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Posted Date: 20 February 2025

doi: 10.20944/preprints202502.1684.v1

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Adaptive Cluster-Based Normalization for Robust TOPSIS in Multicriteria Decision-Making

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Abstract: In multicriteria decision-making (MCDM), methods like TOPSIS play a critical role in evaluating and comparing alternatives across multiple criteria. However, traditional normalization techniques often fall short when faced with data that includes outliers, large variances, or diverse measurement units, leading to rankings that may be skewed or biased. To address these challenges, this paper introduces an adaptive cluster-based normalization approach, applied to a real-world case study of selecting a city to host an international event. The method works by grouping alternatives into clusters based on their similarities in criterion values and applying tailored normalization within each group. This localized approach reduces the impact of outliers and ensures that scaling adjustments are aligned with the unique characteristics of each cluster. When tested in the case study, where cities were evaluated based on cost, infrastructure, safety, and accessibility, the cluster-based normalization method provided more stable and balanced rankings, even in the presence of significant data variability. By improving fairness and adaptability, this approach strengthens TOPSIS's ability to deliver accurate, context-aware decisions, making it an invaluable tool for tackling complex datasets in real-world applications.

Keywords: TOPSIS; adaptive normalization; cluster-based normalization; multicriteria decision-making; outlier mitigation

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1. Introduction

Multi-Criteria Decision-Making (MCDM) methods are essential for solving complex problems where multiple factors need to be considered simultaneously. Among these, the Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS) is widely used due to its ability to rank alternatives effectively based on their proximity to an ideal solution. However, despite its strengths, certain challenges remain, particularly in the areas of clustering and normalization. Traditional clustering methods tend to assign alternatives to rigid categories, even when they share characteristics with multiple groups. This can lead to misclassification and a loss of valuable information, especially in cases where data contains uncertainty or overlaps. Meanwhile, commonly used normalization techniques, such as Min-Max and Z-score, often struggle with large variations in data, skewed distributions, and extreme values, which can distort rankings and introduce bias into decision-making.

To address these limitations, this study proposes two innovative methods aimed at making the TOPSIS framework more flexible, reliable, and easy to use. The first is Clustering Using Fuzzy Numbers and Centroid-Based Distance Allocation, a new clustering approach that allows alternatives to belong to multiple clusters with different degrees of membership. Instead of forcing a strict classification, this method reflects the real-world complexity of data, ensuring that alternatives are grouped based on how closely they relate to different centroids rather than being arbitrarily assigned

 doi:10.20944/preprints202502.1684.v1

to just one. The second innovation is Logarithmic Normalization in TOPSIS, a transformation technique that helps smooth extreme variations, maintain proportional differences, and prevent any single criterion from dominating the rankings. A key advantage of this approach is that, despite its ability to improve stability and accuracy, it is just as simple to apply as Min-Max or Z-score normalization, making it a practical improvement for decision-makers.

By integrating these two methodological advancements, this study enhances both the clustering and normalization processes in TOPSIS, filling key gaps in traditional approaches while keeping the framework straightforward and efficient. The proposed methods are designed to be easy to implement and adaptable, making them suitable for a wide range of applications, including finance, environmental assessment, and industrial decision-making. By improving how alternatives are grouped and ensuring fairer comparisons between criteria, this study offers a more balanced, insightful, and scalable approach to complex decision-making. The following sections provide a detailed explanation of the proposed methods, their theoretical foundations, and how they can be effectively applied.

The rest of this paper is organized as follows: Section 2 reviews the existing literature on clustering and normalization in TOPSIS, highlighting their strengths and limitations and identifying the gaps that this study aims to address. Section 3 outlines the Materials and Methods, providing a detailed explanation of the proposed Clustering Using Fuzzy Numbers and Centroid-Based Distance Allocation approach, as well as the Logarithmic Normalization for TOPSIS, along with their theoretical foundations and implementation process. Section 4 presents a Case Study, demonstrating how these methods can be applied in a real-world decision-making scenario. Section 5 analyzes the Results, comparing the proposed techniques with traditional methods to evaluate improvements in accuracy, robustness, and efficiency. Finally, Section 6 offers the Conclusion, summarizing the key findings, discussing their broader implications, and suggesting possible directions for future research.

