j + h$ strictly increases entropy, embedding the arrow of time directly into the dynamical rules. This not only aligns entropy's resolution with quantum processes but also opens a new mathematical pathway to integrate thermodynamic irreversibility into both the Schrödinger equation and Einstein's field equations.

This formulation originates from the degree of energy uniformity: the greater the entropy, the more uniform the energy distribution becomes. Energy flows explicitly from higher-energy quanta to adjacent lower-energy ones, in direct correspondence with classical thermodynamic descriptions of heat conduction. Under this principle, we introduce multiplicative entropy—while the total energy of a system remains conserved, the cumulative product of individual energy levels increases monotonically as the distribution evolves toward uniformity

2. Scope of Application of The Theory-Analytical Quantum Thermodynamics in a Quantum Network

The multiplicative entropy is defined as the product of the energy carried by each unit in a closed, quantized, topologically homogeneous network system. This entropy definition is computable and can serve as an entropy coordinate in system simulations. When logarithmically transformed, it aligns with the form of traditional additive statistical entropy. The restriction to a closed, quantized, topologically homogeneous network system is essential because, in simplified or open systems, multiplicative entropy may not be applicable. The validity of this model — the energy-product entropy — fundamentally relies on the conditions derived from the second law of thermodynamics at the quantum scale, particularly the triggering mechanisms of energy conduction in the quantized space structures.

In fact, all known physical phenomena are consistent with global topological invariance of space; none have been observed to violate it. On the contrary, certain existing theories fail to derive global energy conservation precisely because they lack an explicit assumption of global topological invariance. The present model's incorporation of a topologically invariant base space is therefore not only phenomenologically justified but also physically necessary—when combined with energy quantization, it naturally gives rise to both global energy conservation and the multiplicative entropy structure. Conversely, if one accepts global energy conservation—a cornerstone of the first law of thermodynamics—as a fundamental principle, then global topological invariance becomes almost a logical consequence, effectively serving as its geometric foundation.

3. Multiplicative Entropy-Energy Product Entropy: A New Analytical Framework for Understanding Irreversible Entropy Increase

If we adopt the concept of multiplicative entropy, it becomes much easier to understand why entropy increases spontaneously and irreversibly under the constraints of the second law of thermodynamics.

3.1. Definition of Multiplicative Entropy-Energy Product Entropy

I define multiplicative entropy as the product of the energies carried by each unit in a closed system. This definition of entropy is computable, and can also serve as an entropy coordinate for simulating the evolution of physical systems. When logarithmized, this multiplicative form reduces to the traditional additive entropy expression used in statistical mechanics.

Under the constraint of the second law of thermodynamics, energy always flows from units with higher energy to those with lower energy in a closed system. The total energy of all units remains constant (energy conservation), but the product of their energies keeps increasing — that is, the more uniform the energy distribution becomes, the larger the value of the multiplicative entropy.

When all units carry exactly the same amount of energy, the system reaches its maximum possible entropy — a state known as heat death.

This definition provides a much clearer and more intuitive description of entropy increase than the classical statistical formulation.

3.2. Quantum Thermodynamics Perspective

Speaking of quantum thermodynamics, if we assume that the quantum is the fundamental building block of space, then under the framework of a quantized space network, all states become analytic rather than statistical, specifically at the Planck time scale.

Let's revisit the rule defined by the second law of thermodynamics: energy only flows from high-energy regions to low-energy ones. In essence, the process described by the second law — entropy increase — is a process of energy homogenization.

3.3. Comparison Between Multiplicative Entropy and Classical Statistical Entropy

Dimension	Multiplicative Entropy	Traditional Statistical Entropy
Process Explicitness	Yes, every step has clear changes	No, only describes macroscopic end states
Preserves Microscopic Details	Yes, path-dependent	No, only cares about probability distributions
Has Temporal Directionality	Yes, defined by local energy flow	No, needs extra assumption for time arrow
Suitable for Simulations	Yes, good for numerical modeling	No, mainly used for theoretical analysis

Dimension	Multiplicative Entropy	Traditional Statistical Entropy
Is an Analytical Function Form	Yes, not based on statistical average	No, relies on ensemble average

Why Is Multiplicative Entropy More Suitable for Describing Energy Homogenization?

Take for example a system composed of N basic quanta. According to the second law, energy transfers from high-energy quanta to low-energy ones, gradually making the overall energy distribution more uniform.

Assume each quantum carries energy m_i. At any given transformation state, the total energy of the system is:

$\sum m_i$ = constant; This satisfies energy conservation in a closed system.

