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Non-Measurability Theory

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Abstract

First, we generalize measure theory by establishing measures on all sets. Secondly, we take two properties of dark energy as the principle of the theory, one is that dark energy has the property of a non-measurable set, and the other is that all all matter and fields are excitation of dark energy. We do deductive reasoning on the basis of these two principles, and the first conclusion is that there is non-measurable matter, and we provide evidence for the existence of such matter. According to the second principle, we can solve the cosmological constant problem and the black hole information paradox. We can get that all geometric quantities in de Sitter space are interval-valued. Therefore, we can introduce quantum intertextuality into the theory of gravity and geometry.Finally,we get quantum gravity and quantum geometry.

Keywords: non-measurable set; quantum gravity; dark energy

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1. Introduction

"The grand aim of all science is to cover the greatest number of empirical facts by logical deduction from the smallest number of hypotheses or axioms". Einstein said in the article [49]. In physics,non-measurability theory is a theoretical framework in which all elementary particles are composed of three-dimensional objects called dark energy. Dark energy itself is non-measurable,some dark energy forms measurable matter, and some forms non-measurable matter(Table 1). We use the properties (especially the Banach-Tarski paradox) of non-measurable sets to study the frontier of physics. The expansion of the universe, the star collapses into a black hole, neutrino oscillations, parity nonconservation, and so on may be all physical manifestations of the Banach-Tarski paradox.

In ancient mathematics, some lengths that could not be measured by rational numbers caused the first mathematical crisis. In order to solve this crisis, we introduced the concept of irrational numbers. Similarly, the axiom of choice implies that there is a non-measurable set. We cannot assign a real number to the nonmeasurable set. To solve the measurement problem, we assign an interval number to each nonmeasurable set, and the property of the interval number can solve the measurement problem of the nonmeasurable set. The mathematical tool for analyzing nonmeasurable sets is interval mathematics ([7–12]). Here we need to use interval arithmetic, interval measure theory, interval probability theory, interval tensor theory and interval differential geometry. Interval arithmetic is a mature branch of mathematics, but other branches of interval mathematics are not mature enough and need us to do a lot of research.

Does nonmeasurable matter exist in our universe? Most people say no, because we can't divide matter into complicated cases.But there is a possibility that there has always been nonmeasurable matter in the universe! At the beginning of the 20th century, the idea that energy was discontinuous revolutionized physics. Energy itself is discontinuous, that's the law of nature.We have a similar idea that ***there exists matter whose mass (energy) value is a nondegenerate interval number.In other words, non-measurable matter has different masses at the same time,we call such matter nonmeasurable matter. We assume that non-measurable matter has all the properties of non-measurable sets.According to the assumptions, we can solve a large number of physics problems***.(An analogy between hadronic physics and theory of nonmeasurable sets can be found in reference [14]. A black hole has different masses at the same time [25], so a black hole is nonmeasurable).This hypothesis is closely related to the following physics topics: (1)The mystery of the missing antimatter, (2) Quantum gravity, (3) The gravitational theory of a single particle, (4) Properties of neutrinos or quarks,(5)Dark matter and dark energy, (6) Hubble constant (7) The energy of gravitational fields, (8) Properties of black holes,(9)The accelerating expansion of the universe,(10)Black Hole Information Paradox,(11)Cosmological constant problem,(12)Big Bang Puzzle. Evidences for the existence of non-measurable matter can be found in section 4 of the article.

We take two properties of dark energy as the principle of the theory, one is that dark energy has the property of a non-measurable set, and the other is that All matter and fields are excitation of dark energy. We do deductive reasoning on the basis of these two principles, and the first conclusion is that there is non-measurable matter, and we provide evidence for the existence of such matter. According to the second principle, there are only dark energy fields in vacuum, and no other quantum fields, which can solve the cosmological constant problem. We can see that energy is not conserved if there is non-measurable matter, which solves the black hole information paradox. We can get that the cosmological constant is not a real number, but an interval number. We can deduce that the Hubble constant of de Sitter spacetime is a non-degenerate interval constant whose value depends on the measurement method, which can explain the irreconcilable gap between the measurements obtained by different measurement methods. And then, we introduce an important principle of quantum mechanics, namely quantum intertextuality, into the theory of gravity and geometry, we get quantum gravity and quantum geometry, and we try to explain strange quantum phenomena in quantum geometry.

"we ought in fact to suppose it, if we can thereby obtain a simpler explanation of phenomena". Riemann said in his famous paper [26].The above hypothesis can explain a large number of physical phenomena very simply, and we think that these two principles may be the truth. Experiment is the only standard to test a theory, so we need to provide more experimental evidence.

2. Preliminaries [4]

Theorem 2.1 (Banach-Tarski Paradox, Euclidean space). (AC).If $A, B \subseteq \mathbb{R}^n (n \geq 3)$ are any bounded sets with nonempty interior, then A and B are equidecomposable.

Theorem 2.2 (Banach-Tarski Paradox, Sphere). (AC).If $A, B \subseteq \mathbb{S}^n (n \geq 2)$ are any bounded sets with nonempty interior, then A and B are equidecomposable.

Theorem 2.3 (Banach-Tarski Paradox, Hyperbolic space). (AC).If $A, B \subseteq \mathbb{H}^n (n \geq 2)$ are any bounded sets with nonempty interior, then A and B are equidecomposable.

Theorem 2.4 (Banach-Tarski Paradox). (AC).Let G be a connected, finite-dimensional non-solvable Lie group acting transitively and continuously on a metric space (X, d) with no isolated points and suppose that G has negligible fixed points. Then any two subsets of X with compact closures and non-empty interiors are G-equidecomposable.

Remark Some conclusions on this topic can be found in References [4,27]

3. Generalized Measure Theory and Measurement Theory

Definition 3.1 $\mathbb{M} = \{I_x^y | x, y \in \mathbb{R}, x \leq y\}$,$I_{x_1}^{y_1} \leq I_{x_2}^{y_2}$ iff $x_1 \leq x_2$ and $y_1 \leq y_2$, $I_{x_1}^{y_1} \sqsubseteq I_{x_2}^{y_2}$ iff $x_2 \leq x_1, y_1 \leq y_2$. $I_{x_1}^{y_1} + I_{x_2}^{y_2} = I_{x_1+x_2}^{y_1+y_2}$, $I_{x_1}^{y_1} \cdot I_{x_2}^{y_2} = I_{\min\{a \cdot b | a \in [x_1, y_1], b \in [x_2, y_2]\}}^{\max\{a \cdot b | a \in [x_1, y_1], b \in [x_2, y_2]\}}$, , $I_0^0 = 0, I_1^1 = 1$. Then I_x^y is called an (finite) interval number. $\langle \mathbb{M}, +, \cdot, \leq, \sqsubseteq, 0, 1 \rangle$ is called ordering interval arithmetic,denoted by $\mathcal{M}$.

Remark $I_{x_1}^{y_1} - I_{x_2}^{y_2} =_{def} I_{x_1}^{y_1} + I_{-y_2}^{-x_2}$.$I_{x_1}^{y_1} \div I_{x_2}^{y_2} =_{def} I_{x_1}^{y_1} \cdot I_{\frac{1}{y_2}}^{\frac{1}{x_2}} (x_2 > 0 \text{ or } y_2 < 0)$.

Definition 3.2 $\mathbb{M}^* = \{I_x^y | x, y \in \mathbb{R}^*, x \leq y\}$,$I_{x_1}^{y_1} \leq I_{x_2}^{y_2}$ iff $x_1 \leq x_2$ and $y_1 \leq y_2$, $I_{x_1}^{y_1} \sqsubseteq I_{x_2}^{y_2}$ iff $x_2 \leq x_1, y_1 \leq y_2$. $I_{x_1}^{y_1} + I_{x_2}^{y_2} = I_{x_1+x_2}^{y_1+y_2}$, $I_{x_1}^{y_1} \cdot I_{x_2}^{y_2} = I_{\min\{a \cdot b | a \in [x_1, y_1], b \in [x_2, y_2]\}}^{\max\{a \cdot b | a \in [x_1, y_1], b \in [x_2, y_2]\}}$, , $I_0^0 = 0, I_1^1 = 1$. Then I_x^y is called a generalized interval number. $\langle \mathbb{M}^*, +, \cdot, \leq, \sqsubseteq, 0, 1 \rangle$ is called generalized ordering interval arithmetic,denoted by $\mathcal{M}^*$.

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Remark $I_{x_1}^{y_1} - I_{x_2}^{y_2} =_{def} I_{x_1}^{y_1} + I_{-y_2}^{-x_2}.I_{x_1}^{y_1} \div I_{x_2}^{y_2} =_{def} I_{x_1}^{y_1} \cdot I_{\frac{1}{y_2}}^{\frac{1}{x_2}}(x_2 \geq 0 \text{ or } y_2 \leq 0).(x_2 = 0 \Rightarrow \frac{1}{x_2} = \infty),(y_2 = 0 \Rightarrow \frac{1}{y_2} = -\infty)$

Definition 3.3 Let X be a set,$\mathcal{A} \subseteq \mathcal{P}(X)$,$\mathcal{A}$ be a algebra ($\sigma-$algebra or Complete Boolean algebra) over X, a mapping $\nu : \mathcal{A} \rightarrow \mathbb{M}^*$ is called an *interval measure* on X if it satisfies the following properties (1) (2) (3) (4). $(X, \mathcal{A}, \nu)$ is called an interval measure space. A mapping $\nu : \mathcal{A} \rightarrow \mathbb{M}^*$ is called a *finitely additive interval measure* on X if it satisfies the following properties(1)(2)(3)(4$'$).$(X, \mathcal{A}, \nu)$ is called a finitely additive interval measure space.

(1)(Non-negativity)$\nu(A) \geq I_0^0$.

(2)(Empty set)$\nu(\emptyset) = I_0^0$.

(3)(Shiffman Property)If$A \bigcap B = \emptyset, \nu(A) = I_{x_1}^{y_1}, \nu(B) = I_{x_2}^{y_2}$, $\min\{x_1 + y_2, x_2 + y_1\} = \bot$, $\max\{x_1 + y_2, x_2 + y_1\} = \top$, then $I_\bot^\top \sqsubseteq \nu(A \cup B)$.

(4)(Countable covering property)For all countable collections $\{A_i\}_{i=1}^\infty$ of pairwise disjoint sets in $\mathcal{A}$, $\nu(\cup_{i=1}^\infty A_i) \sqsubseteq \Sigma_{i=1}^\infty \nu(A_i)$.

