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Dark-Matter Specifications and Gravitational Aspects that Explain Otherwise Seemingly Unexplained Cosmic Data

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Abstract: We provide quantitative explanations for known ratios of dark-matter effects to ordinary-matter effects, and we suggest qualitative explanations for the rate of expansion of the universe and for lumpiness tensions. Our work features a well-defined specification for dark matter. Our work features a new multipole-expansion method that combines Newtonian gravity and special-relativistic interpretations of properties of objects. Our work adds one integer-centric equation to successful popular modeling. Some solutions of the equation help to quantitatively explain dark-matter-to-ordinary-matter ratios and help to suggest insight regarding galaxy formation, eras in the rate of expansion, the Hubble tension, and the S8 tension.

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1. Introduction

This unit discusses context for our work; highlights some aspects of our work; provides perspective about those aspects; and provides perspective that suggests that our work stems from, embraces, does not intrude on, and suggests improvements to successful mainstream cosmology.

1.1. Context for Our Work

This unit discusses opportunities that our work attempts to capture. The discussion de-emphasizes providing references for specific aspects of generally accepted physics in relevant areas such as general physics [1], cosmology [2,3], elementary-particle physics [4,5], relativity [6], and uses of multipole expansions [7]. The discussion de-emphasizes providing references for data for which subsequent units of this paper provide references.

Popular modeling, including the standard cosmological model, approximately fits some cosmic data and leaves unresolved some key aspects [8–10].

One key unresolved aspect features the opportunity to explain changes in the so-called rate of expansion of the universe. The rate of expansion features notions of typical speeds at which neighboring similar large objects move apart from each other. Non-colliding galaxy clusters are such large objects. Popular modeling suggests two relevant multibillion-year eras in the evolution of the universe. During the earlier era, the rate (or typical speed) is positive and decreases as the universe evolves. During the later era, the rate is positive and increases. Popular modeling generally suggests that the later era includes today. Popular modeling that might fit or explain the changes during either one of the two eras seems not to extrapolate to fit or explain the changes during the other era [11].

Notions that the universe might, in the above sense, expand or contract date to the 1920s [12,13]. Popular modeling sometimes associates the two-word term dark energy with forces that might associate with gravity and that might seem to drive objects away from each other.

Our work suggests notions that might explain the inability of popular modeling to bridge between the two multibillion-year eras.

One key unresolved aspect features the opportunity to explain phenomena that popular modeling sometimes associates with the two-word term dark matter. Starting in the 1930s, people observed cosmological phenomena that popular modeling seemed unable to fit or explain based on stuff that people observed electromagnetically [14,15]. Popular modeling associated such phenomena with notions that there was more gravitational attraction (of visible stuff toward other visible stuff) than popular modeling suggested based on the presence of only visible stuff. Some popular modeling suggested that fitting or explaining data could have bases in stuff that people did not see. Here, the word see associates with the phrase observe via electromagnetic radiation. Popular modeling suggests using the two-word term dark matter to name such not-seen stuff. Popular modeling suggests various candidate specifications for dark matter [16]. Either one of the two-word terms ordinary matter and baryonic matter pertains to, generally, the stuff that people do see.

Popular modeling proposes notions, such as MOND (as in MOdified Newtonian Dynamics), that might help fit or explain relevant gravitational phenomena without needing to invoke dark matter [16,17]. Popular modeling proposes new elementary particles, such as axions, or other stuff that might associate with dark matter and, thereby, might help fit or explain relevant gravitational phenomena. Popular modeling suggests various categories, such as SIDM (as in self-interacting dark matter), of candidate types of dark matter stuff [16,18]. Popular modeling associates the notion of self-interacting (as in the SI in SIDM) with the notion of interacting with itself via means other than gravitational means.

Observational work suggests a set of relevant observed ratios of not ordinary-matter effects to ordinary matter effects. Some of the data pertain to galaxy evolution and tend to cluster around ratios of 0+:1, ∼4:1, 5+:1, and 1:0+. Popular modeling might suggest that these ratios associate with gravitational effects. Some of the data regarding galaxy clusters and regarding the observable universe tend to cluster around 5+:1. Popular modeling might suggest that these ratios associate with gravitational effects. One ratio is 1:1. Popular modeling might suggest that this ratio associates with electromagnetic effects that deplete, over billions of years, cosmic microwave background radiation.

Our work accepts the notion that dark matter associates with stuff. We suggest a specification for elementary particles that underlie dark-matter stuff. The specification suggests that about 20-minus percent of dark matter associates with popular modeling notions of SIDM. For the dark matter stuff that associates with popular modeling notions of SIDM, we suggest the acronym SEA-DM (as in Significantly Electromagnetically Active Dark Matter). The specification suggests that about 80-plus percent of dark matter does not associate with popular modeling notions of SIDM. For the dark matter stuff that does not associate with popular modeling notions of SIDM, we suggest the acronym MEA-DM (as in Marginally Electromagnetically Active Dark Matter).

Our work develops notions regarding gravitational interactions between two objects. That portion of our work combines aspects of Newtonian gravity [19], special-relativistic interpretations regarding properties of objects, and multipole-expansion mathematics. We suggest the acronym SPRISENG (as in SPecial-Relativity-Inspired Suggested Extensions to Newtonian Gravity) or, for short, ENG (as in Extended Newtonian Gravity). The Newtonian list of gravitational properties of objects consists of one item, mass. Our extensions to Newtonian gravity extend the list of gravitational properties of objects to include other properties, such as object-internal angular momentum and moments of inertia. The Newtonian list might not associate with notions of gravitational repulsion between two objects. The expanded list of gravitational properties associates with modeling that embraces notions of gravitational repulsion and points to the notion that, in some circumstances, gravitational repulsion between two objects occurs.

Our work does not use MOND, which attempts to account for some gravitational attraction without using the notion of dark matter. ENG suggests an explanation for phenomena, such as increases in the rate of expansion of the universe, that might suggest the relevance of notions of gravitational repulsion. We do not explore possible associations or dissonances between aspects of ENG and aspects of MOND.

We suggest that the combination of our candidate specification for dark matter and our suggested combining of special relativity, Newtonian gravity, and an extended list of gravitational properties of objects explains quantitatively each one of the observed ratios.

We suggest that the combination of our candidate specification for dark matter and our suggested combining of special relativity, Newtonian gravity, and extended list of gravitational properties of objects provides qualitatively useful insight regarding eras in the rate of expansion of the universe, the Hubble tension regarding the rate of expansion, and the S8 tension regarding the large-scale lumpiness of matter.

1.2. Perspective About Our Methods

This unit suggests that our methods feature novel reuses of aspects of popular modeling. This unit highlights such reuses. This unit does not necessarily motivate our choice of methods. Subsequent units discuss motivations, provide details about the methods, use the methods to address opportunities that this paper notes above, and provide references.

Table 1 lists five facets of our work. Discussion immediately below provides further details.

Table 1. Five facets of our work. These facets preview aspects of our work.

Facet	Our work suggests ...
Facet-1	A specification for dark matter.
Facet-2	New, compared to popular modeling, uses for multipole expansions regarding electromagnetic potentials and regarding gravitational potentials.
Facet-3	That the new type of multipole expansions points to gravitational properties (of objects) that attract two objects toward each other and to gravitational properties (of objects) that repel two objects from each other.
Facet-4	That merging into popular modeling the above-discussed suggestions involves adding, to popular modeling, a new equation.
Facet-5	That choosing appropriate solutions of the new equation helps explain cosmic data and can help tune cosmological models.

Facet-1 suggests a specification for dark matter. The specification reuses a popular modeling list of most known elementary particles. The specification reuses aspects of popular modeling that associate with ordinary matter. Each one of our five reuses associates with dark matter. The dark-matter elementary particles are like ordinary-matter elementary particles, but some seemingly simple differences pertain. Merging this dark-matter specification into popular modeling seems to require no new notions other than some notions about modeling electromagnetic interactions and some notions about modeling gravitational interactions.

- We suggest using the three-element term ordinary-matter isomeric set to denote a set of elementary particles that includes all known elementary particles, except the photon. The set includes the Higgs boson, three neutrinos, three charged leptons (including the electron, the muon, and the tau), six quarks, the Z boson, the W boson, and eight gluons.
- We suggest using the two-element term long-range-interaction boson to denote the photon (which popular modeling associates with spin-1), the might-be graviton (which popular modeling associates with spin-2), and any higher-spin elementary bosons that would extend the series that starts with the photon and the graviton.
- The ordinary-matter isomeric set does not include any long-range-interaction bosons. The ordinary-matter isomeric set might include some unfound elementary particles, such as the inflaton (which popular modeling suggests).
- We suggest that nature includes six isomeric sets of elementary particles. One isomeric set is the ordinary-matter isomeric set. Each one of the other five isomeric sets includes one counterpart to each ordinary-matter isomeric-set elementary particle. Each one of those other five isomeric sets does not include any elementary particles that do not have ordinary-matter isomer-set

counterparts. Each one of those other five isomeric sets is like, but not identical to, the ordinary-matter isomeric set.

- Across the six isomeric sets, counterpart particles have the same mass and the same spin. The differences between the sets feature handedness and matches between lepton flavours and lepton masses.
- We use the word isomer to associate with each set.
- One isomer (the ordinary-matter isomeric set) associates with ordinary matter and, for elementary particles that exhibit handedness, with left-handedness. (Spin-0 elementary particles do not exhibit handedness. Zero-mass elementary particles do not exhibit handedness. All other known elementary particles exhibit handedness.) The other five isomers associate with dark matter.
- We use the word stuff to associate with atomic nuclei, atoms, and so forth that exist in nature. For example, the two-element term ordinary-matter stuff includes atomic nuclei that people have observed.
- One dark-matter isomer is identical to the ordinary-matter isomer, except that elementary particles that exhibit handedness exhibit right-handedness. For each of the ordinary-matter isomer and this one dark-matter isomer, the fermion flavour of the lowest-mass charged lepton equals the fermion flavour of the two lowest-mass quarks. We suggest that stuff that has bases in this dark-matter isomer associates with the popular modeling three-element term self-interacting dark matter. (Some observational results [20–22] suggest that some dark matter might comport with popular modeling notions of self-interacting dark matter [16,23]. Some popular modeling results [24–27] point to possible benefits of considering that some dark matter is self-interacting dark matter.) We associate, with the stuff that has bases in this dark-matter isomer, the term significantly-electromagnetically-active dark matter and the acronym SEA-DM.
- We use the one-element term isomer-pair to denote a pair of isomers that differ by handedness and have the same matches between lepton flavours and lepton masses.
- Each one of the remaining four dark-matter isomers associates with an isomer-pair for which one isomer associates with left-handedness and the other isomer associates with right-handedness. For each one of these four dark-matter isomers, the fermion flavour of the lowest-mass charged lepton does not equal the fermion flavour of the two lowest-mass quarks. For stuff that has bases in these four isomers, electromagnetic interactions can have significance within hadron-like particles but might not contribute to substantial effects beyond individual objects such as hadron-like particles or neutron-star-like objects. We suggest that stuff that has bases in these four dark-matter isomers does not associate with the popular modeling three-element term self-interacting dark matter. We associate, with the stuff that has bases in these four dark-matter isomers, the term somewhat-electromagnetically-active dark matter and the acronym MEA-DM.

