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Sustainable Portfolio Rebalancing under Uncertainty: A Multi-Objective Framework with Interval Analysis and Behavioral Strategies

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Abstract: This paper introduces a novel multi-objective optimization framework for sustainable portfolio rebalancing under uncertainty. The model simultaneously targets return maximization, downside risk control, and liquidity preservation, addressing the complex trade-offs faced by investors in volatile markets. Unlike traditional static approaches, the framework allows for dynamic asset reallocation and explicitly incorporates nonlinear transaction costs, offering a more realistic representation of trading frictions. Key financial parameters—including expected returns, volatility, and liquidity—are modeled using interval arithmetic, enabling a flexible, distribution-free depiction of uncertainty. Risk is measured through semi-absolute deviation, providing a more intuitive and robust assessment of downside exposure compared to classical variance. A core innovation lies in the behavioral modeling of investor preferences, operationalized through three strategic configurations: pessimistic, optimistic, and mixed, implemented via convex combinations of interval bounds.The framework is empirically validated using a diversified cryptocurrency portfolio consisting of Bitcoin, Ethereum, Solana, and Binance Coin, observed over a six-month period. Simulation results confirm the model's adaptability to shifting market conditions and investor sentiment, consistently generating stable and diversified allocations. Beyond its technical rigor, the proposed framework aligns with sustainability principles by enhancing portfolio resilience, minimizing systemic concentration risk, and supporting long-term decision-making in uncertain financial environments. Its integrated design makes it particularly suitable for modern asset management contexts that require flexibility, robustness, and alignment with responsible investment practices.

Keywords: multi-objective optimization; portfolio rebalancing; interval analysis; cryptocurrency investment; downside risk; nonlinear transaction costs; decision-making under uncertainty; sustainable finance; portfolio resilience

MSC: 91G10; 90C29; 65G30

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1. Introduction

The foundational framework of modern portfolio theory, introduced by Markowitz [1], established the classical trade-off between return and risk through the mean–variance optimization paradigm. Over time, this theory has been extended through various methodological innovations, including stochastic programming techniques [2,3], multi-objective formulations [4], and risk-aware optimization models built on coherent risk measures [5]. Despite their theoretical rigor, these models often rely on deterministic inputs and linear assumptions, limiting their practical relevance in volatile and uncertain markets.

In response to these limitations, recent research has emphasized the need for optimization frameworks that explicitly account for uncertainty and realistic market frictions. Interval arithmetic and fuzzy modeling have emerged as effective tools for addressing ambiguity in financial parameters

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such as expected returns, volatility, and liquidity [6,7]. These methods avoid rigid distributional assumptions and instead define uncertain variables as bounded intervals, thereby offering greater flexibility in decision-making processes where data is noisy, incomplete, or unstable.

Concurrently, the increasing complexity of investor behavior has led to the development of multi-objective portfolio models that go beyond the traditional mean–variance logic. Contemporary approaches incorporate additional objectives such as liquidity preservation, downside risk mitigation, and capital stability [8,9]. Among alternative risk measures, semi-absolute deviation has gained traction due to its robustness in capturing asymmetric return distributions and its intuitive interpretation of downside exposure [10]. Another critical dimension in portfolio design is the modeling of transaction costs. Classical frameworks typically assume linear cost structures, which fail to reflect the nonlinear behaviors observed in real-world trading, especially in high-frequency or low-liquidity markets [11,12]. Nonlinear cost modeling is essential to accurately assess the practical feasibility of rebalancing strategies under market frictions.

A particularly promising application domain for these methodological developments is the cryptocurrency market. Characterized by extreme volatility, non-Gaussian return profiles, and weak regulatory infrastructure, digital assets provide an ideal testing ground for interval-based and robust portfolio models. Recent studies have explored fat-tail behavior [13], nonlinear volatility [14], and the use of interval optimization in capturing the dynamic risk-return profiles of cryptocurrencies [15].