(4$'$)(Finite covering property)For all finite collections $\{A_i\}_{i=1}^n$ of pairwise disjoint sets in $\mathcal{A}$, $\nu(\cup_{i=1}^n A_i) \sqsubseteq \Sigma_{i=1}^n \nu(A_i)$.

An interval measure on X is called an interval probability measure if (5) (Normalisation) $\nu(X) = I_1^1$.$(X, \mathcal{A}, \nu)$ is called an interval probability space.

A finitely additive interval measure on X is called a finitely additive interval probability measure if (5) (Normalisation) $\nu(X) = I_1^1$.$(X, \mathcal{A}, \nu)$ is called a finitely additive interval probability space.

Remark 1 The Shiffman Property is proved for a general measure space in [1,2].

Remark 2 For two non-measurable sets, which can be combined freely, axioms 3 and 4 give the range of this global measure.

Theorem 3.1 Every interval probability measure(IPM) on a $\sigma-$ algebra is an Interval-valued Probability Measure(IVPM) on the $\sigma-$ algebra.

Proof (1)(2)(3) of IVPM are axioms of IPM.Now we prove (4) of IVPM in IPM.

(4) For every partition of X, $\{A_{k\in K}\}, \{B_{j\in J}\} \subseteq \sigma_X$ such that $A = \cup_{k\in K} A_k$ and$A^c = B = \cup_{j\in J} B_j$,then $i_m(A) \subseteq [\max\{1 - \Sigma_{j\in J} B_j^u, \Sigma_{k\in K} A_k^l\}, \min\{1 - \Sigma_{j\in J} B_j^l, \Sigma_{k\in K} A_k^u\}]$.(Here we use the notation of IVPM, $i_m(A) = [A_l, A_u] \subseteq [0, 1]$).

The above formula can be expressed by the notation in this paper.

$\nu(A) \sqsubseteq I_{\max\{1-\Sigma_{j\in J} B_j^u, \Sigma_{k\in K} A_k^l\}}^{\min\{1-\Sigma_{j\in J} B_j^l, \Sigma_{k\in K} A_k^u\}}$.

The following Proposition can be obtained according to (3) of IPM.

$\nu(A) = I_1^1 - \nu(A^c)$.

So $\nu(A) = I_{A_l}^{A_u} = I_1^1 - \nu(A^c) = I_1^1 - \nu(B) = I_{1-B_u}^{1-B_l}$

The following Proposition can be obtained according to (4) of IPM.

$B^u \leq \Sigma_{j\in J} B_j^u, \Sigma_{j\in J} B_j^l \leq B^l$. $A^u \leq \Sigma_{j\in J} A_j^u, \Sigma_{j\in J} A_j^l \leq A^l$.

So $1 - \Sigma_{j\in J} B_j^u \leq 1 - B^u, 1 - B^l \leq 1 - \Sigma_{j\in J} B_j^l$.

Therefore

$$I_{A_l}^{A_u} = I_{1-B_u}^{1-B_l} \sqsubseteq I_{\max\{1-\Sigma_{j\in J} B_j^u, \Sigma_{k\in K} A_k^l\}}^{\min\{1-\Sigma_{j\in J} B_j^l, \Sigma_{k\in K} A_k^u\}}.$$

Remark A formal definition of Interval-valued Probability Measure(IVPM) is proposed in [3].

Proposition 3.2 If m^* is the Lebesgue outer measure on $\mathbb{R}^n$, m_* is the Lebesgue inner measure on $\mathbb{R}^n$,for every subset S of $\mathbb{R}^n$, $\mu(S) = I_{m_*(S)}^{m^*(S)}$,then μ is an interval measure on $\mathbb{R}^n$.

Proof (1)-(2)and (4) are easy to prove. (3) is proved by Max Shiffman.

Proposition 3.3 If $\overline{d}$ is the upper asymptotic density on $\mathbb{N}$, $\underline{d}$ is the lower asymptotic density on $\mathbb{N}$,for every subset A of $\mathbb{N}$, $\mathbb{D}(A) = I_{\underline{d}(A)}^{\overline{d}(A)}$,then $\mathbb{D}$ is a finitely additive ideal probability measure on $\mathbb{N}$.

Proof It is easy to pove (1)-(2).

(3)This follows from elementary properties from upper and lower limits.

(4) For all finite collections $\{A_i\}_{i=1}^m$ of pairwise disjoint sets in $\mathcal{A}$, $\underline{\lim}_{n\to\infty} \frac{\cup_{i=1}^m A_i(n)}{n} = \underline{\lim}_{n\to\infty}(\frac{A_1(n)}{n} + \frac{A_1(n)}{n} + \cdots + \frac{A_m(n)}{n}) \geq \underline{\lim}_{n\to\infty}\frac{A_1(n)}{n} + \underline{\lim}_{n\to\infty}\frac{A_1(n)}{n} + \cdots + \underline{\lim}_{n\to\infty}\frac{A_m(n)}{n}$. $\overline{\lim}_{n\to\infty} \frac{\cup_{i=1}^m A_i(n)}{n} = \overline{\lim}_{n\to\infty}(\frac{A_1(n)}{n} + \frac{A_1(n)}{n} + \cdots + \frac{A_m(n)}{n}) \leq \overline{\lim}_{n\to\infty}\frac{A_1(n)}{n} + \overline{\lim}_{n\to\infty}\frac{A_1(n)}{n} + \cdots + \overline{\lim}_{n\to\infty}\frac{A_m(n)}{n}$.

Theorem 3.4 Let X be a set,$\mathcal{A}$ be a σ- algebra over X, $\{f_i \mid i \in K\}$ is a set of some probability measures on $(X, \mathcal{A})$, $\{f_i(A)|i \in K\}$ is an interval, $f_K^-(A) = \min\{f_i(A)|i \in K\}$,$f_K^+(A) = \max\{f_i(A)|i \in K\}$. $\nu(A) = I_{f_K^-(A)}^{f_K^+(A)}$,then ν is an interval probability measure on $(X, \mathcal{A})$.

Proof The proof of (1),(2)and (5) is easy.(1)$\nu(\varnothing) = I_0^0$,(2)$\nu(A) \geq I_0^0$,(5)$\nu(X) = I_1^1$.

(3) Case1: $f_K^-(A) + f_K^+(B) \leq f_K^+(A) + f_K^-(B)$,we have

$$\max\{f_i(A)|i \in K\} + \min\{f_i(B)|i \in K\} \leq \max\{f_i(A)|i \in K\} + \max\{f_i(B)|i \in K\} = \max\{f_i(A \cup B)|i \in K\}.$$

$$\min\{f_i(A \cup B)|i \in K\} = \min\{f_i(A)|i \in K\} + \min\{f_i(B)|i \in K\} \leq \min\{f_i(A)|i \in K\} + \max\{f_i(B)|i \in K\}.$$

So $I_{f_K^-(A)+f_K^+(B)}^{f_K^+(A)+f_K^-(B)} \sqsubseteq I_{f_K^-(A\cup B)}^{f_K^+(A\cup B)}$.

Case2: $f_K^+(A) + f_K^-(B) \leq f_K^-(A) + f_K^+(B)$,we have

$$\min\{f_i(A)|i \in K\} + \max\{f_i(B)|i \in K\} \leq \max\{f_i(A)|i \in K\} + \max\{f_i(B)|i \in K\} = \max\{f_i(A \cup B)|i \in K\}.$$

$$\min\{f_i(A \cup B)|i \in K\} = \min\{f_i(A)|i \in K\} + \min\{f_i(B)|i \in K\} \leq \max\{f_i(A)|i \in K\} + \min\{f_i(B)|i \in K\}.$$

So $I_{f_K^+(A)+f_K^-(B)}^{f_K^-(A)+f_K^+(B)} \sqsubseteq I_{f_K^-(A\cup B)}^{f_K^+(A\cup B)}$.

(4)For all countable collections $\{A_i\}_{i=1}^\infty$ of pairwise disjoint sets in $\mathcal{A}$. $\nu(\cup_{i=1}^\infty A_i) = I_{f_K^-(\cup_{i=1}^\infty A_i)}^{f_K^+(\cup_{i=1}^\infty A_i)}$, $\Sigma_{i=1}^\infty \nu(A_i) = \Sigma_{i=1}^\infty I_{f_K^-(A_i)}^{f_K^+(A_i)}$. It is assumed that $f_K^-(\cup_{i=1}^\infty A_i) = f_a(\cup_{i=1}^\infty A_i) = \Sigma_{i=1}^\infty f_a(A_i), a \in K$. So$\Sigma_{i=1}^\infty f_a(A_i) \geq \Sigma_{i=1}^\infty f_K^-(A_i)$. It is assumed that $f_K^+(\cup_{i=1}^\infty A_i) = f_b(\cup_{i=1}^\infty A_i) = \Sigma_{i=1}^\infty f_b(A_i), b \in K$.So $\Sigma_{i=1}^\infty f_b(A_i) \leq \Sigma_{i=1}^\infty f_K^+(A_i)$. Thus,$I_{f_K^-(\cup_{i=1}^\infty A_i)}^{f_K^+(\cup_{i=1}^\infty A_i)} \sqsubseteq I_{\Sigma_{i=1}^\infty f_K^-(A_i)}^{\Sigma_{i=1}^\infty f_K^+(A_i)}$,namely,$\nu(\cup_{i=1}^\infty A_i) \sqsubseteq \Sigma_{i=1}^\infty \nu(A_i)$.

Definition 3.4 Let X be a set,$\mathcal{A} \subseteq \mathcal{P}(X)$,$\mathcal{A}$ be a algebra ($\sigma-$algebra or Complete Boolean algebra) over X,a mapping $\nu : \mathcal{A} \to \mathbb{M}^*$ be an *interval measure* on X, a mapping $\mathcal{F} : \mathcal{A} \to \mathbb{R}^*$ is called a *measurement* on X if it satisfies the following properties (1) (2)

(1) $\mathcal{F}(A) \in \nu(A), \mathcal{F}(B) \in \nu(B), \mathcal{F}(A \cup B) \in \nu(A \cup B)$;

(2) $\mathcal{F}(A \cup B) = \mathcal{F}(A) + \mathcal{F}(B)(A \cap B = \varnothing)$;

Theorem 3.5 $\nu(A) = \{\mathcal{F}(A)|\mathcal{F}$ is a measurement$\}$

Proof Let $\nu(A) = I_{x_1}^{y_1}$, $\nu(B) = I_{x_2}^{y_2}$.