Facet-2 suggests new, compared to popular modeling, uses for multipole expansions regarding electromagnetic potentials and regarding gravitational potentials. Popular modeling expansions feature spatial distributions of properties such as charge (for electromagnetic applications) and mass (for gravitational applications). Our modeling features expansions that pertain to modeling that treats objects as modeling as pointlike. Our modeling features expansions based on selections from the entire set of electromagnetic properties of objects or on selections from the entire set of gravitational properties of objects. For example, for an electromagnetic expansion, one term in the expansion might associate with charge and another term might associate with magnetic moment.

- Popular modeling associates various properties with an object. Properties of objects can include position (relative to the position of another object), velocity (relative to the velocity of another object), electromagnetic properties (such as charge, charge current, and magnetic moment), gravitational properties (such as mass), inertial properties (such as mass), and properties (such as fermion flavour or color charge) that associate with specific sets of elementary particles.
- Within the context of popular modeling regarding perceived-to-be-stationary objects, the electromagnetically associated series (of properties) that starts with charge (for which, in our

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pointlike-object multipole modeling, the word monopole pertains) and magnetic moment (for which, in our pointlike-object multipole modeling, the word dipole pertains) can extend to include, for example, precessing magnetic moment (for which, in our pointlike-object multipole modeling, the word quadrupole pertains). This notion of precessing magnetic moment does not associate with Larmor precession (which associates with interactions with an external magnetic field). The Earth exhibits (non-Larmor) precession of the Earth's magnetic moment.

- Within the context of popular modeling Newtonian gravitation, the gravitationally associated set (of properties) contains just one element, mass (for which, in our pointlike-object multipole modeling, the word monopole pertains). We suggest that, regarding perceived-to-be-stationary objects, a gravitationally associated series (of properties) that starts with mass (monopole) can extend to include (for example) object-internal angular momentum (dipole), moments of inertia (quadrupole), and rotating moments-of-inertia (octupole). We suggest that one can consider object-internal angular momentum to be a gravitational analog to the electromagnetic property of magnetic moment.
- For each one of the above-mentioned electromagnetic properties that does not directly associate with motion relative to an observer and each one of the above-mentioned gravitational properties that does not directly associate with motion relative to an observer, popular modeling might consider an additional property that associates with the word current. Examples include charge current, magnetic-moment current, and mass current. We suggest that, for a perceived-to-be-moving object to model as staying intact, the currents need to associate with one (common) velocity.
- Regarding the perceived values of properties, popular modeling special relativity interrelates pairs of values. Each pair consists of the value of a property for which perceptions of nonzero value can pertain when an object is not moving and the value for the counterpart current. For situations in which an object is not moving, the value that associates with the current is zero.
- Popular modeling includes uses of mathematics that associates with the two-word term multipole expansions. Typically, a multipole expansion provides a series of terms. Each term associates with a contribution toward an approximation for an overall potential that associates with a spatial distribution of a property. Typically, one assumes that the extent of the distribution is bounded. In the case of a bounded distribution, popular modeling can associate the distribution with a notion of one object. If the property is charge, the potential associates with an electrostatic potential. If the property is mass, the potential associates with a gravitational potential. Typically, each term in an expansion associates with a negative-integer power of a radial coordinate, r. A term might feature a factor of r^{-1} or a factor of r^{-2} or so forth. The terms share a common notion of units. Typically, along a straight line that runs from large r to $r = 0$, the accuracy of the overall potential decreases with decreasing r. Typically, one might tend to avoid using a multipole expansion when r is less than the spatial extent of the distribution. Popular modeling includes the notion that the potential can associate with an interaction (via the relevant electrostatic field or gravitational field) between the above-mentioned one object and a second object.
- Our work suggests a possibly novel use of multipole expansions. Modeling can associate with the spatial extent of a source (that contributes to an electromagnetic field or that contributes to a gravitational field) being essentially pointlike. For a multipole expansion that associates with electrostatic potentials, a term that associates with r^{-1} associates with the property of charge. This association with charge comports with popular modeling. Popular modeling associates the word monopole with this association with r^{-1}. We suggest that one term can associate with a property of rotating rigid charge (and can associate with the popular modeling property of magnetic moment) and with the factor r^{-2}. The word dipole associates with this association with r^{-2}. Along a straight line that runs from large r to $r = 0$, adequate accuracy of the overall potential can pertain from r being large to r being small. For a source that exhibits more than one electromagnetic property, as one moves closer to $r = 0$, a term that associates with a higher-order

pole (for example, dipole) can take over (regarding dominance) from a term that associates with a lower-order pole (for example, monopole).

- Each one of the popular modeling notions of multipole expansions and our notions above of multipole expansions might sometimes de-emphasize popular modeling notions that the potential that associates with one point with respect to space-time coordinates might associate with characteristics of the source object that would associate with a time that precedes the time that associates with the one point.
- The discussion above tends to de-emphasize popular modeling notions of properties, such as energy density and momentum density and pressure, that might associate with regions (for example, regions that associate with space-time coordinates) and that might not associate with objects that can model as pointlike.

Facet-3 suggests that the new type of multipole expansions points to gravitational aspects that attract two objects toward each other and to gravitational aspects that repel two objects from each other.

- Popular modeling suggests and our work embraces the notion that a component of gravity that associates with the mass of an object-A and the mass of an object-B associates with a monopole component of force that attracts the two objects toward each other.
- Our work suggests that a dipole component of gravity that associates with the object-internal angular momentum of object-A and the mass of object-B associates with a component of force that repels the two objects away from each other.
- We anticipate suggesting (below) that such dipole repulsion between neighboring, but not colliding, galaxy clusters associates with the onset of the later-multibillion-year-era increases in the rate of expansion of the universe.
- We associate the word pull with components, of gravitational fields, that attract two objects toward each other.
- We associate the word push with components, of gravitational fields, that repel two objects away from each other.
- Our work regarding multipole expansions leaves untouched much successful popular modeling that has bases in general relativity and in properties such as energy density and momentum density. Energy density and momentum density pertain to regions (of space-time coordinates) and not to individual objects that model as pointlike.

Facet-4 suggests that merging into popular modeling the above-discussed suggestions involves adding, to popular modeling, a new equation. The new equation associates with the notion of more than one isomer. Each one of the one popular modeling use of and our five reuses of the set of elementary particles that associates with facet-1 associates with a different one of six isomers. Each solution to the equation involves two positive integers and the requirement that the multiplicative product of the two integers be six. The equation pertains for each electromagnetic property of an object and for each gravitational property of an object. The solutions vary, based on the specific property.

- Compared to popular modeling, one might consider the notion that our work adds a new type of property (of objects), namely isomer.
- Our introduction of the property of isomer adds, compared to popular modeling, the issue of the extent to which single terms in multipole expansions affect objects that associate with just some isomers and not with other isomers.
- Popular modeling suggests that, at least approximately, gravity associates with the notion that each object interacts with every other object. We suggest that the six isomers share one instance of the property of mass. We say that, regarding interactions, the one instance has a reach of six isomers.
- Popular modeling suggests that, at least approximately, electromagnetism associates with the notion that each ordinary-matter object interacts only with other objects that include ordinary

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matter. We suggest that each of the six isomers has its own instance of the property of charge. We say that, regarding interactions, each instance has a reach of one isomer.

- We suggest that, for any one popular modeling electromagnetic or gravitational property (such as charge, magnetic moment, or mass) of an object, the multiplicative product of the number of instances of the property and the number of isomers in the reach of any one instance is six.
- For properties (such as mass) for which the reach is six, we suggest that elementary-particle counterpart magnitudes are equal across isomers.
- For properties (such as charge) for which the reach is less than six, we suggest that elementary-particle counterpart ratios of magnitudes (such as, for lowest-mass charged leptons, the ratio of the magnitude of the charge to the mass) are equal across isomers.

Facet-5 suggests that choosing appropriate solutions of the new equation tunes cosmological models. We suggest that appropriate tuning enables fitting and explaining dark-matter data and dark-energy data that present popular modeling seems unable to adequately fit or explain.

- The equation that associates with the product of numbers of instances and number of isomers in the reach of any one instance being six is a suggested addition to popular modeling.
- We suggest that this addition enables quantitatively explaining ratios of dark-matter effects to ordinary-matter effects.
- We suggest that this addition enables qualitative explanations regarding the rate of expansion of the universe and regarding large-scale tensions.

In general, we suggest the following notions.

- Our work comports with successful popular modeling, reuses aspects of successful popular modeling, extends present popular modeling, and enables capturing physics opportunities that present popular modeling seems not to capture.
- Our work suggests observations that people might want to make to verify or refute aspects of present popular modeling and to verify or refute aspects of our work.
- Our notions of basing a specification for dark matter on isomers and of using multipole expansions to help explain repelling aspects of gravity preceded [28] the availability of enough data to help us adequately hone our work or to suggest that data might lend enough credibility to our approach.
- There now is enough data to suggest that our work provides a credible candidate basis for extending aspects of cosmology that associate with the popular modeling terms dark matter and dark energy.

2. Methods

This unit develops and applies novel methods to fit and explain specific data.

2.1. Cosmic Data that Motivate Our Work

This unit discusses cosmological data that motivate our work. Some data associate with observed ratios of not-ordinary-matter effects to ordinary-matter effects. Some data associate with eras in the rate of expansion of the universe. Some data associate with large-scale lumpiness of stuff. We know of no other line of research that tries to fit all the relevant data.

The following observed ratios of not-ordinary-matter effects to ordinary-matter effects provide a basis for our work.

- 1:1 – Amount of some depletion of cosmic microwave background radiation [29–31].
- 1:0+ – Amounts of stuff in some individual galaxies [32–40].
- 0+:1 – Amounts of stuff in some individual galaxies.
  - Redshifts of more than approximately seven [41,42].
  - Redshifts of approximately six [43].
  - Redshifts of less than six through redshifts of nearly zero [44–51].
- $\sim$ 4:1 – Amounts of stuff in some individual galaxies [52,53].

- 5+:1 – Amounts of stuff in many individual galaxies [32,54].
- 5+:1 – Amounts of stuff in many individual galaxy clusters [54–58].
- 5+:1 – Densities of the universe [59].

We know of no data that would seem to point to other seemingly significant ratios of not-ordinary-matter effects to ordinary-matter effects.

A notion of eras in the rate of expansion of the universe provides a basis for our work.

Popular modeling suggests two observed multibillion-year eras [60–63]. Chronologically, the first multibillion-year era associates with a positive rate of expansion that decreases as time increases. The second multibillion-year era associates with a positive rate of expansion that increases as time increases.

Popular modeling suggests that popular modeling does not adequately explain some aspects regarding the two multibillion-year eras. Popular modeling sometimes associates the two-word phrase Hubble tension with the lack of an adequate explanation.

Popular modeling might provide hints that the second multibillion-year era might be ending [64,65] and that a new era, which would associate with a positive rate of expansion that decreases as time increases, might be starting.

A notion of large-scale lumpiness of stuff provides a basis for our work.