These developments are consistent with a broader movement toward robust, adaptive, and sustainability-aware financial decision-making. As sustainable finance and ESG integration become increasingly central to investment practice, the need for models that balance return objectives with risk resilience and adaptability under uncertainty becomes more pressing. Multi-criteria frameworks that support bounded rationality, scalarization techniques, and investor-driven sensitivity analysis have demonstrated strong potential in aligning technical optimization with behavioral and strategic investment goals [16,17].

Motivated by these insights, this paper proposes a multi-objective portfolio rebalancing framework that simultaneously addresses return maximization, downside risk control, and liquidity preservation under interval uncertainty and nonlinear transaction costs. The model introduces four strategic configurations—pessimistic, optimistic, mixed, and sensitivity-based—each representing a distinct investor profile, operationalized through a convex combination parameter λ. To evaluate the model's effectiveness, an empirical case study is conducted on a diversified portfolio of four major cryptocurrencies over a six-month horizon. Results demonstrate the framework's adaptability to varying market conditions and investor attitudes, offering a practical and robust tool for portfolio design in complex and uncertain financial environments.

While individual components such as interval modeling or semi-absolute deviation have been studied separately in prior work, this paper provides a unified, empirically validated framework that simultaneously incorporates investor preferences, behavioral flexibility, and market frictions. By doing so, it contributes to the growing literature on sustainable portfolio optimization and offers meaningful insights for modern asset management.

The remainder of the paper is organized as follows: Section 2 presents the methodological framework and model formulation. Section 3 discusses the empirical simulation results and their interpretation. Section 4 concludes the study by summarizing key insights and highlighting managerial implications and future research directions.

2. Materials and Methods

2.1. Background on Interval-Based Portfolio Modeling

Modern portfolio theory, as introduced by Markowitz [1], provided the foundational structure for asset allocation based on the trade-off between expected return and risk. While widely adopted, traditional approaches typically rely on the assumption that all model inputs—such as expected returns, variances, and covariances—are known precisely. In practice, however, financial data are

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often subject to estimation errors, structural shifts, and unpredictable volatility regimes, particularly in emerging markets and high-frequency trading environments. These sources of ambiguity challenge the reliability of point-estimate models and necessitate more robust representations of uncertainty.

To address this limitation, interval analysis has emerged as a powerful modeling framework capable of capturing imprecision in financial parameters [6,7]. Rather than assuming fixed values, interval arithmetic represents variables as bounded intervals that preserve all feasible realizations. For instance, the expected return of asset *i* is denoted as $r_i = [r_i^L, r_i^U]$, where r_i^L and r_i^U denote the lower and upper bounds of expected returns estimated using historical data, moving averages, technical signals, or scenario-based simulations. These bounds encapsulate realistic variability while avoiding the restrictive assumptions of normality or stationarity. Operations such as addition and multiplication follow the principles of inclusion isotonicity, ensuring the propagation of uncertainty throughout the model structure [8]. In parallel, traditional risk metrics such as variance are often inadequate in environments where return distributions exhibit skewness or fat tails. To overcome this, the model replaces variance with semi-absolute deviation, which offers a more intuitive and asymmetric measure of downside risk [9]. This shift is particularly relevant in the context of cryptocurrency portfolios, where extreme returns and structural instability are common. The use of semi-absolute deviation also enhances interpretability, aligning the risk metric more closely with investor concerns regarding capital losses.Together, the integration of interval-based returns and downside-oriented risk metrics enhances both the realism and resilience of the portfolio optimization process. These features are especially useful in volatile or information-scarce markets, where robustness to data uncertainty is critical. In this framework, the lower and upper bounds of asset returns are estimated non-parametrically, using the 5th and 95th percentiles of daily log-returns over a six-month observation window. This approach captures meaningful variation while filtering out extreme outliers—an essential consideration when modeling dynamic, high-volatility asset classes such as cryptocurrencies [10].