(1)$\nu(A \cup B) \sqsubseteq \nu(A) + \nu(B)$, so $\forall z \in \nu(A \cup B) \exists x \in \nu(A) \exists y \in \nu(B)(x + y = z)$

(2) If $x_1 + y_2 \leq x_2 + y_1$, then $I_{x_1+y_2}^{x_2+y_1} \sqsubseteq \nu(A \cup B) \sqsubseteq I_{x_1}^{y_1} + I_{x_2}^{y_2}$.

Case 1: $\nu(A \cup B)$ is equal to $I_{x_1}^{y_1} + I_{x_2}^{y_2}$, $\nu(A \cup B) - \nu(B) = I_{x_1}^{y_1} + I_{x_2}^{y_2} - I_{x_2}^{y_2} = I_{x_1+x_2}^{y_1+y_2} - I_{x_2}^{y_2} = I_{x_1+x_2-y_2}^{y_1+y_2-x_2}$,so $\nu(A) \sqsubseteq \nu(A \cup B) - \nu(B)$.

Case 2: $\nu(A \cup B)$ is equal to $I_{x_1+y_2}^{x_2+y_1}$,$\nu(A \cup B) - \nu(B) = I_{x_1+y_2}^{x_2+y_1} - I_{x_2}^{y_2} = I_{x_1+y_2-y_2}^{x_2+y_1-x_2} = I_{x_1}^{y_1}$,we have $I_{x_1}^{y_1} \sqsubseteq I_{x_1}^{y_1}$,so $\nu(A) \sqsubseteq \nu(A \cup B) - \nu(B)$.

If $x_2 + y_1 \leq x_1 + y_2$, then $I_{x_2+y_1}^{x_1+y_2} \sqsubseteq \nu(A \cup B) \sqsubseteq I_{x_1}^{y_1} + I_{x_2}^{y_2}$.

Case 1: $\nu(A \cup B)$ is equal to $I_{x_1}^{y_1} + I_{x_2}^{y_2}$, $\nu(A \cup B) - \nu(B) = I_{x_1}^{y_1} + I_{x_2}^{y_2} - I_{x_2}^{y_2} = I_{x_1+x_2}^{y_1+y_2} - I_{x_2}^{y_2} = I_{x_1+x_2-y_2}^{y_1+y_2-x_2}$,so $\nu(A) \sqsubseteq \nu(A \cup B) - \nu(B)$.

Case 2: $\nu(A \cup B)$ is equal to $I_{x_2+y_1}^{x_1+y_2}$,$\nu(A \cup B) - \nu(B) = I_{x_2+y_1}^{x_1+y_2} - I_{x_2}^{y_2} = I_{x_2+y_1-y_2}^{x_1+y_2-x_2}$,we have $I_{x_1}^{y_1} \sqsubseteq I_{x_2+y_1-y_2}^{x_1+y_2-x_2}$,so $\nu(A) \sqsubseteq \nu(A \cup B) - \nu(B)$.

so $\forall x \in \nu(A) \exists z \in \nu(A \cup B) \exists y \in \nu(B)(x = z - y)$

(3) We have $\forall y \in \nu(B) \exists z \in \nu(A \cup B) \exists x \in \nu(A)(y = z - x)$,this proof is similar to the proof of (2).

So $\nu(A) = \{\mathcal{F}(A)|\mathcal{F}$ is a measurement$\}$

Remark Physical measurement is the selection of a measure from an interval valued probability measure, which is the theoretical foundation for quantum measurements.

4. The Foundation of Non-Measurability Theory

Lemma 4.1 Properties of non-measurable sets(Refer to the real variable function textbook):

(1) Non-measurable sets have no definite measure.

(2) If the intersection of a non-measurable set and a measurable set is an empty set, then the union of the non-measurable set and the measurable set is non-measurable.

(3) A non-measurable set is neither a continuous (connected) nor a discrete set.

(4) The complement of a non-measurable set is non-measurable.

(5) The cardinality of a non-measurable set is uncountable.

(6) (The Shiffman Property)If$A \bigcap B = \emptyset, \nu(A) = I_{x_1}^{y_1}, \nu(B) = I_{x_2}^{y_2}$, $\min\{x_1 + y_2, x_2 + y_1\} = \bot$, $\max\{x_1 + y_2, x_2 + y_1\} = \top$, then $I_{\bot}^{\top} \sqsubseteq \nu(A \cup B)$.

(7) (Finite covering property) For all finite collections $\{A_i\}_{i=1}^n$ of pairwise disjoint sets in $\mathcal{A}$, $\nu(\cup_{i=1}^n A_i) \sqsubseteq \Sigma_{i=1}^n \nu(A_i)$.

(8) (Separation invariance) If $d(A, B) > 0$, then $\nu(A \cup B) = \nu(A) + \nu(B)$.

(9) Various versions of Banach-Tarski Paradox [27].

Definition 4.1 If the mass of a matter is a real number, then the matter is called *measurable matter*. If the mass of a matter is a non-degenerate interval number, then the matter is called *nonmeasurable matter*. If A and B have different masses , A and B are equidecomposable, then we call this phenomenon *the physical manifestation of the Banach-Tarski Paradox.*

Remark Photon,electron, moun,tau are measurable particles. ***Higgs, z boson, w boson, quarks, neutrinos, gravitons, dark matter,black holes, the universe, dark energy gluons,colored particles, magnetic monopoles, Bose-Einstein condensate are most likely non-measurable.*** The expansion of the universe, the star collapses into a black hole, neutrino oscillations, parity nonconservation, and so on are all physical manifestations of the Banach-Tarski paradox.

Property 4.1 The nonmeasurable matter has the properties of three-dimensional nonmeasurable sets.Such as

(1) The mass of a non-measurable matter is a nondegenerate interval number,the value of the mass is dependent on the measurement.

(2) The non-measurable particles have the property of oscillation (the reactants depend entirely on the relative positions of the nonmeasurable particles). For example, neutrinos have a property called neutrino oscillation.In other words,the nonmeasurable matter has the physical manifestation of the Banach-Tarski Paradox.

(3) Non-measurable particles and non-measurable antiparticles behave differently, such as neutrinos and antineutrinos [28], quarks and anti-quarks [29]. Although non-measurable particles and non-measurable antiparticles have the same mass and shape, they have different spins,which causes them to behave differently.This is also a physical manifestation of the Banach-Tarski Paradox.

(4)The non-measurable matters cannot be measured directly, they are detected by indirect measurements. For example, neutrinos, Higgs, etc. are indirectly proved to exist through the products of interaction. its shape is complex and full of wormholes.

(5)The non-measurable matter has a strong ability to penetrate. For example,neutrinos pass through matter easily.

(6)The non-measurable matter is neither continuous nor discrete.

(7)(Unilateral Gravity) Non-measurable matter exhibits unilateral gravity: it generates gravitational fields and curves spacetime, but does not respond to external gravity.

Remark The non-measurable theory is based on two principles, one of which is that dark energy is non-measurable, and the other is that all matter is composed of dark energy.All the conclusions of this paper can be proved on the two principles. the two principles are supported by experimental data, as are the conclusions derived from them.

If you want to talk about the properties of dark energy, then you have to make sure that dark energy exists [5,6]. It is an accepted view in modern physics that fields exist. *We believe that dark energy is the fundamental field in the universe,the dark energy field is non-measurable,other fields are first excitation of the dark energy field, and all particles are second excitation of the dark energy field.*

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Fundamental 4.1(Non-measurability) `Dark energy is non-measurable.`

(Dark energy is a field, it is non-measurable.)

The equation-of-state parameter governs the rate at which the dark energy density evolves. For a perfect, unchanging vacuum energy, we have $w = -1$. Current experimental bounds tell us that $w = -1$ is the central preferred value. Riess team got this statement in the paper [15]: This [20] resulted in $H_0 = 73.8 \pm 2.4 kms^{-1}Mpc^{-1}$, which coupled with the 7-year WMAP results [21] yielded $w = -1.08 \pm 0.10$, or an estimate of the effective number of relativistic species of $Neff = 4.2 \pm 0.7$. Dark energy seems to work in a different way than anything previously observed. The density of anything is the amount of matter in a given volume, and dark energy is an unusual phenomenon because even as the volume of the universe increases with expansion, the density stays the same. As the universe expands, new dark energy is being created all the time, meaning its density remains the same. You can think of it this way, when you get more space, you actually get more dark energy, it's like getting something for nothing. You can explain this if you assume that dark energy is nonmeasurable.

The Banach-Tarski paradox was first proposed by Banach and Tarski in 1924. This theorem states that if the axiom of choice holds, a three-dimensional solid sphere can be divided into finite (non-measurable) parts and then formed into two complete spheres of the same radius simply by rotating and moving them elsewhere.The process can be represented by the following formula $S \to S + S$. A stronger version of this paradox is that any two subsets in a three-dimensional Euclidean space with a non-empty interior are equidecomposable. Dark energy is constantly recombining and decaying, so that more and more dark energy can be obtained.

Fundamental 4.2 (Universality) `All matter and fields are excitation of dark energy.`

(The dark energy field is the fundamental field in the universe, other fields are excitations of the dark energy field.Such as, matter interacts with dark energy to produce gravitational field, electrons interact with dark energy to produce electric field, changing electric field interacts with dark energy to produce changing magnetic field, changing magnetic field interacts with dark energy to produce changing electric field, dark matter is another excitation of dark energy.)

Heisenberg worked on a unified field theory of elementary particles [43]. All the elementary particles are made of the same substance, which we may call energy or universal matter ;they are just different forms in which matter can appear. Heisenberg wrote in his book [44]. If the different parts of the universe were not as the organs of the same body, they would not react one upon the other; they would mutually ignore each other, and we in particular should only know one part. We need not, therefore, ask if Nature is one, but how she is one. Poincare wrote in his book [45]. Einstein attempted to construct a unified field theory in which electromagnetism and gravity would emerge as different aspects of a single fundamental field.

According to QFT,there are many quantum fields in the universe, particles are excitation of their respective fields.A key open question in fundamental physics is the cosmological constant problem.Experimental measurements show that the cosmological constant is extremely small, in contrast with the theoretical prediction, which is some 120 orders of magnitude larger.We can solve this problem by assuming that the existence of more fundamental fields,the vacuum state has only a dark energy field and no other fields or particles.If dark energy is the fundamental field, then the dark energy field would unify the four interactions, and the matter and field as we know it today are just different manifestations of dark energy.

