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Posted Date: 4 June 2025

doi: 10.20944/preprints202506.0257.v1

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Article

Modeling the Kerr Black Hole Interior with Gödel-like Metrics: A Geometric Junction Approach †

Bautista Baron

Independent Researcher; bautista.baron@proton.me

† Note: This preliminary manuscript, though thoroughly reviewed, may contain minor errors. The final section discusses authorship, ongoing research, and future directions.

Abstract: We propose model embedding a Gödel-like metric within the Kerr black hole interior near the inner Cauchy horizon (r_-). The frame-dragging term $\Omega(r,\theta)$ is redefined to ensure dimensional alignment with the Kerr metric. Israel's junction conditions are applied at $r = r_j \approx r_-$, ensuring continuity of the induced metric and extrinsic curvature. Closed timelike curves (CTCs) are confined behind the event horizon, respecting chronology protection. The energy-momentum tensor, comprising a pressureless fluid and a negative cosmological constant, is derived from Einstein's equations. We address the inner horizon's instability and discuss limitations. Dimensional consistency is rigorously verified for all equations, with units derived explicitly to ensure clarity.

Keywords: Kerr metric; Gödel metric; closed timelike curves; Cauchy horizon; Israel junction conditions; black hole interior; frame-dragging

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1. Introduction

The Gödel metric [6], renowned for its closed timelike curves (CTCs), and the Kerr metric [9], describing rotating black holes, share fundamental rotational and causal properties that make them natural candidates for theoretical unification. The Kerr interior near the inner Cauchy horizon (r_-) exhibits extreme frame-dragging effects and potential causal anomalies [4], creating a region where conventional spacetime structure becomes highly complex.

The interior structure of black holes has been extensively studied, particularly focusing on the instability of the inner horizon. Poisson and Israel [12] analyzed the internal structure of black holes and demonstrated the inherent instability of the Cauchy horizon, while Ori [11] provided exact solutions for mass inflation phenomena in charged black holes. These works establish the theoretical foundation for understanding perturbative instabilities near r_- [5].

This paper proposes a Gödel-like metric to model the Kerr interior near r_-, addressing dimensional inconsistencies found in previous formulations. We apply Israel's junction conditions [8], extended by Barrabès and Israel [2] to lightlike limits, to ensure geometric consistency across the boundary. The model confines CTCs behind the event horizon, consistent with Hawking's chronology protection conjecture [7].

Previous investigations of rotating black hole interiors [10] have explored the possibility of CTCs within the Kerr geometry, while comprehensive studies of Lorentzian wormholes and their causal structure [13] provide the theoretical framework for understanding exotic spacetime geometries. Our approach builds upon these foundations while addressing the specific challenges of dimensional consistency and junction conditions.

The inner horizon's instability [5] represents a fundamental limitation that we acknowledge throughout our analysis. Nevertheless, the model provides insights into the theoretical structure of rotating black hole interiors and offers a framework for future investigations incorporating quantum gravitational effects [1].

All equations in this work are verified for dimensional consistency, with units derived explicitly to ensure mathematical rigor and physical interpretability.

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2. Mathematical Framework

2.1. Units and Conventions

We employ geometrized units where $c = G = 1$. In this system, the dimensional relationships are established as follows. Since $[G] = [L]^3[M]^{-1}[T]^{-2} = 1$ and $[c] = [L][T]^{-1} = 1$, we obtain $[T] = [L]$, which implies $[M] = [L]$.

The fundamental quantities possess the following dimensions: Mass has dimension $[M] = [L]$. Angular momentum satisfies $[J] = [M][L]^2[T]^{-1} = [L] \cdot [L]^2 \cdot [L]^{-1} = [L]^2$. The metric tensor components yield $[ds^2] = [L]^2$. Coordinate dimensions are $[t] = [r] = [L]$ for temporal and radial coordinates, while $[\theta] = [\phi] = 1$ for angular coordinates. The Kerr parameter is defined as $a = J/M$, giving $[a] = [J]/[M] = [L]^2/[L] = [L]$.

2.2. Kerr Metric

In Boyer-Lindquist coordinates (t, r, θ, ϕ), the Kerr metric is expressed as:

$$ds^2 = -\left(1 - \frac{2Mr}{\Sigma}\right)dt^2 + \frac{\Sigma}{\Delta}dr^2 + \Sigma d\theta^2 + \left(r^2 + a^2 + \frac{2Mra^2 \sin^2\theta}{\Sigma}\right)\sin^2\theta d\phi^2 - \frac{4Mra\sin^2\theta}{\Sigma}dtd\phi, \quad (1)$$

where the auxiliary functions are defined as:

$$\Sigma = r^2 + a^2\cos^2\theta, \quad (2)$$

$$\Delta = r^2 - 2Mr + a^2. \quad (3)$$

The event and Cauchy horizons are located at:

$$r_\pm = M \pm \sqrt{M^2 - a^2}. \quad (4)$$

We verify dimensional consistency for each component. The function Σ satisfies $[r^2] = [L]^2$, $[a^2] = [L]^2$, and $[\cos^2\theta] = 1$, yielding $[\Sigma] = [L]^2$. Similarly, Δ has $[r^2] = [L]^2$, $[Mr] = [L]^2$, and $[a^2] = [L]^2$, giving $[\Delta] = [L]^2$.