2. Literature Review

Multi-Criteria Decision-Making (MCDM) refers to a set of methodologies that help evaluate and prioritize multiple, often conflicting, factors in decision-making. These approaches play a crucial role in fields like environmental management, engineering, and economics—where complex decisions are a daily reality [1].

One of the most widely used MCDM techniques is the Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS). The core idea behind TOPSIS is simple: the best choice is the one closest to the ideal solution and farthest from the worst possible option. This method stands out for its ease of use and ability to handle both qualitative and quantitative data efficiently [2].

However, like any method, TOPSIS has its limitations. It struggles with data that involves uncertainty, outliers, or high variance, which can impact the consistency and reliability of its rankings. To address this, researchers have explored ways to link input uncertainty to output uncertainty within the TOPSIS framework, shedding light on the challenges of interpreting these uncertainties in real-world decision-making [3].

By refining these techniques, decision-makers can gain more confidence in their results, making MCDM methods even more valuable across different industries [4].

Comparative studies have been conducted between TOPSIS and other MCDM methods such as VIKOR, PROMETHEE, and AHP. For example, a study evaluated four different MCDM methods—AHP, TOPSIS, ELECTRE III, and PROMETHEE II—in the context of sewer network group decision problems, providing insights into their applicability and effectiveness [5–7].

Traditional clustering methods, such as K-Means and Hierarchical Clustering, have long been fundamental tools for grouping similar data points in decision-making models. Their efficiency and ease of implementation make them popular choices, but they come with notable limitations—particularly when dealing with uncertainty, complex data distributions, and overlapping classifications. These methods rely on crisp boundaries, meaning each data point is strictly assigned

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to one cluster, which can lead to inaccurate or overly simplistic groupings in real-world scenarios where data is often ambiguous and multidimensional [8].

To address these shortcomings, fuzzy clustering techniques, most notably Fuzzy C-Means (FCM), offer a more flexible alternative. Unlike traditional clustering, FCM allows data points to belong to multiple clusters with varying degrees of membership, providing a more nuanced and adaptable classification. This makes it particularly valuable in medical diagnosis, image segmentation, and customer profiling, where real-world data does not always fit neatly into predefined categories [9].

However, while FCM improves clustering accuracy and adaptability, it introduces significant computational complexity. Compared to K-Means, which operates with a relatively simple iterative approach, FCM requires more intensive calculations due to its membership probability updates and objective function optimization. The method involves iteratively minimizing an objective function, which can become computationally expensive, particularly for large datasets with high dimensionality. This complexity not only increases processing time but also demands greater computational resources [10].

Another major barrier to adoption is the need for programming and algorithmic expertise. Implementing fuzzy clustering methods, especially in large-scale applications, requires familiarity with programming languages such as Python, R, or MATLAB, as well as specialized libraries like scikit-fuzzy or Fuzzy Logic Toolbox. Unlike traditional clustering algorithms, which can often be applied using built-in software tools with minimal coding, FCM and other fuzzy clustering methods require manual parameter tuning (such as selecting the optimal fuzziness coefficient m) and careful preprocessing to ensure meaningful results [11].

Additionally, FCM is highly sensitive to initialization—poorly chosen initial centroids can lead to suboptimal clustering, requiring advanced techniques like genetic algorithms or particle swarm optimization to improve results. As a result, while fuzzy clustering offers greater accuracy and flexibility, its application demands more expertise, computational power, and algorithmic refinement [12].

Recent research is focused on reducing the computational overhead of fuzzy clustering, exploring hybrid models that integrate deep learning and optimization algorithms to automate parameter selection and enhance performance. These advances aim to make fuzzy clustering more accessible and scalable, bridging the gap between its theoretical advantages and practical usability in complex, real-world decision-making [13].

Normalization is a critical step in Multi-Criteria Decision-Making (MCDM) processes, ensuring that criteria with different scales are comparable [14].

Common normalization methods include: Min-Max Normalization: This method rescales data to a fixed range, typically [0,1], but is sensitive to outliers, which can distort the normalized values. Z-Score Normalization: This technique standardizes data based on mean and standard deviation, assuming a normal distribution, which may not hold true for all datasets. Vector Normalization: Often used in TOPSIS, this method normalizes data by dividing each criterion value by the Euclidean norm of the vector. While effective, it may not always preserve proportional differences between criteria.

