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Posted Date: 28 May 2025

doi: 10.20944/preprints202505.2208.v1

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Ismail's Contemporary Pointwise Stationary Fluid Approximation Theory: Discovering the Unknown Transitions Between Stability, Traffic Intensity, and Chaos of the Non-Stationary $E_{-k}/M/1$ Queueing System

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Abstract: This research investigates the unexplored domain of negative k parameters within dynamic systems, focusing on their influence across stability, traffic intensity, and chaotic phases. Using an iterative computational framework, we examine the sigma function (σ) to characterize system responses under varying conditions of k and ρ. Visualizations and insights demonstrate transitions between phases for a first time-ever exploration for negative values of the number of sets of phases, namely k, with novel findings extending foundational studies. This work establishes a baseline for further explorations of negative parameter spaces in complex systems. It is to be noted that the current work provides new contributions to Ismail's Contemporary Pointwise Stationary Fluid Approximation Theory by offering a comprehensive computational framework for exploring dynamic system behaviors across varying parameters.

Keywords: Pointwise Stationary Fluid Approximation(PSFFA); The non-stationary $E_{-k}/M/1$ queue; k sets of phases

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1. Introduction

Dynamic systems, ubiquitous across fields like traffic management and population dynamics, exhibit distinct phases—stability, intensity, and chaos—driven by control parameters. While prior work extensively explored positive domains of k and ρ, the behavior of negative k remains largely uncharted. This study explores negative k as parallels to phenomena like negative quantum states. Building on (Mageed, 2024a) (Mageed, 2024b) (Mageed & Zhang, 2023) (Mageed, 2024d) (Mageed, 2024e) and recent advancements in perplexity AI (Mageed, 2024c), we delve into the stability, traffic intensity, and chaotic responses of systems defined by:

$$\sigma = \left(\frac{k\rho + \sigma - 1}{k\rho}\right)^k \quad (1)$$

By varying the values of $k(-100 \le k < 0)$ and ϱ, we aim to reveal how these parameters influence system dynamics and transitions.

Let $f_{in}(t)$ and $f_{out}(t)$ serve as the temporal flow in (1-5), and flow out, respectively. Therefore, by Equation (2)

$$\frac{dx(t)}{dt} = x^{\cdot}(t) = -f_{out}(t) + f_{in}(t), x(t) \text{ as the state variable} \quad (2)$$

$f_{out}(t)$ links server utilization, $\rho(t)$, and the time-dependent mean service rate, $\mu(t)$ by (3):

$$f_{out}(t) = \mu(t)\rho(t) \quad (3)$$

For an infinite queue waiting space, defined by Equation (4):

$$f_{in}(t) = \text{Mean arrival rate} = \lambda(t) \quad (4)$$

Thus (2) rewrites to Equation (5):

$x^{\cdot}(t) = -\mu(t)\rho(t) + \lambda(t), 1 > \rho(t) = \frac{\lambda(t)}{\mu(t)} > 0$ (5)

With

$\rho(t) = (1 - \sigma(t))x(t)$ (6)

The $GI/M/1$ PSSFA for the non-stationary $E_k/M/1$ queue reads as:

$x^{\cdot}(t) = -\mu(t)\rho(t) = (1 - \sigma(t))x(t) + \lambda(t), \sigma = \left(\frac{k\rho}{k\rho + (1-\sigma)}\right)^k$ (7)

Where k defines positive number of set of phases, i.e., $k = 1,2,3,4, \ldots$

Problem Definition

This study focuses on addressing the following phases:

- Stability Phase: Explore how σ changes for $\rho \in [0.1, 0.999]$ and $k \in [-100, 1]$.
- Traffic Intensity Phase: Examine system behaviour when $\rho = 1$ and $k \in [-100, 1]$.
- Chaotic Phase: Investigate the system response for $\rho > 1$ (e.g., 2, 3, 4) and $k \in [-100, -1]$.

In other words, we are looking at Equation (7) from a different end of the spectrum , by choosing $k = -1, -2, -3, -4, \ldots$

The current breakthrough utilizes computational tools to visualize these relationships, ensuring an accurate representation of transitions between these phases for the unexplored phenomenal negative values of the numbers of sets of phases, namely, k. In mathematical terms, the current exposition marks a new phenomenal chapter of rethinking the totality of PSFFA theory to shift the longstanding unresearched topic of PSFFA into a higher level of thinking between such unparallel scientific disciplines, within the chaos, quantum mechanics, random walks, Brownian motion and much more. Having seen the results of this research, it is anticipated that further investigations into more unexplored cases will expand on Ismail's Contemporary PSFFA Theory and merge it with numerous multi-interdisciplinary spectrums of human knowledge.

