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Brief Report

Maxwell–Lodge Effect and Electric Fields of a Solenoid

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Abstract: Maxwell–Lodge effect is the observed electric field (or electromotive force) outside a long solenoid, where the magnetic field is essentially zero. Proposed explanations for the Maxwell–Lodge effect include non-local induction (the magnetic field inside the solenoid generating an electric field outside), the magnetic vector potential acting as a physical quantity that produces the electric field outside the solenoid, or Weber's force resulting from moving electrons. In this paper, we calculate the electric fields of a solenoid using the electric field theory of Weber's electrodynamics. We show that the solenoid not only generates the conventional magnetic field inside the solenoid but also produces velocity- and acceleration-related electric fields both inside and outside the solenoid. This explanation, based on the electric field theory of Weber's electrodynamics, is simpler and more straightforward.

Keywords: Maxwell-Lodge effect; Electric field; Weber's force; Field theory; Solenoid

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1. Introduction

The Maxwell–Lodge effect [1,2] refers to the observed electric field outside a long solenoid with a slowly varying current. According to classical electromagnetism, the Coulomb electric field inside and outside a non-resistive solenoid is zero because there is no net charge. For a resistive solenoid, there exists a non-zero charge distribution along the solenoid, and thus a non-zero electric field [3]. However, this electric field is small and should be very similar for both constant and slowly varying currents. Therefore, the detected electric field in the case of a slowly varying current—but not for a constant current—must be due to induction.

For a long solenoid with constant current, the magnetic field inside the solenoid is uniform, while the magnetic field just outside is nearly zero. However, when the current varies slowly, the assumption of a zero magnetic field outside leads to inconsistencies in Maxwell's equations (as well as in extended electrodynamics) [4,5]. Solutions to Maxwell's equations under these conditions reveal both an induced electric field and a very small magnetic field outside the solenoid [6]. It is generally accepted that these fields can be treated as quasi-stationary, with negligible electromagnetic radiation.

According to Maxwell's equations [7], a changing magnetic field generates an electric field, and by the principle of locality in physics, this must occur locally. In the case of a slowly varying current, the magnetic field inside the solenoid changes, while the magnetic field outside remains essentially zero. This makes it difficult for classical electromagnetism to explain the detected electric field outside the solenoid unless one assumes a non-local effect, i.e., the varying magnetic field inside somehow causes the electric field outside [8].

Even though the magnetic field outside is zero, the magnetic vector potential is not [9]. Since the time-varying magnetic vector potential contributes to the electric field, it is proposed that the observed field outside is caused by this variation. While this explanation seems to uphold the locality principle, it assumes that the magnetic vector potential is physically real—a point still debated in classical theory [2]. Many studies [2,9–12] support the physical reality of magnetic vector potential, though the traditional view treats it as merely a mathematical construct.

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A pioneering study explained the induced electromotive force outside a long solenoid using Weber's force [13], avoiding the controversial reality assumption of a physical vector potential. In this paper, we continue along this path, using the field theory of Weber's electrodynamics [14] to derive the electric fields of a solenoid. We demonstrate that electron motion in the current produces magnetic and electric fields both inside and outside the solenoid.

2. Electric Fields of Weber's Electrodynamics

In the electric field theory of Weber's electrodynamics [14], the force that a charge Q exerts on another charge q can be written in the following form (equation 1) for any given reference frame. Let Q have velocity v_i and acceleration a_i, and q have velocity u_i and acceleration w_i. The distance between the two charges is r, and n_k is the unit vector pointing from charge Q to charge q. Here, ε_0 is the vacuum permittivity and c is the speed of light.

$$F_k = q\left[E_k + u_i M_{ikj} u_j + u_i B_{ik} + \tilde{E}_k + w_i L_{ik} + \bar{\bar{E}}_k\right] \qquad (1)$$

Where:

$$\begin{aligned}
E_k &= \frac{Q n_k}{4\pi\varepsilon_0 r^2} \\
M_{ikj} &= \frac{E_k}{c^2}\left(\delta_{ij} - \frac{3}{2} n_i n_j\right) \\
B_{ik} &= \frac{E_k}{c^2}\left(-2\delta_{ij} + 3 n_i n_j\right) v_j \\
\tilde{E}_k &= v_i \frac{E_k}{c^2}\left(\delta_{ij} - \frac{3}{2} n_i n_j\right) v_j \\
L_{ik} &= \frac{E_k r}{c^2} n_i \\
\bar{\bar{E}}_k &= \frac{-E_k r}{c^2} n_i a_i \qquad (2)
\end{aligned}$$

In this formulation:

- E_k is the classical Coulomb electric field,
- B_{ik} is equivalent to the magnetic field,
- $\tilde{E}_k$ is the electric field due to the charge Q (source) velocity,
- $\bar{\bar{E}}_k$ is the electric field due to the charge Q (source) acceleration,
- M_{ikj} is the electric field acting on the charge q (receiver) velocity,
- L_{ik} is the electric field acting on the charge q (receiver) acceleration.