Popular modeling tries to explain observed large-scale lumpiness of stuff. Popular modeling suggests that popular modeling overestimates large-scale lumpiness [66–73]. Popular modeling sometimes associates the two-element phrase S8 tension with some such overestimates.

2.2. Suggestions that Underlie Our Work

This unit discusses suggestions that we make and that underlie our work.

We suggest that the not-ordinary-matter effects associate with dark matter. We suggest that the appearances, in the relevant ratios, of approximate integers have significance.

The second 1 in the 1:1 ratio associates with effects of ordinary-matter hydrogen atoms. These atoms deplete, via hyperfine transitions, cosmic microwave background radiation. We suggest that the first 1 in the 1:1 ratio associates with dark-matter hydrogen-like atoms. Popular modeling associates ordinary matter with a set of elementary particles. We suggest that, for our work, the ordinary-matter set includes all known elementary particles except the photon. We suggest that these dark-matter hydrogen-like atoms associate with a set of elementary particles that has similarities to, but does not equal, the ordinary-matter set of elementary particles.

We use the word isomer to denote such a set of elementary particles.

Based on the prevalence of 5+:1 ratios, we suggest that nature includes six isomers. We number the isomers. Isomer-0 associates with ordinary matter. Each one of isomer-1 through isomer-5 associates with dark matter. Based on the pluses in the 5+:1 ratios, we suggest that four of the dark-matter isomers are less like isomer-0 than is the one dark-matter isomer that associates with the first 1 in the ratio 1:1.

We suggest that one can extend notions of Newtonian gravity by considering possible parallels between gravitation and electromagnetism. Electromagnetism includes, in a list of properties of objects, both charge and magnetic moment. Newtonian gravitation considers mass, which might be an analog to charge, but not object-intrinsic angular momentum, which might be an analog to magnetic moment. We suggest that parallels to special relativistic notions regarding contributions by objects to electromagnetic fields point to the notion that contributions to gravitational fields that associate with the object-intrinsic angular momentum of one object and with the mass of a second object associate with dilution or sign-reversal regarding a net gravitational attraction that might otherwise associate just with the attraction that associates with the masses of the two objects.

We suggest that decreases in the rate of expansion associate with gravitational components of force that pull galaxy clusters toward each other. We suggest that increases in the rate of expansion associate with gravitational components of force that push galaxy clusters away from each other.

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We suggest that aspects related to isomers, gravitational components of force that pull objects toward each other, and gravitational components of force that push objects away from each other can help explain the Hubble tension and the S8 tension.

2.3. Multipole Expansions that Combine Newtonian Gravity and Special Relativity

This unit develops a new type of multipole expansion. This unit suggests using the new type of multipole expansion to merge notions from Newtonian gravity and notions about special-relativistic interpretations of properties of objects. This unit suggests using the term SPRISENG (as in SPecial-Relativity-Inspired Suggested Extensions to Newtonian Gravity) or, for short, the term ENG (as in Extended Newtonian Gravity). This unit suggests using ENG to gain insight regarding two-body dynamics. This unit suggests that, whereas Newtonian gravity directly considers only the property of mass, ENG directly considers at least the properties of mass and spin (as in object-internal angular momentum). This unit suggests that, whereas Newtonian gravity admits components of forces that pull objects toward each other and might seem to exclude components of forces that push objects away from each other, based on ENG, one can consider that some contributions to gravitational interactions associate with pull and that some contributions to gravitational interactions associate with push. This unit discusses an electromagnetic so-called sub-case-2rr+ that analyzes, via special relativity, aspects that interrelate perceived values of charge and perceived values of magnetic moment. This unit suggests that a gravitational analog to sub-case-2rr+ points to the notion that, for two adequately close together objects, the dominant component of the gravitational interaction can associate with the spin (or object-internal angular momentum) of one object, the mass of the other object, and the notion of push. We anticipate that push components of forces between galaxy clusters can explain the notion that the rate of expansion of the universe can increase. We anticipate that push components of forces between solar-system sized objects can help explain data about ratios, that pertain to galaxies, of dark-matter effects to ordinary matter effects.

This unit features (in the following order) an introduction, a summary of results that suffice for supporting work in subsequent units, and a derivation of those results.

Introduction

We discuss interactions between an object-A and an object-P. The A in object-A associates with the two-word term active properties. Popular modeling associates active properties with the notion of properties about which fields, such as electromagnetic fields and gravitational fields, convey information. The P in object-P associates with the two-word term passive properties. Popular modeling associates passive properties with interactions, by object-P, with fields that associate with objects, such as object-A, other than object-P.

Our discussion assumes that each object models as pointlike. Pointlike associates with notions that the two objects are far enough apart that modeling can assume that each property associates with a point (and not necessarily a nonzero volume). Our discussion de-emphasizes popular modeling notions of, for example, mass density and energy density.

Our discussion assumes, for an object, that one point associates with all of the properties that associate with the object.

Our discussion assumes that, for Newtonian modeling, Equation (1) associates with the radial dependence of the contribution, to a total potential that associates with an object-A, of the potential that associates with a property of object-A. r denotes the distance from object-A. n_r is a positive integer. For $n_r \geq 2$, $V(r)$ can vary based on angular coordinates.

$$V(r) \propto r^{-n_r} \tag{1}$$

For example, $n_r = 1$ for the electromagnetic property of charge and for the gravitational property of mass.

Our discussion uses the 3-vector v_A to symbolize the velocity that associates with the motion of object-A relative to the position of object-P.

Our discussion features two sets of properties. The two-word term intrinsic properties associates with one of the sets. In effect, object-P can sense that intrinsic properties of object-A can have nonzero values when $v_A = 0$. Examples of electromagnetic intrinsic properties include charge and magnetic moment. Mass is an example of a gravitational intrinsic property. The word current associates with the other one of the sets. For each intrinsic property, there is a property for which the name associates with a phrase of the form intrinsic-property current. For $v_A = 0$, object-P senses a zero value for each property, of object-A, for which the name includes the word current. Charge current (or, in popular modeling, current) is an example of an electromagnetic current. For each one of charge and charge current, popular modeling provides that $n_r = -1$. We suggest that each intrinsic property and its counterpart current share one value of n_r.

Our discussion focuses on the extent to which an accelerometer that is part of object-P would sense gravitational attraction toward object-A or gravitational repulsion away from object-A. (We suggest as an aside that popular modeling might associate our notion of gravitational attraction with the three-word term gravitational centripetal forces. Popular modeling might associate our notion of gravitational repulsion with the four-word term gravitational centrifugal pseudo forces.)

Summary

Table 2 lists some gravitational object-intrinsic properties that an object-P might infer about an object-A.

Table 2. Some gravitational object-intrinsic properties that an object-P might infer about an object-A. For each nonzero-valued intrinsic property, there is a contribution (to an overall potential) that has radial characteristics that associate with $V(r) \propto r^{-n_r}$. r denotes the distance from object-A. For a property for which $n_k > 1$, non-constant angular-dependence pertains regarding that property's contribution to an overall $V(r)$. Uses of the words monopole, dipole, and so forth associate with values of n_r and echo popular modeling uses of the words. Regarding mass, one set of three independent parameters associates with position. Also, the word scalar associates with zero-tensor. Regarding angular momentum, one set of three independent parameters associates with position. Another set of three independent parameters associates with the three components of the angular momentum 3-vector. Also, the word vector associates with one-tensor. Regarding moments of inertia, one set of three independent parameters associates with position. Another set of three independent parameters associates with the three on-diagonal components of the 3-by-3 symmetric moments-of-inertia tensor. Another set of three independent parameters associates with three off-diagonal components of the 3-by-3 symmetric moments-of-inertia tensor. The case of rotating moments of inertia adds, compared to the case of moments of inertia, one set of three parameters that associate with the angular velocity 3-vector that associates with the rotation of object-A. We suggest that the notion of four-tensor pertains. Throughout this table, n_k equals the number of sets of three parameters. EM abbreviates the word electromagnetic. NNR denotes not necessarily relevant for our discussion. We anticipate de-emphasizing the notion of possibly relevant intrinsic-property hexadecapole potentials.

One-body intrinsic property	n_k	n_r	Potential	Notes re potential	EM analog
Mass	1	1	Monopole	Zero-tensor	Charge
Angular momentum	2	2	Dipole	One-tensor	Magnetic moment
Moments of inertia	3	3	Quadrupole	Two-tensor	NNR
Moments-of-inertia rotation	4	4	Octupole	Three-tensor	NNR
NNR	5	5	Hexadecapole	Four-tensor	NNR

We suggest that Table 2 associates with a new type of gravitational multipole expansion. In each of popular modeling multipole expansions and our multipole expansions, an expansion features a series of terms. Each term contributes via addition or subtraction to a notion of an overall spatial potential. In popular modeling, a multipole expansion tends to have a basis in a spatial distribution

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of one property such as charge or mass. Our multipole expansions feature one object that models spatially as pointlike and physically as having at least one nonzero-valued property.

Popular modeling suggests that the object-property of mass is always nonnegative. In the context of two-body interaction potentials, the contribution that associates with the mass of object-A interacting with the mass of object-P associates with a pull component of force.

For each intrinsic property, there is also a property for which the name is the one-body-intrinsic-property name followed by the word current. The notion of current associates with a set of three velocity parameters. The n_k for a current exceeds by one the n_k for the counterpart intrinsic property. The n_r that associates with a current equals the n_r that associates with the counterpart intrinsic property. (We note as an aside that we suggest that a continuation to an n_k that exceeds by two the n_k for the counterpart intrinsic property would associate with a nonzero acceleration and with the notion that, from the perspective of object-P, object-A would model as part of a system, of objects, for which the system does not necessarily include object-P.)

Table 3 lists some contributions, by an object-A, to gravitational forces, as perceived by an object-P. Table 3 extends Table 2.

Table 3. Some contributions, by an object-A, to gravitational forces, as perceived by an object-P. The value of each property is the value as perceived by object-P. Pull associates with notions of attraction of object-P toward object-A. Push associates with notions of repulsion of object-P away from object-A. RSD abbreviates the three-word term radial spatial dependence that associates with the component of force. The values of the exponents for RSD items echo the popular modeling notion that a force can associate with the gradient of a potential. Type associates with relevant intrinsic properties. Notions of monopole, dipole, and so forth echo popular modeling use of such terms. For example, a row for which the RSD is r^{-2} associates with a potential, that associates with object-A, for which $n_r = 1$. However, for a current property, the velocity-related 3-vector associates with an extra (compared to for the counterpart intrinsic property) notion of angular dependence. For example, for the second row in the table, the force varies based on the angle with respect to the velocity-related 3-vector that associates with the object-A mass current, even though, for the first row, the force does not vary based on angular coordinates.