These conventional normalization techniques can encounter challenges when dealing with highly skewed data, extreme values, and non-linear distributions, potentially impacting the fairness and accuracy of decision-making outcomes. For instance, the choice of normalization method can significantly influence the ranking of alternatives in MCDM methods, highlighting the importance of selecting appropriate techniques for specific decision contexts [15,16].

Logarithmic transformation is a mathematical tool used to handle non-linear data and compress large numerical variations. By applying a logarithmic function, data can be transformed to reduce skewness, manage outliers, and stabilize variance. This transformation preserves relative differences and diminishes the dominance of extreme values, making it advantageous in various fields, including statistics, finance, and machine learning. Despite its benefits, logarithmic normalization remains

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underutilized in MCDM methods like TOPSIS. Integrating logarithmic normalization into TOPSIS could enhance ranking stability and decision accuracy, particularly in datasets with high variance [17].

A study introduced a new logarithmic normalization method in game theory, demonstrating its effectiveness in segregating normalized values more efficiently than other methods, suggesting potential applications in MCDM [18].

Current literature indicates a lack of studies combining fuzzy clustering and centroid-based distance allocation within MCDM frameworks [19,20].

Additionally, despite the advantages of logarithmic normalization in handling data with high variance, its application in TOPSIS and other MCDM models is limited [21].

Addressing these gaps presents an opportunity to improve decision-making processes by developing a unified framework that integrates these methods, thereby enhancing both clustering and ranking accuracy [22].

Future research could focus on developing hybrid models that combine fuzzy clustering techniques with logarithmic normalization within MCDM frameworks, potentially leading to more robust and accurate decision-making tools.

3. Materials and Methods

In this section, we introduce a new methodological approach that enhances both clustering and normalization within the TOPSIS framework, while ensuring that the process remains straightforward and easy to implement. The proposed methods, Clustering Using Fuzzy Numbers and Centroid-Based Distance Allocation and the integration of Logarithmic Normalization in TOPSIS, address key limitations in traditional techniques. By introducing a more flexible clustering process and an adaptive normalization approach, these methods allow for a more accurate representation of real-world data variability, improving decision-making outcomes.

One of the fundamental challenges in decision models is that traditional clustering methods tend to assign alternatives to rigid categories, even when the data suggests a more nuanced classification. This can lead to misinterpretations, particularly when dealing with uncertainty or overlapping data points. To overcome this limitation, we propose Clustering Using Fuzzy Numbers and Centroid-Based Distance Allocation, which introduces a degree of membership for each alternative within a cluster instead of enforcing a strict assignment. This method acknowledges that alternatives may exhibit characteristics of multiple clusters, leading to a more precise, meaningful, and interpretable grouping of data.

Similarly, conventional normalization techniques such as Min-Max and Z-score often fail to handle datasets with large numerical variations, highly skewed distributions, or extreme outliers. These issues can distort rankings in TOPSIS, as criteria with significantly larger values may disproportionately influence the final results. To address this, we propose the integration of Logarithmic Normalization in TOPSIS, which effectively smooths extreme variations, preserves relative differences, and ensures a more balanced influence across criteria. One of its most compelling advantages is that, despite its effectiveness in handling complex data distributions, it remains as easy to apply as traditional normalization methods, making it an accessible yet powerful enhancement for decision-makers.

The strength of our proposed methodology lies in its ability to enhance accuracy, robustness, and adaptability while maintaining ease of implementation. Both the fuzzy clustering approach and logarithmic normalization are designed to seamlessly integrate into existing decision-making workflows without adding computational complexity. By introducing greater flexibility in clustering and a more adaptive approach to normalization, this study provides a scalable, practical, and efficient framework for improving multi-criteria decision-making. The following sections provide an in-depth explanation of how these methods work and their practical applications.

3.1. Clustering Using Fuzzy Numbers and Centroid-Based Distance Allocation

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To group alternatives into meaningful clusters, we implement a fuzzy clustering approach. This method involves the following steps:

Step 1: Representation of Alternatives with Fuzzy Numbers - Each alternative's criteria's values are converted into fuzzy numbers *(a, b, c)* representing the lower bound, central value, and upper bound, respectively. This allows for a more flexible representation of uncertainty in the decision-making process.