To obtain the total field from multiple source charges, we sum the contributions of each individual charge. To compute the force on a single receiver charge, we apply Equation (1) using its properties (charge, velocity, and acceleration) and the total electric field resulting from the superposition of all source charges.

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3. Long Solenoid with Constant Current

In this section, we analyze a circular electric current in a solenoid centered at the origin, as shown in Figure 1a. Our goal is to calculate the electric fields at point A on the x-axis, as depicted in Figures 1b and 1c.

Assume that the current is constant. The positive charges in the solenoid are stationary, while the negative charges (electrons) move with a velocity $\vec{v}$ along the circular path, experiencing an acceleration $\vec{a}$ that points towards the z-axis (Figure 1b).

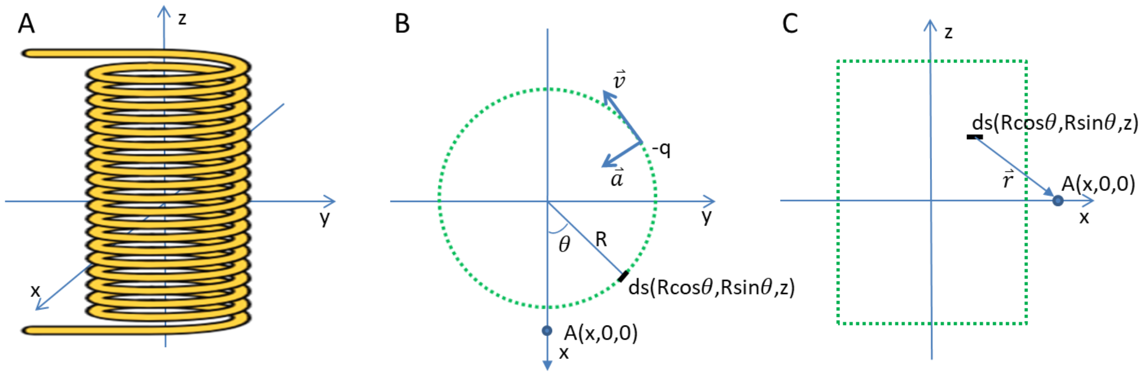

Figure 1. A) Schematic diagram of the solenoid; B) Top-down view of the solenoid with radius R; C) Side view of the solenoid, showing the electrical action at point A (located at x, 0, 0) due to a current segment ds.

Given the coexistence of positive and negative charges in the solenoid, the terms E_k, M_{ikj}, and L_{ik} will be zero due to the cancellation of the positive and negative charge contributions. Therefore, we focus on calculating the terms B_{ik}, $\tilde{E}_k$, and $\bar{\bar{E}}_k$ from the moving negative charges in the solenoid.

Let's begin by describing the current segment $ds = Rd\theta \cdot dz$, where θ is the angle around the solenoid's circular path. The vector $\vec{r}$ is pointing from the segment ds to point A, with a unit vector n_i. The surface density of negative charge is $-\rho$, where ρ is the electron charge density.

The vector $\vec{r}$, its magnitude and unit vector are given by:

$$\vec{r} = (x - R\cos\theta\,, -R\sin\theta\,, -z)$$

$$r = |\vec{r}| = \sqrt{(x - R\cos\theta)^2 + (R\sin\theta)^2 + z^2}$$

$$n_i = \frac{1}{r}(x - R\cos\theta\,, -R\sin\theta\,, -z) \qquad (3)$$

Next, the differential Coulomb electric field dE_k due to the current segment is:

$$dE_k = \frac{ds \cdot (-\rho n_k)}{4\pi\varepsilon_0 r^2} \qquad (4)$$

The velocity and acceleration components of the moving negative charges (electron) are given by:

$$v_j = v(-\sin\theta\,, \cos\theta\,, 0)$$

$$a_i = \frac{v^2}{R}(-\cos\theta\,, -\sin\theta\,, 0) \qquad (5)$$

Using the above expressions for v_j and a_i, we proceed to calculate the magnetic field and electric field components resulting from these moving electrons. From Equations (2, 3, 4, 5), the differential magnetic field contribution from the current segment is:

$$dB_{ik} = \frac{ds\cdot(-\rho v)}{4\pi\varepsilon_0 r^5 c^2}\begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{pmatrix} \quad (6)$$

