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Article

Collatz Trees: A Structural Framework for Understanding the 3x+1 Problem

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Abstract

The Collatz Conjecture remains one of the most enduring unsolved problems in mathematics, despite being based on an extraordinarily simple rule. Given any natural number n, the conjecture posits that repeatedly applying the operation—dividing by 2 if even, or multiplying by 3 and adding 1 if odd—will eventually result in the number 1. This paper develops a structural perspective by proposing the *Collatz Tree* as a framework to organize and visualize natural numbers. Each *branch* is the geometric ray $\{k \cdot 2^b\}_{b\geq 0}$ for an *odd core* k, and the *trunk* is the ray from 1. We introduce a trunk–branch indexing that bijects $\mathbb{N}_{\geq 1}$ with $\mathbb{Z}_{\geq 0} \times \mathbb{Z}_{\geq 0}$, providing a two-dimensional coordinate system on $\mathbb{N}_{\geq 1}$ adapted to Collatz dynamics. Algebraically, we encode a three-way map T built from affine pieces $E(x) = \frac{3}{2}x$, $O_1(x) = \frac{3x+1}{2}$, $O_2(x) = \frac{3x+1}{4}$, and analyze finite compositions $F_W(x) = a_W x + c_W$ associated to words W in $\{E, O_1, O_2\}$. We prove strong *obstructions* to finite cycles and, via a parity-based *rotation-free* argument, establish the *absence of nontrivial finite cycles for* T. We then formulate a *bridge conjecture* relating cycles of T to cycles of the standard accelerated map $A(n) = (3n+1)/2^{\nu_2(3n+1)}$ on odd integers, and outline a *coverage program* based on the inverse Collatz tree. These latter parts are presented as a research program and conjectural framework rather than as completed proofs. The aim is to separate rigorously proved structural facts from conjectural components, and to organize the global Collatz problem into two pillars: cycle-freeness (proved here for T) and coverage (reachability in the inverse tree), which remains open. *Note.* Here, the three-way map T is an *auxiliary refinement* of the usual Collatz step that deterministically selects one of E, O_1, O_2 according to $\nu_2(3n+1)$ (and parity). Thus "absence of nontrivial finite cycles for T" is a structural result about this refinement, not a direct proof of global convergence for the original Collatz map. We separate the rigorously proved structural facts (cycle-freeness for T) from conjectural components (the Bridge and Coverage programs) on purpose.

Keywords: collatz conjecture; directed tree; geometric sequence; reverse computation; natural numbers

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1. Decomposing All Natural Numbers into Geometric Sequences

1.1. Background and Objective

We express $\mathbb{N}_{\geq 1}$ as a collection of rays parameterized by odd cores and powers of two, providing a structural stage for Collatz dynamics.

1.2. Definitions and Goals

Let

$$S = \{(2a+1) \cdot 2^b \mid a, b \in \mathbb{Z}_{\geq 0}\}.$$

We show $S = \mathbb{N}_{\geq 1}$ and the representation is unique.

1.3. Prime Factorization and Classification

Every $n \in \mathbb{N}_{\geq 1}$ decomposes uniquely as

$$n = 2^b \cdot k, \qquad b \in \mathbb{Z}_{\geq 0},\ k \text{ odd}.$$

1.4. *Exhaustion of Odd Numbers*

Any odd k is $k = 2a + 1$ with $a \geq 0$, giving $1, 3, 5, 7, \ldots$.

1.5. *Exhaustion of Even Parts*

For each odd k, the ray $k, 2k, 4k, \ldots$ exhausts the even multiples of k.

1.6. *Construction of S and Uniqueness*

By the above, every $n = (2a + 1)2^b$ with $a, b \geq 0$. If

$$(2a + 1)2^b = (2a' + 1)2^{b'},$$

then $(2a + 1)/(2a' + 1) = 2^{b'-b}$, forcing $a = a'$ and $b = b'$ since the left side is odd rational and the right is a power of two. Hence $S = \mathbb{N}_{\geq 1}$ bijectively.

1.7. *Remarks from the Collatz Perspective*

For odd k, $3k + 1$ is even and belongs to some ray $(2a' + 1)2^{b'}$. This exhibits *inter-branch* connections. However, the assertion that *every* number lies on a finite *forward* path to 1 (global convergence) is a separate issue (coverage) and is not implied by the mere classification $S = \mathbb{N}_{\geq 1}$.

Takeaway of Chapter 1. We obtain a clean, bijective *indexing* of $\mathbb{N}_{\geq 1}$ by odd core and 2-adic height, furnishing a coordinate system on which later structural/affine arguments are staged.

2. The Structure of the Collatz Tree

2.1. *Definition (Branches and Trunk)*

Define the *trunk* $T_0 = \{1 \cdot 2^b : b \geq 0\}$ and for each odd $k \geq 3$ the *branch* $B_k = \{k \cdot 2^b : b \geq 0\}$. These rays partition $\mathbb{N}_{\geq 1}$ disjointly.