Object-A property	n_k	Object-P property	Force	RSD	Type
Mass	1	Mass	Pull	r^{-2}	Monopole
Mass current	2	Mass	Push	r^{-2}	Monopole
Angular momentum	2	Mass	Push	r^{-3}	Dipole
Angular-momentum current	3	Mass	Pull	r^{-3}	Dipole
Moments of inertia	3	Mass	Pull	r^{-4}	Quadrupole
Moments-of-inertia current	4	Mass	Push	r^{-4}	Quadrupole
Moments-of-inertia rotation	4	Mass	Push	r^{-5}	Octupole
Moments-of-inertia-rotation current	5	Mass	Pull	r^{-5}	Octupole
Mass	1	Angular momentum	Push	r^{-3}	Dipole
Mass current	2	Angular momentum	Pull	r^{-3}	Dipole
Angular momentum	2	Angular momentum	Pull	r^{-4}	Quadrupole
Angular-momentum current	3	Angular momentum	Push	r^{-4}	Quadrupole
Moments of inertia	3	Angular momentum	Push	r^{-5}	Octupole
Moments-of-inertia current	4	Angular momentum	Pull	r^{-5}	Octupole
Moments-of-inertia rotation	4	Angular momentum	Pull	r^{-6}	Hexadecapole

We anticipate discussing the notion that, when applied to gravitational interactions between galaxy clusters, two rows of Table 3 can associate with the beginning of the recent multibillion-year phenomena of increasing rate of expansion of the universe. One row pertains to object-A mass and object-P angular momentum. The other row pertains to object-A angular momentum and object-P mass. Each one of the two rows features the notion of push. Each one of those two rows associates with use of a gravitational analog of aspects of an electromagnetic so-called sub-case-2rr+ (which we discuss below) that analyzes, via special relativity, aspects that interrelate perceived values of charge

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and perceived values of magnetic moment. For the gravitational analog to sub-case-2rr+, mass is analogous to charge and angular momentum is analogous to magnetic moment.

Much popular modeling regarding the Hubble tension and the S8 tension features applications of general relativity. We anticipate trying to associate our pointlike properties (such as mass) with other pointlike properties (such as energy) and with regional properties (such as energy density).

Derivation

The following are key themes for this discussion.

- In Newtonian physics, the values of intrinsic properties (such as mass and charge) of an object-A are invariant to special-relativistic notions of the motion of the object with respect to an observer of that object (or to a reference frame) and are invariant to special-relativistic notions of the relative motion of that object-A with respect to another object-P.
- In special relativity, the values of an intrinsic property of an object-A vary, based on a choice of an observer, a frame of reference, or the motion relative to an object-P. The magnitudes of the variations, away from values that associate with a frame of reference in which object-A is at rest, associate with a nonzero velocity and with nonzero values for the object-A intrinsic-property current that associates with the intrinsic property of object-A.
- Our work considers mathematics that associates with notions that aspects related to such an intrinsic-property current can subtract from aspects related to the special-relativistic value of an intrinsic property and, thereby, recover the Newtonian-physics value of the intrinsic property.

We discuss Newtonian gravity.

Newton developed Equation (2) to describe an aspect of the motion of object-P [19]. G is the gravitational constant. m_A is the mass of object-A. Mass is a scalar property. m_P is the mass of object-P. r is the scalar distance that object-P is away from object-A. $\bigtriangledown$ is the gradient operator. $\bigtriangledown$ produces a 3-vector from a scalar. F_P is the force that object-P feels. F_P is a 3-vector. Object-P might sense effects of that force via an accelerometer that is part of object-P.

$$Gm_A m_P(-\nabla(1/r)) = F_P \tag{2}$$

Equation (2) describes, for example, motion in which one body orbits the other body. Newtonian physics includes the notion that each of the two objects can exhibit nonzero motion. In Newtonian physics, each one of m_A and m_P is invariant to a special-relativistic choice of a frame of reference. We use the notation $m_{X,rest}$ to denote the rest mass of object-X. Regarding special relativity, a rest property of object-X associates with a frame of reference in which object-X is not moving. For Newtonian gravity, we suggest that Equations (3) and (4) pertain in all frames of reference.

$$Gm_{A,rest}m_{P,rest}(-\nabla V) = F_P \tag{3}$$

$$V = 1/r \tag{4}$$

We discuss some aspects of special relativity, and we suggest some extensions to those aspects.

In the following equations, the following notions pertain. A subscript denotes the rank of the relevant tensor. For example, one denotes a 1-tensor (as in a 3-vector). $|\cdots|^2$ denotes the mathematical notion of a tensor dot product. E_0 associates with special-relativistic notions of energy. m_0 associates with special-relativistic notions of rest mass. p_1 associates with special-relativistic notions of momentum. We associate $|p_1|^2$ with the square of a motion-related energy that an observer might associate with nonzero linear momentum. These equations do not explicitly specify factors for converting properties, such as momentum or angular momentum, into energies.

Equation (5) echoes aspects of special relativity.

$$|E_0|^2 = |m_0|^2 + |p_1|^2 \tag{5}$$

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We associate $|s_1|^2$ with the square of a spin-related energy that an observer might associate with nonzero (intrinsic to the object) angular momentum. We suggest that transiting from zero intrinsic angular momentum to nonzero intrinsic angular momentum associates with adding energy. We suggest Equation (6).

$$|E_0|^2 = |m_0|^2 + |p_1|^2 + |s_1|^2 \tag{6}$$

We suggest that transiting from modeling that associates with a spherically symmetric object to modeling that associates with a non-spherically-symmetric object associates with an equation of the form that Equation (7) shows. E_2 corrects for a lack of spherical symmetry and associates with the moments-of-inertia tensor. In effect, p_3 corrects p_1. s_3 corrects s_1.

$$|E_0|^2 + |E_2|^2 = |m_0|^2 + |p_1|^2 + |s_1|^2 + |p_3|^2 + |s_3|^2 \tag{7}$$

Equation (8), with $m_0 = m_{A,rest}$, provides perspective regarding developing a multipole expansion for the $m_{A,rest}$ in Equation (3).

$$|m_0|^2 = +|E_0|^2 - |p_1|^2 - |s_1|^2 + |E_2|^2 - |p_3|^2 - |s_3|^2 \tag{8}$$

Equations such as Equation (8) are quadratic with respect to properties of objects.

In popular modeling, dynamics equations, such as Equation (3), are linear with respect to properties of objects.

Popular modeling notions regarding electromagnetism provide perspective for developing a multipole expansion, that is linear with respect to properties, for the $m_{A,rest}$ in Equation (3).

We discuss electromagnetism.

We engage in this discussion to develop context for subsequent discussion regarding gravitation. People developed special relativity based on popular modeling techniques that pertain to electromagnetism. The relevant popular modeling techniques are familiar to many physicists.

Equations (4) and (9) pertain in all frames of reference. ε_0 denotes the vacuum electric permittivity. q denotes charge.

$$-(1/(4\pi\varepsilon_0))q_{A,rest}q_{P,rest}(-\nabla(1/r)) = F_P \tag{9}$$

Popular modeling developed special relativity to help describe observations regarding motions of objects that have nonzero charges.

Popular modeling provides the following equations. E_A denotes the electric field that object-P associates with object-A. v_P denotes the velocity of object-P. B_A denotes the magnetic field that object-P associates with object-A. Each one of E_A, v_P, and B_A is a 3-vector. The scalar potential ϕ_A and the vector potential A_A combine to form a 4-vector. $\partial.../\partial t$ denotes a partial derivative with respect to time. The equations are invariant with respect to a choice of a frame of reference. Values for a variable in an equation can vary, based on the choice of a frame of reference.

$$F_P = q_P(E_A + (v_P \times B_A)) \tag{10}$$

$$E_A = -\nabla\phi_A - \partial A_A/\partial t \tag{11}$$

$$B_A = \nabla \times A_A \tag{12}$$

We discuss interpretations with respect to the rest frame that associates with object-P. (We note as an aside that choosing this rest frame associates with standardizing some notions regarding the time t, though this paper does not necessarily directly take advantage of such standardization. Choosing this rest frame associates with standardizing notions of lengths, such as distances r away from object-A.)

Per previous remarks, this paper de-emphasizes notions that distances r may pertain to present times for object-P and earlier times for object-A. We suggest that, for the purposes of this paper, time delays regarding the propagation of information about object-A are not necessarily adequately important to warrant more careful attention to the delays. Equation (13) pertains.

$$v_P = 0 \tag{13}$$

Thus, based on Equation (10), the value of B_A is not relevant regarding F_P. However, Equation (11) suggests that A_A can still have relevance regarding F_P.

The symbol v_A denotes the velocity, of object-A, in the rest frame of object-P. v_A is a 3-vector. We use the symbol μ_A to denote the magnetic moment that object-P associates with object-A. μ_A is a 3-vector.

We discuss a so-called case-1.

For case-1, we assume that $q_{A,rest} \neq 0$ and that $\mu_{A,rest} = 0$.

Popular modeling suggests that the following equations pertain. I_A is the charge current (or current of charge) that associates with object-A. I_A is a 3-vector. I_A associates with both the charge q_A and the velocity v_A. $A_{A,I}$ is a 3-vector. $A_{A,I}$ contributes to A_A.

$$\phi_A \propto q_A / r \tag{14}$$

$$A_{A,I} \propto I_A / r \tag{15}$$

Popular modeling suggests that, if object-P would infer that the magnitude of v_A is nonzero, object-P would infer that $|\nabla \phi_A| / |\nabla \phi_{A,rest}|$ exceeds one, that $q_A / q_{A,rest}$ exceeds one, and that $|E_A| / |E_{A,rest}|$ exceeds one.

We suggest that, to maintain Equation (11) and parallels to Newtonian gravity, one can select a new (additive) contribution $A_{A,v}$ to A_A to satisfy Equation (16).

$$-\nabla \phi_{A,rest} = E_A = -\nabla \phi_A - \partial A_{A,v} / \partial t \tag{16}$$

$$A_A = A_{A,I} + A_{A,v} \tag{17}$$

We suggest that we can consider (for purposes of this discussion) using popular modeling for which each one of q_A, ϕ_A, v_A, I_A, and $A_{A,I}$, is a constant with respect to the time t.

Object-A models as pointlike. One way to effect Equation (11) features using Equation (18).

$$A_{A,v} = (-\nabla \phi_A - (-\nabla \phi_{A,rest}))t \tag{18}$$

The $A_{A,v}$ that Equation (18) provides has a radial spatial dependence of $1/r$ and does not vary based on angular coordinates. Except at $r = 0$ (which is not physically relevant), the contribution to B_A is $\nabla \times A_{A,v}$, which is zero.

Regarding Equations (14) and (15), q_A is the charge that object-P infers for object-A and I_A is the charge current that object-P infers for object-A. Based on the factor $1/r$ in Equation (14), popular modeling associates the word monopole with the q_A-related contribution to the overall potential that associates with F_P. Based on the factor $1/r$ in Equation (15), we associate the word monopole with the I_A-related contribution to the overall potential that associates with F_P.

We suggest that, from the perspective of object-P, effects that associate with nonzero charge current I_A dilute effects that associate with nonzero charge q_A. The magnitude of the dilution depends on the magnitude of I_A. The magnitude of the dilution does not depend on the direction of I_A.

The discussion above does not consider the notion that object-A might have non-pointlike structure or other non-pointlike characteristics.

We anticipate discussing cases other than case-1.