References [22–24] provide strong evidence for this hypothesis. Matter is converted into vacuum energy (dark energy), as the matter falls through the event horizon.

Quantum fluctuations also provide strong evidence for this hypothesis.Our universe is a quantum fluctuation of vacuum [36,37].

The team [38] determined that matter makes up about 31.5 percent of the total amount of matter and energy in the universe, and dark energy makes up 68.5 percent. About 20 percent of total matter is made up of regular matter, which includes stars, galaxies, atoms and life, while about 80 percent is made up of dark matter, whose mysterious nature is unknown but which may consist of subatomic

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particles that have yet to be discovered. Everything in the universe is made up of dark energy, and dark energy itself is nonmeasurable, so from the point of view of probability, most matter should be nonmeasurable, and only a few can form measurable matter, which can also explain that most things in the universe are unknown, and almost everything in the universe is dark matter and dark energy.

"*we ought in fact to suppose it, if we can thereby obtain a simpler explanation of phenomena*". Riemann said in his famous paper [26].

Remark Dark energy is the most elementary object in the universe.Dark energy itself is nonmeasurable,dark energy can produce more dark energy,some dark energy forms measurable matter, and some forms nonmeasurable matter.

Corollary 4.3 There is non-measurable matter in the universe.

Remark Much of the matter in the universe may be non-measurable, and we provide some examples for Corollary 4.1 below.

Table 1

non-measurable matter	**measurable matter**
Higgs	photon,
Z boson	electron
W boson	moun
quark	tau
neutrino	neutron
graviton	proton
gluon	atom
magnetic monopole	molecule
dark matter	planet
black hole	star
Bose-Einstein condensate	neutron star
fractionated electrons	comet
anyon	satellite
⋮	⋮

Remark Table 1 describes the possible non-measurable matter in the universe.

Evidence 4.1 Quarks are non-measurable particles.

(1)The mass of a quark depends on how you measure it,a quark has different masses at the same time.Because quarks can never be free, the mass of quarks can only be represented by it in some kind of interaction. So the mass of quarks is not unique, but depends on the way they are confined. Any quark mass should be footmarked to indicate the method by which the value of that mass was obtained.

(2)The fact that a system can have different potential energies (masses) is a physical manifestation of the Banach-Tarski Paradox.The mass comes from the interaction, of which 99 percent comes from the strong interaction. If the relative position between quarks and gluons changes, then the mass changes, which is the physical manifestation of the Banach-Tarski Paradox. Quarks and gluons are non-measurable particles, so we think of quarks and gluons as non-measurable sets, and if the relative positions of the non-measurable sets change, then the measure of the whole changes. For example, protons and $\Delta+$ particles have the same composition, but their masses are not the same, which can be explained by the Banach-Tarski Paradox.

(3)Quarks and anti-quarks behave differently. [28]

(4) B. W. Augenstein [14] draws an extensive analogy between quarks and non-measurable sets.

Evidence 4.2 Neutrinos are non-measurable particles.

(1) We look at neutrinos differently, and we assume that there is only one type of neutrino, just as there is only one type of photon, which explains a lot of strange things about neutrinos. Then a neutrino has different masses at the same time,its mass depends on the measurement.

(2) "Oscillations" [17–19] refer to the way neutrinos change flavor as they travel. A neutrino born as one flavor (electron, muon, or tau neutrino) will eventually morph into the other varieties-and the

probability of appearing as a different flavor depends on how far it has gone.Particles are objects that can move, rotate and boost. The reactant depends on where you messasure it,Neutrinos constantly change their position relative to the detector as they travel.Neutrinos do not change as they travel, which is the physical manifestation of the Banach-Tarski Paradox.The three types of neutrinos are different manifestations of the same non-measurable particle.

(3) Beta decay is another important piece of evidence that neutrinos are non-measurable particles. β decay is the most familiar weak interaction, which is the interaction of nucleus, neutrinos and electrons.Neutrinos are non-measurable particles, and the products of their reactions depend on the relative positions of nucleus, neutrinos and electrons, so parity is not conserved in weak interaction.The non-measurability of neutrinos has led to all sorts of strange experimental phenomena about them.The θ-τ puzzle is the different decay modes of the same particle. In other words, the θ-τ puzzle is the physical manifestation of the Banach-Tarski Paradox.The different spins of cobalt-60 [31] lead to different combinations, which lead to different reactants.

(4) Neutrinos and antineutrinos behave differently [29]. Neutrinos and antineutrinos are also different representations of the same particle [35], and they behave differently for non-measurable particles with different spins, which is the physical manifestation of the Banach-Tarski paradox.In other words, a particle with a left spin is a neutrino, and the same particle with a right spin is an antineutrino. Neutrinos and antineutrinos are just different behaviors of the same particle, and because neutrinos are non-measurable, they behave differently.

Evidence 4.3 The energy of the gravitational field is non-measurable.

The energy of the gravitational field cannot be locally defined. In general relativity, there is no covariant stress-energy tensor for the gravitational field because gravity itself is geometrized (as space-time curvature). Instead, the energy of gravity must be described through pseudotensors (such as the Einstein pseudotensor or the Landau-Lifshitz pseudotensor), which depend on the choice of coordinate system. In a local inertial frame (e.g., for a freely falling observer), these pseudotensors vanish, causing the gravitational energy to “disappear,” whereas in non-inertial frames (e.g., for a stationary observer), the pseudotensors are non-zero. In terms of measurability, the energy of the gravitational field is fundamentally non-measurable, as its specific value depends on the chosen measurement framework. This constitutes direct theoretical evidence for the intrinsic non-measurability of the gravitational field itself.

Evidence 4.4 Dark matter is non-measurable.

(1)Theory predicts there must be dark matter in AGC 114905 [16], but our observations suggest there isn’t. Here’s one explanation: Dark matter is non-measurable, its mass is interval valued and depends on how it is measured.

(2) The massive relic galaxy NGC 1277 is dark matter deficient [42]. Here’s one explanation: Dark matter is non-measurable, its mass is interval valued and depends on how it is measured.

(3)Dark matter is extremely difficult for detectors to detect.

Evidence 4.5 Black holes are non-measurable.

(1)A Black hole has different masses at the same time (the ref [25] provides strong evidence for this phenomenon).A star with a specific mass collapses into a black hole with many different masses at the same time. In other words, the star was measurable matter, the star collapsed into a black hole and became non-measurable matter. The matter collapses and splits into a lot of non-measurable matter, which recombines to form black hole.Black holes are likely to be dark energy stars [22],the inside of a black hole has a different cosmological constant than the outside.So,the formation of a black hole is the physical manifestation of the Banach-Tarski Paradox.

(2)If mass growth of black holes only occurred through accretion or merger, then the masses of these black holes would not be expected to change much at all.But the gravitating mass of a black hole can increa se with the expansion of the universe independently of accretion or mergers.So, black holes are most likely dark energy stars, and the non-measurability of dark energy could explain the mystery of black hole mass growth. [32,33] provide the first observational evidence linking black holes to dark

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energy.This paper provided a theoretical explanation for mass growth of black holes. A black hole is dark energy, dark energy is constantly decaying and recombining to produce more dark energy, which is the physical manifestation of the Banach-Tarski Paradox.

Evidence 4.6 The universe is non-measurable.

Is the expansion of the Universe accelerating? All signs point to yes [33]. The expansion of the universe was propelled by a repulsive gravitational force generated by an exotic form of matter.Here, the expansion of the universe is nothing more than a physical manifestation of the Banach - Tarski paradox. Viewing the universe as a non-measurable set, the decomposition and reorganization of its non-measurable subsets could drive cosmic expansion. Galaxies within it may be regarded as measurable components, where the distance between two galaxies changes over time. When the rate of such replication increases, it could explain the accelerated expansion of the universe.

Evidence 4.7 W boson is non-measurable.

The mass of the W boson is dependent on the measurement method [39,40]. The Fermilab Tevatron collider is a proton-antiproton collider,the Large Hadron Collider (LHC) is a proton-proton collider,different measurement methods get different results,the two measurements are incompatible.The measurement [39] of CDF II detector is in tension with the prediction of the standard model.

Evidence 4.8 H boson is non-measurable.

The mass of the H boson is dependent on the measurement method. "As shown in Table V, the measured values of the Higgs boson mass for the $H \to \gamma\gamma$ and $H \to ZZ^* \to 4\iota$channels are $125.98 \pm 0.42(stat) \pm 0.28(syst) GeV$ and $124.51 \pm 0.52(stat) \pm 0.06(syst)$ GeV, respectively. The compatibility between the mass measurements from the two individual channels is at the level of 2.0σ corresponding to a probability of 4.8%." [41].

Evidence 4.9 Bose-Einstein condensate is non-measurable.

The superconducting state of electrons and the superfluid state of helium-4 can have a very strong ability to penetrate matter, which indicates that the Bose-Einstein condensate is very likely to be non-measurable matter.

In 1957, J. Bardeen, L. Cooper, J.R. Schrieffer (BCS) wrote down a wave function that they showed could accurately describe all the properties of superconductors. However, it is disturbing that this wave function does not have U(1) symmetry, and more specifically, the wave function does not have a single fixed number of electrons, but rather a combination of states with different numbers of electrons. The absence of a definite number of electrons means that the number of electrons is non-measurable, which is a theoretical proof that the Bose-Einstein condensate of electrons is non-measurable.

Evidence 4.10 Fractionated electrons is non-measurable.

The evidence that electrons can recombine appears in the fractional quantum Hall effect. The fractional quantum Hall effect allows for the ballistic transport of fractional electrons, which is not allowed in known theories, but in our theory, this phenomenon has the same interpretation as the superconductivity mechanism: Electrons are composed of non-measurable particles, which can be recombined into a new particle at high magnetic and low temperatures, so that fractional electrons can appear.

"Recently, however, interest in fractionated electrons has exploded, because it turns out that they have a kind of collective memory. To put this more concretely: After you move them around one another, their subsequent behavior reliably reflects how you treated them. Because of this "memory," fractional electrons-known as anyons-are promising ingredients for building up and storing quantum information, and ultimately for making quantum computer".Frank Wilczek mentioned in an article [47]. The phenomenon is most likely the physical manifestation of the Banach-Tarski Paradox.If fractionated electrons are non-measurable, then the phenomenon can be explained by Banach-Tarski Paradox.

