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A Quantitative Revelation in Equity Valuation: The P/E Ratio is a Degenerate Case of the Potential Payback Period (PPP): Why Settle for the Limited when a Comprehensive Model Exists?

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A Quantitative Revelation in Equity Valuation: The P/E Ratio is a Degenerate Case of the Potential Payback Period (PPP) Why settle for the limited when a comprehensive model exists?

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Abstract: This article introduces the Potential Payback Period (PPP) as a generalized, risk- and growth-adjusted metric for equity valuation. In contrast to the traditional Price-to-Earnings (P/E) ratio, which assumes static earnings and neglects the time value of money, the PPP accounts for expected earnings growth and discount rates, thus providing a more dynamic and realistic measure of a stock's investment appeal. By applying L'Hôpital's Rule to the PPP formula, we demonstrate that the P/E ratio emerges as a limiting or degenerate case of PPP in two scenarios: (1) when the earnings growth rate converges to the discount rate, and (2) when both growth and discount rates are zero—an idealized static world. This theoretical result recontextualizes the P/E ratio within a broader, mathematically grounded framework and offers significant implications for valuation theory and portfolio management. PPP serves not merely as a substitute for P/E, but as its logical and quantitative extension.

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1. Introduction

The Price-to-Earnings (P/E) ratio has long served as a cornerstone metric in equity valuation. Its intuitive simplicity and ease of calculation have made it a staple for both professional analysts and retail investors. Yet despite its ubiquity, the P/E ratio offers only a narrow snapshot of valuation, failing to incorporate critical dynamics such as earnings growth and risk. This article introduces the Potential Payback Period (PPP) as a generalized and dynamic alternative to the P/E ratio, demonstrating that the traditional P/E is a degenerate case of PPP in two limiting scenarios: (1) when earnings growth converges to the discount rate, and (2) when both growth and discount rates are zero—an idealized static world. The theoretical framework is formalized through the application of L'Hôpital's Rule, revealing a deeper mathematical and financial relationship between these two valuation metrics.

2. Theoretical Framework: From P/E to PPP

The P/E ratio is defined simply as:

$$\frac{P}{E}$$

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Where P is the current stock price and E is the current earnings per share (EPS). It implicitly assumes static earnings and ignores both the risk-adjusted discount rate and earnings growth.

More specifically, the P/E ratio can be interpreted as the number of years it would take for constant annual earnings (EPS) to recover the initial investment (P), assuming no discounting and no earnings growth. However, this interpretation breaks down in a world where earnings are expected to grow or where future earnings must be discounted to reflect time and risk. This highlights the need for a more general metric.

The Potential Payback Period (PPP), by contrast, incorporates the time value of money, the expected annual growth rate, and a discount rate, usually derived via the Capital Asset Pricing Model (CAPM). The PPP answers the question: "**How many years will it take for discounted, growing earnings to equal the current price?**"

The PPP is calculated using the following formula:

$$PPP = \frac{\log\left[\frac{P}{E}\cdot\frac{g-r}{1+r}+1\right]}{\log\left(\frac{1+g}{1+r}\right)}$$

Where:

- $\frac{P}{E}$ is the current Price-to-Earnings ratio.
- g is the expected annual growth rate of earnings.
- r is the discount rate, representing the investor’s required rate of return.

This expression adjusts for both growth and risk, offering a refined lens through which to view investment recovery time. Unlike the P/E ratio, PPP can adapt to different macroeconomic conditions and company-specific growth trajectories. It generalizes the payback concept by integrating financial realities that static ratios like P/E cannot capture, including compounding earnings and risk-adjusted discounting. In doing so, PPP provides a more accurate and intuitive framework for investment decision-making.

3. Degenerate Cases and the Role of L'Hôpital's Rule

A critical insight arises when we consider the behavior of PPP under limiting conditions. In both cases below, the PPP formula becomes indeterminate and requires the use of **L'Hôpital's Rule** to resolve the limit.