To start extending the discussion to include non-pointlike aspects of object-A, we note that popular modeling provides that charges add. We use q_{+A} to denote a sum, that object-P would perceive, of the charges of positively charged sub-objects of object-A. We use q_{-A} to denote a sum, that object-P would perceive, of the charges of negatively charged sub-objects of object-A. Equation (19) pertains.

$$q_A = q_{+A} + q_{-A} \tag{19}$$

We discuss case-2rr, which associates with rigid rotation, within object-A, of spatially distributed charge. We discuss two sub-cases.

Sub-case-2rr+ associates with a lack of negatively charged sub-objects, with a lack of a magnetic moment other than one that would associate with positively charged sub-objects of object-A, and with the rigid rotation of a spatially distributed charge q_{+A} around an axis that includes the point that associates with pointlike modeling for object-A. Object-P senses that object-A has a nonzero magnetic moment $\mu_A = \mu_{+A}$. μ_{+A} denotes the magnetic moment that associates with the rigid rotation of a spatially distributed charge q_{+A}. The magnetic moment μ_{+A} associates with a new (compared to discussion above) contribution $A_{+A,\mu}$ to A_A. An angular velocity ω_{+A} associates with the rotation. The nonzero magnitude of ω_{+A} associates with the motions of positive-charged components of object-A. Per case-1, the moving elements of charge associate with dilutions of the magnitude, as perceived by object-P, of q_{+A}. The orientation of ω_{+A} and the directions of the (instantaneously linear) motions of individual elements of charge are not relevant to the magnitude of the dilution. Popular modeling and we associate the word dipole with the property of magnetic moment. We suggest that the magnetic-moment current associates with a means for compensating, for situations in which $v_A \neq 0$, for some otherwise miscounting that would associate with motions (of elements of the overall charge q_{+A}) for which the (instantaneously linear) angular motions are not perpendicular to the linear motion of object-A.

For the sub-case-2rr−, one reverses (compared to sub-case-2rr+) the sign that associates with each subscript that includes a sign. Sub-case-2rr− suggests a new (compared to discussion above) contribution $A_{-A,\mu}$ to A_A.

Case-2th associates with the thermal motion of charged sub-objects of object-A and with two sub-cases.

Sub-case-2th+ associates with a lack of negatively charged sub-objects, with a lack of a magnetic moment other than one that would associate with positively charged sub-objects of object-A, and with the thermal motion of positively charged sub-objects of object-A. As with case-2rr+, object-P senses that object-A has a new (compared to case-1 and to case-2rr) nonzero electromagnetic property. The new property associates with a new (compared to discussion above) contribution $A_{+A,therm}$ to A_A. Per case-1, the moving elements of charge associate with dilutions of the magnitude, as perceived by object-P, of q_{+A}. We suggest associating the word dipole with the new electromagnetic property. We suggest that the new-property current associates with a means for compensating, for situations in which $v_A \neq 0$, for some otherwise miscounting that would associate with motions (of elements of the overall charge q_{+A}) for which the (instantaneously linear) angular motions are not perpendicular to the linear motion of object-A.

For the sub-case-2th−, one reverses (compared to sub-case-2th+) the sign that associates with each subscript that includes a sign. Sub-case-2th− suggests a new (compared to discussion above) contribution $A_{-A,therm}$ to A_A.

Case-3 associates with $q_{-A} < 0$, with $q_{+A} > 0$, with $q_A = q_{-A} + q_{+A} = 0$, and with $\mu_A = \mu_{-A} + \mu_{+A} \neq 0$. Quadrupole effects that associate with magnetic-moment current detract, for situations in which $v_A \neq 0$, from dipole effects that associate with magnetic moment. The magnetic-moment current effects associate with differences between μ_A and $\mu_{A,rest}$. One can consider that effects that associate with the dipole intrinsic property of magnetic moment do not affect the monopole intrinsic property of charge, which has a value of zero. (We note as an aside that we do not explore the extent to which case-3 might prove useful regarding considering popular modeling notions that there might be more than one popular modeling definition of magnetic moment for case-3 situations and that the definitions might not be equivalent regarding special-relativistic transformations [74].)

The discussion above provides four cases (case-1, case-2rr, case-2th, and case-3) in which $n_k + 1$-effects that associate with values of a current dilute n_k-effects that associate with the effects that associate with the counterpart intrinsic property. The two relevant intrinsic properties are charge and magnetic moment.

The discussion above provides two cases (case-2rr and case-2th) in which the effects that associate with values of an $n_k + 1$ intrinsic property dilute effects that associate with an n_k intrinsic property. For the case-2rr, the n_k intrinsic property is charge and the $n_k + 1$ intrinsic property is magnetic moment. For the sub-case-2th+, the n_k intrinsic property is charge and the $n_k + 1$ intrinsic property associates with the thermal motions of sub-objects that have positive charges.

The discussion above does not directly involve notions of effects that bind multiple sub-objects of object-A to form object-A. For example, if object-A contains multiple positively charged sub-objects and no negatively charged sub-objects, the work does not consider aspects of how object-A avoids disintegration via electromagnetic repulsion between the positively charged sub-objects.

We return to discussing gravitation.

Differences between electromagnetism and gravitation include the following. For an object, the gravitational scalar property (mass) is positive, whereas the electromagnetic scalar property (charge) can be negative, zero, or positive. (Our work generally de-emphasizes discussing zero-mass objects.) For an object, the masses of any sub-objects of an object-A contribute additively to the mass of object-A, whereas, for electromagnetism, the positive charges contribute additively and the negative charges contribute subtractively. Popular modeling associates gravitation with notions of a spin-2 field, whereas popular modeling associates electromagnetism with notions of a spin-1 field.

We suggest that the differences are not relevant for the purposes of extrapolating from discussion above regarding electromagnetism to discussion below regarding gravitation. We suggest that our work has some parallels to work that popular modeling associates with the word gravitoelectromagnetism [75–78].

Per the first and last columns in Table 2, we suggest that mass provides a gravitational analog to charge. We suggest that angular momentum provides a gravitational analog to magnetic moment. (We note as an aside that, in popular modeling, zero-mass objects such as photons can associate with nonzero angular momenta. We note as an aside that the notion of zero mass and nonzero angular momentum might provide a gravitational analog to case-3, which features the notion of zero charge and nonzero magnetic moment.) Table 2 also suggests names for two more gravitational properties. For other than zero-mass objects, each one of the four named gravitational properties associates with mass. Angular momentum associates with object-A internal motions of mass. Moments of inertia associates with a lack of spherical symmetry regarding the object-A internal mass. (One might note that such a lack of spherical symmetry might associate with popular modeling notions of a non-monopole contribution to a distribution of mass. We note as an aside that popular modeling might or might not associate a non-monopole contribution that might associate with a would-be dipole mass distribution that is oblate or oval. We de-emphasize discussing possible associations between angular momentum and such non-sphericity.) Moments-of-inertia rotation associates with rotation of object-A.

Each one of the four gravitational intrinsic properties that Table 2 names associates with mass. One might suppose that our discussion of gravitational properties of an object-A does not include gravitational analogs of case-3. However, popular modeling suggests the possibility that nature includes gravitons. Gravitons would have masses of zero and angular momenta that have magnitudes of $2\hbar$, in which $\hbar$ denotes Planck's constant. These notions seem not to be incompatible with the notion that we can consider that the mass of object-A associates with the sum of the masses of nonzero mass components of object-A and that we can consider that phenomena that associate with binding those components to make object-A do not necessarily associate, in our work, with the mass of object-A.

Regarding object-A, paralleling discussion above regarding case-2rr, we suggest that effects of nonzero angular momentum detract from effects of mass and that effects of nonzero moments-of-inertia rotation detract from effects of moments of inertia. We suggest that effects of moments of inertia augment effects of mass.

Regarding an interaction between object-A and an object-P for which mass is the only significantly nonzero property, object-A mass associates with a pull component of force on object-P and object-A moments of inertia associates with a pull component of force on object-P. We suggest that for each one of nonzero object-A angular momentum and nonzero object-A moments-of-inertia rotation, one can consider that a push component of force affects object-P.

Paralleling our discussion regarding electromagnetism, the effects of a gravitational intrinsic-property current dilute the effects of the counterpart gravitational intrinsic property. We suggest that the net effect retains the sense (pull or push) of the gravitational intrinsic property.

Similar sense-retaining relationships between contributions that associate with one gravitational intrinsic property of object-A and contributions that associate with another gravitational intrinsic property of object-A do not pertain if the two relevant n_k differ by an odd number. For example, regarding interactions with object-P mass, for a case of adequately close together object-A and object-P, push effects (for which the Table 3 RSD is r^{-3}) that associate with object-A angular momentum (for which $n_k = 2$) can dominate pull effects (for which the Table 3 RSD is r^{-2}) that associate with object-A mass (for which $n_k = 1$). In that case, object-P associates the net effect with repulsion away from object-A.

We suggest that Table 2, Table 3, and our discussion related to those two tables pertain.

2.4. Cosmic Clumping of Stuff

This unit suggests that, from the perspective of dominant multipole aspects of gravitational forces, the formation and evolution of smaller cosmic objects occur more quickly than do the formation and evolution of larger cosmic objects.

We assume, for discussion purposes, that, at some time in the evolution of the universe, stuff (ordinary matter and dark matter) had a uniform spatial distribution. We assume that, at that time, bunches of stuff were moving away from each other. We de-emphasize notions related to properties that associate with the word current.

We consider Newtonian notions. We consider two spatially non-overlapping, similarly-sized spherical regions of stuff. The amount of stuff in each region scales as the cube of the radius of the region. The monopole contribution to the gravitational force that one region exerts on the other region scales inversely as the square of the distance between the centers of the two regions. If one scales the two radii and the one distance similarly, the monopole aspect of gravitational force scales as $r^3 r^3 r^{-2} = r^4$. Dipole aspects scale as $r^3 r^3 r^{-3} = r^3$. Quadrupole aspects scale as $r^3 r^3 r^{-4} = r^2$. Octupole aspects scale as $r^3 r^3 r^{-5} = r^1$. Hexadecapole aspects scale as $r^3 r^3 r^{-6} = r^0$.

We suggest that the following notions pertain for Newtonian models and for other relevant popular modeling models.

We suggest notions that might associate with observed and possible clumping, at various scale sizes, in the history of the universe. Hexadecapole attraction might be scale-invariant and might not necessarily be adequately relevant for this discussion of clumping. Across a range of sizes, octupole repulsion was dominant at some time. Smaller clumps transited a sequence, of octupole repulsion to quadrupole attraction and onward, faster than did larger clumps. Typically, pairs of neighboring solar-system-sized clumps transited to dominance by monopole attraction before pairs of neighboring galaxy-sized clumps transited to dominance by monopole attraction. Typically, pairs of neighboring galaxy-sized clumps transited to dominance by monopole attraction before pairs of neighboring galaxy-cluster-sized clumps might transit to dominance by monopole attraction.

2.5. Eras in the Rate of Expansion of the Universe

This unit suggests that eras in the rate of expansion of the universe associate with multipole aspects of gravitational interactions between galaxy clusters.