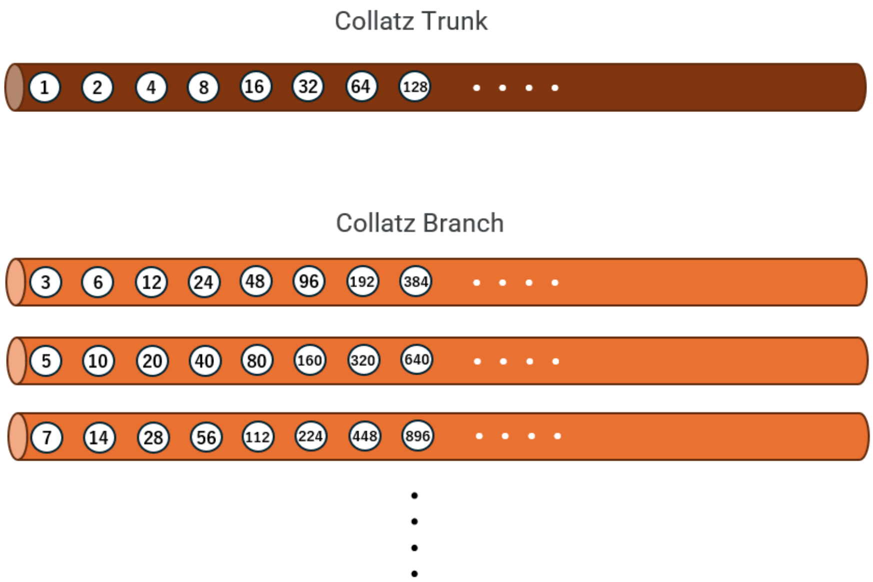

Figure 1. Trunk and branches (schematic; reverse orientation when embedded into the inverse graph: edges point to preimages).

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2.2. *Branch–Branch Links via* $3k + 1$

Given odd k, $3k + 1$ is even and decomposes as $(2a' + 1)2^{b'}$, indicating where the branch from k can *merge* into another branch/trunk in forward dynamics. This shows linkage patterns but *does not* by itself prove global coverage of the tree by reverse generation.

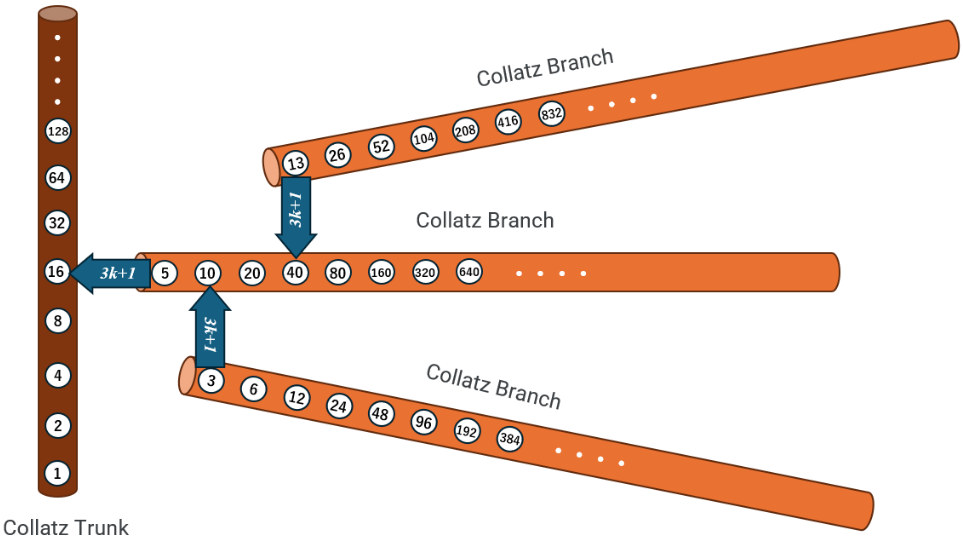

Figure 2. Branch connections (schematic; reverse orientation: edges point to preimages).

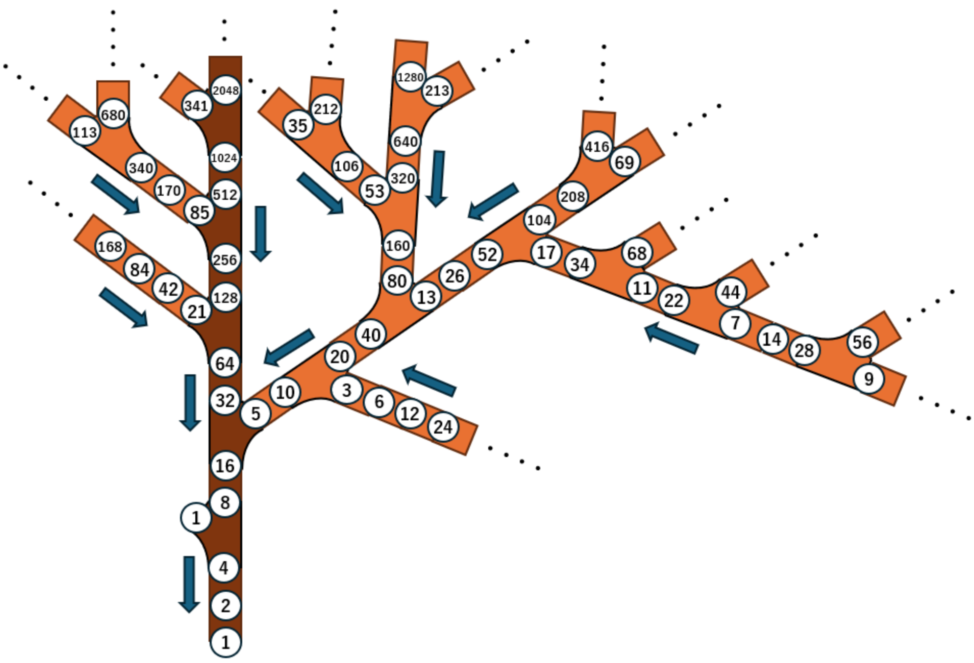

Figure 3. Collatz execution units (forward Collatz tree; edges follow the usual $3n + 1$ iteration).

2.3. *Forward vs. Reverse Orientation*

Let the standard forward map be

$$f(n) = \begin{cases} n/2, & n \text{ even}, \\ 3n + 1, & n \text{ odd}. \end{cases}$$

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The *forward* graph (edges $n \to f(n)$) is a functional digraph (out-degree 1). It is *not* acyclic due to the trivial $1 \to 4 \to 2 \to 1$ three-cycle; nontrivial finite cycles are excluded later at the level of the three-way auxiliary map T.