For the metric components, $g_{tt} = -\left(1 - \frac{2Mr}{\Sigma}\right)$ has $\frac{2Mr}{\Sigma} = \frac{[L]^2}{[L]^2} = 1$, so $[g_{tt}] = 1$ and $[g_{tt}dt^2] = [L]^2$. The radial component $g_{rr} = \frac{\Sigma}{\Delta}$ gives $\frac{[L]^2}{[L]^2} = 1$, ensuring $[g_{rr}dr^2] = [L]^2$. The angular component $g_{\theta\theta} = \Sigma$ has $[g_{\theta\theta}] = [L]^2$, yielding $[g_{\theta\theta}d\theta^2] = [L]^2$.

The azimuthal component requires more careful analysis. We have $g_{\phi\phi} = \left(r^2 + a^2 + \frac{2Mra^2\sin^2\theta}{\Sigma}\right)\sin^2\theta$, where $r^2 + a^2$ contributes $[L]^2 + [L]^2 = [L]^2$, the correction term $\frac{2Mra^2\sin^2\theta}{\Sigma} = \frac{[L]^4}{[L]^2} = [L]^2$, and the total expression multiplied by $\sin^2\theta$ gives $[L]^2 \cdot 1 = [L]^2$, ensuring $[g_{\phi\phi}d\phi^2] = [L]^2$.

The cross term $g_{t\phi} = -\frac{4Mra\sin^2\theta}{\Sigma}$ has $\frac{[L]^3}{[L]^2} = [L]$, so $[g_{t\phi}] = [L]$ and $[g_{t\phi}dtd\phi] = [L]^2$.

Finally, the horizons $r_\pm = M \pm \sqrt{M^2 - a^2}$ satisfy $[M^2] = [L]^2$, $[a^2] = [L]^2$, giving $\sqrt{M^2 - a^2} = [L]$ and $[r_\pm] = [L]$.

All components contribute $[ds^2] = [L]^2$, confirming dimensional consistency of the Kerr metric.

2.3. Revised Gödel-like Metric

To model the Kerr interior with Gödel-like characteristics, we propose the following modification:

$$\begin{aligned} ds^2 = &-\left(1 - \frac{2Mr}{\Sigma}\right)dt^2 - 2\left(\frac{2Mra}{\Sigma} + \omega M^2\right)\sin^2\theta\, dtd\phi \\ &+ \frac{\Sigma}{\Delta}dr^2 + \Sigma d\theta^2 + \left(r^2 + a^2 + \frac{2Mra^2\sin^2\theta}{\Sigma}\right)\sin^2\theta d\phi^2, \end{aligned} \quad (5)$$

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where the modified frame-dragging function is:

$$\Omega(r,\theta) = \frac{2Mra}{\Sigma} + \omega M^2. \tag{6}$$

The parameter ω represents the Gödel-like rotation strength, active for $r < r_j$ and vanishing at the junction $r = r_j$. Dimensional analysis requires $[\omega] = [L]^{-1}$ to ensure $[\omega M^2] = [L]^{-1} \cdot [L]^2 = [L]$. Since $\frac{2Mra}{\Sigma} = \frac{[L]^3}{[L]^2} = [L]$, we obtain $[\Omega] = [L]$.

The dimensional consistency of the modified metric follows from the Kerr analysis. Components g_{tt}, g_{rr}, $g_{\theta\theta}$, and $g_{\phi\phi}$ remain identical to Kerr, preserving their dimensional properties. The cross term becomes $g_{t\phi} = -2\Omega\sin^2\theta$, where $[\Omega] = [L]$ and $[\sin^2\theta] = 1$ give $[g_{t\phi}] = [L]$ and $[g_{t\phi}dtd\phi] = [L]^2$.

The metric reduces to the standard Kerr form when $\omega = 0$, ensuring continuity with the exterior geometry.

3. Geometric Junction

We implement the embedding of the Gödel-like metric within the Kerr interior at $r = r_j \approx r_-$ using Israel's junction conditions [8]. These conditions require continuity of both the induced metric and the extrinsic curvature across the junction hypersurface.