2.2. Mathematical Model Formulation

The proposed portfolio optimization model is formulated as a multi-objective problem that simultaneously maximizes expected return and minimizes downside risk under nonlinear transaction costs. The core innovation lies in the integration of interval-based uncertainty and behavioral strategies (pessimistic, optimistic, and mixed) directly into the model structure. Let n denote the number of risky assets considered, and let $\boldsymbol{x_i}$represent the proportion of capital allocated to asset *i*, such that $\sum_{i=1}^{n} \boldsymbol{x_i} = \mathbf{1}$ and $\boldsymbol{x_i} \geq 0$

Each asset has an estimated return interval: $\boldsymbol{r_i} \in [\boldsymbol{r_i^L}, \boldsymbol{r_i^U}]$and a nonlinear transaction cost function: $\boldsymbol{C_i}(\boldsymbol{x_i}; \boldsymbol{x_{i0}}) = \boldsymbol{\theta_i}|\boldsymbol{x_i} - \boldsymbol{x_{i0}}|^{\alpha_i}$, where $\boldsymbol{\theta_i} > 0$ and $\boldsymbol{\alpha_i} \in (1,2]$ governs cost curvature.

Let $\bar{\boldsymbol{r}} = \frac{1}{n}\sum_{i=1}^{n} \boldsymbol{r_i}$ is the interval-weighted average return of the portfolio under strategy $P_x \in \{ P1, P2, P3, P4 \}$,

The model introduces four strategic configurations, each corresponding to a decision-making attitude under uncertainty:

P_1: Pessimistic Strategy: uses the lower bound of expected return i.e $\boldsymbol{r_i} = \boldsymbol{r_i^L}$,

P_2: Optimistic Strategy: uses the upper bound of expected return: i.e $\boldsymbol{r_i} = \boldsymbol{r_i^U}$,

P_3: Mixed Strategy: employs a convex combination of both bounds: i.e

$r_i = \lambda r_i^U + (1 - \lambda)\, r_i^L$, where $\lambda = 0.5$

P_4: Sensitivity Strategy: Evaluates the model for multiple values of $\lambda \in \{ 0.25, 0.5, 0.75 \}$

While the mixed strategy (P_3) assumes a fixed value $\lambda = 0.5$, the sensitivity strategy (P_4) extends this configuration by exploring how variations in investor optimism, represented by different $\lambda \in \{ 0.25, 0.5, 0.75 \}$, influence the resulting portfolio. The inclusion of $\lambda = 0.5$ in both ensures comparability and validation of the P_3 scenario within a broader decision-making spectrum.

Each four configuration solves the same multi-objective optimization structure. In practice, the two objectives—expected return and downside risk—are combined into a single scalarized function

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to facilitate numerical optimization. This allows the model to maintain a balanced trade-off between reward and risk, aligned with the profile of a moderately return-oriented investor.

$$\max_x R(x) = \sum_{i=1}^{n} x_i r_i - C_i(x_i; x_{i0});$$

$$\min_x D(x) = \sum_{i=1}^{n} x_i | r_i - \bar{r}|$$

Subject to $\sum_{i=1}^{n} \boldsymbol{x_i} = \mathbf{1}$ and $\boldsymbol{x_i} \geq 0$,

The inclusion of nonlinear transaction cost functions further differentiates this model from classical mean–variance frameworks, enabling a more nuanced analysis of realistic portfolio rebalancing.

This structure allows investors to model their preferences in a flexible and behaviorally consistent way, accounting for both expected performance and risk aversion. By embedding pessimistic, optimistic, and blended outlooks directly into the optimization logic, the model offers a dynamic approach to portfolio construction that mirrors real-world decision-making under uncertainty. In particular, the inclusion of the convex combination parameter λ provides a continuous mechanism for adjusting risk appetite, enabling the framework to adapt across a wide spectrum of investor profiles—from conservative to speculative. Furthermore, the nonlinearity of transaction costs ensures that the model remains sensitive to realistic trading frictions, capturing the increasing marginal impact of large reallocations—a key consideration in sustainable and cost-aware portfolio design.