The *reverse* (preimage) graph rooted at 1, with edges to preimages under f, is a true DAG: levels increase with each application of a reverse step.

2.4. Tree Language

When drawing a *reverse* BFS tree rooted at 1, each node is assigned a unique *parent by construction* (though a number may have up to two preimages as graph children). Connectivity of *every* node to 1 in the forward sense is equivalent to *coverage* of the reverse tree, which is equivalent to the Collatz convergence; see Theorem 4 for a formal statement.

3. Trunk–Branch Indexing of the Natural Numbers

Definition 1 (Odd core, 2-adic valuation). *For* $n \in \mathbb{N}_{\geq 1}$, *write uniquely* $n = \mathrm{odd}(n) \cdot 2^{\nu_2(n)}$ *where* $\mathrm{odd}(n)$ *is odd and* $\nu_2(n) \in \mathbb{Z}_{\geq 0}$ *is the exponent of* 2 *in* n.

Definition 2 (Trunk and branches). *The trunk is* $T_0 = \{1 \cdot 2^b : b = 0, 1, 2, \dots\} = 1, 2, 4, 8, \dots$. *For any odd* $k \geq 3$, $B_k = \{k \cdot 2^b : b = 0, 1, 2, \dots\}$. *Then* $\{T_0\} \cup \{B_k : k \text{ odd} \geq 3\}$ *is a disjoint partition of* $\mathbb{N}_{\geq 1}$.

Definition 3 (Indices). *Order the odd numbers as* $1, 3, 5, 7, \dots$. *Assign the branch index* $\mathrm{br}(odd) = (odd - 1)/2 \in \mathbb{Z}_{\geq 0}$, *so that* $\mathrm{br}(1) = 0$ *and* $\mathrm{br}(3) = 1$, $\mathrm{br}(5) = 2$, *etc. Define the height* $\mathrm{ht}(n) = \nu_2(n)$. *Set*

$$\mathrm{Idx} : \mathbb{N}_{\geq 1} \to \mathbb{Z}_{\geq 0} \times \mathbb{Z}_{\geq 0}, \qquad \mathrm{Idx}(n) = \big(\mathrm{br}(\mathrm{odd}(n)), \mathrm{ht}(n)\big), \quad \mathrm{Idx}^{-1}(i, b) = (2i+1) \cdot 2^b.$$

Theorem 1 (Complete classification). *The map* $n \mapsto \mathrm{Idx}(n)$ *is a bijection from* $\mathbb{N}_{\geq 1}$ *onto* $\mathbb{Z}_{\geq 0} \times \mathbb{Z}_{\geq 0}$.

Proof. Uniqueness of $\mathrm{odd}(n)$ and $\nu_2(n)$ is immediate; disjointness/exhaustiveness of rays follows. □

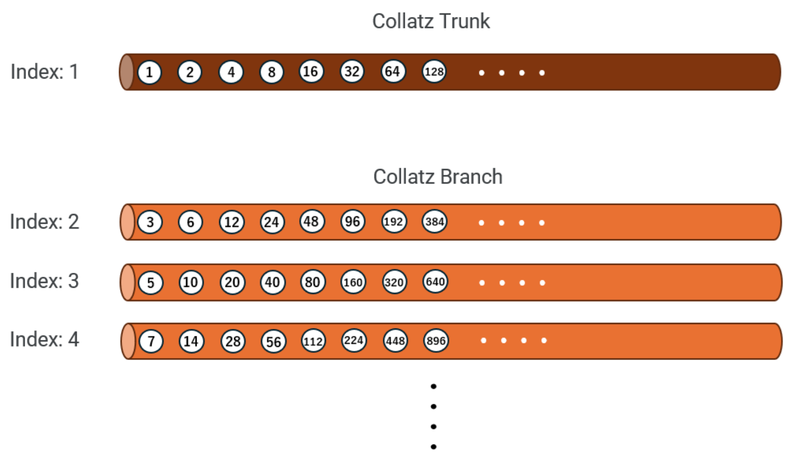

Figure 4. Trunk–branch indexing (schematic; reverse orientation in the inverse graph).

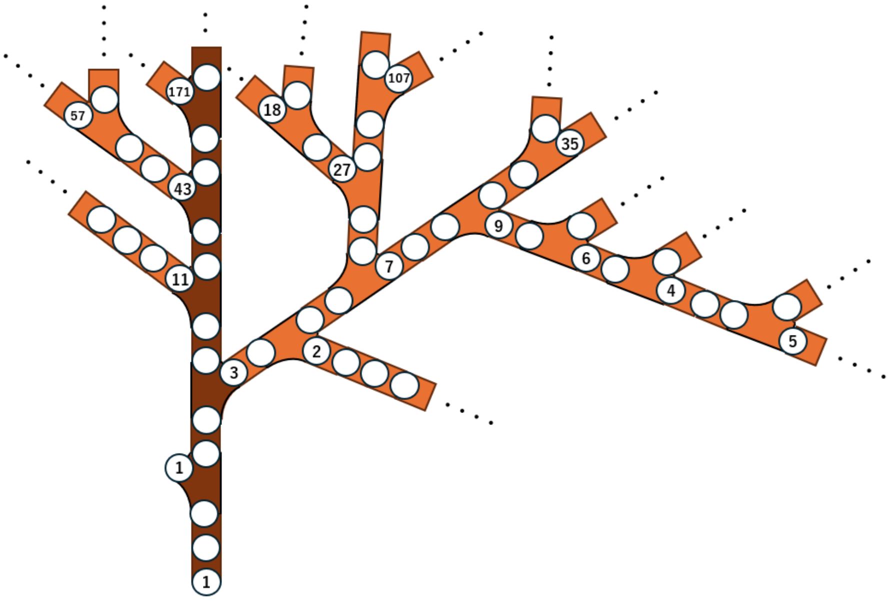

Figure 5. Indexed reverse tree (schematic; edges point to preimages).