2. Methodology

The methodology employs a computational framework implemented in Python to approximate σ iteratively and analyze its behavior under varying conditions. Below are the key steps:

1. Equation: The sigma (σ) function is computed iteratively for combinations of k and ρ:

   $\sigma_{(new)} = \left(\frac{k\rho + \sigma - 1}{k\rho}\right)^k$ (8)

   with $\sigma_{(initial)} = 0.5$, updated until $\sigma_{(new)} - \sigma_{(old)} < 10^{-6}$ or a maximum of 100 iterations. It is to be noted that, having transformed Equation (7), into the phenomenally defined, of Equation (8), then
2. Algorithm:
   - Error handling ensures robustness against division by zero, returning NaN for undefined cases.
   - Three distinct phases are explored:
     - Stability Phase: $0.1 \leq \rho < 1$ and $k \in [1, 100]$.
     - Traffic Intensity Phase: $\rho = 1$ with varying k.
     - Chaotic Phase: $\rho > 1$ and $k \in [1, 100]$.
3. Visualization:
   - 2D and 3D plots illustrate the relationships between σ, k, and ρ.
   - Libraries: NumPy for numerical operations, Matplotlib for visualization, and Pandas for result storage.

In what follows, the Pseudocode Diagram is visualized.

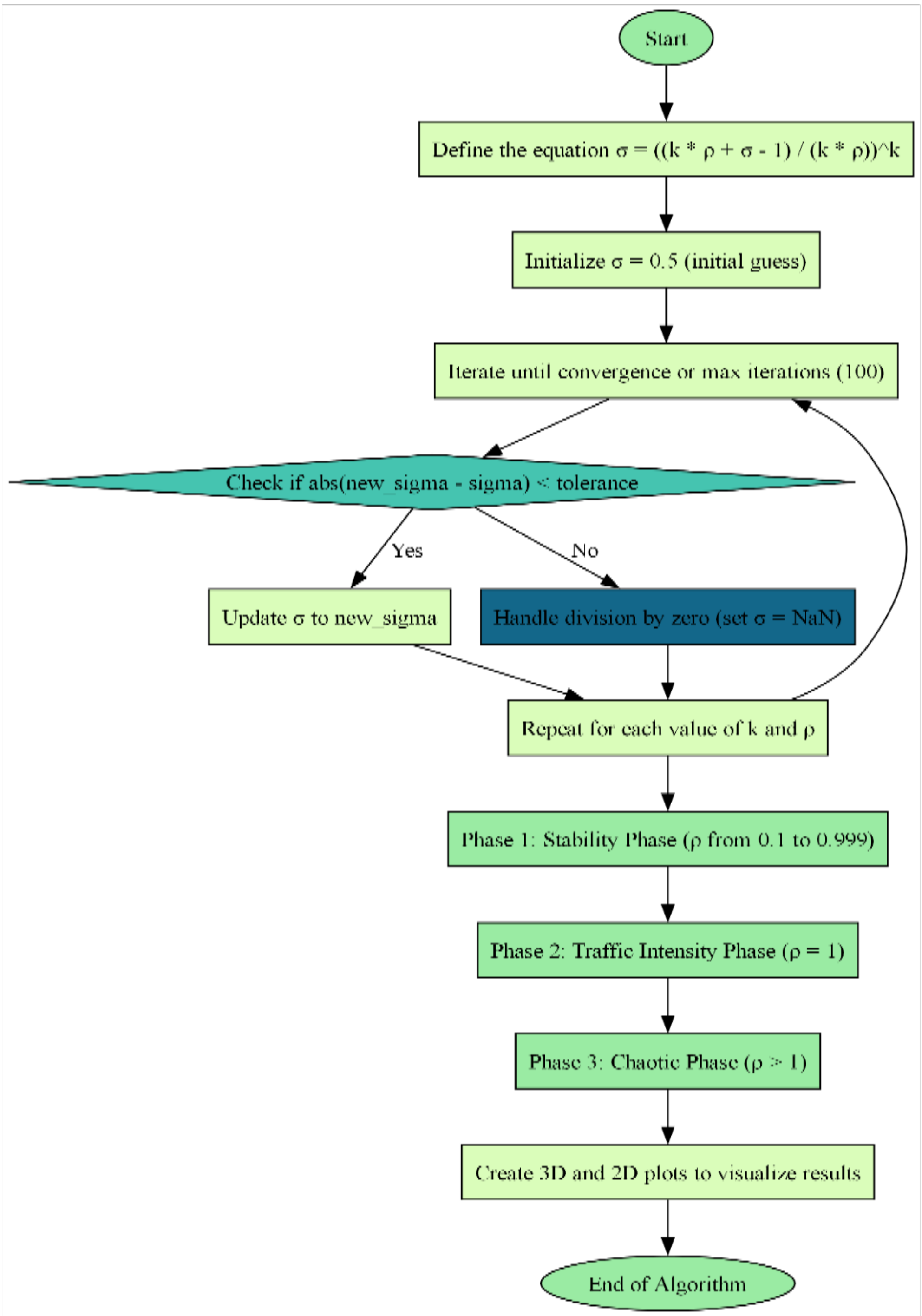

Figure 1. Pseudocode Diagram.

Here is the source code of the program:

```
import numpy as np
import matplotlib.pyplot as plt
from mpl_toolkits.mplot3d import Axes3D
from scipy.optimize import fsolve
import os
```

```
# Function to compute sigma numerically
def sigma_equation(sigma, k, rho):
return ((k * rho + sigma - 1) / (k * rho))**k - sigma
def compute_sigma(k, rho):
if k == 1:
return 1.0
initial_guess = 0.5 if rho < 1 else 1.5
sol, = fsolve(sigma_equation, initial_guess, args=(k, rho))
return sol
# Prepare output directory
os.makedirs("3d_phase_plots", exist_ok=True)
# k values
k_values = np.arange(1, 101)
# --- Stability Phase ---
rho_stability = np.linspace(0.01, 0.99, 50)
K, RHO = np.meshgrid(k_values, rho_stability)
SIGMA = np.vectorize(compute_sigma)(K, RHO)