Where the components b_{ij} are given by the following expressions:

$$b_{11} = 2\sin\theta\, r^2 x - 2\sin\theta\, r^2 R\cos\theta + 6x^2\sin\theta\, R\cos\theta - 3x\sin\theta\, R^2 cos^2\theta - 3x^3\sin\theta$$
$$b_{12} = -2sin^2\theta r^2 R - 3xsin^2\theta R^2\cos\theta + 3x^2 sin^2\theta R$$
$$b_{13} = -2\sin\theta\, r^2 z - 3x\sin\theta\, R\cos\theta\, z$$
$$b_{21} = -2\cos\theta\, r^2 x + 2cos^2\theta r^2 R + 3x^2 sin^2\theta R - 3xsin^2\theta R^2\cos\theta$$
$$b_{22} = 2\sin\theta\, r^2 R\cos\theta - 3xsin^3\theta R^2$$
$$b_{23} = 2\cos\theta\, r^2 z - 3xsin^2\theta Rz$$
$$b_{31} = 3x^2\sin\theta\, z - 3x\sin\theta\, R\cos\theta\, z$$
$$b_{32} = -3xsin^2\theta zR$$
$$b_{33} = -3x\sin\theta\, z^2 \quad (7)$$

We integrate the components along the solenoid, and then we obtain B_{ik}:

$$B_{ik} = \begin{pmatrix} 0 & B_{12} & 0 \\ B_{21} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad (8)$$

In this equation, all other components become zero due to symmetry, except for B_{12} and B_{21}, for which we perform numerical integration. The parameters for the numerical integration are predefined as follows: the solenoid radius $R = 0.1m$; the solenoid length is 10 m; the electron surface density $\rho = 8.5\times 10^{26} m^{-2}$; the electron charge is $1.6\times 10^{-19} C$; the electron drift velocity is $4\times 10^{-4} m/s$; the speed of light $c = 3\times 10^8 m/s$; and the vacuum permittivity $\varepsilon_0 = 8.854\times 10^{-12} F/m$.

The result of the integration is plotted in Figure 2a. Inside the solenoid, the B_{12} and B_{21} components are uniform. Outside the solenoid, the B_{12} and B_{21} components are zero. This outcome is consistent with the magnetic field predicted by classical electromagnetism.

Similarly, from equations (2,3,4,5), we can derive differential electric field $d\tilde{E}_k$:

$$d\tilde{E}_k = \frac{\rho v^2\cdot ds}{8\pi\varepsilon_0 r^5 c^2}\begin{pmatrix} \tilde{e}_1 & \tilde{e}_2 & \tilde{e}_3 \end{pmatrix} \quad (9)$$

Where:

$$\tilde{e}_1 = -2r^2 x + 2r^2 R\cos\theta + 3x^3 sin^2\theta - 3x^2 sin^2\theta R\cos\theta$$
$$\tilde{e}_2 = 2\sin\theta\, r^2 R - 3x^2 sin^3\theta R$$
$$\tilde{e}_3 = 2r^2 z - 3x^2 sin^2\theta z \quad (10)$$

We integrate these components along the solenoid to obtain $\tilde{E}_k$. Due to symmetry, all components become zero except for $\tilde{E}_1$, which is computed through numerical integration.

$$\tilde{E}_k = \begin{pmatrix} \tilde{E}_1 & 0 & 0 \end{pmatrix} \quad (11)$$

Again, from equation (2,3,4,5), we can derive differential electric field $d\bar{\bar{E}}_k$:

$$d\bar{\bar{E}}_k = \frac{\rho v^2\cdot ds}{4\pi\varepsilon_0 r^3 c^2 R}\begin{pmatrix} \bar{\bar{e}}_1 & \bar{\bar{e}}_2 & \bar{\bar{e}}_3 \end{pmatrix} \quad (12)$$