Step 2: In this step, the cluster centroids are determined qualitatively by the user, identifying the optimal ranking for each criterion within the cluster. This process employs fuzzy numbers, represented as values *(a, b, c)* ranging between 0 and 1. These results will allow us to measure the distance between the cluster centroids, which represent the ideal position for each cluster based on the selected criteria—and the scores of each alternative. Beyond this primary purpose, the resulting centroids are also used to determine the criterion weights, which will later be applied in the TOPSIS method. Formula (1) calculates the centroid representing the optimal position within a given cluster.

$$C_{cwj} = \left(\frac{a_w + b_w + c_w}{3} \right)_j \quad (1)$$

Where a_w, b_w, c_w the fuzzy number components representing the optimal position within a given cluster j.

Step 3: Determination of Alternatives Centroids - Using an fuzzy clustering approach, the centroids of each alternative are determined. The centroid C_{Aij} for each alternative i is computed as:

$$C_{Aij} = \left(\frac{a_i + b_i + c_i}{3} \right)_j \quad (2)$$

where a_i, b_i, c_i are the fuzzy number components of alternative i in the criterion j.

This formula is applied to all alternatives across all considered criteria to establish the ranking of the alternatives.

Step 4: The new decision matrix, obtained from Step 3, is normalized using the cost (lower is better) and benefit (higher is better) formulas (Formulas 2 and 3).

$$N_i = \frac{\max(X) - X_i}{\max(X) - \min(X)} \quad (3)$$

$$N_i = \frac{X_i - \max(X)}{\max(X) - \min(X)} \quad (4)$$

Where N_i is the normalized value of the alternative i, X_i is the original value of the alternative i, $\max(X)$ is the maximum value in the criterion and $\min(X)$ is the minimum value in the criterion. These formulas scale the values between 0 and 1, ensuring a fair comparison between alternatives while maintaining the meaning of cost and benefit criteria.

Step 5: Calculation of Distance to Cluster Centroids - The distance between each alternative centroid and each cluster centroid is calculated using the Euclidean distance formula:

$$D_{iw} = \sqrt{\sum_{j=1}^{m} \left(C_{Aij} - C_{Cwj} \right)^2} \quad (5)$$

where D_{iw} is the distance between the centroid of alternative i and the centroid of the optimal position within cluster w.

Step 4: Assignment of Alternatives to Clusters - Each alternative is assigned to the cluster with the smallest distance to its centroid. This process involves calculating the distance between each alternative and all cluster centroids. The alternative is then assigned to the cluster with the closest centroid, ensuring it is grouped with the most similar alternatives based on the given criteria, formula (6).

$$C_i = \arg\min_w D_{iw} \quad (6)$$

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where C_i is the cluster assigned to alternative i and $\arg\min_w$ selects the cluster w that minimizes the distance.

3.2. Logarithmic Normalization: An Adaptive Approach for TOPSIS

Normalization is a fundamental step in Multi-Criteria Decision-Making (MCDM) methods such as TOPSIS (Technique for Order of Preference by Similarity to Ideal Solution). It ensures that criteria with different units and scales can be meaningfully compared. Traditional normalization methods, such as Min-Max and Z-score, effectively rescale data but may not be suitable for datasets with high variance, extreme outliers, or non-linear distributions. In such cases, logarithmic normalization emerges as an alternative technique that dynamically adjusts to data distributions, making it particularly effective for handling data spanning multiple orders of magnitude. By compressing large numerical variations while amplifying smaller differences, this approach ensures a more balanced contribution of criteria to the final decision, preserving the relative ranking among alternatives. Logarithmic normalization is mathematically expressed as shown in equation (7):

$$X^{'} = \frac{\log(X) - \log(X_{\min})}{\log(X_{\max}) - \log(X_{\min})} \quad (7)$$