Evidence 4.11 Anyons are non-measurable.

"In a series of experiments, the researchers at Google observed the behavior of these non-Abelian anyons and how they interacted with the more mundane Abelian anyons. Weaving the two types of particles around one another yielded bizarre phenomena–particles mysteriously disappeared,

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reappeared and shapeshifted from one type to another as they wound around one another and collided. Most importantly, the team observed the hallmark of non-Abelian anyons: when two of them were swapped, it caused a measurable change in the quantum state of their system- a striking phenomenon that had never been observed before" [48]. The phenomenon is most likely the physical manifestation of Banach-Tarski Paradox.If anyons are non-measurable, then the phenomenon can be explained by Banach-Tarski Paradox.

Corollary 4.4 Energy is not conserved.

If there is non-measurable matter, then energy is not conserved, because the total energy of the non-measurable matter depends on how the parts of it are put together. We now extend the energy value from real values to interval values, the interval energy value of an object depends on the way all the parts of the object are put together, so the energy is no longer conserved on interval values.The law of energy change is described by the axiom of interval value measure,which is also a generalization of the law of energy conservation.According to the axiom of interval measures, if there is no decomposition-recombination involved, it appears that energy is conserved, but if there is decomposition-recombination involved, Banach-Tarski paradox phenomenon occurs, and energy is not conserved.

Corollary 4.5 Information entering the black hole is lost.

"Energy is not conserved if and only if information entering the black hole is lost ", which comes from an article by Leonard Susskind.Because dark energy is non-measurable,it means that energy is no longer conserved. So the reversibility of information no longer holds.

A framework for problem solving 1 (The mystery of the missing antimatter:Banach-Tarski mechanism) The following Banach-Tarski mechanism may provide an answer to the mystery of the disappearance of antimatter. Let us consider a non-measurable particle of this type, whose particle and antiparticle are the same particle. According to the Banach-Tarski paradox, different combinations of non-measurable particles may produce different products. In experiments [28], the researchers observed that measurements of the oscillations of experimental neutrinos and antineutrinos showed that neutrinos have a higher probability of oscillating than antineutrinos, which means that more matter is produced.This means that the answer to the puzzle is hidden in a non-measurable particle, which is the same particle as the antiparticle.Dark matter may have such particles in abundance that matter and antimatter disappear, leaving a surplus of matter.

A framework for problem solving2(The non-measurability interpretation of mass) We knew that gauge field mathematics was beautiful, but there was a flaw: the gauge particles associated with it could only have a mass of zero, which would result in all the elementary particles in the Standard Model having a mass of zero. In non-measurability theory, the W boson, Z boson and gluon are all non-measurable, the mass values of these elementary particles have the interval valued form with [b, a]$(a > b)$,and the mass of these particles depends on the measurement.

The fact that a system can have different potential energies (masses) is a physical manifestation of the Banach-Tarski Paradox.If the relative position of the system material changes, then the potential energy will change, this is due to the existence of a large number of non-measurable objets in the system,if the relative position of non-measurable objects changes,then the mass of the system will change.

A framework for problem solving3(Big Bang Puzzle): Instead of introducing an inflation field, we can use the properties of dark energy to solve this problem. Dark energy has the properties of a non-measurable set (including the Banach-Tarski paradox). At the beginning of the universe, there was only dark energy in the universe, which had a very high temperature and pressure, and violent interactions among various subsets of dark energy. Dark energy was decomposed and recombined at a very high rate, and a lot of dark energy was copied, resulting in a rapid increase in the density of the universe. In order to ensure that the density of dark energy remained constant at interval values, the universe needed exponential expansion. Since replication is preserving the original properties, this explains the high flatness and temperature uniformity of the universe.

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5. On the Hypotheses Which Lie at the Bases of Geometry

“Still more complicated relations may exist if we no longer suppose the linear element expressible as the square root of a quadric differential. Now it seems that the empirical notions on which the metrical determinations of space are founded, the notion of a solid body and of a ray of light, cease to be valid for the infinitely small. We are therefore quite at liberty to suppose that the metric relations of space in the infinitely small do not conform to the hypotheses of geometry; and we ought in fact to suppose it, if we can thereby obtain a simpler explanation of phenomena".Riemann said in his famous paper [26].

“In fact we go even further and claim that even for an entangled pair of particles, in a quantum theory of gravity there must be a Planckian bridge between them, albeit a very quantummechanical bridge which probably cannot be described by **classical geometry**" [46]. In a very interesting attempt to connect the geometry of space-time and fundamental properties of quantum mechanics Maldacena and Susskind proposed the so-called ER=EPR conjecture.It is highly likely that the new geometry we are discussing may give an explanation to ER=EPR.

By adding quantum intertextuality to gravitational theory and geometry, we can reach quantum gravity and quantum geometry.We know that Riemannian geometry is a mathematical tool for general relativity, but it cannot explain the phenomena at the Planck scale. We need to generalize Riemannian geometry, but the generalization must explain quantum mechanical phenomena. Quantum contextuality is a principle of quantum mechanics, which states that the values of some physical quantities are uncertain and depend on the measurement methods. To unify general relativity and quantum mechanics, introducing quantum contextuality into general relativity is a worthwhile idea.We assume that there is a quantum matter that has many mass values, depending on how it is measured. This matter we call non-measurable matter,so we have introduced quantum contextuality into the gravity theory, making general relativity and quantum mechanics organically merged together. This hypothesis also explains many strange phenomena in particle physics and has received some experimental support. But we can also deduce the existence of non-measurable matter through the properties of two dark energies. We take these two assumptions as the principles of this theory, which naturally leads to the conclusion that space-time is non-measurable, and physical quantities about space-time also have quantum contextuality, such as distance, volume, angle and other geometric concepts that depend on measurement methods.This is a possible way to unify general relativity and quantum mechanics.

The following are deductive conclusions of the two fundamentals(non-measurability and universality of dark energy), and these conclusions are supported by cosmological data and modern quantum mechanical experiments.

Definition 5.1 Let$I_i \subseteq R$.$\{f(x, a_1, \cdots, a_n) = 0 | a_1 \in I_1, \cdots, a_n \in I_n\}$ is called an interval equation on R. $\{x \in R \mid \exists a_1 \in I_1 \cdots \exists a_n \in I_n(f(x, a_1, \cdots, a_n) = 0)\}$ is called the solution set of the interval equation.

Remark The following equations can be regarded as interval equations.

Corollary 5.1 The cosmological constant Λ is a non-degenerate interval constant.

Proof $\{\rho_\Lambda = \frac{c^4}{8\pi G}\Lambda | \rho_\Lambda \in I\}$. Dark energy is nonmeasurable, so I is a nondegenerate interval constant. c and G are real constat.Therefore, cosmological constant is a nondegenerate interval constant.

Remark The conclusion is the starting point for quantum gravity.

In order to establish a rigorous foundation for the theory of gravity, we need to generalize Riemann geometry,we use interval tensor theory to establish interval Riemannian geometry.So,the theory of gravity moves from scalar theory to tensor theory and finally to interval tensor theory.

The set of some probability measures forms a mathematical structure (Theorem 3.4).Similarly,the set of some Riemannian manifoles forms a mathematical object, and the set of Riemannian metric on these manifoles forms an interval tensor. We call such a set of structures an interval manifold, and the set of Riemannian measures an interval Riemannian metric.

Definition 5.2 Let **A** be a cluster of order-n tensors, a_{ij} be the component of a($a \in \mathbf{A}$). If$\{a_{ij}|a \in \mathbf{A}\}$ is an interval number, then **A** is called a *generalized interval tensor*.If **A** has only one element, it is called *a degenerate generalized interval tensor*, otherwise it is called *a nondegenerate generalized interval tensor*.

Remak Here the concept of generalized interval tensor is a generalization of the concept of interval tensor in ref [7,8]. Sometimes, generalized interval tensors are simply written as interval tensors,but the meaning of the interval tensor here is different from the reference [7,8].

Definition 5.3 Let T be a cluster of Riemannian manifolds on the smooth manifold M, G be the cluster of Riemannian metrics in T . If the components of the Riemannian metrics g of T form a interval tensor at each point, then we call $\{\langle M, g\rangle|g \in G\}$ an interval Riemannian manifold.

If the components of the Riemannian metrics of T form a degenerate interval tensor at each point of M, and we call $\{\langle M, g\rangle|g \in G\}$ a measurable manifold(Riemannian manifold).Otherwise,we call $\{\langle M, g\rangle|g \in G\}$ a nonmeasurable manifold.

Using this interval metric tensor, we can calculate the length of the curve, the distance between two points, and curvature. In the degenerate case, the interval metric tensor is exactly the Riemannian metric. In the non-degenerate case, the value of a geometric quantity defined by ds may be a nondegenerate interval number.

The above concepts can be extended to pseudo-Riemannian manifolds.

According to Einstein's vacuum field equation $R_{\mu\nu} = \Lambda g_{\mu\nu}$ and Corollary 5.1, we naturally come to the following conclusion:

Corollary 5.2 The de Sitter metric $g_{\mu\nu}$ is a nondegenerate interval metric.

Proof We use the inflationary coordinate system, $ds^2 = -dt^2 + a^2(t)[dx^2 + dy^2 + dz^2]$(Where $a(t) = \exp(l^{-1}t)$,$\Lambda = 3l^{-2}$),. Λ is a nondegenerate interval constant, $\exp(2l^{-1}t)$ is a monotone function of l, so the metric $g_{\mu\nu}$ is a nondegenerate interval metric.

Theorem 5.3 All geometric quantities in de Sitter space are interval-valued.

Proof Let the cosmological constant Λ take all values in a closed interval $[\Lambda_1, \Lambda_2]$, where $\Lambda_1 > 0$. We aim to prove that all geometric quantities of de Sitter space (e.g., curvature tensors, scalar curvature, geodesic distances, volume elements) are interval-valued, i.e., their values span closed intervals determined by $\Lambda \in [\Lambda_1, \Lambda_2]$.