In this table we use a 1-based display index $I(k) = \mathrm{br}(k) + 1 = (k+1)/2$.

Table 1. Trunk –Branch Indexing (sample). Here $I = (k+1)/2$.

Odd k	Index (I)	Next Index (rule)	Parity-based trend	Transition factor
1	1	–	–	–
3	2	3	increase	$(3/2)I$
5	3	1	decrease	$(3I+1)/2$
7	4	6	increase	$(3/2)I$
9	5	4	decrease	$(3I+1)/4$
11	6	9	increase	$(3/2)I$
13	7	3	decrease	$(3I+1)/2$
15	8	12	increase	$(3/2)I$
17	9	7	decrease	$(3I+1)/4$
19	10	15	increase	$(3/2)I$

Lemma 1 (Index dynamics and monotonicity of O1)**.** *Every $x \in \mathbb{N}_{\geq 1}$ can be written uniquely as $x = k \cdot 2^b$ with an odd core $k \geq 1$ and $b \geq 0$. Under the three maps*

$$E(x) = \frac{3}{2}x,$$
$$O_1(x) = \frac{3x+1}{2} \quad (\textit{if } x \equiv 3 \pmod 4),$$
$$O_2(x) = \frac{3x+1}{4} \quad (\textit{if } x \equiv 1 \pmod 4).$$

the (k,b)-state evolves as

$$E:\ (k,b) \longmapsto (3k,\ b-1) \qquad \textit{for } b \geq 1,$$
$$O_1:\ (k,0) \longmapsto \left(\frac{3k+1}{2}, 0\right) \qquad \textit{for } k \equiv 3 \pmod 4\ (\Leftrightarrow\ \nu_2(3k+1) = 1),$$
$$O_2:\ (k,0) \longmapsto \left(\mathrm{odd}(\tfrac{3k+1}{4}),\ \nu_2(\tfrac{3k+1}{4})\right) \qquad \textit{for } k \equiv 1 \pmod 4\ (\Leftrightarrow\ \nu_2(3k+1) \geq 2).$$

Proof. A direct check. For O_1 we have $(3k+1)/2 - k = (k+1)/2 \geq 1$. □

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4. Affine Word Method: Obstructions to Nontrivial Finite Cycles

We now introduce a three-way map T whose branches are affine maps, and analyze its finite compositions. The focus of this section is on rigorous algebraic obstructions to nontrivial cycles of T. Expository roadmap.

We first record several algebraic lemmas. A historical/sketch-type criterion is stated only for context and is *not* used later. The definitive, self-contained proof of cycle-freeness for T is Theorem 2 in §4.3; readers may safely skip directly there.

4.1. Definition of the three-way map T

Define $T : \mathbb{N}_{\geq 1} \to \mathbb{N}_{\geq 1}$ by

$$T(n) = \begin{cases} \frac{3}{2}\,n, & n \text{ even}, \\ \frac{3n+1}{2}, & n \text{ odd and } \nu_2(3n+1) = 1, \\ \frac{3n+1}{4}, & n \text{ odd and } \nu_2(3n+1) \geq 2. \end{cases}$$

Thus T is a deterministic three-way refinement of the usual Collatz step.

Lemma 2 (Well-definedness and totality of T)**.** *For every $n \in \mathbb{N}_{\geq 1}$, exactly one of the three branches of T applies, and the image $T(n)$ is an integer.*

Proof. If n is even, then $E(n) = \frac{3}{2}n \in \mathbb{N}_{\geq 1}$. If n is odd, write $r = \nu_2(3n+1) \geq 1$. When $r = 1$ we use $O_1(n) = \frac{3n+1}{2} \in \mathbb{N}_{\geq 1}$; when $r \geq 2$ we use $O_2(n) = \frac{3n+1}{4} \in \mathbb{N}_{\geq 1}$. Exactly one of $\{r = 1,\ r \geq 2\}$ holds, hence the rule is deterministic and total on $\mathbb{N}_{\geq 1}$. □

Layer terminology. We call a state n *odd-layer* if $\nu_2(n) = 0$ and *even-layer* otherwise. We also speak of a *return to the odd layer* when the orbit first hits $\nu_2 = 0$ again.

4.2. Elementary Steps and Words

Introduce affine maps

$$E(x) = \frac{3}{2}x, \qquad O_1(x) = \frac{3x+1}{2}, \qquad O_2(x) = \frac{3x+1}{4}.$$

Definition 4 (Admissibility and cyclic admissibility)**.** *Let $W = s_\ell \circ \cdots \circ s_1$ with $s_i \in \{E, O_1, O_2\}$. Given $x \in \mathbb{N}_{\geq 1}$, we say that W is* admissible at x *if the forward evaluation applies each s_i to the current integer according to the defining side-conditions: E only at even inputs; O_1 only when $\nu_2(3 \cdot current + 1) = 1$; and O_2 only when $\nu_2(3 \cdot current + 1) \geq 2$. We call W* cyclically admissible *if it is admissible at every point along a full period of a putative cycle and returns to the initial state.*

Remark 1. *In cycle arguments we tacitly restrict to cyclically admissible words. Purely formal words violating the side-conditions are irrelevant to actual T-orbits.*

For any finite word W over $\{E, O_1, O_2\}$, the composition is

$$F_W(x) = a_W x + c_W, \qquad a_W = \frac{3^{\,m+o_1+o_2}}{2^{\,m+o_1+2o_2}},$$

where $m = \#E$, $o_1 = \#O_1$, $o_2 = \#O_2$.