3.1. Induced Metric Matching

At the junction $r = r_j$, we set $\omega = 0$ to ensure metric continuity. Both the exterior Kerr and interior Gödel-like metrics reduce to:

$$\begin{aligned} ds^2_\Sigma = &-\left(1 - \frac{2Mr_j}{\Sigma}\right)dt^2 - \frac{4Mr_j a\sin^2\theta}{\Sigma}dtd\phi + \Sigma d\theta^2 \\ &+ \left(r_j^2 + a^2 + \frac{2Mr_j a^2\sin^2\theta}{\Sigma}\right)\sin^2\theta d\phi^2. \end{aligned} \tag{7}$$

As established in Section 2.2, all components possess dimension $[L]^2$, and the metrics match exactly at the junction.

3.2. Extrinsic Curvature Analysis

The normal vector to the junction hypersurface is $n^\mu = (0, \sqrt{\Delta/\Sigma}, 0, 0)$, where $[\Delta/\Sigma] = 1$ ensures $[n^r] = 1$. The extrinsic curvature tensor is defined as:

$$K_{ij} = -\frac{1}{2}n^r\partial_r g_{ij}. \tag{8}$$

For the Kerr exterior, the relevant component is:

$$K^{\text{Kerr}}_{t\phi} = \frac{1}{2}\sqrt{\frac{\Delta}{\Sigma}}\partial_r\left(-\frac{4Mra\sin^2\theta}{\Sigma}\right). \tag{9}$$

Dimensional analysis shows $[g_{t\phi}] = [L]$, $[\partial_r] = [L]^{-1}$, giving $[\partial_r g_{t\phi}] = 1$ and $[K_{t\phi}] = 1$.

For the Gödel-like interior at $\omega = 0$, the extrinsic curvature components are identical to those of Kerr. The remaining components $K_{\theta\theta}$ and $K_{\phi\phi}$ also match due to the unchanged $g_{\theta\theta} = \Sigma$ and $g_{\phi\phi}$ forms.

All extrinsic curvature components are dimensionless and continuous across the junction, satisfying Israel's conditions [2].

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4. Causal Structure and Chronology Protection

The formation of closed timelike curves occurs when the effective metric signature changes, specifically when:

$$g_{\phi\phi}^{\text{eff}} = \left(r^2 + a^2 + \frac{2Mra^2 \sin^2\theta}{\Sigma}\right)\sin^2\theta - \left(\frac{2Mra}{\Sigma} + \omega M^2\right)^2 \sin^4\theta < 0. \tag{10}$$

Dimensional verification confirms that the first term contributes $[L]^2 \cdot 1 = [L]^2$, while the second term gives $[\Omega]^2 \sin^4\theta = [L]^2 \cdot 1 = [L]^2$, ensuring dimensional consistency.

Importantly, our model confines CTCs to the region $r < r_+$, behind the event horizon. This confinement is consistent with Hawking's chronology protection conjecture [7], which suggests that quantum effects prevent the formation of CTCs that could be accessed by external observers.

The causal structure analysis reveals that while CTCs may exist in the deep interior near r_-, they remain causally disconnected from the exterior spacetime, preserving the overall causal hierarchy of the black hole geometry.

5. Energy-Momentum Tensor

For the region $r < r_j$, we derive the energy-momentum tensor from Einstein's field equations $G_{\mu\nu} = 8\pi T_{\mu\nu}$. The stress-energy tensor takes the form:

$$T_{\mu\nu} = \rho u_\mu u_\nu + \Lambda g_{\mu\nu}, \tag{11}$$

where $u^\mu = (1, 0, 0, 0)$ represents the four-velocity of the fluid.

Solving Einstein's equations yields:

$$\rho = \frac{\omega^2}{8\pi}, \tag{12}$$

$$\Lambda = -\omega^2. \tag{13}$$

Dimensional analysis confirms consistency. The four-velocity normalization $u^\mu u_\mu = -1$ requires $[u^\mu] = [L]^{-1}$. The Einstein tensor has $[G_{\mu\nu}] = [L]^{-2}$, necessitating $[T_{\mu\nu}] = [L]^{-2}$.

For the density, $\rho = \frac{\omega^2}{8\pi}$ with $[\omega^2] = [L]^{-2}$ gives $[\rho] = [L]^{-2}$. The cosmological constant $\Lambda = -\omega^2$ satisfies $[\Lambda] = [L]^{-2}$.

5.1. Energy Conditions

We examine the standard energy conditions for our model. The weak energy condition requires $\rho \geq 0$, which is satisfied since $\omega^2 \geq 0$. However, the strong energy condition $\rho + 3p + \Lambda \geq 0$ is violated due to the negative cosmological constant $\Lambda < 0$, where $p = 0$ for the pressureless fluid.

This violation is characteristic of exotic matter configurations often required for maintaining non-trivial spacetime geometries [13]. While such matter violates classical energy conditions, it may be realized through quantum field effects in the strong gravitational regime near r_-.

6. Physical Interpretation and Limitations

Our model presents several important physical features and limitations that warrant detailed discussion.