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Where:

$$\bar{\bar{e}}_1 = -x^2 \cos\theta + xR + xRcos^2\theta - R^2 \cos\theta$$
$$\bar{\bar{e}}_2 = xR \sin\theta \cos\theta - R^2 \sin\theta$$
$$\bar{\bar{e}}_3 = x \cos\theta\, z - Rz \qquad (13)$$

We integrate these components along the solenoid to obtain $\bar{\bar{E}}_k$. Due to symmetry, all components become zero except for $\bar{\bar{E}}_1$, which is computed using numerical integration.

$$\bar{\bar{E}}_k = \begin{pmatrix} \bar{\bar{E}}_1 & 0 & 0 \end{pmatrix} \qquad (14)$$

The integrated $\tilde{E}_1$ and $\bar{\bar{E}}_1$ components are plotted in Figure 2b. Inside the solenoid, $\tilde{E}_1$ is zero, while $\bar{\bar{E}}_1$ is non-zero and increases with radius. Outside the solenoid, both $\tilde{E}_1$ and $\bar{\bar{E}}_1$ are non-zero; however, their sum is zero.

For a charge q located at point A, with velocity u_i and acceleration w_i, the force exerted on it can be calculated using equation (1):

$$F_k = q\left[E_k + u_i M_{ikj} u_j + u_i B_{ik} + \tilde{E}_k + w_i L_{ik} + \bar{\bar{E}}_k\right] = q u_i B_{ik} + \tilde{E}_k + \bar{\bar{E}}_k \quad (15)$$

The force term $q u_i B_{ik}$ is consistent with the Lorentz force derived from classical magnetic theory. However, the terms $\tilde{E}_k$ and $\bar{\bar{E}}_k$ can only be obtained from the field theory of Weber's electrodynamics.

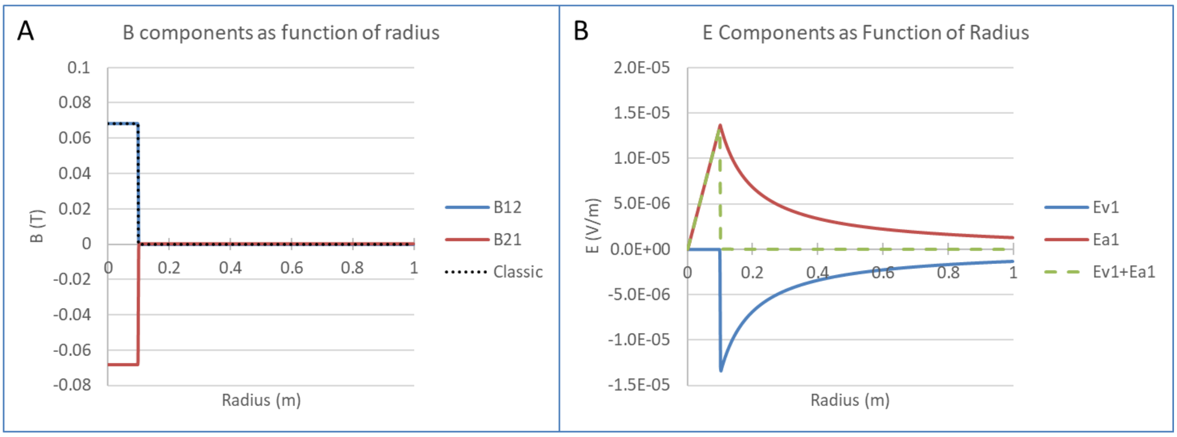

Figure 2. A) B components (B_{12}, B_{21}) and the magnetic field from classical electromagnetism as a function of radius; B) E components $\tilde{E}_1$ (Ev1), $\bar{\bar{E}}_1$ (Ea1), and their summation (Ev1 + Ea1) as a function of radius.

4. Long Solenoid with Slowly Varying Current

Here, we consider a solenoid with a slowly varying current (Figure 3a). The solenoid has the same geometry and parameters as the one in Section 3 (Figure 1), except that it has an acceleration component in the tangential direction. We will not repeat the analysis of the electric fields due to the current itself. Instead, we will only analyze the electric field contributed by this tangential acceleration component.

$$a_i = a_0(-\sin\theta, \cos\theta, 0) \qquad (16)$$

From equation (2,3,4,16), we can calculate differential electric field $d\bar{\bar{E}}_k$ due to this tangential acceleration component:

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$$d\bar{\bar{E}}_k = \frac{\rho a_0 \cdot ds}{4\pi\varepsilon_0 r^3 c^2}(\bar{\bar{e}}_1 \quad \bar{\bar{e}}_2 \quad \bar{\bar{e}}_3) \qquad (17)$$

Where:

$$\bar{\bar{e}}_1 = -x^2 \sin\theta + xR\sin\theta\cos\theta$$
$$\bar{\bar{e}}_2 = xRsin^2\theta$$
$$\bar{\bar{e}}_3 = x\sin\theta\, z \qquad (18)$$

We integrate them along the solenoid, resulting in $\bar{\bar{E}}_k$. Here, all other components become zero due to symmetry except for $\bar{\bar{E}}_2$, for which we perform numerical integration.

$$\bar{\bar{E}}_k = \begin{pmatrix}0 & \bar{\bar{E}}_2 & 0\end{pmatrix} \qquad (19)$$

The parameters we used are the same as those in Section 3, with the addition that $a_0 = 4 \times 10^{-4} m/s^2$. This represents the maximum acceleration of a sinusoidal current at 1 Hz, assuming a maximum current of $4 \times 10^{-4} m/s$. The integration outcome is plotted in Figure 3b. Inside the solenoid, $\bar{\bar{E}}_2$ increases linearly with radius. Outside the solenoid, $\bar{\bar{E}}_2$ decreases with radius. This outcome is consistent with the electric field derived from classical electromagnetism, assuming an infinitely long solenoid.

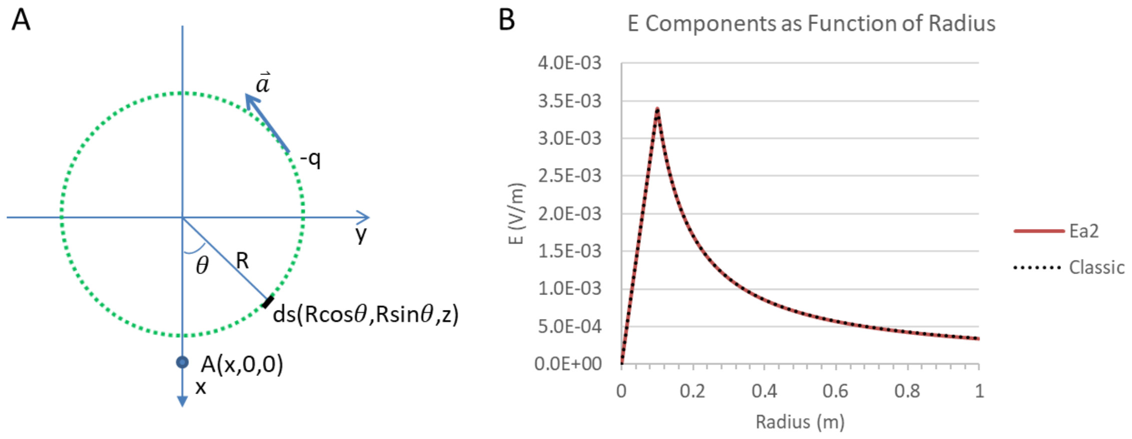

Figure 3. A) Top-down view of the solenoid with radius R; B) E component $\bar{\bar{E}}_2$ (Ea2) and the electric field from classical electromagnetism as a function of radius.

5. Short Solenoid with Constant Current

Here, we consider a scenario in which the solenoid is short and the current is constant. The solenoid length is reduced to 0.01 m, and all other parameters are the same as those in Section 3. The calculated B and E components are plotted in Figure 4.

The B components are no longer uniform inside the solenoid. Outside the solenoid, the B components reverse polarity and gradually decrease with radius (Figure 4a). This behavior is consistent with classical electromagnetism. However, classical electromagnetism predicts a zero electric field both inside and outside the solenoid. In contrast, the field theory of Weber's electrodynamics predicts non-zero E components. Compared with the long solenoid case, the E components are dominated by the velocity-related component $\bar{E}_1$. The acceleration-related component $\bar{\bar{E}}_1$ is relatively small (Figure 4b), and the two components do not cancel each other outside the solenoid.

