Review

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A Mathematical View on the Coordinate Distortion Induced by Gravitational Waves

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A Mathematical View on the Coordinate Distortion Induced by Gravitational Waves

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Abstract: We develop a theoretical framework to analyze how coordinate systems deform under the influence of gravitational waves. This model treats coordinates as nodes in a discrete spatial graph, with inter-node distances perturbed by time-varying distortions due to passing waves. A direct proportionality is proposed between the distortion and the energy density of the wave, analogous to strain measured in interferometric detectors. We also relate this to the geodesic deviation equation from general relativity and propose directions for extending the model with continuous differential geometry and perturbation theory.

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1. Introduction

Gravitational waves (GWs), predicted by Einstein's general theory of relativity, are ripples in the curvature of space-time generated by accelerated mass distributions, such as binary black holes or neutron star mergers. As these waves propagate, they cause transient distortions in the geometry of space-time itself, which can manifest as changes in distances between test masses or coordinate markers. These perturbations form the basis of detection in gravitational wave observatories like LIGO and Virgo [1].

In this work, we present a discrete geometric model that captures the coordinate distortions caused by gravitational waves. Unlike the standard continuous tensorial approach, we represent coordinates as discrete points in a spatial graph and investigate how wave-induced metric changes affect the effective "lengths" between these nodes.

2. Discrete Coordinate Framework

Let the spatial domain be discretized as a set of points (nodes):

$$\Sigma = \{x^0, x^1, \dots, x_n\}$$

Define a length functional over this set:

$$\Gamma(t) = \Sigma \,||x_{i+1}(t) - x_i(t)||$$

This length $\Gamma(t)$ represents the total path length between boundary coordinates under the influence of a time-dependent gravitational wave field.

3. Metric Perturbations and Time Evolution

In linearized general relativity, gravitational waves are modeled as small perturbations h_μν)to the flat Minkowski metric η_μν:

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, |h_{\mu\nu}| \ll 1$$

For a transverse-traceless (TT) gauge, the only non-zero components are $h_+(t)$ and $h_\times(t)$, corresponding to the two GW polarizations.

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4. Energy Content of Gravitational Waves

The energy flux carried by gravitational waves per unit area is given by the Isaacson stress-energy tensor in the high-frequency limit:

$$E_{GW} = (c^3 / 32\pi G) \langle \dot{h}_+{}^2 + \dot{h}_\times^2 \rangle$$

5. Geodesic Deviation and Physical Interpretation

The distortion of spatial distances under gravitational wave influence can also be understood using the geodesic deviation equation:

$$D^2\xi^\mu / d\tau^2 = -R^\mu_{\nu\rho\sigma} u^\nu \xi^\rho u^\sigma$$

6. Example: Triplet Configuration

Consider a simple configuration of three spatial points (x_0, x_1, x_2). When a gravitational wave passes through, the central point x_1 experiences the maximal displacement at time t′, creating a symmetric distortion.

7. Graph-Based Representation

Let the total event duration be:

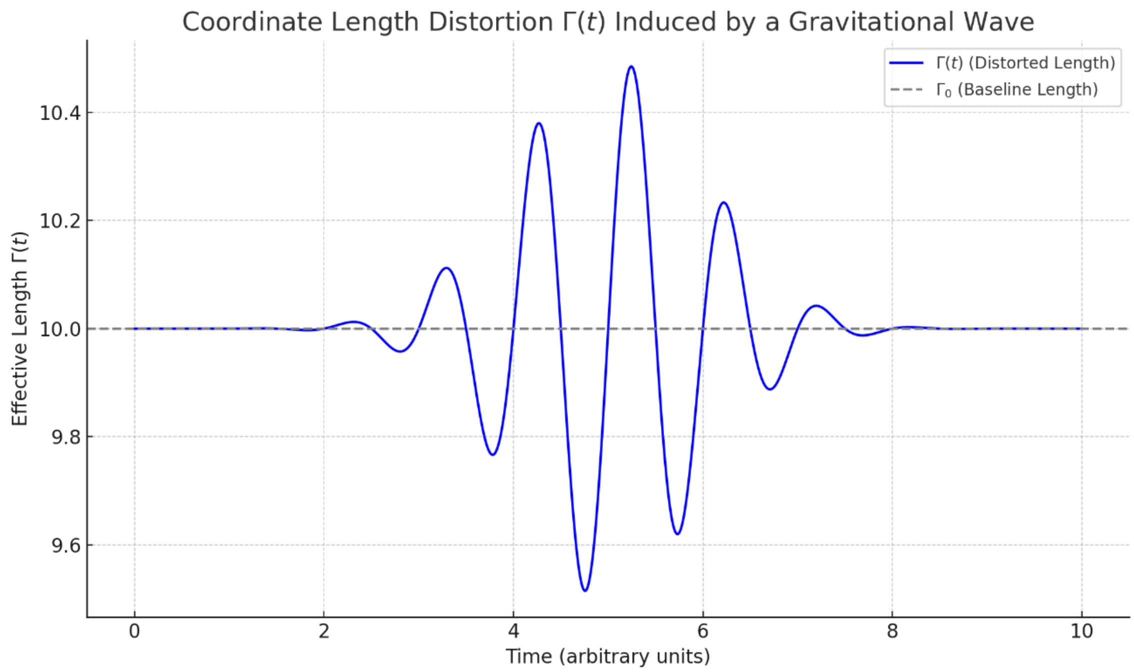
$$\tau = \{t_1, t_2, \dots, t_k\}$$

Then the integrated distortion is:

$$\partial\Gamma(\tau) = \Sigma\, \Gamma(t_i), \text{ and } \Delta\Gamma_{total} = \partial\Gamma(\tau) - k\Gamma^0$$

8. Conclusions and Outlook

We have introduced a graph-based, discrete geometric model for understanding coordinate deformation caused by gravitational waves. This model captures the essence of space-time perturbations in a simplified setting and relates naturally to known physical and mathematical frameworks.



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