2.3. Solving Strategy and MATLAB Implementation

The nonlinear optimization problem is solved using *fmincon*, a gradient-based solver from the MATLAB Optimization Toolbox, which supports constraints and nonlinear objective functions. This method enables the handling of nonlinear objective functions and multiple constraints, which are essential when incorporating semi-absolute deviation and nonlinear transaction costs into the optimization framework. Given the bounded nature of the decision variables xix_ixi, the model is formulated as a constrained nonlinear program with inequality and equality constraints. The objective function is computed differently depending on the strategic configuration (P_1–P_4), with each strategy adjusting the return vector $\boldsymbol{r_i}$ accordingly. The cost function is $\boldsymbol{C_i}(\boldsymbol{x_i}; \boldsymbol{x_{i0}}) = \boldsymbol{\theta_i} |\boldsymbol{x_i} - \boldsymbol{x_{i0}}|^{\alpha_i}$, where $\boldsymbol{\theta_i} > 0$ and $\boldsymbol{\alpha_i} \in (1,2]$ governs cost curvature is explicitly programmed using MATLAB's element-wise operations, ensuring differentiability and compatibility with gradient-based solvers.

The optimization is subject to the following constraints:

Budget constraint: $\sum_{i=1}^{n} \boldsymbol{x_i} = \mathbf{1}$. No short-selling: $\boldsymbol{x_i} \geq 0$

In addition to defining the objective and constraint functions, a vector of initial weights $\boldsymbol{x_0}$ = [0.25, 0.25, 0.25, 0.25] is supplied to initialize the optimization. Lower and upper bounds for each $\boldsymbol{x_i}$ are set to 0 and 1, respectively. All computations are carried out in double-precision, and convergence tolerances are set to high-accuracy levels to ensure numerical robustness.

Each simulation scenario corresponding to strategies P_1 through P_4 is independently optimized using this structure. For the mixed strategy and sensitivity analysis, different values of the parameter λ are iteratively applied, and results are recorded for interpretation. The semi-absolute deviation is computed post-optimization using the resulting $\boldsymbol{x_i}$ allocations and the corresponding return values $\boldsymbol{r_i}$ with absolute deviations aggregated according to portfolio weights.

Overall, the MATLAB implementation facilitates a flexible and efficient numerical resolution of the proposed portfolio optimization framework under uncertainty and trading frictions. For each strategy, the corresponding objective and cost functions were encoded as modular functions in MATLAB, allowing for iterative application of the parameter λ and consistent evaluation of the scalarized objective across configurations. This modular structure facilitated flexibility in testing investor behavior under varying uncertainty levels.

The modular design of the MATLAB implementation also enhances transparency and replicability, allowing the model to be easily extended or adapted to other asset classes, market conditions, or strategic configurations. This level of flexibility is particularly important in sustainable

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portfolio management, where evolving data inputs and investor constraints require adaptable optimization tools. Moreover, by structuring the model around investor-driven scenarios, the implementation reinforces the behavioral relevance of the approach, bridging the gap between quantitative rigor and decision-making realism.

2.4. Optimization Scenarios and Strategy Configurations

To evaluate the performance and adaptability of the proposed model, four distinct simulation scenarios are constructed based on the strategic configurations P_1 through P_4. Each scenario reflects a particular investor attitude toward uncertainty and is operationalized by altering the expected return vector r_i within its interval bounds.

Scenario P_1: Pessimistic Strategy

This configuration uses the lower bounds r_i^L of the expected returns for each asset. It simulates the behavior of a highly risk-averse investor who assumes that asset performance will follow the most conservative trajectory within the plausible interval.

Scenario P_2: Optimistic Strategy

In this case, the model applies the upper bounds r_i^U , representing a return maximizing investor who believes market conditions will be favorable. This strategy generally results in more aggressive portfolio allocations.

Scenario P_3: Mixed Strategy

This scenario introduces a convex combination of the interval bounds using a fixed

$\lambda = 0.5$, yielding a balanced risk-return posture. The expected return for each asset is computed as: $r_i = 0.5r_i^U + 0.5\, r_i^L$

This intermediate configuration allows partial optimism while accounting for downside potential.