where $X^{'}$ represents the normalized value, X is the original value, and $X_{\min}$ and $X_{\max}$, denote the minimum and maximum values within a given criterion. This transformation is particularly beneficial in decision-making scenarios where some criteria exhibit exponential growth patterns, such as financial metrics, environmental indicators, and energy consumption data. By using a logarithmic scale, the influence of extreme values is harmonized, ensuring that all criteria contribute meaningfully to the decision-making process in TOPSIS for instance. A key strength of logarithmic normalization is its adaptive nature. Unlike fixed-range methods, it automatically adjusts to varying data magnitudes, dynamically scaling values to ensure fair comparisons across criteria. This makes it particularly effective for datasets with highly skewed distributions or large numerical differences. Moreover, it enhances decision stability, reducing the dominance of criteria with disproportionately large values while ensuring that smaller values remain distinguishable. Another advantage of logarithmic normalization is its ability to enhance differentiation among alternatives. By redistributing values in a way that emphasizes proportional differences, it ensures that the ranking process in TOPSIS remains representative and reliable, even when dealing with highly dispersed datasets. This is particularly beneficial in cases where criteria exhibit non-linear relationships, allowing for a more accurate reflection of each alternative's performance. The implications of logarithmic normalization in TOPSIS are significant. By integrating this approach, rankings become more stable and reflective of real-world conditions, ensuring that decision-making processes remain robust and interpretable. Given its ability to balance differences across criteria without distorting rankings, logarithmic normalization offers an advanced scaling technique that aligns well with diverse decision-making scenarios. Despite its many advantages, logarithmic normalization has yet to be widely integrated into the TOPSIS framework, presenting an exciting opportunity for innovation. By introducing this approach, we can create a more adaptive way of handling criteria with high variance, non-linear distributions, and sensitivity to outliers. This integration helps improve ranking stability, ensures fairer comparisons between alternatives, and strengthens the overall decision-making process. What makes logarithmic normalization even more appealing is its ease of implementation. While it effectively balances data distribution and minimizes the impact of extreme values, it remains just as simple to apply as Min-Max or Z-score normalization. This means that decision-makers can benefit from its advantages without facing additional computational complexity or implementation challenges.

4. Case Study

This case study focuses on selecting the best city to host an international event planned for two years from now. The decision is complex, requiring an evaluation of key factors such as costs,

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logistics, attendee experience, and the event's overall impact. To simplify this process, a dataset of potential host cities was analyzed and grouped into clusters based on economic profiles, infrastructure quality, safety, and accessibility. Table 1 provides an overview of the cities, each represented by a letter for clarity. These cities were carefully assessed, with descriptions highlighting their unique strengths and challenges. This structured approach offers valuable insights, helping decision-makers identify the most suitable location to ensure the event's success.

Table 1. Potential Cities for Selection as Hosts of an International Event.

City (Letter)	City	Description
City A	Hanoi, Vietnam	Low-cost city with functional infrastructure, ideal for regional events.
City B	Kathmandu, Nepal	Highly affordable but with limited infrastructure and moderate safety.
City C	Tokyo, Japan	Exceptional infrastructure, safety, and accessibility; high-cost city.
City D	Singapore, Singapore	Similar quality to Tokyo with slightly lower costs.
City E	Kuala Lumpur, Malaysia	Balanced city with high safety, accessibility, and moderate costs.
City F	Bangkok, Thailand	More affordable than Kuala Lumpur, with slightly lower safety scores.
City G	Colombo, Sri Lanka	Affordable with growing infrastructure and moderate accessibility.
City H	Manila, Philippines	Slightly higher cost with challenges in infrastructure and safety.
City I	Seoul, South Korea	High safety and good infrastructure, though slightly less accessible.
City J	Hong Kong, China	Highly accessible and secure, with costs similar to Tokyo.
City K	Ho Chi Minh City, Vietnam	Good safety and accessibility, with moderately higher costs.
City L	Jakarta, Indonesia	High accessibility and safety with balanced, moderate costs.

The cities are assessed based on four key criteria, each playing a vital role in determining their suitability as a host for the international event:

1. Cost (C1): The estimated total expense of hosting the event, measured in millions of dollars. This criterion reflects the financial feasibility of each city and its potential impact on the event's budget.
2. Infrastructure (C2): A score ranging from 1 to 10, indicating the quality of venues, transportation systems, accommodations, and other facilities essential for hosting an event of this scale.
3. Safety (C3): An index, also ranging from 1 to 10, that measures the city's overall safety, including crime rates, political stability, and preparedness for emergencies. A higher score indicates a safer environment for attendees.
4. Accessibility (C4): Another score from 1 to 10, representing the city's connectivity and ease of access. This includes the availability of international and domestic flights, efficient public transportation, and road networks.

To simplify the decision-making process, the cities under consideration have been grouped into three clusters based on shared characteristics such as cost, infrastructure, safety, and accessibility. Each cluster represents a distinct category of cities, helping decision-makers narrow their focus and evaluate options more effectively.

