Laffer Curve: One of the First Mathematical Descriptions

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Article

Laffer Curve: One of the First Mathematical Descriptions

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Abstract: The article proves the priority of the author to obtain in 1994 one of the first mathematical descriptions of the Laffer curve, which is also claimed by the Georgian economist Vladimir Papava, who published the same result in 1996. The author proved for the first time that in a free market, the total profit deducted to the budget from all enterprises of a certain territory is proportional to the product of the tax rate and its natural logarithm. It follows that the maximum of this function on the unit interval of the variable value of the tax rate is equal to the reciprocal of the base of the natural logarithm, that is, 0.368 or 36.8%. To confirm the found optimum, it is proposed to conduct a full-scale economic experiment. The pattern found theoretically is proposed to be called "the market law of optimal taxation of enterprises" (the law of the inverse base of the natural logarithm).

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Foreword

In the early 90s of the 20th century, not knowing about the existence of the Laffer curve, we came up with its first, as we later found out, rather rigorous mathematical description by publishing a small article on the subject in the Kharkov (Ukraine) journal "Business Inform" [1]. Later, we learned from the monograph [2] that the result we obtained in 1994 was rediscovered in 1996 by the Georgian economist V.G. Papava [3], who in many of his subsequent works called the resulting function describing the Laffer curve the entropy function, since it reminded him of the Shannon entropy measure, but for some reason with reference not to Claude Shannon but to the work of A.M. Yaglom and I.M. Yaglom (1973) [4].

This function in the form of the product of the tax rate and its natural logarithm with a negative coefficient was written by the author without any conclusion or justification, after which it was confirmed that the function under consideration satisfies the characteristic properties of the Laffer curve: at tax rates $t=0$ and $t = 1$ (corresponding to a tax rate of 100 %), the function under consideration is equal to zero, and between points *0* and *1* there is a maximum of this function (fiscal Laffer point of the second kind) [3].

We managed to obtain this function on the basis of economically justified approximations, but since V.G. Papava in his works constantly emphasised the priority of his result obtained in 1996, we decided to declare our priority of this result as the first mathematical description of the Laffer curve obtained in 1994 on the basis of the apparatus of mathematical analysis and economically justified considerations of free market functioning. The following is an English translation of our 1994 paper [1] with our clarifications in square brackets.

Optimal taxation of enterprises in a market economy

Currently, corporate income tax is established empirically, without any theoretical justification for its value. We propose a rationale for this tax for the conditions of a free market economy (free creation and withering away of market structures). Let us introduce a variable value of corporate

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income tax in fractions of a unit ($0 \le X \le 1$) and make it dependent on the number of operating enterprises in a given region ($0 \le n \le \infty$) [this parameter is called a tax rate measured as a fraction of a unit]. For the introduced function $X(n)$, it is easy to determine its boundary values $X(0) = 1$, $X(\infty) = 0$. Indeed, if all profits ($X = 1$) are taken from enterprises, their number will tend to zero (the situation which the functioning of market structures is impossible); if all profits ($X = 0$) are left to enterprises, then their number will grow rapidly (for mathematical approximation and further optimization, we assume $n = \infty$). These boundary estimates allow us to introduce the following approximation function

$$X(n) = exp(-\alpha n), \quad (1)$$

where $\alpha = \text{const} > 0$ is some parameter. Then the function inverse to (1) takes the form

$$n(X) = (-1/\alpha) lnX \; . \quad (2)$$

Let us denote the profit averaged over all enterprises as Po, then the total profit deducted to the budget from all enterprises will be determined from the expression

$$P(X) = PoXn(X) = PoX(-1/\alpha) lnX \; . \quad (3)$$

Since the non-negative [sign-positive] function $P(X)$ on the boundaries of the [closed] interval of its definition $[0,1]$ takes zero values, it is clear that it reaches its maximum within the interval, that is, there is some intermediate optimal value of income tax, leading to the maximum total profit deducted to the budget. We determine this optimal value of X by equating to zero the first derivative of expression (3) $dP(X)/dX = 0$, from which we obtain $lnX = -1$ or $X = 1/e = 0.368$, where e is the base of the natural logarithm. It is important that the optimal value of X does not depend on the approximation parameter α.