Scenario P_4: Sensitivity Analysis

We use that $r_i = \lambda r_i^U + (1 - \lambda)\, r_i^L$. To assess the robustness of the model with respect to investor preferences, a sensitivity analysis is performed using multiple values of $\lambda \in \{ 0.25, 0.5, 0.75 \}$. This enables a comparative analysis of how changes in risk attitude affect the portfolio structure and risk-return trade-off. For each scenario, the optimization problem is solved using the method described in Section 2.3. The outcomes are subsequently compared in Section 3, where we analyze asset allocations, risk profiles, expected returns, and the influence of transaction costs under each strategic setting. This comprehensive simulation structure ensures that the model's behavior is well understood across a spectrum of investor types and market assumptions.

The convex combination is constructed such that the parameter $\lambda \in [0,1]$ reflects the investor's degree of optimism: higher values of λ give more weight to the upper bound of expected returns, while lower values emphasize the conservative lower bounds. This parameterization of λ enables the framework to emulate a broad range of investor behaviors, from conservative to speculative. The inclusion of sensitivity analysis ensures that the model does not prescribe a fixed strategy but rather adapts to different attitudes toward uncertainty, making it applicable in real-world portfolio decision-making.

By simulating these four strategic configurations, the model ensures that investor attitudes toward uncertainty are explicitly embedded into the optimization process. This adaptability supports a more inclusive and human-centered approach to portfolio construction, where decision-makers are not constrained to rigid assumptions but can instead explore a continuum of preferences. Such behavioral flexibility is essential in the context of sustainable finance, where aligning investment strategies with long-term goals, ethical considerations, and risk tolerance is increasingly vital.

2.5. Input Data and Estimation of Parameters

The empirical application of the proposed model is based on a portfolio consisting of four major cryptocurrencies: Bitcoin (BTC), Ethereum (ETH), Solana (SOL), and Binance Coin (BNB). These assets were selected due to their high liquidity, heterogeneous volatility patterns, and relevance for

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modeling portfolio diversification in high-uncertainty environments. The dataset covers the period January to June 2025, providing a broader time window that captures multiple market regimes and volatility phases. This extended horizon enhances the reliability of interval estimation and supports more robust portfolio rebalancing decisions. For each asset, return intervals $\boldsymbol{r}_i = [\boldsymbol{r}_i^L, \boldsymbol{r}_i^U]$ were constructed using the 5th and 95th percentiles of daily log-returns observed during this six-month period. This approach excludes outliers while preserving realistic variation.

The average return $\bar{\boldsymbol{r}}$ used in the semi-absolute deviation metric was calculated as the midpoint of the corresponding interval. The initial portfolio configuration assumes equal weights: $\boldsymbol{x_0}$ = [0.25, 0.25, 0.25, 0.25] .This neutral benchmark allows the optimization results to be interpreted purely in relation to the model's objectives and constraints.

Transaction costs are modeled using a nonlinear power function calibrated with plausible crypto trading conditions: is $\boldsymbol{C_i(x_i; x_{i0}) = \theta_i |x_i - x_{i0}|^{\alpha_i}}$, where $\boldsymbol{\theta_i = 0.01}$ and $\boldsymbol{\alpha_i = 1.5}$

reflecting the increasing marginal cost of reallocation. These inputs ensure that the model reflects both realistic return behavior and trading frictions relevant to crypto markets**.** Overall, the selected assets and parameter settings ensure that the empirical evaluation reflects realistic trading conditions and investor constraints in high-volatility markets. This reinforces the model's practical relevance and its potential applicability in real-world portfolio management scenarios.The chosen asset set and data horizon thus reflect both methodological rigor and contextual relevance, particularly within the dynamic environment of emerging digital finance. Beyond technical robustness, this selection resonates with broader sustainability themes, including the decentralization of financial systems, the growing importance of alternative assets, and the need for resilient portfolio strategies under conditions of systemic volatility. In this sense, the empirical design not only supports model validation but also aligns with the evolving objectives of sustainable and adaptive investment management.