1. Preliminaries: Geometry of de Sitter Space De Sitter space is a maximally symmetric solution of Einstein's field equations with a positive cosmological constant Λ. Its metric in flat slicing coordinates is: $ds^2 = -dt^2 + e^{2Ht}(dx^2 + dy^2 + dz^2)$, $H = \sqrt{\frac{\Lambda}{3}}$. Key geometric quantities include: Curvature radius: $L = \sqrt{3/\Lambda}$, Hubble parameter: $H = \sqrt{\Lambda/3}$, Ricci tensor: $R_{\mu\nu} = \Lambda g_{\mu\nu}$, Scalar curvature: $R = 4\Lambda$, Geodesic distances: Derived from the metric.

2. Continuity of Geometric Quantities with Respect to Λ

Theorem (Extreme Value Theorem): If $f : [a, b] \to \mathbb{R}$ is continuous, then f attains its minimum and maximum on $[a, b]$, and its range is the closed interval $[f_{\min}, f\max]$.

All geometric quantities in de Sitter space are continuous functions ofΛ. Since $\Lambda \in [\Lambda_1, \Lambda_2]$ is a closed interval, the range of each quantity is also a closed interval.

2.1 Curvature Radius L: $L(\Lambda) = \sqrt{\frac{3}{\Lambda}}$. Continuity: $L(\Lambda)$ is continuous for $\Lambda > 0$. Monotonicity: $L(\Lambda)$ is strictly decreasing in Λ. Interval Value: For $\Lambda \in [\Lambda_1, \Lambda_2]$: $L \in \left[\sqrt{\frac{3}{\Lambda_2}}, \sqrt{\frac{3}{\Lambda_1}}\right]$.

2.2 Hubble Parameter H: $H(\Lambda) = \sqrt{\frac{\Lambda}{3}}$. Continuity:$H(\Lambda)$ is continuous for $\Lambda \geq 0$. Monotonicity: $H(\Lambda)$ is strictly increasing inΛ. Interval Value: For$\Lambda \in [\Lambda_1, \Lambda_2]$: $H \in \left[\sqrt{\frac{\Lambda_1}{3}}, \sqrt{\frac{\Lambda_2}{3}}\right]$.

2.3 Scalar Curvature R: $R(\Lambda) = 4\Lambda$. Continuity: Linear function, hence continuous. Monotonicity: Strictly increasing in Λ. Interval Value: For $\Lambda \in [\Lambda_1, \Lambda_2]$: $R \in [4\Lambda_1, 4\Lambda_2]$.

2.4 Ricci Tensor Components $R_{\mu\nu}$: $R_{\mu\nu} = \Lambda g_{\mu\nu}$. Continuity: Components are linear in Λ, hence continuous. Interval Value: For $\Lambda \in [\Lambda_1, \Lambda_2]$, each component satisfies: $R_{\mu\nu} \in [\Lambda_1 g_{\mu\nu}, \Lambda_2 g_{\mu\nu}]$.

2.5 Geodesic Distances: Consider the physical distance between two comoving points in flat slicing coordinates: $D(t) = e^{Ht}D_0, D_0 = \sqrt{(\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2}$. Dependence on Λ: $H = \sqrt{\Lambda/3}$.

Continuity: $D(t)$ is continuous in Λ as $H(\Lambda)$ is continuous. Interval Value: For $\Lambda \in [\Lambda_1, \Lambda_2]$: $D(t) \in \left[D_0 \exp\left(t\sqrt{\frac{\Lambda_1}{3}}\right), D_0 \exp\left(t\sqrt{\frac{\Lambda_2}{3}}\right)\right]$.

2.6 Volume Element:

The spatial volume element in flat slicing coordinates is: $dV = e^{3Ht} dx\,dy\,dz$. Dependence on Λ: $H = \sqrt{\Lambda/3}$. Continuity: Exponential function is continuous in Λ. Interval Value: For $\Lambda \in [\Lambda_1, \Lambda_2]$: $dV \in \left[\exp\left(3t\sqrt{\frac{\Lambda_1}{3}}\right) dx\,dy\,dz, \exp\left(3t\sqrt{\frac{\Lambda_2}{3}}\right) dx\,dy\,dz\right]$.

3. General Case: Arbitrary Geometric Quantity

Let $\mathcal{G}$be any geometric quantity in de Sitter space (e.g., curvature invariants, connection coefficients, Killing vectors). By construction: $\mathcal{G}$ is derived algebraically or differentially from the metric $g_{\mu\nu}$, which depends smoothly on Λ. Smoothness of $g_{\mu\nu}$ in Λ implies $\mathcal{G}$ is continuous in Λ.

By the Extreme Value Theorem, $\mathcal{G}(\Lambda)$ maps the closed interval $[\Lambda_1, \Lambda_2]$ to a closed interval $[\mathcal{G}(\Lambda_1), \mathcal{G}(\Lambda_2)]$. 4. Conclusion:

For de Sitter space with $\Lambda \in [\Lambda_1, \Lambda_2]$: 1. All geometric quantities are continuous functions of Λ. 2. By the Extreme Value Theorem, their ranges are closed intervals. 3. Monotonicity (where applicable) ensures the intervals are strictly ordered.

Thus, all geometric quantities in de Sitter space are interval-valued when Λ is restricted to a closed interval. This result holds universally due to the smooth dependence of the metric on Λ.

Remak This result can be generalized to space-time with matter.

Corollary 5.4 *The probability of a particle appearing in space is interval valued, and the specific value depends on the measurement.*

Experimental evidence of Corollary 5.4 The probability value of particle occurrence in a given measurable region is interval valued, and the probability value is dependent on measurement. This is consistent with interpretation of quantum mechanics.Born's interpretation of Wave function says that the square of the modulus of the wave function is a measure of the probability of finding the particle at that position.In quantum mechanics, we can't ask what is the probability of a particle appearing at that position, the probability depends on the measurement.

The measure of a region of space is non-measurable, so the probability of a particle appearing in a given region is uncertain and dependent on the measurement, which is confirmed by quantum mechanics.In the two-slit interference experiment, the probability of the particle appearing on the screen depends on the measurement.

Corollary 5.5 *The Hubble constant is interval valued, and different methods may obtain different values.*

Experimental evidence of Corollary 5.5 Hubble constant values for several teams can be found in the references of the paper [15] by Riess team. It is not that the measurements are not accurate enough, but that the constant themselves is a non-degenerate interval number, and the constant itself have different values under different measurements.Recently, Riess team [15] "find a 5σ difference with the prediction of H_0 from Planck CMB observations under" CDM, with no indication that the discrepancy arises.

6. Interpreting Quantum Phenomena in Non-Measurable Space-Time

In quantum mechanics, we cannot talk about the probability of a particle appearing in a region of space, only about the probability of a particle being detected in a region of space, because there is no definite probability of a particle before it is measured. Now, we can use interval probability to talk about the probability of a particle appearing in space, and the specific probability value is dependent on the measurement..

Geometric interpretation of quantum phenomena *The concept of non-measurable sets appears in measure theory. If the inner and outer measures of a set are not equal, then the set is non-measurable. Using the axiom of choice we can prove that non-measurable sets exist. The essence of quantum phenomena exists in the two-slit interference experiment. We provide a geometric model for the two-slit interference experiment. Suppose A square hole of area 1, and we divide the square into A and B, and both A and B are non-measurable sets, in* *which case each part does not have a definite measure. A particle passes through this square hole, so now we have to ask what is the probability that this particle passes through A? Obviously,A has no definite measure, so the probability of a particle passing through A is also uncertain. This geometric model can give a reasonable explanation to the two-slit interference experiment. In quantum mechanics, we cannot talk about the probability of a particle appearing in a place, because there is no definite probability, the specific probability value depends on the assembly of the detector, we can only talk about the probability of the particle being detected in this place by the detector. We can just think of A and B as two gaps in the double-slit experiment, because A or B have no definite measure. In addition, the square hole can be divided into two non-measurable parts with extreme complexity, making A and B indistinctly distinguishable, and A and B entangled. In the general discussion, if you think of a particle as a point, the particle actually has a size, so that the particle passes through B when it passes through A, and the particle passes through A when it passes through B, which means that the particle can pass through both A and B at the same time. From the classical point of view, there is a distance between the gaps in the two-slit experiment, but if spacetime is assumed to be nonmeasurable, then spacetime is full of extremely complex holes, and the two gaps can be connected through this wormhole.*

Because slit A is non-measurable, the probability from slit A to a point C on the screen is uncertain, possessing an interval value-in other words, this event is non-measurable. Similarly, the event from slit B to point C is also non-measurable. Therefore, we need to employ interval probability measures (Defination 3.3) to handle the calculations here, analogous to the Banach-Tarski paradox, where the overall volume depends on the relative positions of the non-measurable parts. Here, the probability of reaching point C from the two slits depends on the phase difference. Ultimately, we obtain an interval probability measure (Proposition 6.1), and the measurement by the detector corresponds to selecting a specific probability measure (Defination 6.3) from this interval probability measure.

In this way, this geometric model can also be used to explain quantum entanglement, two entangled particles have a strong coupling effect after separation, the whole has a definite physical quantity, but the part of the physical quantity is uncertain. Particles that appear to be separated are actually connected through wormholes. According to the Banach-Tarski paradox, the measure of the whole depends on the combination of non-measurable parts, and quantum entanglement is exactly the opposite of the Banach-Tarski paradox, quantum entanglement is that the parts of the object move, but the physical quantity of the whole does not change.

Definition 6.1 *(Quantum entanglement) The non-measurable parts of multiple particles recombine to form a new whole, and the quantum property of the whole does not change during the motion of these parts, we say that these particles are quantum-entangled. If the physical quantity of the whole changes when the parts are moved, we say that these parts are independent, and the change is described by the Banach-Tarski Paradox.*

Remark 1 According to the non-measurability theory, particles are composed of non-measurable parts, and the so-called multi-particle quantum entanglement is the recombination of the non-measurable parts of these particles into a whole. Because spacetime is non-measurable, seemingly separated particles are actually connected to each other through non-measurable spacetime, their distances are non-measurable, and each part of the whole moves relative to each other while keeping the physical properties of the whole unchanged.

Remark 2 Quantum entanglement can occur between elementary particles, which means that elementary particles are composed of non-measurable parts.

Remark 3 Measurable matter is made up of non-measurable matter, and measurable matter has fixed physical properties precisely because of the quantum entanglement between the non-measurable parts. The universe is expanding precisely because so much matter in the universe is independent.