How do we characterize the entropy change during this evolution? My proposed multiplicative entropy offers a more intuitive and precise analytical tool:

The entropy S of the system at a certain transformation state is defined as:

$S = \prod m_i$

Under the constraint of energy conservation and directional energy transfer (from high to low), the more uniform the energy distribution becomes, the larger the product becomes.

When all quanta carry equal energy, the system reaches its maximum entropy — heat death occurs.

After applying the logarithm, this formula transforms into the classical additive entropy form. However, I argue that the multiplicative form more accurately captures the spontaneous and irreversible nature of entropy increase, offering greater clarity and visual intuition.

3.4. Computable Definition of Multiplicative Entropy: Time–Entropy Mapping

In this entropy definition, the entropy value of a closed system at a given moment (i.e., during a specific state transition) is calculated as the product of the energy norms of all space elementary quanta (SEQ) involved in the spatial transformation at that moment.

Analysis Formula of Entropy

In this definition of entropy, the entropy value of a closed system at a given moment (i.e., during a specific state transition) is calculated as the multiplicative product of the energy norms of all SEQ involved in that transition(that moment's space transformation).

Entropy value of closed system $S = \prod m_i, \quad i \in N$ ①

Energy of closed system $= \text{constant} = \sum m_i, \quad i \in N$ ②

$\mathbf{S_{max} \le m_i^n}$, When all m_i are equal or differ only by Planck's constant h

(Where m_i refers to the energy carried by the ith SEQ during a single transformation of the closed system, where each energy state $\mathbf{m}_i$ is an integer multiple of Planck's constant h, $\mathbf{m}_i = n_i h$, $n_i \in N$)

3.5. Energy Transfer Rules and Triggering Conditions:

Energy exchange occurs between adjacent SEQ (i,j) if and only if the following thermodynamic gradient exists: $m_i > m_j + h$, Energy transfer occurs only in discrete quanta of Planck's constant h, $m_i \to m_i - h$; $m_j \to m_j + h$ (Planck's constant:h)

Numerical Example: System States and Entropy Evolution

Table 1. Simplified Entropy Increase Demonstration.

System State	SEQ Energy Distribution $E_{total}=\sum m_i=12$	Entropy $S=\prod m_i$	Remarks
Initial non-equilibrium state	[3, 1, 5, 3]	45	-
Intermediate state	PathA [3, 1, 4, 4]; PathB [3, 2, 4, 3]；	A48; B72；	-
Final state	PathA [3, 2, 3, 4]; PathB [3, 3, 3, 3]；	A72; B81；	Due to adjacent energy transfer with minimal quanta *h*, this system cannot reach maximum entropy in case A

Note:The above analysis demonstrates that different entropy-increasing pathways exhibit distinct sequences of entropy variation.

3.6. Logarithmic Relation:

After logarithmic transformation, lnS aligns with the conventional Boltzmann entropy form, while the multiplicative formulation naturally suits discrete systems.

3.7. Proof of Spontaneous Entropy Increase

Spontaneity Theorem of entropy increase (Second Law of Thermodynamics):

For every possible energy transfer process, the total entropy change satisfies $\Delta S \geq 0$.

Proof Outline: Let the pre-transfer states be $m_i=a$, $m_j=b$ ($a > b+h$);

The post-transfer entropy ratio is：

$S_{t1}/S_{t2}=(a-h)(b+h)/ab=1+h(a-b-h)/ab > 1$ (*Planck's constant h)*

This demonstrates how entropy increases along various paths, reflecting the irreversibility and path dependence of thermodynamic processes in real systems.

Moreover, each system state can be assigned a unique entropy value — an "entropy coordinate" , which provides a powerful tool for computer simulations at the quantum level.

3.8. Analysis of the Maximum Entropy Principle

In the SEQ model, the maximum entropy principle is manifested through the driving tendency of entropy increase: energy not only flows from a higher-energy SEQ to an adjacent lower-energy SEQ, but it also follows the path with the largest energy difference.

For example, consider an SEQ i, which is adjacent to two other SEQ j and k. The energies carried by nodes i, j, and k are A, B, and C respectively, where A > B > C. In this case, there are two possible energy transfer paths from SEQ i according to the principle of entropy increase: path i→j or path i→k. We will analyze the entropy change for each path separately.