Lemma 3 (No $a_W = 1$ for nonempty words)**.** *If W is nonempty, then $a_W \neq 1$.*

Proof. If $a_W = 1$, then $3^{m+o_1+o_2} = 2^{m+o_1+2o_2}$, which forces $m = o_1 = o_2 = 0$. □

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Lemma 4 (Odd numerator for $1 - a_W$).

$$1 - a_W = \frac{2^{m+o_1+2o_2} - 3^{m+o_1+o_2}}{2^{m+o_1+2o_2}},$$

and the numerator is odd.

Remark 2 (On fixed points with $m \geq 1$). *We do not exclude integral fixed points solely by $x = c_W/(1 - a_W)$; instead, cycle-freeness will follow from the odd-layer tour in §4.3.*

Lemma 5 (Odd-layer words ($m = 0$)). *Let W be a nonempty word over $\{O_1, O_2\}$. If $o_1 = 0$ (all O_2), then F_W is a contraction and the only integer fixed point is $x = 1$.*

4.3. Alternative Proof of Cycle-Freeness (Rotation-Free via O_1 and the Odd Layer)

For a word $W = s_\ell \circ \cdots \circ s_1$ with $s_i \in \{E, O_1, O_2\}$, put

$$W(x) = \frac{3^{e(W)}x + B(W)}{2^{n(W)}}, \qquad B(W) \in \mathbb{Z}. \tag{1}$$

Here $e(W)$ and $n(W)$ count, with multiplicity, the number of odd-branch multipliers 3 and the total power of 2 in the denominator along the evaluation, respectively. When W is cyclically admissible (Def. 4), these counts are uniquely determined by W.

Then any period-1 point satisfies

$$\left(2^{n(W)} - 3^{e(W)}\right) x = B(W). \tag{2}$$

The coefficient on the left is always odd.

Lemma 6 (Parity pinned by the first symbol). *For any nonempty W with first symbol s_1,*

$$B(W) \equiv \begin{cases} 0 \pmod 2, & s_1 = E, \\ 1 \pmod 2, & s_1 \in \{O_1, O_2\}. \end{cases}$$

This parity is invariant under further extensions of W.

Proof of Lemma 6. Any finite word $W = s_\ell \circ \cdots \circ s_1$ can be written in the normalized form

$$W(x) = \frac{3^{e(W)}x + B(W)}{2^{n(W)}}, \qquad B(W) \in \mathbb{Z},$$

where $e(W), n(W)$ are the accumulated exponents of 3 and 2 along the evaluation.

Base step (length 1). A direct check gives

$$E(x) = \frac{3x + 0}{2} \Rightarrow B = 0 \equiv 0 \pmod 2, \qquad O_1(x) = \frac{3x+1}{2}, \quad O_2(x) = \frac{3x+1}{4} \Rightarrow B = 1 \equiv 1 \pmod 2.$$

Inductive step. Assume $U(x) = \dfrac{3^e x + B}{2^n}$ with B mod 2 already fixed by the *first* symbol of U. Prepending $s \in \{E, O_1, O_2\}$ yields

$$E \circ U(x) = \frac{3^{e+1}x + 3B}{2^{n+1}}, \qquad O_1 \circ U(x) = \frac{3^{e+1}x + (3B + 2^n)}{2^{n+1}}, \qquad O_2 \circ U(x) = \frac{3^{e+1}x + (3B + 2^n)}{2^{n+2}}.$$

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Hence the new constant term is

$$B' = \begin{cases} 3B, & s = E, \\ 3B + 2^n, & s \in \{O_1, O_2\}. \end{cases}$$

Because $3 \equiv 1 \pmod 2$ and $2^n \equiv 0 \pmod 2$, we have $B' \equiv B \pmod 2$. Therefore the parity of B is determined by the *first* symbol and remains unchanged under any further left extensions. Together with the base step, the claim follows. □

Lemma 7 (Any cycle uses both O_2 and E)**.** *If a forward cycle exists, then it contains at least one O_2 and at least one E.*

Proof. (i) Suppose a cycle contains no E. Then the orbit never leaves the odd layer. If it uses only O_2, the induced affine factor is a strict contraction on $\mathbb{N}$, hence the only fixed point is 1 (Lemma 5); this contradicts nontriviality. If it uses O_1 at least once, then on odd cores the map $k \mapsto (3k+1)/2$ is strictly increasing (Lemma 1), so periodicity is impossible. Thus any nontrivial cycle must contain E.

(ii) Suppose a cycle contains no O_2. If it consists only of E, the orbit never returns to the odd layer, hence it cannot close a cycle. If it uses O_1, then whenever the orbit is on the odd layer, the odd core strictly increases again by Lemma 1, so periodicity is impossible. Therefore any nontrivial cycle must also contain O_2. □

Theorem 2 (Cycle-freeness (odd-layer tour argument))**.** *There is no nontrivial cycle for $T = \{E, O_1, O_2\}$ on $\mathbb{N}_{\geq 1}$.*

Remark 3 (What Theorem 2 does *not* claim)**.** *The theorem rules out nontrivial cycles for the auxiliary map T. It does not, by itself, imply global convergence of the original Collatz iteration. Sections 5–6 separate the conjectural bridge and the coverage program explicitly.*