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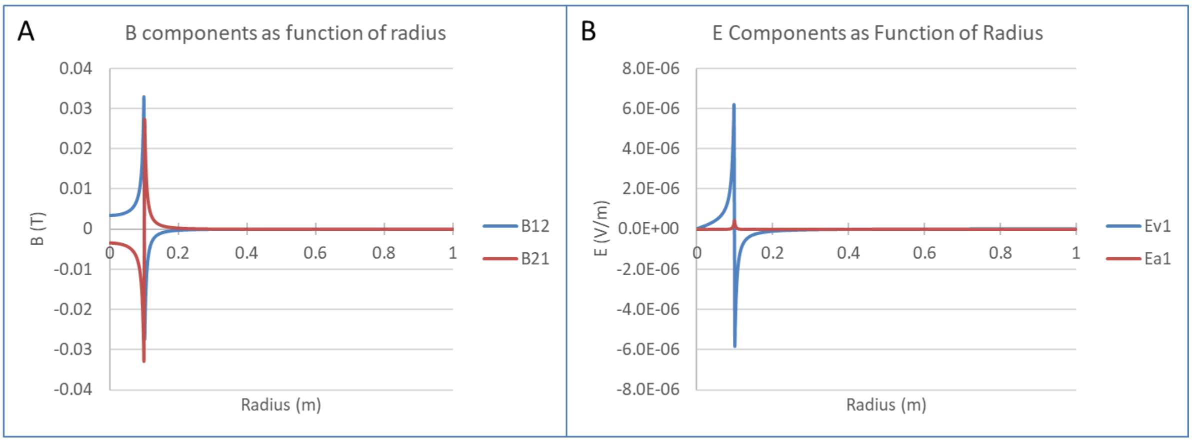

Figure 4. A) B components (B_{12}, B_{21}) as a function of radius; B) E components $\tilde{E}_1$ (Ev1), $\bar{\bar{E}}_1$ (Ea1) as a function of radius.

6. Discussion

In classical electromagnetism, a time-varying electric field generates a magnetic field, and a time-varying magnetic field generates an electric field. The electric and magnetic potentials are typically not regarded as physical quantities, but rather as mathematical tools for calculating electric and magnetic fields. Furthermore, under the principle of locality, field generation must occur locally.

However, in the case of a long solenoid with a slowly varying current, this conventional understanding faces challenges. The magnetic field outside the solenoid is essentially zero, while the magnetic vector potential remains non-zero. To explain the induced tangential electric field outside the solenoid, one must either abandon the principle of locality or assume that the magnetic vector potential is a physical entity. Yet, there is no consensus on which interpretation is correct. Moreover, the explanation based on the magnetic vector potential raises a further question: what generates the magnetic vector potential outside the solenoid? If it is generated by the magnetic field inside the solenoid, this again implies non-locality.

In contrast, the explanation provided by the field theory of Weber's electrodynamics is simpler and more direct. The solenoid wire contains stationary positive charges and moving electrons (negative charges). The combined effects of these charges produce electric fields, which can have six components. Since the positive and negative charges are balanced, three of these components are zero. The remaining three non-zero components include the B component (equivalent to the magnetic field), a velocity-dependent E component, and an acceleration-dependent E component. Unlike in classical electromagnetism, these components are independent of each other. The electric field is not generated by the magnetic field; rather, all components are generated directly by the motion of electrons in the solenoid.

The electric fields in Weber's electrodynamics are non-local, similar to the Coulomb field produced by a stationary charge. The apparent contradiction between non-locality ("action at a distance") and the principle of locality can be reconciled, as discussed in detail in a previous paper [15]. Essentially, the electric fields in Weber's electrodynamics can form waves due to the presence of virtual charges (many-body effects) in a polarizable vacuum [16,17].

In this paper, the current variation is slow (1 Hz), so radiative effects can be neglected. However, when the frequency of current variation is high (>6 kHz), observed electromotive force (emf) deviates from the linear Weber force prediction [13]. Explaining this deviation may require incorporating radiation effects.

For a long solenoid, the velocity-related and acceleration-related electric fields cancel each other outside the solenoid. This may be one reason why previous experiments failed to detect the velocity-related electric field [18]. In the case of a short solenoid, the velocity-related electric field is not canceled by the acceleration-related component. However, this field is quite weak and may be difficult to detect.

7. Conclusions

In this paper, we provided a simple and straightforward explanation of the Maxwell-Lodge effect using the electric field theory of Weber's electrodynamics. The tangential electric field is generated directly by the moving electrons in the solenoid, rather than being induced by the magnetic field or magnetic vector potential. Additionally, we demonstrated the different components of the electric field both inside and outside long and short solenoids, highlighting their similarities and differences compared to classical electromagnetism.

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