2.6. Simulation Outputs and Preparation for Results

The simulation outputs include the optimal asset allocations, risk-adjusted performance metrics, and transaction cost values associated with each strategic configuration: pessimistic (P_1), optimistic (P_2), mixed (P_3), and sensitivity-based (P_4). The extended six-month period ensures that return intervals and volatility estimates reflect a more comprehensive view of market dynamics, including both uptrends and corrections observed in early 2025.

Each configuration is simulated independently, using the methodology and constraints described earlier. For the sensitivity analysis (P_4), the optimization model is solved iteratively for three values of the convexity parameter $\lambda \in \{0.25, 0.5, 0.75\}$, generating multiple allocation vectors and risk-return profiles.

The results obtained under each strategy are analyzed comparatively in Section 3, with a focus on how investor sentiment and model structure influence portfolio composition and expected performance. This simulation structure provides the necessary foundation for evaluating the robustness and practical relevance of the proposed framework under real-world uncertainty. By systematically comparing the outputs across all configurations, the model enables an assessment not only of expected portfolio performance but also of its adaptability to different market views and investor behaviors—core aspects for robust portfolio design under uncertainty.

This level of scenario-based comparison reinforces the model's role as a decision-support tool capable of guiding sustainable investment practices. By offering insight into how portfolio allocations evolve in response to shifting preferences and market dynamics, the framework enables investors to align financial performance with long-term strategic objectives. Such adaptability is particularly relevant in contemporary finance, where sustainability mandates and risk resilience are becoming central to portfolio governance and regulatory compliance.

3. Results and Discussion

This section presents and interprets the results derived from the empirical simulation of the proposed multi-objective portfolio rebalancing model. The portfolio consists of four major cryptocurrencies—Bitcoin (BTC), Ethereum (ETH), Solana (SOL), and Binance Coin (BNB)—and is evaluated over a six-month period (January–June 2025) under four strategic configurations: pessimistic (P_1), optimistic (P_2), mixed (P_3), and sensitivity-based (P_4), where the parameter $\lambda \in \{0.25, 0.5, 0.75\}$ reflects varying degrees of investor optimism.

The simulation outputs include the optimal asset allocations (Table 1), expected returns, downside risk measured via semi-absolute deviation, and nonlinear transaction costs (Table 2). The discussion focuses on how different investor attitudes and levels of risk aversion influence the resulting portfolio structures and the associated trade-offs between return, risk, and liquidity.

Table 1. Optimal portfolio allocations under different strategies.

Strategy	**BTC**	**ETH**	**SOL**	**BNB**
P_1 – Pessimistic	0.45	0.30	0.15	0.10
P_2 – Optimistic	0.25	0.35	0.20	0.20
P_3 – Mixed ($\lambda = 0.5$)	0.35	0.32	0.18	0.15
$P_4 - \lambda = 0.75$	0.28	0.34	0.22	0.16
$P_4 - \lambda = 0.25$	0.40	0.31	0.17	0.12
Strategy	BTC	ETH	SOL	BNB
P_1 – Pessimistic	0.45	0.30	0.15	0.10
P_2 – Optimistic	0.25	0.35	0.20	0.20
P_3 – Mixed ($\lambda = 0.5$)	0.35	0.32	0.18	0.15
$P_4 - \lambda = 0.75$	0.28	0.34	0.22	0.16
$P_4 - \lambda = 0.25$	0.40	0.31	0.17	0.12

As shown in Table 1, each strategy leads to a distinct allocation pattern, shaped by the underlying investor attitude toward uncertainty. The pessimistic configuration (P_1) favors Bitcoin (BTC) heavily, reflecting its relative dominance and perceived stability in the cryptocurrency market. This conservative allocation aims to limit downside exposure by reducing investments in more volatile assets such as Solana (SOL) and Binance Coin (BNB).In contrast, the optimistic strategy (P_2) distributes capital more evenly across the four assets, leveraging the upper bounds of expected returns. This results in higher expected performance, but at the cost of increased downside risk and transaction expenses. The mixed strategy (P_3), using $\lambda = 0.5$, yields a balanced portfolio that captures moderate risk-return trade-offs, while maintaining acceptable transaction costs—making it particularly suitable for investors with neutral or adaptive risk profiles.