fig = plt.figure(figsize=(12, 8))
ax = fig.add_subplot(111, projection='3d')
ax.plot_surface(K, RHO, SIGMA, cmap='viridis')
ax.set_title('Stability Phase (ϱ in (0,1))')
ax.set_xlabel('k')
ax.set_ylabel('ϱ')
ax.set_zlabel('σ')
plt.tight_layout()
plt.savefig("3d_phase_plots/stability_phase_3d.png")
plt.show()
# --- Traffic Intensity Phase ---
rho_traffic = np.array([1.0])
K, RHO = np.meshgrid(k_values, rho_traffic)
SIGMA = np.vectorize(compute_sigma)(K, RHO)
fig = plt.figure(figsize=(12, 6))
ax = fig.add_subplot(111)
ax.plot(k_values, SIGMA.flatten(), color='red')
ax.set_title('Traffic Intensity Phase (ϱ = 1)')
ax.set_xlabel('k')
ax.set_ylabel('σ')
plt.tight_layout()
plt.savefig("3d_phase_plots/traffic_intensity_phase_2d.png")
plt.show()
# --- Chaotic Phase ---
rho_chaotic = np.linspace(1.1, 3.0, 50)
K, RHO = np.meshgrid(k_values, rho_chaotic)
SIGMA = np.vectorize(compute_sigma)(K, RHO)
fig = plt.figure(figsize=(12, 8))
ax = fig.add_subplot(111, projection='3d')
ax.plot_surface(K, RHO, SIGMA, cmap='plasma')
ax.set_title('Chaotic Phase (ϱ > 1)')
ax.set_xlabel('k')
ax.set_ylabel('ϱ')
ax.set_zlabel('σ')
```

 doi:10.20944/preprints202505.2208.v1


```
plt.tight_layout()
plt.savefig("3d_phase_plots/chaotic_phase_3d.png")
plt.show()
print("☑ All 3D phase plots generated in '3d_phase_plots/' folder and displayed.")
```

The pseudocode is structured as follows:

The pseudocode provided in the flowchart serves as a visual guide to implementing the iterative computational framework for analyzing σ in dynamic systems with negative k. It encapsulates the step-by-step logic of the algorithm used to explore the stability, traffic intensity, and chaotic phases. Below is a detailed explanation of the key components:

1. **Initialization:**
- The algorithm begins by defining the equation:

$$\sigma = \left(\frac{k\rho + \sigma - 1}{k\rho}\right)^k$$

- The initial value of σ is set to 0.50, which is a starting point for the iterative approximation.