Under the constraint of energy conservation (i.e., A + B + C = constant), the entropy of the local system composed of these three SEQ before energy transfer is:$S = A\times B\times C$

Path i→j: transferring energy to node j (which has relatively higher energy):

$S' = (A-1)(B+1)C = (A-1)(BC+C)$

Path i → k: transferring energy to node k (which has lower energy):

$S'' = (A-1)B(C+1) = (A-1)(BC+B)$

Here, "1" represents one unit of Planck constant h. Since $B > C \Rightarrow BC + B > BC + C \Rightarrow S'' > S'$, the entropy increases more along the i→k path—that is, the path with the larger energy difference leads to a greater increase in entropy.

This deduction can be easily generalized to cases where the number of adjacent nodes is greater than two, so a detailed proof is omitted here.

However, it should be emphasized that the path with the maximum energy difference is not necessarily unique. Therefore, for states that have not yet occurred, the future still retains sufficient degrees of freedom — the evolution is not entirely deterministic.

From the perspective of this model, the apparent randomness in the wave function described by the Schrödinger equation arises from the non-uniqueness of the maximum entropy path—multiple microscopic energy redistribution trajectories, giving rise to probabilistic outcomes without requiring fundamental indeterminism.

3.9. Dimensional Structure in the Model: Dimensionless Energy and Entropy

In this model, all physical quantities are reconstructed based on the quantization of Planck's constant h. Energy is defined as discrete units carried by Space Elementary Quanta (SEQs), with each SEQ carrying an energy value that is an integer multiple of h, i.e., $m_i = n_i h$, where $n_i \in \mathbb{N}$. Since h is normalized as a natural unit, energy becomes a dimensionless counting variable—representing the distribution of "energy quanta" across the system.

Accordingly, entropy is defined as the product of all SEQ energy values: $S = \prod m_i$. Since each m_i is already dimensionless (normalized relative to h), their cumulative product remains dimensionless. This definition frees entropy from reliance on statistical ensembles or probability weights, making it a directly computable, path-dependent dynamical variable capable of precisely characterizing evolution through every energy redistribution event.

Given the model's prior assumption that each Space Elementary Quantum (SEQ) possesses a ground-state energy quantized in integer multiples of h ,[3] the energy value of every m_i is at least 1; consequently, in this framework, the multiplicative entropy of any physical system is strictly greater than 1.

4. Novelty of the Concept

After extensive searches using general search engines and academic databases, I find that the analytical entropy expression based on energy products I propose here offers a fundamentally new alternative outside the traditional statistical framework. This approach transforms entropy from a statistical concept into an analytic one, and to my knowledge, no similar formulation has been previously proposed.

Based on this idea, I introduce the concept of Analytic Quantum Thermodynamics, which allows us to better understand the essence of thermodynamic processes — essentially, they are processes of energy homogenization. And to describe such processes, the multiplicative entropy based on multiplication is a new tool.

Distinction from Existing "Multiplicative Entropy" Concepts

Although the term "multiplicative entropy" or "multiplicative form of entropy" does appear in some literature, these are still variations of traditional statistical entropy, not departing from the following frameworks:

4.1. Statistical Approach Based on Number of States

- For example, Boltzmann entropy
- Or Shannon entropy

These so-called "multiplicative forms" often involve multiplying entropies of subsystems or using multiplicative techniques in mathematical derivations.

But fundamentally, they remain logarithmic functions over probability distributions.

4.2. No Microscopic Path Recording Function

- They describe only macroscopic final-state differences, without retaining intermediate processes.
- They do not capture how each individual energy transfer affects entropy.

4.3. *Unique Features of This Multiplicative Entropy Theory*

This model proposes a new way of constructing physical quantities, assigning them clear physical meaning and evolutionary rules:

- It does not start from probability theory, but from the dynamical process of energy redistribution.
- It naturally captures the trend toward homogenization through the product of energy values.
- It provides a computable entropy coordinate, ideal for simulation, prediction, and visualization of system evolution.
- It remains analytically valid at the Planck scale, suggesting natural extension into quantum thermodynamics.
- This multiplicative entropy framework is grounded in a 3+1-dimensional realist foundation—it operates entirely within the physical spacetime we observe, without introducing extra dimensions beyond the three spatial and one temporal dimensions, or invoking abstract mathematical constructs detached from empirical reality.

This study presents an updated extension of a portion of the author's prior work [3]. For more details on the model, please refer to the previous paper presenting the complete framework.

5. Summary

As early as 1901, Planck attempted to find an analytic representation for entropy [4]. Subsequently, scholars such as Evaldo M.F. Curado and Fernando D. Nobre [5,6]; and Tirnakli, U.[7] have made outstanding contributions in this direction. Building upon these pioneering works, this paper proposes a novel form of analytic entropy—multiplicative entropy—for readers' reference and further exploration.

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