Thus, the maximum contribution to the budget from the totality of all enterprises operating in a free market economy in a certain region will take place at an income tax equal to 36.8% [here we mean the interest rate of income tax]. The resulting optimal tax should be considered as an upper limit value that should not be exceeded, and which should be strived for in stages as the market economy system develops in a given region.

For example, a possible variant of a taxation strategy that stimulates the development of market structures can be presented as follows: in the first year, corporate income tax - 20%, in the second - 25%, in the third – 30%, in the fourth - 36 - 37% (theoretically found optimum). Deviation from the found optimum in any direction leads to a decrease in the funds received by the budget. To refine and concretize this theoretical optimum, we can assume the following experiment. The total number of market structures [meaning commercial enterprises] is fixed under the existing, rather high level of taxation, with the calculation of the allocation of part of their income (profit) to the budget.

After that, with a uniform step (2 - 4%), the income tax is reduced. After each tax cut, enough time (1-1.5 years) is allowed to allow the process of building up market structures to reach a quasi-stationary level. Immediately after that, calculations are made of the total number of market structures and their total return to the budget. This full-scale economic experiment will make it possible to specify the optimal taxation for the conditions of a particular region, and it is advisable to carry it out in the conditions of the created free economic zones. The pattern found theoretically is proposed to be called “the market law of optimal taxation of enterprises” (the law of the inverse base of the natural logarithm).

The material was provided on March 13, 1994.

Afterword

Thus, in this work, as well as later in the work of V. G. Papava, the Laffer fiscal point of the second kind turned out to be a constant value, independent of specific economic conditions. In order to clarify this, we suggested conducting large-scale economic experiments, for example, for free economic zones, which were much talked about after the collapse of the USSR. Examples of such

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experiments are shown, where the tax rate approaches the supposed optimum both from below and from above.

The model we propose was considered for an arbitrary closed territory (region) and not at the level of the country, which is not important. Since in this territory commercial firms operating in a free market were considered, it was decided, for the sake of simplicity and without loss of generality, to limit ourselves to the consideration of income tax only. However, in the general case it is necessary to consider the aggregate (average) tax rate.

Since the work of V.G. Papava 1996 [3], in which his priority result on the entropy model describing the Laffer curve was obtained, is in the public domain, we decided to look at this result. The work itself is devoted to the development of the Georgian economy in the context of shock therapy and post-communist transformation. The entropy model result is given as an auxiliary in the Appendix: Laffero - Keynesian synthesis. This appendix begins with the words: "Let's assume that the Laffer curve is described by the function $T = - Ntlnt$". In this expression, t is the tax rate, and it coincides with our expression (3) up to notation. The author does not provide any explanations for the derivation of the rationale for this expression, but only analyzes its properties, finding the maximum value of this expression.

In this article, the author does not call this model entropy. In his further works, apparently trying to somehow justify this model, the author defines it in the class of entropy models [2, P.78], referring for some reason to the monograph by A.M. Yaglom and I.M. Yaglom in 1973 [4], although it is well known that Claude Shannon (1916 – 2001) first came to the concept of information entropy in his famous 1948 paper on the mathematical theory of communication. Indeed, the structure of expression (3) resembles the Shannon information-entropy measure, but this measure is introduced for random variables that take n independent random values Xi with probabilities Pi ($i = 1,\ldots,n$), and it sums the products of these probabilities by their logarithms. It is unlikely that Shannon's measure of informational entropy is related to our problem, since we must assume that there is a set of random events of tax rates with certain probabilities.

The original text of our 1994 article was relatively recently posted on the ResearchGate platform https://www.researchgate.net/publication/352932074_Optimalnoe_nalogooblozenie_predpriatij_v_usloviah_rynocnoj_ekonomiki_V_MoskovkinBiznes_Inform_-_1994_-_No_15_-_S_17 and in the open-access electronic archive of the Belgorod National Research University http://dspace.bsuedu.ru/handle/123456789/43709

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