Proposition 6.1 Let $\{\Psi_i | i \in Q\}$ be the set of all normalized quantum pure states, and $\{P_i\}$ be a set of non-negative real numbers satisfying $\sum_i P_i = 1$. Then the set of all possible detection probabilities in region A for mixed states:

$$\left\{ \int_A \sum_i P_i |\Psi_i(\mathbf{r},t)|^2 d\mathbf{r} \;\middle|\; \{\Psi_1, P_1; \Psi_2, P_2; \cdots ; \Psi_i, P_i; \cdots\} \text{ is a mixed state} \right\}$$

forms an interval.

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Proof

Let $a_i = \int_A |\Psi_i(\mathbf{r},t)|^2 d\mathbf{r}$, then each $a_i \in [0,1]$. For any mixed state $\rho = \sum_i P_i |\Psi_i\rangle\langle\Psi_i|$, the detection probability in region A is:

$$\text{Prob}\,(A;\rho) = \sum_i P_i a_i$$

Let $a_{\min} = \min\{a_i\}$ and $a_{\max} = \max\{a_i\}$.

First, we prove $\text{Prob}\,(A;\rho) \in [a_{\min}, a_{\max}]$: Since $\sum_i P_i = 1$ and $P_i \geq 0$, we have:

$$\sum_i P_i a_i \geq \sum_i P_i a_{\min} = a_{\min}, \sum_i P_i a_i \leq \sum_i P_i a_{\max} = a_{\max}$$

Therefore, $\{\,\text{Prob}\,(A;\rho)\} \subseteq [a_{\min}, a_{\max}]$.

Second, we prove $[a_{\min}, a_{\max}] \subseteq \{\,\text{Prob}\,(A;\rho)\}$: Let $\Psi_{\min}$ and $\Psi_{\max}$ be the pure states that achieve the minimum and maximum a_i values, respectively. Consider the family of mixed states consisting only of these two states:

$$\rho(\lambda) = \lambda|\Psi_{\max}\rangle\langle\Psi_{\max}| + (1-\lambda)|\Psi_{\min}\rangle\langle\Psi_{\min}|, \lambda \in [0,1]$$

The corresponding detection probability is:

$$\text{Prob}\,(A;\rho(\lambda)) = \lambda a_{\max} + (1-\lambda)a_{\min}$$

As λ varies continuously from 0 to 1, $\text{Prob}\,(A;\rho(\lambda))$ varies continuously from $a_{\min}$ to $a_{\max}$, thus covering all values in $[a_{\min}, a_{\max}]$.

In conclusion, $\{\,\text{Prob}\,(A;\rho)\} = [a_{\min}, a_{\max}]$, which is a closed interval.

Defination 6.2 (Probability interpretation of the nonmeasurable state) Because space is non-measurable, the measure of a region of space is interval-valued. The probability of a particle appearing in a region of space is related to the region measure. The probability of a particle appearing in a region of space is also interval valued. $\{\int_A \Sigma_i P_i \mid \Psi_i(r,t) \mid^2 dr \mid \Psi_1, P_1; \Psi_2, P_2; \cdots; \Psi_i, P_i; \cdots$ is a mixed state$\}$ = Interval probability of particles appearing at time t in area A.

Defination 6.3 (Quantum measurement) *We can construct a probability measure on the space according to the mixed state, and all such probability measures form an interval probability measure. The so-called quantum measurement is to choose a probability measure from the interval probability measure.*

Electron model 6.1 According to our hypothesis, electrons are composed of more fundamental particles that are non-measurable. It is a very simple fact that electrons are subject to coulomb forces in an electric field, and moving electrons are subject to Lorentz forces in a magnetic field, and the reason why this is so is that we dare to propose such a model. The electron consists of three non-measurable more elementary particles, a non-measurable particle with an electric charge, a non-measurable particle with an N-order magnetic charge, and a non-measurable particle with an S-order magnetic charge, which are quantum-entangled. That is to say, magnetic monopoles exist in electrons.

Pairing mechanism of electron pairs 6.2 For superconducting materials at high pressure or low temperature, the n-order (S-order) magnetic charge of one electron is entangled with the S-order (N-order) magnetic charge of another electron, which can explain the electron pairing mechanism.

7. Quantum Contextuality of Gravity Equations

Corollary 7.1(Quantum Contextuality) `Gravity equations are quantum contextuality.`

Since dark energy is non-measurable, some physical parameters are non-degenerate interval numbers. So gravity equations are no longer a tesor equation, but a cluster of tensor equations.

Matter is converted into vacuum energy (dark energy), as the matter falls through the event horizon[22-24].The interior of a black hole is a vacuum with a different cosmological constant, and the universe before the Big Bang was a vacuum with a different cosmological constant. The deeper principle behind this principle may be the interconversion between matter and dark energy,dark

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energy particles are probably the most fundamental particles in the universe.Using this assumption, we can solve the problem that general relativity breaks down at the singularity.The star that formed the black hole collapsed into dark energy and nothing else, so the energy tensor of matter is zero,the vacuum field equation can describe the interior of a black hole. Similarly, a vacuum field equation with a different cosmological constant could describe the universe before the Big Bang.

The following equations are in accordance with the non-measurability and universality of dark energy.

Corollary 7.2 (Interval tensor equation for quantum gravity in the vacuum)

$$\{R_{\mu\nu} = \Lambda g_{\mu\nu} | \Lambda \in I\}$$

Remark1 The universe within a black hole or prior to the Big Bang contained only dark energy, devoid of matter, but with a distinct cosmological constant. Therefore, this equation can describe the state of the universe in black holes or during the pre-Big Bang era.

Corollary 7.3 (Interval tensor equation for quantum gravity)

$$\{R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \kappa (T_{\mu\nu} - \frac{\Lambda}{\kappa} g_{\mu\nu}) | \Lambda \in I, T_{\mu\nu} \in \mathbf{T}\}.$$

where I is an interval constant,**T** is an interval tensor.

Remark1 This is a cluster of Einstein field equations, and this equation is an interval equation, and studying the solution set is the only way to understand the physical meaning of this equation.If the solution set is a nondegenerate generalized interval tensor, then the space-time is nonmeasurable.The basic concept of interval equations can be found in Reference [13].

Remark2 The interval tensor equation for quantum gravity has generalized covariance. Because of physical considerations, Einstein insisted that the right-hand side ofthe field equations can be written as the sum of quantities representing the energy-momentum of matter and the energy- momentum of the gravitational field.However, such an equation does not have generalized covariance.Using interval tensors to describe gravitational field can solve this problem.

Remark3 This theory can explain not only the macroscopic phenomena of general relativity but also the microscopic phenomena of quantum mechanics.There is no doubt that this equation can explain the macroscopic phenomena of general relativity.The remaining problem is to derive the Schrodinger (Dirac) equation from the equation of quantum gravity.

Remark4 The pseudo energy-momentum tensor of the gravitational field depends on the coordinate system, which means that the energy of the gravitational field depends on the measurement. That is, the gravitational field energy is nonmeasurable.

Remark5 In a quantum state, the particle has a definite position and velocity under a measurement, and therefore a definite energy-momentum tensor under the measurement(Although the particle doesn't have a definite momentum when a particle has a definite position, the particle will have a momentum under the measurement).The set of all energy-momentum tensor of the quantum state is described in terms of an interval tensor.So we've solved the gravity problem of a single particle.

8. Background Independent Quantum Theory on de Sitter Space

In Feynman's path integral, if one compares a set of neighboring paths, the phases or positions in the cycle will differ greatly. This means that the waves associated with these paths will almost exactly cancel each other out. However, for some sets of neighboring paths the phase will not vary much between paths. The waves for these paths will not cancel out such paths correspond to Bohr's allowed orbits.In general, *these orbits that cannot be cancelled are called allowed orbits*. The principle of least action says that the actual path taken by the system is an extremum of S, and a reasonable interpretation of the Feynman path integral can only be that particles are coming from allowed orbits, which are geodesics in some nonmeasurable spacetime.

Here's an analogy that will allow us to build a background independent quantum theory.

Allowed orbits in Minkowski space-time $\simeq$ ***Geodesics in non-measurable spacetime***.

We try to build a background independent quantum theory, making quantum mechanics dependent on gravity.There is a specific cosmological constant under each measurement, so we get a de Sitter space, in which there may be a unique geodesic between two points. If there is a geodesic between two points in a non-measurable de Sitter space, there are many different geodesics at the same time. We construct a background independent quantum theory by analogy with Feynman's path integral theory, the geodesics of non-measurable spacetime are similar to the allowed orbits of Minkowski spacetime.

Definition 8.1 If $K(x', t'; x_0, t_0) = A\sum_{\mathcal{C}\in K} e^{\frac{i}{\hbar}S^{\mathcal{C}}_{Cl}}$,then $K(x', t'; x_0, t_0)$ is called the propagator on the nonmeasurable de Sitter space.

Where $\mathcal{C}$ is a measurement, K is the set of all measurements, A is the normalization constant, $L^{\mathcal{C}} = \frac{1}{2}mg^{\mathcal{C}}_{\mu\nu}\dot{x}^{\mu}\dot{x}^{\nu} - V$(**x** is spatial coordinates, and it is a function of time t), $S^{\mathcal{C}}_{Cl} = S[x^{\mathcal{C}}_{Cl}(t)] = \int_{t_0}^{t'} L^{\mathcal{C}}dt$,$x^{\mathcal{C}}_{Cl}(t)$ is a geodesic between the two points under the measurement $\mathcal{C}$. $\{g^{\mathcal{C}}_{\mu\nu}|\mathcal{C} \in K\}$ is the solution set of the equation $\{R^{\mathcal{C}}_{\mu\nu} = \Lambda^{\mathcal{C}}g^{\mathcal{C}}_{\mu\nu}|\mathcal{C} \in K\}$. The value of $e^{\frac{i}{\hbar}S^{\mathcal{C}}_{Cl}}$ is defined as follows

(1)If there's a geodesic $x^{\mathcal{C}}_{Cl}(t)$ between the two points under the measurement $\mathcal{C}$, then $e^{\frac{i}{\hbar}S^{\mathcal{C}}_{Cl}} = e^{\frac{i}{\hbar}S[x^{\mathcal{C}}_{Cl}(t)]} = e^{\frac{i}{\hbar}\int_{t_0}^{t'} L^{\mathcal{C}}dt}$.

(2)If there is no geodesic between the two points under the measurement $\mathcal{C}$,then $e^{\frac{i}{\hbar}S^{\mathcal{C}}_{Cl}} = 0$.

Definition 8.2 If $P(x', t'; x_0, t_0) = |K(x', t'; x_0, t_0)|^2$, then $P(x', t'; x_0, t_0)$ is called the probability going from (x_0, t_0) to(x', t').