Case 1: g → r

When the earnings growth rate approaches the discount rate , both the numerator and denominator of the PPP formula approach zero:

$$\lim_{g\to r} PPP = \frac{\log\left[\frac{P}{E}\cdot\frac{g-r}{1+r}+1\right]}{\log\left(\frac{1+g}{1+r}\right)}$$

This is an indeterminate form of type 0/0. Applying L'Hôpital's Rule:

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$$\lim_{g \to r} PPP = \lim_{g \to r} \frac{\frac{d}{dg} \log\left[\frac{P}{E} \cdot \frac{g-r}{1+r} + 1\right]}{\frac{d}{dg} \log\left(\frac{1+g}{1+r}\right)}$$

Differentiating numerator and denominator:

$$= \frac{\frac{P}{E} \cdot \frac{1}{1+r}}{\left(\frac{P}{E} \cdot \frac{g-r}{1+r} + 1\right)} \div \frac{1}{1+g}$$

At the point where **g = r**, this simplifies to:

$$PPP = \frac{P}{E}$$

Thus, **the P/E ratio emerges as a degenerate case of PPP** when earnings growth equals the discount rate. Economically, this corresponds to a condition where earnings are neither growing nor shrinking in real, risk-adjusted terms.

Case 2: g = 0 and r = 0

In the hypothetical case where both the growth rate and the discount rate are zero, the PPP formula becomes:

$$PPP = \frac{\log\left[\frac{P}{E} \cdot 0 + 1\right]}{\log(1)} = \frac{\log(1)}{\log(1)} = \frac{0}{0}$$

Again, we encounter an indeterminate form. Using L'Hôpital's Rule:

$$\lim_{g \to 0, r \to 0} PPP = \frac{\frac{P}{E}}{1} = \frac{P}{E}$$

This confirms that even in a zero-growth, zero-discount world—an abstract but instructive benchmark—the P/E ratio is recovered as the limiting form of PPP. In such a world, earnings are perfectly constant in nominal terms and investors are indifferent to time, making the traditional P/E interpretation valid by construction.

4. Implications for Stock Valuation

These results reframe the traditional P/E ratio within a broader valuation context. Instead of viewing P/E as an isolated metric, analysts can now interpret it as a special case within the PPP

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continuum. The PPP adapts dynamically to different growth and risk conditions, making it inherently more robust and informative.

Key implications:

- **Growth-adjusted valuation**: Unlike P/E, PPP differentiates between high-growth and low-growth stocks relative to their risk levels.
- **Risk-sensitive interpretation**: PPP explicitly incorporates the investor's required rate of return. **Time-based intuition**: PPP reflects the discounted payback period, aligning more closely with intuitive investment reasoning.
- **Limiting clarity:** The degenerate cases ($\mathbf{g \to r}$ and $\mathbf{g = r = 0}$) reveal the traditional P/E as an idealized boundary of the more general PPP.

5. Applications in Portfolio Management

The PPP's sensitivity to earnings growth and risk makes it a powerful tool for modern portfolio management. Investors and fund managers can:

- **Rank stocks** by PPP to prioritize faster-payback opportunities.
- **Compare valuations** across sectors with differing risk and growth profiles.
- **Integrate PPP** into multi-factor models, enhancing predictive accuracy beyond legacy metrics.

By recognizing the P/E ratio as merely a degenerate case within the PPP framework, portfolio strategies can evolve toward more nuanced, risk-adjusted, and forward-looking methodologies.

6. Conclusion

The recognition that the P/E ratio is a **degenerate case** of the Potential Payback Period represents a profound **quantitative revelation** in financial analysis. Using L'Hôpital's Rule, we have demonstrated that the PPP formula seamlessly reduces to the P/E ratio under limiting conditions—when growth equals the discount rate or both are zero.

This insight elevates PPP from a theoretical construct to a **superior valuation framework**, one that incorporates the core financial realities of growth, risk, and time. As financial markets grow increasingly complex and forward-looking, the PPP provides the adaptability and interpretive power that modern valuation demands.

The time has come to move beyond static ratios. By generalizing the P/E ratio through the lens of PPP, we gain a clearer, more consistent, and economically grounded understanding of equity value.

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