Proof. Assume a cycle exists. By Lemma 7 it has both O_2 and E. Start at an odd occurrence whose next step is O_2; write k for its odd core. Then $(2^n - 3^e)k = \widetilde{B}$ with $\widetilde{B}$ odd. Rotating the same tour to start at the even occurrence after that O_2 yields $(2^n - 3^e)k^* = \widetilde{B}^*$ with $\widetilde{B}^*$ even, forcing k^* even—contradiction. □

5. Bridge to the Accelerated Collatz Map: A Conjectural Framework

Remark 4 (Compatibility via coefficient matching)**.** *An accelerated step $A(x) = \frac{3x+1}{2^{\nu_2(3x+1)}}$ consists of one odd reduction followed by a total 2-adic division of size $r = \nu_2(3x+1)$. While T uses $E(x) = \frac{3}{2}x$ on even layers (not literal halving), one can* encode *the same (e, E) exponents by a word over $\{O_1, O_2, E\}$ with matching counts (coefficient matching). This is the sense in which our Bridge program relates hypothetical A-cycles to T-cycles.*

5.1. Cycles of the Accelerated Map

Let $A : \{\, n \in \mathbb{N}_{\geq 1} \mid n \text{ odd} \,\} \to \{\, n \in \mathbb{N}_{\geq 1} \mid n \text{ odd} \,\}$ be

$$A(n) = \frac{3n+1}{2^{\nu_2(3n+1)}}.$$

Suppose $(x_0, \ldots, x_{L-1})$ is a hypothetical cycle of A, with $r_j = \nu_2(3x_j + 1)$ and

$$E = \sum_{j=0}^{L-1} r_j, \qquad e = L.$$

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Then there exists $C \in \mathbb{Z}$ with

$$(2^E - 3^e)\,x_0 = C, \qquad D := 2^E - 3^e \neq 0,\ \gcd(3, D) = 1. \tag{3}$$

5.2. Matching Coefficients on the T-Side

For a word W over $\{E, O_1, O_2\}$ with counts (m, o_1, o_2), we have

$$a_W = \frac{3^{\,m+o_1+o_2}}{2^{\,m+o_1+2o_2}}.$$

If W satisfies

$$m = e, \qquad m + o_1 + 2o_2 = E,$$

then $a_W = \frac{3^e}{2^E}$ and

$$1 - a_W = \frac{2^E - 3^e}{2^E} = \frac{D}{2^E}.$$

Definition 5 (Coefficient-matched word)**.** *Given (E, e) from an A-cycle, a word W is* coefficient-matched *if $m = e$ and $m + o_1 + 2o_2 = E$.*

Adjacent-swap effect on c_W.

$$\begin{aligned} E \circ O_1(x) &= \frac{9x+3}{4}, & O_1 \circ E(x) &= \frac{9x+2}{4}, & \Delta c &= \frac{1}{4}, \\ E \circ O_2(x) &= \frac{9x+3}{8}, & O_2 \circ E(x) &= \frac{9x+2}{8}, & \Delta c &= \frac{1}{8}, \\ O_1 \circ O_2(x) &= \frac{9x+5}{8}, & O_2 \circ O_1(x) &= \frac{9x+5}{8}, & \Delta c &= 0. \end{aligned}$$

Thus, swapping E to the left of O_i increases c_W by 2^{-n_i} (with $n_i \in \{1, 2\}$). This local move makes it feasible to tune $c_W \pmod{D}$.

Lemma 8 (Local swap reachability mod D)**.** *Let $D = 2^E - 3^e$ and let W be any coefficient-matched word (so that $a_W = 3^e/2^E$). Performing the adjacent swap $E \circ O_1 \leftrightarrow O_1 \circ E$ increases the constant term c_W by 2^{-1}, and $E \circ O_2 \leftrightarrow O_2 \circ E$ increases c_W by 2^{-2}, when measured before the final normalization by 2^E. Consequently, the set of residues of c_W mod D reachable from a given coefficient-matched word contains the subgroup generated by $\{2^{-1}, 2^{-2}\}$ in $(\mathbb{Z}/D\mathbb{Z})$.*

Remark 5. *Because $\gcd(3, D) = 1$, tuning c_W mod D reduces to understanding the subgroup generated by the 2-adic increments above. A full classification is not required here, but this mechanism underlies the Bridge Conjecture (Theorem 3) by making the fixed-point equation $x = c_W/(1 - a_W)$ congruentially solvable.*

5.3. Bridge Conjecture

Conjecture 3 (Equivalence of cyclicity)**.** *Suppose the accelerated map A admits a nontrivial finite cycle of length $L \geq 1$. Then there exist a coefficient-matched word W over $\{E, O_1, O_2\}$ and an $x \in \mathbb{N}_{\geq 1}$ such that $F_W(x) = x$, and the T-orbit of x follows exactly the step pattern encoded by W. In particular, any nontrivial cycle of A induces a nontrivial cycle of T.*

Corollary 1 (Conditional absence of nontrivial cycles for A)**.** *If Theorem 3 holds, then the accelerated map $A(n) = (3n+1)/2^{\nu_2(3n+1)}$ has no nontrivial finite cycles.*

Proof. By Theorem 3, a nontrivial cycle of A would induce a nontrivial cycle of T, which contradicts Theorem 2. □

Theorem 4 (Reduction to convergence)**.** *For A, the following are equivalent:*

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1. *Every positive integer reaches* 1 *in finitely many steps (global convergence).*
2. *The inverse Collatz tree rooted at* 1 *covers all positive integers (reachability/coverage).*

Proof. This is the standard argument for functional digraphs: global convergence is equivalent to every node being in the basin of the component containing 1, which is equivalent to coverage in the reverse (preimage) tree. □

6. Coverage of the Inverse Collatz Tree: A Program

Assuming cycle-freeness for the accelerated map, global convergence reduces to coverage of the inverse tree (Theorem 4). We do not claim a proof; rather we outline a program.