We suggest the following notions regarding the rate of expansion of the universe. If there was an inflationary epoch [79–82], octupole gravitational repulsion between neighboring proto galaxy clusters played an important role early in the inflationary epoch. Quadrupole gravitational attraction between neighboring proto galaxy clusters played a dominant role around the start of the first multibillion-year era. Dipole gravitational repulsion between neighboring galaxy clusters played a dominant role around the start of the second multibillion-year era. Monopole gravitational attraction could play a dominant role around the start of an era that would succeed the second multibillion-year era.

2.6. Instances of Properties of Objects, Plus Reaches of Contributions to Fields

This unit suggests that two integers, a number of instances and a reach per instance, associate with each electromagnetic property of objects and with each gravitational property of objects. This unit suggests a formula that interrelates, for each property, the number of instances and the reach per instance. This unit suggests that determining numbers of instances and reaches per instance is key to our fitting and explaining cosmic data and to our providing insight about cosmology tensions.

Popular modeling suggests that ordinary matter does not, at least much, sense electromagnetic phenomena that might associate with dark matter. We suggest that each isomer associates with its own instance of the electromagnetic monopole property, which is charge. We say that a monopole contribution to the electromagnetic field has a reach of one isomer. An object that associates just with one isomer might sense monopole electromagnetic contributions that associate with other same-isomer objects. An object that associates just with one isomer would not sense monopole electromagnetic contributions that associate just with objects that do not associate with the same isomer.

Popular modeling suggests that ordinary matter and dark matter sense each other gravitationally. We suggest that all six isomers associate with a common instance of the gravitational monopole property, which is mass. We say that a monopole contribution to the gravitational field has a reach of six isomers. An object that associates just with one isomer senses monopole gravitational contributions that associate with objects that associate just with any one isomer or that associate with more than one isomer.

We use the symbol n_{in} to denote the number of instances of a property. We use the symbol $R_{/in}$ to denote the reach of an instance of the property. The reach is a number of isomers. Each one of n_{in} and $R_{/in}$ is a positive integer.

We suggest that, for each electromagnetic property and for each gravitational property, Equation (20) pertains.

$$n_{in} \cdot R_{/in} = 6 \tag{20}$$

Some aspects of our attempts to fit data associate with suggesting numbers of instances and reaches per instance for aspects, of gravitational fields, that associate with gravitational properties of objects. Some aspects of our attempts to provide insight regarding cosmological tensions use numbers of instances and reaches per instance for aspects, of gravitational fields, that associate with gravitational properties of objects. Some aspects of our attempts to fit data associate with suggesting numbers of instances and reaches per instance for aspects, of electromagnetic fields, that associate with electromagnetic properties of objects.

2.7. Galaxy Formation and Galaxy Evolution

This unit suggests galaxy formation scenarios that seem to explain seemingly naturally preferred known ratios, for galaxies, of dark-matter presence to ordinary-matter presence.

The solar system that includes the Earth associates with the notion of a solar-system-size clump of stuff that associates with just one isomer. Our solar system is not spatially spherical.

We suggest that many other such one-isomer, not-spatially-spherical, somewhat-solar-system-sized clumps exist and generally date to early in the history of the universe. We suggest that such clumps associate with nonzero moments of inertia. Discussion above suggests that moments of inertia is a quadrupole gravitationally attractive property.

We suggest that, during some period early in the history of the universe, quadrupole gravitational attraction dominated regarding interactions between neighboring same-isomer, solar-system-sized clumps. We assume that the relevance of same-isomer associates with there being six instances of the property of moments of inertia and with each instance associating with a reach of one isomer. We suggest that solar-system-sized clumps clumped to form the halos of galaxies.

We suggest that the discussion above explains 1:0+ ratios that pertain to some early galaxies and 0+:1 ratios that pertain to some early galaxies.

We suggest that some later 1:0+ galaxies and some later 0+:1 galaxies retain their ratios from early in the evolution of the universe.

Many later galaxies are 5+:1 galaxies. We suggest that many 5+:1 galaxies resulted from mergers of smaller, previous galaxies. We suggest that such mergers associate with monopole gravitational attraction, which discussion above associates with one instance and a reach of six isomers. We suggest that the earliest mergers that led to a 5+:1 galaxy could have been mergers that involved 1:0+ galaxies and 0+:1 galaxies.

The discussion above seems to explain all the galaxy-related ratios that we discuss above, except for the $\sim$ 4:1 ratio.

For each of some $\sim$ 4:1 galaxies, we suggest the following scenario. The galaxy started as a 0+:1 galaxy. Reach-2 repelling dipole contributions to gravity drove away some ordinary-matter stuff and the stuff that associated with one dark-matter isomer. Reach-6, attracting monopole contributions to gravity attracted remaining nearby stuff. The galaxy evolved to a ratio of $\sim$ 4:1.

For each of some $\sim$ 4:1 galaxies, we suggest the following scenario. The galaxy started as a 1:0+ galaxy. Reach-2 repelling dipole contributions to gravity drove away some dark-matter stuff but essentially no ordinary-matter stuff. Reach-6 attracting monopole contributions to gravity attracted remaining nearby stuff. The galaxy evolved to a ratio of $\sim$ 4:1.

We suggest that the gravitational property of angular momentum associates with the notion of three instances and with the notion of a reach per instance of two isomers.

2.8. Similarities and Differences Between Isomers of Elementary Particles

This unit suggests similarities and differences between the six isomers of elementary particles.

Table 4 suggests a numbering scheme for the six isomers, specifications for the one ordinary-matter isomer and the five dark-matter isomers, and aspects of the stuff that associates with each isomer.

Table 4. Matches between masses and flavours, for isomers of elementary fermions. The symbol l_{isomer} denotes the isomer number. The symbol $l_{isomer-pair}$ denotes the isomer-pair number. Handedness associates with whether the relevant handed elementary particles are left-handed or right-handed. For each row, the quarks column assigns the three flavour numbers in the order of increasing geometric-mean mass, with the geometric mean associating with the masses for the two quarks that are relevant to the flavour. For each row, the leptons column assigns the three flavor numbers in the order of increasing mass for the one charged lepton that is relevant to the flavour. The stuff column identifies stuff made from the isomer as OM, as in ordinary matter, or DM, as in dark matter. Isomer-3 stuff evolves similarly to isomer-0 (or ordinary-matter) stuff. The acronym SIDM abbreviates the popular modeling term self-interacting dark matter. The acronym MEA-DM abbreviates our term marginally-electromagnetically-active dark matter. The stuff that associates with MEA-DM interacts electromagnetically with itself marginally, perhaps mostly via the magnetic moments of zero-charge objects. The popular modeling notion of SIDM does not associate with MEA-DM stuff. The acronym SEA-DM abbreviates our term significantly-electromagnetically-active dark matter. The stuff that associates with SEA-DM interacts electromagnetically with itself on a par with OM stuff interacting electromagnetically with OM stuff. The stuff that associates with SEA-DM interacts electromagnetically somewhat with OM stuff. The popular modeling notion of SIDM associates with SEA-DM stuff.

l_{isomer}	$l_{isomer-pair}$	Handedness	Flavours - quarks	Flavours - leptons	Stuff
0	0	Left	1, 2, 3	1, 2, 3	OM
1	1	Right	1, 2, 3	3, 1, 2	DM (MEA-DM)
2	2	Left	1, 2, 3	2, 3, 1	DM (MEA-DM)
3	0	Right	1, 2, 3	1, 2, 3	DM (SEA-DM)
4	1	Left	1, 2, 3	3, 1, 2	DM (MEA-DM)
5	2	Right	1, 2, 3	2, 3, 1	DM (MEA-DM)

We suggest that the fermion flavour-and-mass pairings for isomer-1, isomer-2, isomer-4, and isomer-5 led to stuff that associates with those isomers forming stable counterparts to isomer-0-stuff neutrons and to stuff that associates with isomer-1, isomer-2, isomer-4, and isomer-5 not forming significant numbers of counterparts to isomer-0-stuff atoms. We use the one-element term alt-isomer to designate an isomer other than isomer-0 and isomer-3. For each one of the six isomers, a charged baryon that includes exactly three flavour-3 quarks is more massive than the counterpart, within the same isomer, zero-charge baryon that includes exactly three flavour-3 quarks. For example, the hadron that includes just two tops and one bottom has a larger total mass than does the hadron that includes just one top and two bottoms. Per Table 4, alt-isomer flavour-3 charged leptons are less massive than isomer-0 flavour-3 charged leptons. When flavour-3 quark states are much populated, the stuff that associates with an alt-isomer converts more charged baryons to zero-charge baryons than does the stuff that associates with isomer-0. Eventually, regarding the stuff that associates with the alt-isomer, interactions that entangle multiple W bosons result in the stuff that associates with the alt-isomer having more counterparts to isomer-0 neutrons and fewer counterparts to isomer-0 protons than does the stuff that associates with isomer-0. The sum of the mass of a counterpart-to-isomer-0 proton and the mass of an alt-isomer flavour-1 charged lepton exceeds the mass of a counterpart-to-isomer-0 neutron. Compared to isomer-0 neutrons, alt-isomer neutrons scarcely decay.

2.9. The Pluses in 5+:1 Ratios of Dark-Matter Effects to Ordinary-Matter Effects

This unit suggests explanations for the pluses in the 5+:1 ratios of dark-matter effects to ordinary-matter effects.

We suggest that nature might include electromagnetic properties (of objects) that would associate with reach-6. Below, discussion related to Table 5 provides a possible example. Based on the reach-6 properties, early in the evolution of the universe, more electromagnetic energy would flow from isomer-pair-0 stuff to isomer-pair-1 stuff than would flow from isomer-pair-1 stuff to isomer-pair-0 stuff. Similarly, based on the reach-6 properties, early in the evolution of the universe, more electromagnetic energy would flow from isomer-pair-0 stuff to isomer-pair-2 stuff than would flow from isomer-pair-2 stuff to isomer-pair-0 stuff.

We suggest that the electromagnetic energy flow imbalances would result in the existence of more isomer-pair-1 stuff than isomer-pair-0 stuff and in the existence of more isomer-pair-2 stuff than isomer-pair-0 stuff. We suggest that the flows would not disturb a one-to-one ratio of presence of isomer-3 stuff to presence of isomer-0 stuff.

We suggest that these energy flow imbalances might have produced (or, at least, contributed to) the pluses in the 5+:1 ratios of dark-matter presence to ordinary-matter presence.

Our work does not necessarily rule out the possibility that some portions of (or the entireties of) the pluses in the 5+:1 ratios of dark-matter presence to ordinary-matter presence associate with axions or with other popular modeling suggestions regarding the nature of dark matter.

2.10. *Hyperfine Depletion of Cosmic Microwave Background Radiation*

This unit suggests that our work explains the observed depletion of cosmic microwave background radiation for which half of the depletion associates with hyperfine transitions in hydrogen atoms.

Regarding the observed depletion of cosmic microwave background radiation, popular modeling suggests that the second 1 in the 1:1 ratio associates with hyperfine effects of ordinary-matter hydrogen atoms. We suggest that the first 1 in the 1:1 ratio associates with hyperfine effects of hydrogen-like atoms that associate with isomer-3.