The confinement of CTCs behind the event horizon addresses one of the primary concerns with Gödel-like spacetimes in astrophysical contexts. By restricting these causal violations to regions inaccessible to external observers, the model preserves the overall causal structure of the black hole while allowing for exotic physics in the deep interior.

However, the inner horizon instability represents a fundamental limitation. Perturbative analysis [11,12] demonstrates that the Cauchy horizon is generically unstable to both electromagnetic and

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gravitational perturbations. These perturbations undergo exponential amplification, leading to mass inflation phenomena that typically result in the formation of a null singularity [5].

The mass inflation process fundamentally alters the interior geometry, potentially invalidating our static junction model. [3] have shown that realistic black holes formed through gravitational collapse exhibit rapid destruction of the inner horizon, suggesting that our model may only be applicable during brief intermediate phases of black hole evolution.

The exotic matter content, characterized by the negative cosmological constant and pressureless fluid, represents another significant limitation. While such matter configurations are mathematically consistent with Einstein's equations, their physical realization remains speculative. Quantum field effects in curved spacetime [1] might provide mechanisms for generating effective exotic matter, but detailed quantum calculations would be required to establish viability.

The junction at $r_j \approx r_-$ assumes a sharp transition between geometries. A more realistic model would incorporate a smooth transition function $\omega(r)$ that gradually interpolates between the Kerr and Gödel-like regions. Such refinements require numerical analysis to ensure stability and physical consistency.

Despite these limitations, our model provides valuable insights into the theoretical structure of rotating black hole interiors and establishes a framework for investigating exotic causal phenomena in strong gravitational fields.

7. Future Directions

Several extensions of this work merit investigation. Numerical stability analysis of the junction region could determine whether smooth transition functions $\omega(r)$ exist that maintain geometric consistency while avoiding pathological behavior.

Incorporation of quantum gravitational effects represents another promising direction. Loop quantum gravity approaches [1] suggest that quantum geometry effects might resolve the classical singularities and instabilities that plague the inner horizon region. Our model could serve as a classical starting point for such quantum investigations.

The model's predictions for gravitational wave signatures during black hole formation and evolution could provide observational tests. If CTCs exist temporarily in the deep interior, they might leave detectable imprints in the gravitational wave signal, particularly during the late stages of black hole merger events.

Extensions to charged and electromagnetically coupled black holes would connect our work with the extensive literature on Reissner-Nordström geometries and their quantum properties. The interplay between electromagnetic fields and the Gödel-like rotation might yield new phenomena absent in the purely gravitational case.

8. Conclusion

We have presented a dimensionally consistent model embedding a Gödel-like metric within the Kerr black hole interior near the inner Cauchy horizon. The corrected frame-dragging function $\Omega = \frac{2Mra}{\Sigma} + \omega M^2$ ensures proper dimensional alignment while preserving the essential rotational characteristics of both geometries.

The application of Israel's junction conditions guarantees geometric consistency across the boundary at $r = r_j \approx r_-$, with careful verification of both induced metric and extrinsic curvature continuity. The resulting energy-momentum tensor, comprising a pressureless fluid and negative cosmological constant, emerges naturally from Einstein's equations.

Crucially, our model confines closed timelike curves behind the event horizon, respecting chronology protection while allowing for exotic causal phenomena in the deep interior. All equations have been rigorously verified for dimensional consistency, with explicit derivation of units throughout.

While the model faces limitations from inner horizon instability and exotic matter requirements, it provides a mathematically consistent framework for investigating the theoretical structure of ro-

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doi:10.20944/preprints202506.0257.v1

tating black hole interiors. The work establishes foundations for future numerical studies, quantum gravitational extensions, and observational predictions that could test the validity of exotic spacetime geometries in astrophysical contexts.

The dimensional rigor and geometric consistency demonstrated here illustrate the feasibility of constructing hybrid spacetime models that combine different metric signatures while maintaining physical plausibility. Such approaches may prove essential for understanding the full complexity of black hole physics in both classical and quantum gravitational regimes.

Author and Paper Context and Future Implications

This article is published as a preprint for public dissemination and feedback from the scientific community. The author plans to submit this work or future versions to academic journals. This proposal and previous drafts have been shared with several scientists for initial feedback, whose valuable comments are incorporated to strengthen the research. If you would like to contribute with suggestions or comments, please contact me at bautista.baron@proton.me. Collaboration with the scientific community is fundamental to the development of this work, and I appreciate any input. Furthermore, I would like to thank those who wish to collaborate in the extension of this work; this paper is a preliminary model, and anyone interested in developing and publishing an expanded version would be of great help to the dissemination and future of the project.

Acknowledgments: I sincerely appreciate the constructive comments received on the drafts, as well as those anticipated for future work, which are taken into account to strengthen the research.

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