6.1. Inverse Graph Definition

For $n \in \mathbb{N}_{\geq 1}$, define preimages

$$\mathcal{C}(n) = \{\, 2n \,\} \cup \left\{ \tfrac{n-1}{3} \;\middle|\; n \equiv 1 \pmod 3,\ \tfrac{n-1}{3} \text{ odd} \right\}.$$

This yields a DAG rooted at 1.

Definition 6 (Coverage)**.** *The inverse Collatz tree is* covering *if every* $n \in \mathbb{N}_{\geq 1}$ *appears at some depth from the root* 1*.*

6.2. Local Branching and Odd Cores

Lemma 9 (Two-Child Criterion)**.** *If* $n \equiv 4 \pmod 6$*, then* $\mathcal{C}(n) = \{2n, (n-1)/3\}$ *with* $(n-1)/3$ *odd; otherwise* $\mathcal{C}(n) = \{2n\}$*.*

Proposition 1 (Descent at branching)**.** *For* $n \equiv 4 \pmod 6$*, the odd core of* $(n-1)/3$ *is* $\leq (n-1)/3 \leq \frac{2}{3}n$*.*

Proposition 2 (Infinitely many potential descent levels when $3 \nmid k$)**.** *If the odd core* k *is not a multiple of* 3*, the levels* $k \cdot 2^b \equiv 4 \pmod 6$ *occur infinitely often in* b*.*

6.3. A Coverage Conjecture

Conjecture 5 (Coverage Conjecture)**.** *The inverse Collatz tree rooted at* 1 *is covering.*

7. Related Work

The affine-composition viewpoint ($3^m/2^E$) is classical (Lagarias; Terras). Our contribution is to combine (i) trunk–branch indexing and inverse-tree structure, (ii) a complete loop-elimination for the three-way map T, and (iii) a conjectural bridge transporting hypothetical A-cycles into T.

Data Availability Statement: Figures can be regenerated using the Python in the Appendix (for visualization; not a proof of coverage).

Appendix A. Python Code for Reverse Collatz Tree Visualization

This script visualizes structure up to a finite cutoff and is *not* a proof of coverage.

Listing A1: Reverse Collatz Tree (visualization only)

```
import networkx as nx
import matplotlib.pyplot as plt

def generate_tree(limit=250):
    G = nx.DiGraph()
    G.add_node(1, level=0)
    queue = [(1, 0)] # (node, level)
```

```python
    visited = set([1])

    while queue:
        n, level = queue.pop(0)

        # Rule 1: multiply by 2 (always a preimage)
        child1 = 2 * n
        if child1 <= limit and child1 not in visited:
            G.add_edge(n, child1) # reverse edge: parent -> preimage
            G.nodes[child1]['level'] = level + 1
            queue.append((child1, level + 1))
            visited.add(child1)

        # Rule 2: inverse of 3n+1; require odd preimage
        if n % 2 == 0 and (n - 1) % 3 == 0:
            child2 = (n - 1) // 3
            if child2 % 2 == 1 and child2 > 0 and child2 not in visited:
                G.add_edge(n, child2)
                G.nodes[child2]['level'] = level + 1
                queue.append((child2, level + 1))
                visited.add(child2)

    return G

def draw_tree(G):
    levels = nx.get_node_attributes(G, 'level')
    pos = {}
    level_widths = {}

    for node, level in levels.items():
        level_widths.setdefault(level, []).append(node)

    for level, nodes in level_widths.items():
        xs = range(-len(nodes) + 1, len(nodes), 2)
        for x, node in zip(xs, nodes):
            pos[node] = (x, level)

    plt.figure(figsize=(12, 10))
    nx.draw(G, pos, with_labels=True, node_size=700,
            node_color='peru', edge_color='black',
            font_size=10, font_weight='bold', alpha=0.85)
    plt.show()

if __name__ == "__main__":
    G = generate_tree(limit=250)
    draw_tree(G)
```

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Appendix B. Python-Generated Tree Visualizations (Illustrative Only)

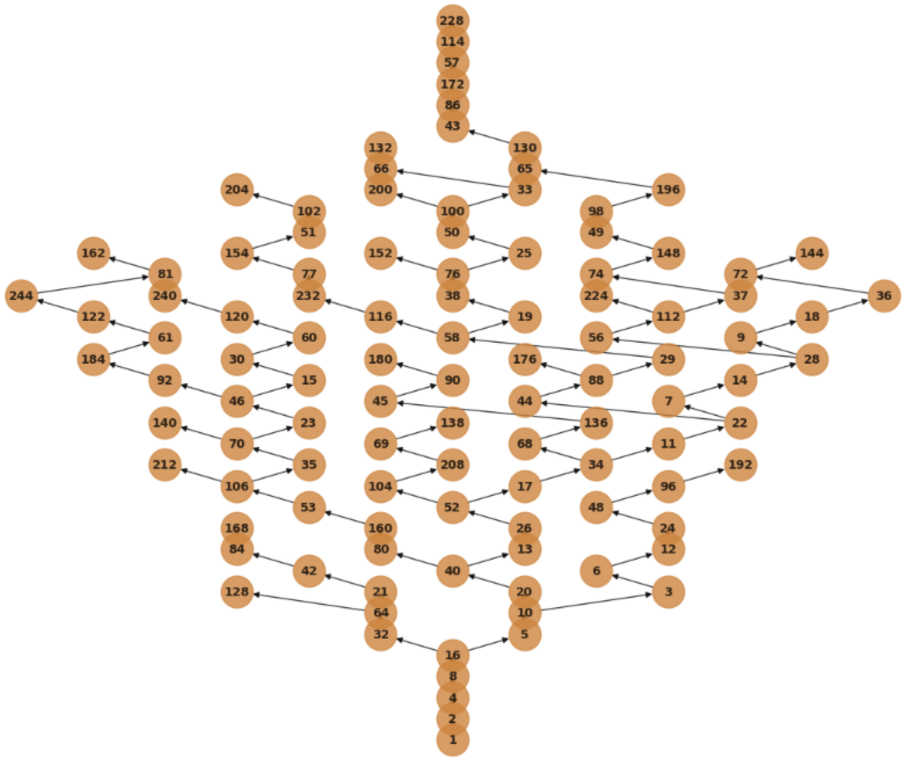

Figure A1. Reverse Collatz tree generated programmatically (limit = 250).