We suggest that above-discussed notions of two-body electromagnetism might not suffice to ascribe a number of instances and a reach per instance that would associate with atomic-state transitions. For example, a hydrogen-atom object-A might not model as a one-body system but might model as a two-body system, with one-body being the atomic nucleus and one-body being the electron cloud.

We suggest that the reach for hyperfine-transition phenomena might be either two or six. A reach of two can suffice to explain the notion that isomer-3 hydrogen-like atoms can absorb, via hyperfine transitions, energy that associates with contributions, by ordinary-matter stuff, to electromagnetic fields. A reach of six might be appropriate, given that four dark-matter isomers might not underlie an adequately significant number of hydrogen-like atoms.

2.11. *Collisions of Galaxy Clusters*

This unit suggests that our specification for dark matter might not necessarily be incompatible with data about collisions of galaxy clusters.

Our specification for dark matter might not necessarily be incompatible with data about collisions, such as the Bullet Cluster collision, of two galaxy clusters [83,84]. Popular modeling suggests two types of trajectories for stuff. Most dark matter, from either one of the clusters, exits the collision with trajectories that are consistent with having interacted just gravitationally with the other cluster. Ordinary matter stars, from either cluster, exit the collision with trajectories that are consistent with having interacted just gravitationally with the other cluster. However, ordinary matter intergalactic medium, from either cluster, lags the cluster's ordinary matter stars and dark matter. Popular modeling suggests that the ordinary matter intergalactic medium interacted electromagnetically with the other cluster's ordinary matter intergalactic medium, as well as gravitationally with the other cluster. We suggest that the four alt-isomers do not form much electromagnetically active intergalactic medium. We suggest that further analysis of data might help determine the validity of our notion that isomer-3 would form electromagnetically interactive intergalactic medium.

2.12. *Instances and Reaches per Instance for Gravitational and Electromagnetic Properties*

This unit summarizes aspects regarding instances and reaches per instance.

Table 5 suggests instances and reaches per instance for some gravitational properties and for some electromagnetic properties.

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Table 5. Instances and reaches per instance for some gravitational properties and for some electromagnetic properties. Reach/instance denotes reach, in number of isomers, per instance. G2BF denotes gravitational two-body force. The gravitational instances, reaches, and G2BF information pertain for interactions with the mass of a second object. The electromagnetic instances, reaches, and two-body-force information pertain for interactions with the charge of a second object. The three-word term internally moving mass includes various cases, including angular momentum, other gravitational parallels to electromagnetic sub-case-2rr+, and gravitational parallels to electromagnetic sub-case-2th+. We suggest that the popular modeling notion of angular momentum can associate with the second row in this table. TBD denotes to be determined. NR denotes not relevant.

Type of property	n_k	Intrinsic property	Instances	Reach/instance	G2BF
Gravitational	1	Mass	1	6	Pull
Gravitational	2	Internally moving mass	3	2	Push
Gravitational	3	Moments of inertia	6	1	Pull
Gravitational	4	Rotating moments-of-inertia	TBD	TBD	Push
Electromagnetic	1	Charge	6	1	NR
Electromagnetic	2	Stellar thermal radiation	6	1	NR
Electromagnetic	2	Magnetic moment	TBD (6)	TBD (1)	NR
Electromagnetic	TBD	Hyperfine state	TBD (3 or 1)	TBD (2 or 6)	NR

For mass, the notion of one instance associates with the notion that gravitation, to a first approximation, attracts all stuff to all stuff.

For internally moving mass (which includes the case of object-internal angular momentum), a notion of three instances (or, equivalently, reaches per instance of two isomers) might help explain ratios of ∼4:1 for some galaxies. The related reach of two isomers might associate with dipole repulsion that drives away stuff that associates with the isomer that, via the notion of isomer-pairs, pairs with the galaxy's originally dominant one isomer. A notion of three instances, compared to six instances for moments of inertia, might help explain some notions that popular modeling might not adequately well extrapolate from equations of state for in early in the first multibillion-year era to equations of state for the second multibillion-year era. (Discussion below, related to Equation (21), provides further information.) A notion of three instances, compared to six instances for moments of inertia, might help explain some notions that popular modeling overestimates some large-scale clumping of stuff.

For moments of inertia, the notion of six instances associates with the notion that each one of many galaxies likely originally formed based on stuff that associates mainly just with one isomer.

For rotating moments-of-inertia, eventually data about the inflationary epoch might help suggest a number of instances.

For charge, the notion of six instances associates with the notion that ordinary matter does not see much light that dark matter produces.

For stellar thermal radiation, we suggest that case-2th pertains. We suggest that the notion of six instances associates with the notion that ordinary matter does not see much light that dark matter produces.

For magnetic moment, each one of electromagnetic, intrinsic, and $n_k = 2$ pertains. Based on these matches with stellar thermal radiation, we suggest that, for magnetic moment, each one of six instances and a reach per instance of one pertains. Perhaps, a notion of six instances ties to the notion that ordinary matter does not feel much electromagnetism that dark matter produces.

For hyperfine-state transitions, data suggest that, practically, a reach of two pertains. We suggest that four isomers do not form many adequately-similar-to analogs to hydrogen atoms. We suggest that, theoretically, the reach per instance for hyperfine-state transitions might be two or might be six. We suggest considering popular modeling notions that suggest that a current associates with a time-rate-of-change regarding a counterpart intrinsic property. A hyperfine state associates with interactions between the magnetic moment of an atomic nucleus and the magnetic moment of an atomic electron cloud. Our discussions above tend to de-emphasize intrinsic properties that might associate with modeling for an object-A that models as having more than one component. We suggest, based on the following notions, that $n_k = 3$ for the hyperfine state. The magnetic moment of the atomic nucleus associates with two 3-vectors, one for position and one for magnetic moment. The magnetic moment of the atomic electron cloud associates with two 3-vectors, one for position and one for magnetic moment. Modeling can assume that the two positions are the same. There are three, as in four minus one, independent 3-vectors. $n_k = 3$ associates with the count of three independent 3-vectors.

For each intrinsic property, there is an associated current property. For each associated current property, we suggest that the number of instances of the current property equals the number of instances of the intrinsic property. The notions of such equalities might be compatible with or seem to extrapolate from notions related to special relativity.

In general, we suggest that people might want to consider using present and future data to suggest numbers of instances for each one of various properties.

2.13. Single-Object Properties and Regional Properties

This unit discusses possible associations between our notions of single-object properties and general-relativistic notions of regional properties such as energy density.

Our work above has bases in properties of individual objects. Popular modeling includes two properties, energy and momentum, of objects that our work above scarcely mentions.

We suggest that, from a standpoint of object-P, each gravitational intrinsic property of object-A contributes nonnegatively to a pointlike energy that object-P can associate with object-A. The mass of object-A can associate with a ground-state energy of object-A. Nonzero internally moving mass associates with energy that is above ground-state energy. We suggest that nonzero moments of inertia associates with additional energy. And so forth.

Similarly, we suggest that each one of mass current, angular-momentum current, and so forth contributes a pointlike momentum that object-P can associate with object-A.

General relativity has bases in a stress-energy tensor that has bases in properties (such as densities) that associate with regions.

Table 6 suggests associations between ENG properties of objects and general relativistic properties that associate with regions.

Table 6. Associations between ENG properties of objects and general relativistic properties that associate with regions. We suggest that these associations might suffice for discussions in this paper.

ENG properties (of objects)	General-relativistic properties (of regions)
Energy	Energy density
Energy minus rest mass	Pressure
Momentum	Momentum density and energy flux
Momentum minus mass-current	Momentum flux and shear stress

2.14. The Hubble Tension

This unit suggests that our work provides a qualitative explanation for the Hubble tension.

Popular modeling estimates for a Hubble constant that would associate with the early universe suggest a Hubble constant that is significantly less than estimates for a Hubble constant that would associate with the recent universe. Estimates tend to associate with Equation (21) [85]. c denotes the speed of light. ρ denotes energy density and is nonnegative. P denotes pressure and is nonpositive.

$$-(c\rho + 3P) \tag{21}$$

Popular modeling suggests that an increasing rate of expansion associates with Equation (21) evaluating to a positive number. A decreasing rate of expansion associates with Equation (21) evaluating to a negative number.

We suggest that interactions between galaxy clusters provide a basis for the rate of expansion. Regarding around the beginning of the multibillion-year era of decreasing rate, attraction (between neighboring galaxy clusters) that associates with $n_k = 3$ two-body gravitational interactions dominates regarding changes in the rate of expansion. Regarding around the beginning of the multibillion-year era of increasing rate, repulsion (between neighboring galaxy clusters) that associates with $n_k = 2$ two-body gravitational interactions dominates regarding changes in the rate of expansion.

Table 7 suggests dominant contributions to pressure, relevant to popular modeling rate-of-expansion calculations, at selected times in the evolution of the universe. In Table 7, each one of the two numerically specified pairs of one n_{in} and one $R_{/in}$ comports with Table 5 and with data about galaxy evolution.

Table 7. Dominant contributions to pressure, relevant to popular modeling rate-of-expansion calculations, at selected times in the evolution of the universe. The information about dominant property, n_k, gravitational instances n_{in}, and reaches per instance $R_{/in}$ pertains for interactions with the mass of a second object. TBD denotes to be determined. We do not know of enough data to determine the two TBD integers. NR denotes not relevant. For the possible future era, we suggest that the property of pressure loses significance compared to the significance of energy density.

Approximately at the start of ...	Dominant property	n_k	n_{in}	$R_{/in}$
A possible future era	(None)	NR	NR	NR
The multibillion-year era of increasing rate	Internally moving mass	2	3	2
The multibillion-year era of decreasing rate	Moments of inertia	3	6	1
Start of inflation	Moments-of-inertia rotation	4	TBD	TBD

We suggest that a reach per instance of one for pressure associates with the start of the multibillion-year era of decreasing rate of expansion of the universe. We suggest that a reach per instance of two for pressure associates with the start of the multibillion-year era of increasing rate of expansion of the universe. We suggest that extrapolating from so-called early-universe pressures would lead to underestimations of so-called later-universe pressures. Underestimations of later-universe pressures would associate with underestimations of later-universe values of the Hubble constant.

2.15. The S8 Tension

This unit suggests that our work provides a qualitative explanation for the S8 tension.

We suggest that large-scale lumpiness associates, at least in part, with an $n_k = 4$-through-$n_k = 2$ sequence that we suggest above regarding galaxy evolution and regarding the rate of expansion of the universe. We suggest that popular modeling would underestimate repulsion that we associate with $n_k = 2$. We suggest that the underestimate associates with overestimates, by popular modeling, of large-scale lumpiness.

3. Results

This unit summarizes notions that our methods fit otherwise seemingly unexplained cosmic data, including data about dark-matter effects, the rate of expansion of the universe, and galaxy evolution. This unit summarizes suggestions that our work provides insight regarding cosmology tensions.

We suggest a novel relationship between dark-matter stuff and ordinary-matter stuff. Dark matter and ordinary matter share a notion of six isomers of a set of most known elementary particles. One isomer underlies ordinary-matter stuff. Each one of the five other isomers underlies some dark-matter stuff. Table 4 provides details.