Remak1 Similarly, we can establish quantum mechanics on other non-measurable Spaces.The theory established here can change our quantum theory, we can get a background independent quantum theory.

Remark2 The problem that remains is to derive Schrodinger equation or Dirac equation from this new quantum theory.

Remark3 There is a specific cosmological constant under each measurement, so this quantum theory depends on the cosmological constant.

Remark4 In general relativity, particles travel along geodesics. In quantum mechanics, particles come from every possible path at the same time. This is an obvious contradiction. In non-measurability theory, there are many different geodesics between two points at the same time, which resolves the contradiction between quantum mechanics and general relativity, the particles come along the geodesic at the same time.

9. Falsifiable Theoretical Predictions

In fact, there are far more theoretical predictions that can be falsified, and experimental data from modern physics have actually provided a lot of evidence, except that people think it is a measurement error of the system, which is actually a property of non-measurable matter.This theory can provide a unified and simple explanation for a wide range of physical phenomena, so the theory has a high probability of being true.

Falsifiable theoretical predictions9.1 *Under identical source, energy spectrum, propagation distance, and environment, if two detectors based on fundamentally different physical principles are used to simultaneously measure the same neutrino beam, and a statistically significant discrepancy is observed in the ratio of neutrino flavor events that cannot be explained by corrections from the Standard Model, it would provide strong support for the hypothesis that "there is only one type of neutrino".*

Falsifiable theoretical predictions9.2 *The mass of non-measurable particle is interval valued, and the specific real value depends on the measurement method.*

Falsifiable theoretical predictions9.3 *Non-measurable particles have an effect similar to the oscillation of neutrinos.*

Falsifiable theoretical predictions9.4 *The measured value of the gravitational wave signal depends on the detection method, and different detection methods may record differences in the amplitude or frequency interval of the same event.*

Falsifiable theoretical predictions9.5 *The non-measurability of the dark energy field causes its self-interaction to generate local interval energy density regions, which may appear as "clusters" or "holes".This can affect the formation of large-scale structures and appear in cosmological observations as anomalous gravitational lensing effects or inhomogeneity in the distribution of galaxy clusters.*

Falsifiable theoretical predictions9.6 *Dark energy is non-measurable and drives the accelerating expansion of the universe. If the inflation of the early universe was also dominated by dark energy, then the exponential expansion of the inflationary period may be a direct result of the non-measurable nature of dark energy. The Banach-Tarski paradox allows for the generation of infinite volumes from finite volumes, which is similar to the rapid spatial expansion of inflation. The energy of inflation field comes from the non-measurable recombination of dark energy, and its interval energy value leads to the uncertainty of inflation rate, which is manifested by the difference of expansion speed in different regions.*

10. Unilateral Gravity

10.1. Concept and Fundamental Principle

Within the framework of the non-measurability theory, the gravitational interaction between matter and spacetime is no longer constrained by reciprocity. When certain forms of matter enter a non-measurable state, their coupling to measurable spacetime undergoes a fundamental transformation, exhibiting a property known as **unilateral gravity**.

Unilateral gravity refers to a state in which **non-measurable matter continues to generate gravitational fields, curve spacetime, and influence the motion of measurable matter, yet does not respond to external gravitational fields**. In other words, such matter acts as a *one-way gravitational source*: it *exerts* gravity but does not *feel* it.

10.2. Physical Implications and Cosmological Significance

The existence of unilateral gravity has profound implications. It provides a natural explanation for the non-collapsing nature of dark matter: non-measurable matter can shape spacetime curvature and account for observed galactic rotation curves and gravitational lensing effects, yet it will not collapse under gravity into compact structures. Moreover, gravitational asymmetry implies that measurable matter will be attracted by non-measurable matter, but the reverse interaction does not occur — a property that could fundamentally reshape the evolution of cosmic structures.

From a theoretical standpoint, unilateral gravity suggests that the *generation* of gravity and the *response* to gravity are fundamentally distinct processes.

10.3. Predictions

The unilateral gravity hypothesis leads to a series of testable predictions:

1. **Non-collapsing mass halos:** Structures composed predominantly of non-measurable matter should remain diffuse and stable, even in regions where measurable matter undergoes gravitational collapse. This directly predicts the persistent halo-like distributions observed in galaxies and clusters.
2. **One-sided gravitational acceleration:** In regions dominated by non-measurable matter, measurable test particles should experience acceleration without a corresponding recoil or mass redistribution from the source.
3. **Modified large-scale dynamics:** Cosmological structure formation simulations incorporating unilateral gravity predict slower mass aggregation rates and a smoother matter distribution on large scales compared to ΛCDM expectations.
4. **Vacuum curvature anomalies:** Non-measurable fields may induce spacetime curvature even in regions apparently devoid of measurable matter, leading to detectable deviations in weak lensing or cosmic microwave background anisotropies.

10.4. Experimental and Observational Evidence

While direct detection of non-measurable matter remains beyond current technological capabilities, several lines of indirect evidence align with the unilateral gravity framework:

- **Dark matter halo morphology:** Observations show that dark matter forms extended, non-collapsing halos around galaxies, consistent with the prediction of gravitational "one-way" influence.
- **Bullet Cluster dynamics:** The separation of gravitational lensing centers from baryonic matter suggests the presence of gravitational sources unaffected by normal gravitational drag.
- **Ultra-diffuse galaxies:** Systems with large mass-to-light ratios but extremely low central densities support the idea of matter exerting gravity without clustering.
- **Gravitational vacuum curvature:** Recent weak lensing surveys and cosmic shear measurements hint at excess curvature in regions lacking visible mass, potentially attributable to non-measurable sources.
- **Creep effect:** The "unilateral gravity-like" phenomenon of superfluid – the creep effect: One of the most astonishing properties of superfluid helium is its ability to defy gravity, climbing up the walls of a container until it spills out. The traditional explanation relies on the combined effects of its zero viscosity and the surface film. However, within the framework of non-measurability theory, this can be interpreted as a kind of ｜ignoring" or "reconstruction" of the directionality of gravity at the microscopic scale.
  Mechanism explanation: When helium enters the superfluid state (BEC state), a portion of its substance (the macroscopic quantum wave function) enters a non-measurable state. In this state, as predicted by the theory, its response to the external gravitational field is greatly weakened or becomes "uncertain". Although the superfluid as a whole is still affected by gravity and remains at the bottom of the container, the superfluid component in the surface film layer, due to the van der Waals interaction with the container wall, has its "non-measurable" property preferentially excited. This allows this part of the superfluid to move almost without energy consumption in the direction perpendicular to gravity, and its motion is dominated by secondary forces such as surface tension, thus exhibiting the seemingly gravity-defying creep behavior. This can be regarded as an emergent manifestation of the unilateral gravity principle of "not feeling gravity" at the microscopic, low-energy scale.

11. Summary

The foundation of Non-measurability theory is made up of two basic principles. The first principle is that dark energy is non-measurable, and the second is thatAll matter and fields are excitation of dark energy. From the point of view of the field, the dark energy field is non-measurable, and all matter is excited by the dark energy field.

These two principles can be used to solve fundamental problems in physics, such as: quantum gravity, unified field theory, cosmological constants, Hubble constants, neutrinos, high-temperature superconductivity mechanisms, fractional charges, energy problems of gravitational fields, gravitational problems of single particles, the mystery of black hole mass growth, the mystery of the disappearance of antimatter, the properties of dark matter and dark energy, and so on.

The concept of non-measurable sets comes from the theory of real variable functions, the most famous theorem about this concept is the Banach-Tarski paradox,, the mathematics is very beautiful, unfortunately, mathematics and physics here have diverged ways, people generally believe that it is impossible to get non-measurable matter in the real world, which requires extremely complex segmentation to get non-measurable matter, but there is also a case: the most basic matter in the universe is non-measurable, and other matter is made up of this most basic matter. This coincides with our current physical discovery that most of the matter in the universe is unknown.

Dark energy has a strange property: as the universe expands, the density of dark energy remains constant, it's very much like something came out of nothing. As long as it is assumed that dark energy

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is non-measurable, it has the property of non-measurable sets, that the dark energy is excited into various matters, and the matter disappears into a vacuum, that is to say, the dark energy is constantly reorganizing and decomposing, according to the Banach-Tarski paradox, this situation is possible, the space is getting bigger and bigger, and the dark energy is also increasing, and the opposite is also possible, which can also be explained by the nature of the non-measurable set.

For the second principle, all matter in the universe is made up of dark energy, Heisenberg once proposed that all matter is made up of the same entity, and unfortunately, there was no concept of dark energy in his time. This principle asserts that this fundamental substance is dark energy. In the case of black holes, this is the end of all matter, where all matter becomes dark energy. There are several articles that try to prove that black holes are made up of dark energy. In turn, black holes produce all matter. In quantum field theory, the elementary particle is the excitation of its corresponding field, and this excitation comes from the vacuum, but the quantum field theory believes that the vacuum is the ground state of the quantum field, and this assertion believes that these fields are the most basic, and that an electron field is needed in order to generate electrons, and an electromagnetic field is needed in order to generate photons, and so on, quantum field theory believes that these fields are ubiquitous, and there are as many fields as there are elementary particles. To use an analogy, quantum field theory seems to say that in order to make a steamed bun, there needs to be flour everywhere in the universe. In fact, in order to make a steamed bun, you only need to have flour in your little place, and this flour is also made.This metaphor is used to say that the quantum field discussed in quantum field theory is not fundamental, but is made up of a more fundamental field, which we think of as dark energy.

The first conclusion is that there is non-measurable matter in the universe. Neutrinos, quarks, Gluons, superfluids, W bosons, Z bosons, magnetic monopoles, Higgs, black holes, Cooper pairs, fractional electrons, dark matter are most likely non-measurable.This conclusion can explain a wide range of physical phenomena.

According to the first principle, the cosmological constant is not a real value, but an interval value. It can be deduced that the concepts of space-time and geometry are quantum intertextuality. We incorporate quantum intertextuality into the theory of gravity and geometry, so we get a quantum theory of space-time and geometry.

We can deduce that all geometric quantities in de Sitter space are interval-valued.

Finally, the equations of quantum gravity can explain not only gravitational phenomena but also quantum phenomena. In addition, if dark energy is the most fundamental field, then the field equation of dark energy is sufficient to unify all fields, which is also the direction of our research.

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