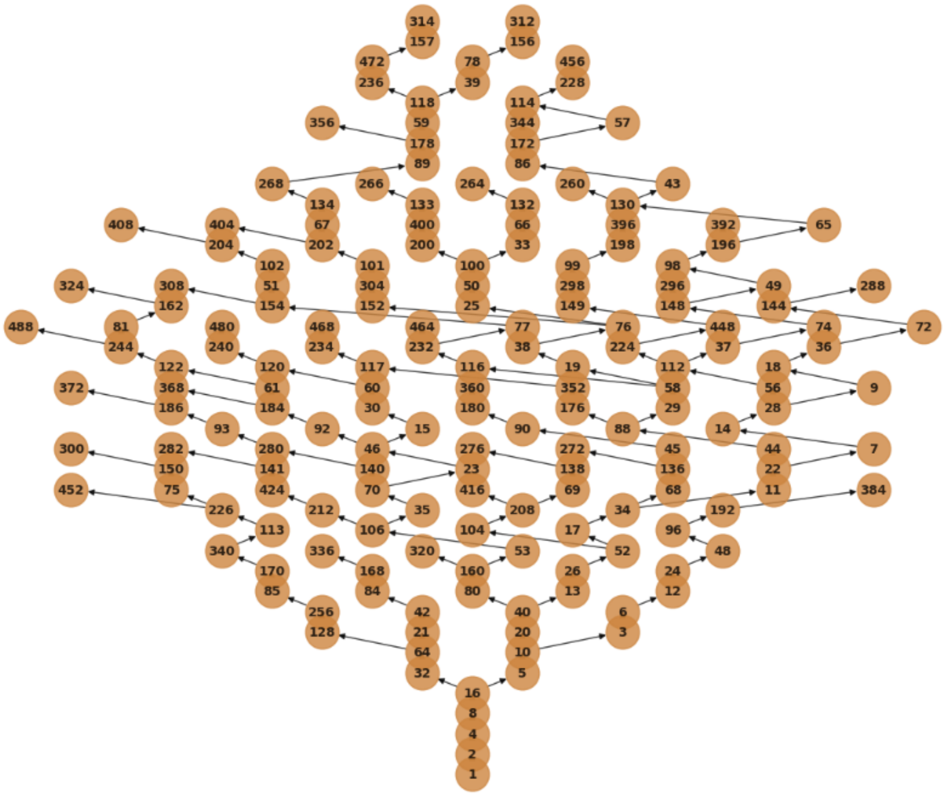

Figure A2. Reverse Collatz tree generated programmatically (limit = 500).

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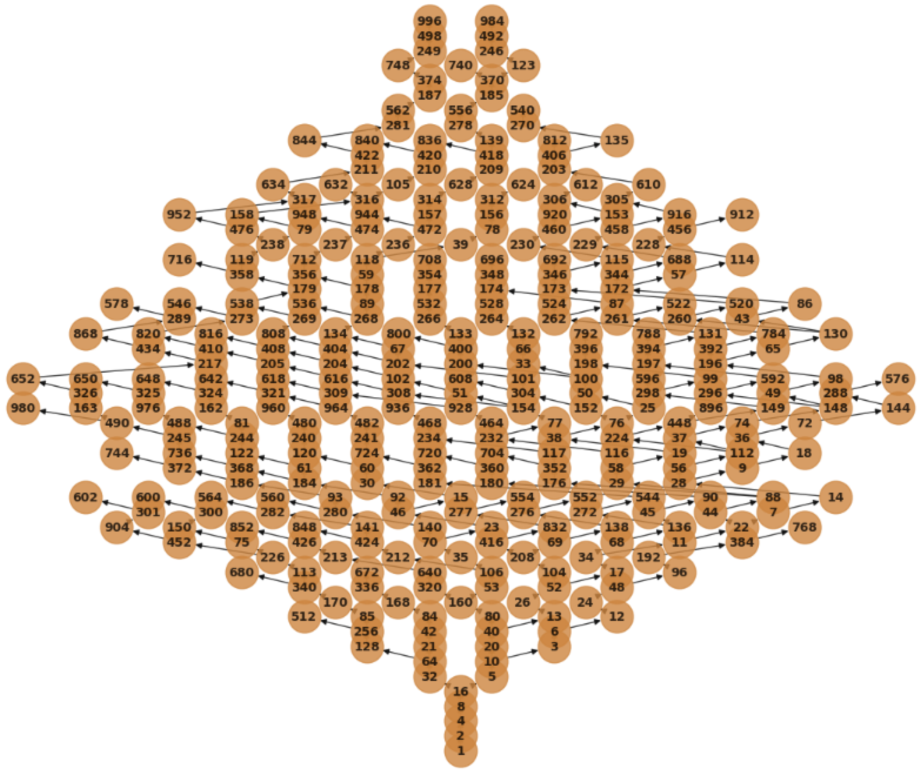

Figure A3. Reverse Collatz tree generated programmatically (limit = 1000).

References

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