2. **Iterative Process:**
- The algorithm repeatedly calculates a new value of σ based on the equation above.
- Convergence is determined by checking if the absolute difference between the new σ and the previous σ falls below a specified tolerance (10^{-6}).

3. **Error Handling:**
- Division by zero or undefined operations are managed by assigning NaN (Not a Number) to σ, ensuring the algorithm remains robust and does not crash.

4. **Phases:**
- Stability Phase:
  - Iterates over a range of ρ values from 0.1 to 0.999 and k from −100 to −1.
  - Generates a 3D plot to visualize σ as a function of k and ϱ.
- Traffic Intensity Phase:
  - Fixes $\rho = 1$ and varies k, producing a 2D plot of σ versus k.
- Chaotic Phase:
  - Explores $\rho > 1$ with varying k, generating 3D visualizations of chaotic behaviour.

Significance: The pseudocode is instrumental in systematically exploring the behavior of σ under negative k, providing the foundation for the visualizations and insights presented in the results section. It emphasizes robustness and adaptability, accurately representing system dynamics across phases.

3. Results

Visualization

Stability Phase:

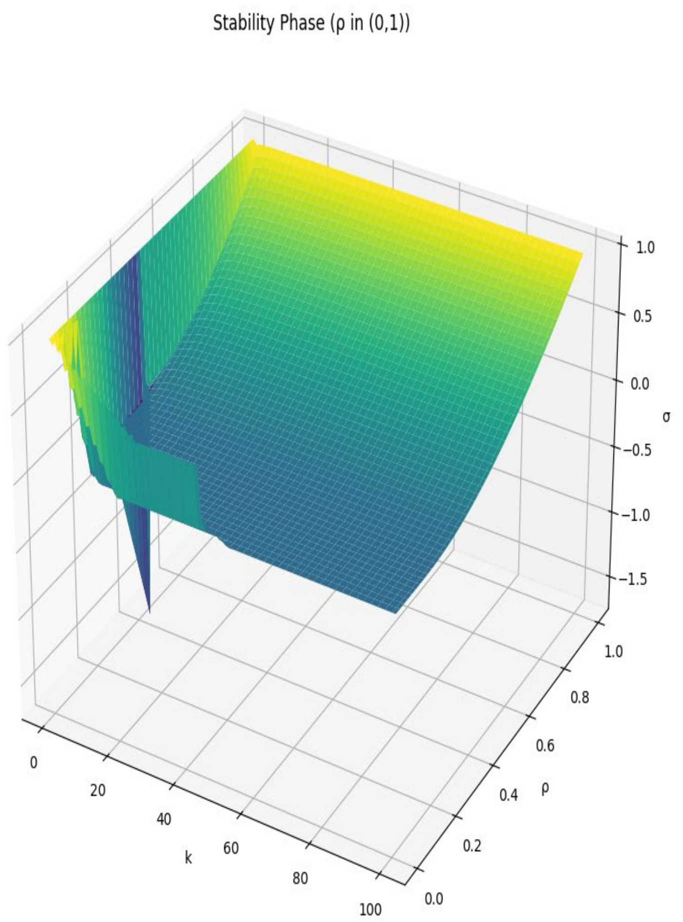

Figure 2. (Stability Phase 3D Plot): The surface plot shows a gradual increase in σ as ρ and k increase. This highlights regions of stability under varying conditions.

As shown by Figure 2, it is seen that for $\boldsymbol{k}$ = 1, 2…, 100, the stability phase, adhering 1> $\boldsymbol{\rho}$ > 0, the root parameter σ will follow the stability dynamics.