We suggest a novel use of multipole-expansion mathematics. Multipole aspects associate with gravitational properties of a single object or with electromagnetic properties of a single object. Tables 2 and 3 provide details.

To fit cosmic data, we need to determine the so-called reaches (which are positive-integer numbers of isomers) of multipole aspects that associate with instances of properties of a single object. We suggest that each reach comports with Equation (20).

We suggest values for some reaches. Table 5 provides details.

The suggested reaches might suffice to explain the otherwise seemingly unexplained cosmic data.

The reaches suggest quantitative explanations for some otherwise seemingly unexplained data regarding dark-matter effects, galaxies, and galaxy evolution. Discussion related to Table 5 summarizes details.

Our work might suffice to qualitatively explain eras in the rate of expansion of the universe and to qualitatively explain the Hubble tension. Discussion related to Table 7 provides details.

Our work might suffice to qualitatively explain aspects regarding large-scale lumpiness of stuff, including the S8 tension. Gravitational aspects of Table 5 underlie the explanations.

Table 8 lists phenomena that our work seems to help explain.

Table 8. Phenomena that our work seems to help explain. DM:OM denotes ratios of dark-matter effects to ordinary-matter effects. CMB denotes cosmic microwave background radiation. The second column suggests approximate characterizations of explanations that we suggest. The rightmost two columns point to some of our bases for the explanations. SPRISENG abbreviates SPecial-Relativity-Inspired Suggested Extensions to Newtonian Gravity.

Phenomena	Explanation	Isomers	SPRISENG
Dark-matter elementary particles	Quantitative	x	
Dark-matter stuff	Quantitative	x	
DM:OM densities of the universe	Quantitative	x	
DM:OM regarding some galaxy clusters	Quantitative	x	
Galaxy evolution and DM:OM regarding some galaxies	Quantitative	x	x
DM:OM regarding some depletion of CMB	Quantitative	x	
Dark-energy gravitational phenomena	Qualitative	x	x
Eras in the rate of expansion of the universe	Qualitative	x	x
Hubble tension	Qualitative	x	x
S8 tension	Qualitative	x	x

Future work might evolve some of the qualitative explanations from qualitative toward quantitative.

4. Discussion

This unit discusses notions of how our work fits with and might help enhance popular modeling that pertains to cosmology. This unit suggests some possible opportunities for observational or experimental work that might help regarding verifying or refuting aspects of our work.

4.1. General Relativity

This unit discusses relationships between our work and some applications of general relativity.

Our work and applications of general relativity have some seeming similarities or compatibilities and some seemingly possible differences or seemingly possible incompatibilities.

One might note differences between the situations for which our notions might add insight and situations for which general relativity seems to add insight. For example, our notions might seem not to have direct use for detailed calculations regarding collisions or mergers involving objects such as black holes or neutron stars. Also, our notions might seem not to have adequately novel direct use for detailed calculations regarding situations for which general relativity seems to have satisfied so-called precision tests [86].

One might want to explore notions that our work regarding instances and reaches points to situations for which modeling based on general relativity might not be adequately accurate. For example, to what extent might general relativity not suffice in situations for which the isomeric composition of stuff varies between regions? (To date, precision tests might seem to associate with just one isomer, the ordinary-matter isomer. To date, many cosmological applications might seem to associate with just one isomer-mixture ratio, namely 5+:1.) Also, to what extent might general relativity not suffice in situations for which significant effective reaches might vary with time? (For example, regarding modeling regarding the rate of expansion of the universe, an equation of state that works well regarding early in the first multibillion-year era might not extrapolate to work adequately well regarding the recent multibillion-year era. Near the beginning of the first multibillion-year era, the dominant reach per instance regarding pressure would be one. Near the beginning of the second multibillion-year era, the dominant reach per instance regarding pressure would be two.)

4.2. *Our Work and the Term λCDM*

This unit discusses relationships between our work and the terms CDM and λ that associate with the popular modeling term λCDM, as in λCDM model of cosmology.

People discuss successes of and possible problems with λCDM-cosmology popular modeling [8].

Our work suggests that popular modeling notions of CDM (as in cold dark matter, but not as in collisionless dark matter) can pertain regarding (perhaps all) dark matter. Our work suggests that popular modeling notions of collisionless dark matter can pertain regarding somewhat more than 80 percent of dark matter and do not pertain regarding somewhat less than 20 percent of dark matter.

Our work does not necessarily directly address the extent to which popular modeling notions of λ (as in a cosmological constant) pertain. However, λ associates with the use of general relativity and some of our previous remarks might shed light on possible limitations regarding the adequate applicability of general relativity.

4.3. *Our Work and the Standard Cosmological Model*

This unit discusses relationships between our work and the standard cosmological model.

Our work tries to fit about ten data clusters or data points. Some of the data clusters or data points are approximate ratios of dark-matter effects to ordinary-matter effects. Some of the data clusters or data points associate with inflection points regarding the rate of expansion of the universe. Each one of some of the data clusters associates with numerous observations. The inflection point regarding the start of the second multibillion-year era in the rate of expansion of the universe provides an example. One of the data points associates with some depletion of cosmic microwave background radiation and with one observation.

Our work seems to offer viable candidate explanations for the about ten data clusters or data points.

Our work does not try to suggest an alternative to the standard cosmological model.

Our work might offer notions that can inform understanding, refining, or replacing aspects of the standard cosmological model. For example, our notions regarding a candidate specification for dark matter might lead to refining aspects of the standard cosmological model.

4.4. *Inflation and Pre-Inflation Cosmology*

This unit discusses possible relationships between our work and cosmology that might pertain during and before the inflationary epoch.

Our work suggests that the beginning of the inflationary epoch [87] might associate with dominance by octupole gravitational repulsion.

Our work suggests that, before the inflationary epoch, dominance by either quadrupole gravitational attraction or hexadecapole gravitational attraction might have led to a compressing, of stuff or energy, that might associate with popular modeling notions of a big crunch. For the case of dominance by hexadecapole gravitational attraction, one might need to suggest a mechanism that associates with a transition from a big crunch to a big bang [88]. Without such a mechanism, our modeling might suggest that a next (after a big crunch) era would associate with 32-pole (as in dotriacontapole or triacontadipole) gravitational repulsion. A candidate mechanism might be Pauli exclusion effects that would associate with fermion objects. The fermion objects might be fermion elementary particles.

Assuming hexadecapole gravitational attraction pertained, we suggest that the reach per instance for each one of hexadecapole gravitational attraction and octupole gravitational repulsion might be six. Consequences of those reaches being six might associate with notions that, throughout the known evolution of the universe, large-scale densities of stuff tend to be, to a first approximation, spatially uniform [89].

4.5. Bounce Cosmology

This unit discusses possible relationships between our work and bounce cosmology.

Popular modeling suggests the notion of bounce cosmology [90,91].

Our work suggests that, eventually, monopole gravitational attraction might lead to the slowing down of the rate of expansion that we suggest associates with gravitational push components of forces between galaxy clusters.

However, possibly, popular modeling will identify structures larger than galaxy clusters for which people will discuss notions of rates of expansion or rates of contraction. To the extent that such larger structures follow our notions of multipole gravity, perhaps dipole repulsion dominates now and for some time into the future.

We choose not to speculate regarding whether a slowdown (associated with galaxy clusters or, perhaps later, with larger objects) might eventually associate with a positive rate of contraction (as in a negative rate of expansion).

4.6. Suggestions for Observational Work

This unit suggests some opportunities, to which our work points, for verifying or refuting aspects of our work, pinpointing opportunities for observational or experimental work, and so forth.

We suggest considering possible synergies between galaxy-evolution studies and our work. To what extent does our work comport with early-universe galaxy-formation and galaxy-evolution? Did nature form at least as many 1+:0 (or, dark-matter) galaxies as 0+:1 (or, ordinary-matter) galaxies? To what extent does it seem reasonable that today's 1+:0 galaxies and 0+:1 galaxies maintained those ratios from early in the evolution of the universe? To what extent do our suggested scenarios for the formation of $\sim$4:1 galaxies comport with nature?

We suggest considering observational work, regarding the aftermath of galaxy-cluster collisions, that could help verify or refute our notions of significantly-electromagnetically-active dark matter. To what extent does isomer-3 intergalactic medium exist and lag other dark-matter stuff?

We suggest considering observational (and perhaps even experimental) work that could help verify refute or extend a table, such as Table 5. Which reach pertains for hyperfine phenomena? Which reach or reaches pertain for atomic transitions that are not hyperfine transitions? To what extent might data suggest a reach for magnetic moment? To what extent do the reaches for $n_k + 1$ current properties comport with the reaches for the counterpart n_k intrinsic properties?

We suggest considering experimental work that might detect dark matter of the types that we suggest.

We suggest considering work that would test general relativity for circumstances in which our work would suggest that dominant reaches change temporally or spatially.

4.7. Suggestions for Enhancing Popular Modeling

This unit suggests some opportunities, to which our work points, for enhancing popular modeling.

We suggest considering how to evolve popular modeling to embrace notions that, while much dark matter might qualify as collisionless dark matter, significant amounts of dark matter might not qualify as collisionless dark matter.

We suggest considering how to incorporate notions regarding instances and reaches per instance into popular modeling and into numerical simulations.

We suggest considering the extent to which popular modeling might benefit by using the series gravitational scalar potential, gravitational vector potential, gravitational two-tensor potential, and so forth.

5. Conclusions

We suggest explanations for approximately ten cosmological phenomena for which the standard cosmological model seems not to provide adequately precise explanations. Some of the phenomena associate with the terms dark matter, dark energy, rate of expansion of the universe, galaxy formation, Hubble tension, and S8 tension. The explanations have bases in our notions of isomers of elementary particles and in our notions of special-relativity-inspired suggested extensions to Newtonian gravity.

The following remarks provide some details.

We try to explain some cosmological data clusters or data points that the standard cosmology model seems not to explain adequately well. Some of the data clusters or data points associate with ratios of dark-matter effects to ordinary-matter effects. Some of the data clusters or data points associate with large-scale notions, such as the rate of expansion of the universe and large-scale clumping of stuff.

Our methods feature two novel concepts.

One concept is that dark matter and ordinary matter have a common basis in six isomers of a set of known elementary particles. One isomer associates with ordinary matter. Five isomers associate with dark matter. One dark-matter isomer underlies stuff that is a right-handed complement to ordinary-matter stuff, which associates with the notion of left-handedness. The other four dark-matter isomers underlie stuff that comports with popular modeling notions of collisionless dark matter.

The other concept is that a new use of multipole-expansion mathematics points, for interactions between two objects, to aspects of gravitational interactions. Our multipole notions feature multiple properties of a single object that models as pointlike, whereas popular modeling multipole expansions tend to feature spatial distributions of one property. Gravitational monopole, quadrupole, and hexadecapole intrinsic-property aspects associate with gravitational attraction between two objects. Gravitational dipole and octupole intrinsic-property aspects associate with gravitational repulsion between two objects.

We combine the two concepts and thereby suggest new insight regarding and explanations for the cosmological data clusters and data points. We suggest bases for some so-called cosmology tensions.

We suggest notions about how our methods and results might help people understand and improve popular modeling notions regarding various aspects of the standard cosmological model.

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