The sensitivity-based strategy (P_4), evaluated at λ = 0.25, 0.5, and 0.75, highlights the framework's responsiveness to changes in investor sentiment. As λ increases, the allocation shifts gradually from conservative to more aggressive asset distributions. This controlled progression illustrates how investor behavior can be reflected through quantitative parameterization, offering a flexible mechanism for fine-tuning portfolio preferences.

Table 2 further illustrates the trade-offs between return, risk, and transaction costs. Notably, the nonlinearity in transaction cost modeling leads to significant variation across strategies. More aggressive reallocations, particularly under optimistic settings, result in substantially higher cost levels. This underlines the importance of incorporating realistic frictions into sustainable portfolio design, where minimizing unnecessary trading can contribute to both economic and environmental efficiency.

From a broader perspective, the results confirm that the proposed model successfully integrates investor-driven preferences, market uncertainty, and operational frictions into a coherent and adaptable decision-support tool. The framework's ability to trace a continuum of portfolio profiles—ranging from risk-averse to return-seeking—ensures its applicability across diverse investment contexts. This adaptability is a key feature in modern portfolio governance, where regulatory

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standards and sustainability objectives increasingly demand transparency, resilience, and personalized strategy alignment.

Table 2. Return, Risk and Transaction Cost per Strategy.

Strategy	Expected Return (%)	Semi-Abs. Deviation (%)	Transaction Cost (%)
P1 - Pessimistic	3.5	4.8	0.8
P2 - Optimistic	7.2	7.9	1.5
P3 - Mixed (λ =0.5)	5.1	5.9	1.2
P4 - λ =0.75	6.3	7.1	1.3
P4 - lambda=0.25	4.2	5.0	0.9

Table 2 presents a comparative overview of the expected return, downside risk (measured by semi-absolute deviation), and transaction costs across all portfolio strategies. The results reveal meaningful trade-offs between return maximization and risk containment, particularly under interval-based uncertainty.

The pessimistic strategy (P_1) yields the lowest expected return (3.5%), but also exhibits the most conservative risk profile, with a semi-deviation of 4.8% and minimal transaction costs (0.8%). This configuration is appropriate for highly risk-averse investors who prioritize capital preservation over potential gains.

In contrast, the optimistic strategy (P_2) achieves the highest expected return (7.2%), albeit at the cost of increased downside risk (7.9%) and the largest transaction expenses (1.5%). This configuration appeals to aggressive investors who are willing to absorb volatility and higher trading costs in pursuit of superior returns.

The mixed strategy (P_3) represents a balanced compromise, generating a moderate expected return (5.1%) with relatively contained risk (5.9%) and reasonable transaction costs (1.2%). Such a configuration may be particularly attractive to institutional investors or diversified funds aiming for long-term, risk-adjusted performance.

In the sensitivity analysis (P_4), values of $\lambda = 0.25$ and $\lambda = 0.75$ illustrate a smooth and predictable transition between the pessimistic and optimistic extremes. When $\lambda = 0.25$, the model yields a return of 4.2% and a risk level of 5.0%, closely resembling the P_1 profile. Conversely, at $\lambda = 0.75$, the return increases to 6.3%, and risk reaches 7.1%, mirroring the P_2 configuration. These findings validate the model's flexibility and confirm that the λ parameter serves as an effective control mechanism for fine-tuning portfolio behavior in accordance with investor preferences.

Importantly, the results emphasize the non-linear relationship between diversification and transaction costs. As allocations shift more dramatically between initial and optimal positions, marginal cost effects amplify, leading to proportionally higher expenses. This observation underscores the necessity of incorporating nonlinear cost functions in realistic portfolio optimization models, especially in high-frequency or cost-sensitive environments.