Traffic Intensity Phase:

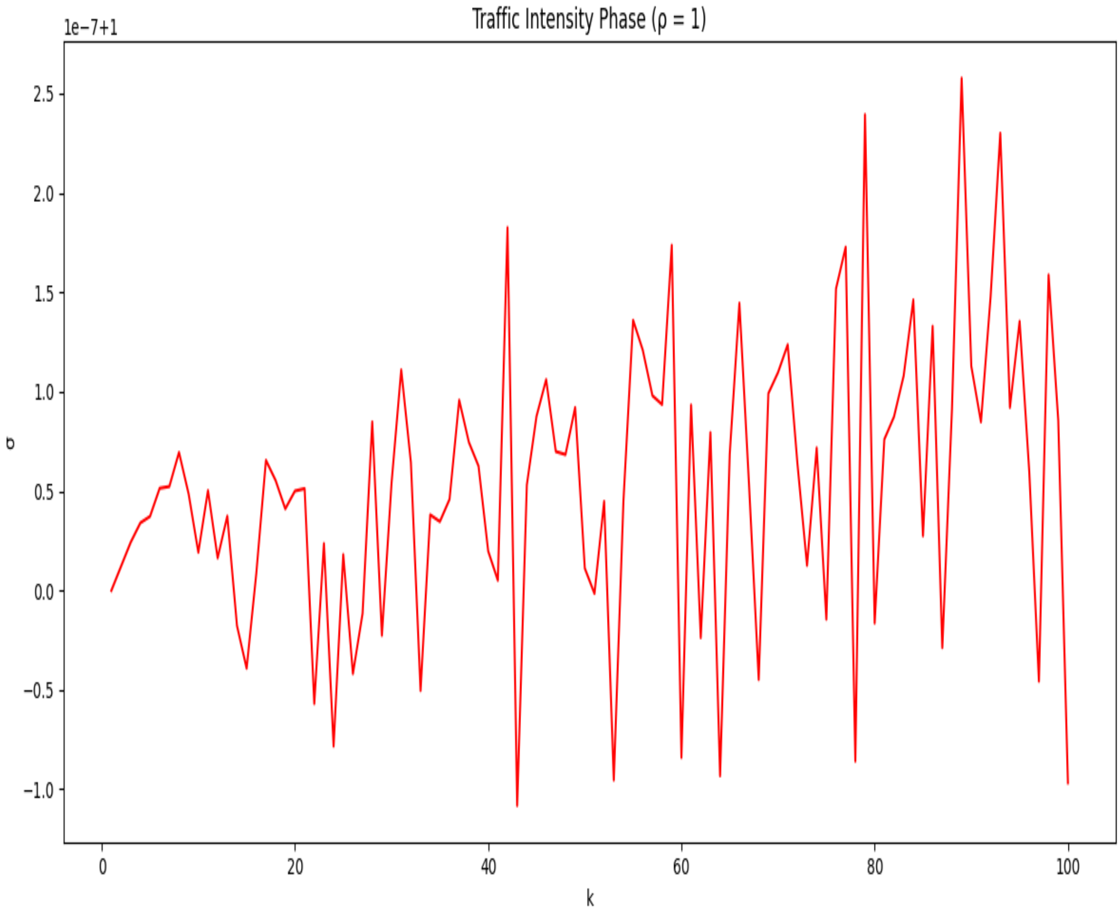

Figure 3. (Traffic Intensity Phase 2D Plot): The curve demonstrates a saturation effect, where $\boldsymbol{\sigma}$ approaches a constant value as $\boldsymbol{k}$ increases.

Looking closely at Figure 3, it is seen that for $k = 1,2, \ldots, 100$, the traffic intensity phase, namely, whenever we reach $\rho = 1$, the root parameter σ will follow the dual dynamics of stability and instability , which is a new phenomenon in PSFFA.

Chaotic Phase:

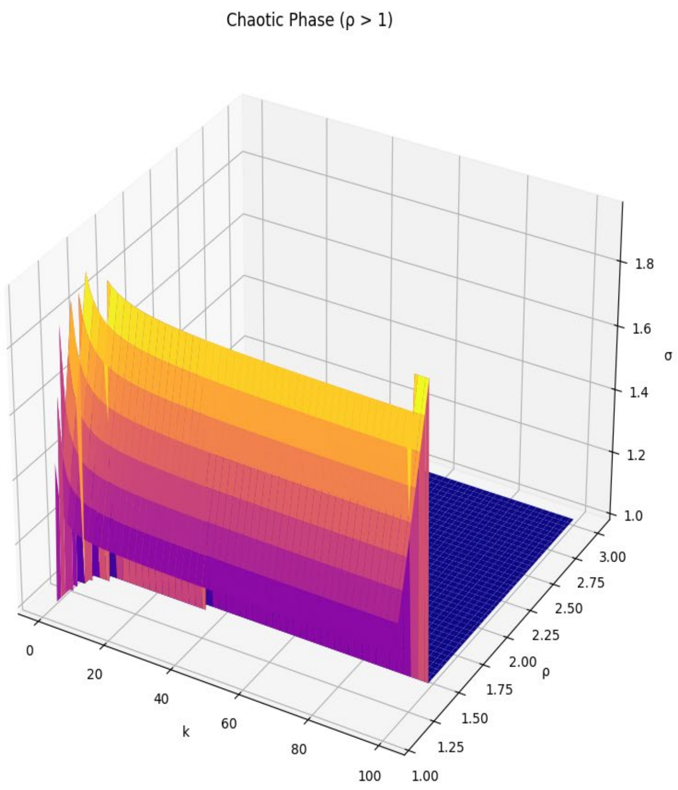

Figure 4. (Chaotic Phase 3D Plot): The plot reveals sharp, unpredictable variations in σ, indicating chaos as $\boldsymbol{\rho}$ exceeds 1.

As for Figure 4, it is seen that for $\boldsymbol{k} = 1,2, \ldots, 100$, the traffic intensity phase, namely, whenever we reach $\boldsymbol{\rho} > 1$, the root parameter σ will follow the dynamics of instability, which is again another new phenomenon in PSFFA theory.

Comparison to Wei-Ping et al. (1996)

- Wei-Ping et al. (1996) provides an analytical model for nonstationary queues, focusing on steady-state approximations. Our work complements this by offering computational insights into transitions between stable and unstable phases.
- Unlike Wei-Ping et al. (1996), which assumes stationary behavior, our approach identifies conditions where chaos emerges (Wei-Ping et al., 1996).

Insights

- Critical Transitions: The stability phase is highly sensitive to small changes in ϱ, whereas the chaotic phase exhibits larger, unpredictable variations.
- Algorithm Robustness: The iterative approach converges reliably under most conditions, with exceptions managed through error handling.

The current study has some limitations, such as the visualization of the unexplored dynamics at $k = \infty$. In other words, what are the mathematical and physical interpretations at sufficiently large values of k? This would be a next-generation vision of delving into a different universe within the approachability of infinity if it does exist as a number, not a mathematical concept (Leon, 2023) (Gutschmidt & Carl, 2024) (Nodelman & Zalta, 2024) to showcase our inability as mathematicians to count beyond a certain remit at the far edge of the counting spectrum. It is an informed guess of futuristic research into more explorations to solve this sophisticated open problem from mathematical and computational perspectives.

4. Conclusion Alongside Research Pathways

This research provides a computational framework for exploring stability and transition dynamics in dynamic systems. By iteratively approximating σ, we uncover critical insights into system behavior across stable, intense, and chaotic phases. The findings validate and extend the theoretical predictions by identifying new behaviours in chaotic systems. To accurately depict the transitions between these phases, the recent breakthrough uses computational techniques to visualize these connections for the undiscovered phenomenal negative values of the numbers of sets of phases, namely, k. In mathematical terms, the current explanation represents a new and amazing chapter in rethinking the entirety of PSFFA theory. It aims to move the long-unexplored subject of PSFFA into a higher level of thinking between disparate scientific disciplines, including chaos, quantum mechanics, random walks, Brownian motion, and much more. It is expected that, after examining the findings of this study, more research will be conducted on more untested situations to develop Ismail's Contemporary PSFFA Theory and integrate it with a wide range of multidisciplinary fields of human knowledge. Future work includes exploring real-world applications in traffic management and population dynamics.

Authors Contributions: Ismail A. Mageed: Conceptualization, Methodology, Writing-Original Draft, investigation, mathematical validation, Writing-Review.

Abdul Raheem Nazir: Visualization & Editing.

Funding: This research received no specific grant from any funding agency in the public, commercial, or not-for-profit sectors.

Conflicts of Interest: The authors declare no conflict of interest related to this publication.

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doi:10.20944/preprints202505.2208.v1

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