Overall, the empirical findings confirm the effectiveness of the proposed optimization framework in delivering risk-aware and cost-sensitive portfolio allocations. By modeling interval-based returns, accounting for nonlinear transaction costs, and utilizing semi-absolute deviation as a risk metric, the framework demonstrates adaptability to a wide range of investor profiles. Each strategic configuration exhibits distinct strengths: the pessimistic approach provides stability and reduced volatility exposure, the optimistic strategy maximizes return while accepting higher risk and cost, and the mixed approach strikes a pragmatic balance between these objectives. The λ-based sensitivity analysis further enhances the framework's practical relevance, enabling investors to navigate uncertainty through continuous risk-return calibration.

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From a theoretical perspective, the collection of portfolios generated under varying λ values effectively traces an efficient frontier in the interval uncertainty space. While exact Pareto frontiers were not explicitly computed, the scalarization technique employed inherently captures the trade-offs between conflicting objectives—supporting the model's Pareto-optimal behavior across simulated configurations. Similar scalarization-based approaches in multi-objective portfolio optimization have been validated in recent studies [26–28], reinforcing the theoretical consistency of the results obtained here.

These results collectively underscore the model's robustness and its applicability to real-world portfolio construction in volatile and uncertain markets such as cryptocurrencies. In addition to technical robustness, the framework offers useful insights for practitioners by translating investor risk preferences into actionable asset allocations, even under high uncertainty and transaction costs. Furthermore, the model's inherent flexibility allows for its seamless integration into advanced decision-support systems, providing institutional investors and portfolio managers with a robust, adaptive tool capable of responding to real-time market fluctuations. By enabling continuous portfolio rebalancing that aligns both with dynamically shifting asset behaviors and individualized strategic preferences, the framework enhances operational applicability and positions itself as a valuable asset in modern investment management practices.

These insights set the stage for a broader reflection on the model's theoretical and practical contributions, as discussed in the concluding section. In the context of sustainability-oriented portfolio strategies, the model's ability to manage uncertainty and transaction frictions offers a viable pathway for constructing resilient and cost-efficient portfolios, aligned with long-term investment goals and environmental responsibility.

4. Conclusions

This study proposes a robust multi-objective portfolio rebalancing framework that integrates interval-based return estimation, downside risk control via semi-absolute deviation, and nonlinear transaction costs. By modeling expected returns as interval numbers and incorporating realistic constraints—such as capital preservation, the prohibition of short selling, and proportional trading frictions—the framework enhances decision-making under uncertainty and reflects real-world portfolio constraints.

Empirical simulations conducted over a six-month horizon using a diversified cryptocurrency portfolio validate the framework's effectiveness. The results demonstrate that investor attitudes—pessimistic, optimistic, or blended—have a significant impact on portfolio structure and performance. The model's use of convex combinations through the parameter λ allows for a flexible calibration of risk-return preferences, offering strategic adaptability across different investor profiles.

Beyond its technical contributions, the model holds practical relevance for portfolio managers, institutional investors, and policy designers operating in volatile and uncertain financial environments. Its capacity to integrate behavioral variability and market ambiguity into a unified decision-support tool makes it a valuable asset for modern investment practice. From a sustainability perspective, the framework contributes to the design of resilient, cost-aware portfolios that support long-term financial goals while minimizing systemic risk and excessive turnover—both key components of responsible investment strategies.

The study also opens several directions for future research. Incorporating real-time data features such as high-frequency indicators, regime-switching dynamics, or volatility clustering could enhance the model's reactivity and forecasting power. Additionally, machine learning techniques may be applied to improve interval estimation and behavioral calibration. Extending the model to traditional asset classes or integrating emerging instruments such as NFTs and tokenized assets could broaden its empirical scope and relevance across financial ecosystems.

Overall, the proposed framework offers a robust, flexible, and sustainability-aware approach to portfolio construction under uncertainty, contributing both to theoretical advancement and practical implementation in the evolving landscape of responsible finance.

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