1. Cluster 1: Cost-Effective Cities with Moderate Infrastructure. This cluster includes budget-friendly cities, making them an attractive choice for events with tighter financial constraints. Their lower costs allow resources to be allocated to other areas, such as marketing or enhancing

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attendee experiences. These cities also have the potential to attract high attendance from local or regional participants due to their affordability. However, they face some challenges. Infrastructure may require temporary upgrades to accommodate the needs of an international audience, and safety and accessibility scores are often moderate, demanding careful planning to ensure a successful event.

In the following, the values presented for each criterion are analyzed and discussed using fuzzy numbers, which represent the optimal values for each criterion within this cluster. These fuzzy values indicate the most desirable levels for cost, infrastructure, safety, and accessibility, providing a degree of flexibility rather than rigid, fixed values. By applying fuzzy logic, this approach acknowledges that real-world city classifications involve gradual transitions rather than strict categorizations, allowing for a more nuanced and adaptable evaluation of urban characteristics.

   1. Cost (C1): This factor represents the financial affordability of the city. Since this cluster focuses on cost-effective locations, the cost should be as high as possible (fuzzy number 0.9, 1, 1). A higher rating means the city is more budget-friendly in terms of living expenses, business operations, and overall affordability.
   2. Infrastructure (C2): This refers to the quality and availability of public services, transportation, and essential facilities. Cities in this cluster should have a moderate level of infrastructure (fuzzy number 0.5, 0.6, 0.7). This means they provide basic amenities but might require improvements in areas like roads, public transportation, healthcare, and digital connectivity.
   3. Safety (C3): This criterion evaluates how secure the city is for residents, businesses, and visitors. These cities should have moderate safety levels (fuzzy number 0.4, 0.5, 0.6). While they are generally safe, they may have certain areas that require extra precautions, such as higher crime rates or specific security concerns.
   4. Accessibility (C4): This factor assesses how well-connected the city is both regionally and internationally. The cities in this cluster should have moderate accessibility (fuzzy number 0.4, 0.5, 0.6). They typically have good regional connectivity through local transportation networks but might lack direct access to global travel hubs, such as major international airports or high-speed rail links.

2. Cluster 2: High-Investment Cities with World-Class Infrastructure. Cities in this cluster are renowned for their exceptional infrastructure, including state-of-the-art venues, premium accommodations, and robust transportation networks. These cities are ideal for events that aim to project prestige or cater to high-profile attendees. High safety and accessibility scores further ensure a smooth and secure experience for participants. However, these advantages come with significant costs, which can impact profitability or restrict participation. Careful budgeting and strong justifications to stakeholders are essential to address these challenges. The following analyzes and discusses the values for each criterion in Cluster 2 using fuzzy numbers, which define the optimal range for cost, infrastructure, safety, and accessibility within this category.
   1. Cost (C1): Should be as low as possible (fuzzy number 0.05, 0.1, 0.12) because these are expensive cities, making budget management a challenge. The lower the rating, the higher the cost of living, business operations, and general expenses.
   2. Infrastructure (C2): Should be as high as possible (fuzzy number 0.8, 0.95, 1) to ensure world-class facilities. This includes cutting-edge public transportation, advanced healthcare systems, efficient digital connectivity, and modern urban planning.
   3. Safety (C3): Should be as high as possible (fuzzy number 0.95, 0.95, 1) since these cities are known for their stability and security. Low crime rates, strong law enforcement, and a secure environment make them attractive for businesses and residents alike.
   4. Accessibility (C4): Should be as high as possible (fuzzy number 0.7, 0.95, 1) to ensure global connectivity. These cities have major international airports, excellent public transit systems, and strong infrastructure to host international conferences and business events.
3. Cluster 3: Balanced Cities with a Mix of Features. This cluster includes cities that strike a strong balance between affordability and quality, offering good infrastructure, high safety ratings, and excellent accessibility at reasonable costs. Their versatility makes them ideal for events that seek to combine cost-effectiveness with a high-quality experience for attendees. While these cities may not be as affordable as those in Cluster 1 or have infrastructure as advanced as those in Cluster 2, their overall balance makes them strong contenders for hosting successful events. Choosing

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between similarly balanced options in this cluster might require additional considerations, but their high safety and accessibility scores enhance the experience for all participants. The following analyzes and discusses the values for each criterion in Cluster 3 using fuzzy numbers, which define the optimal range for cost, infrastructure, safety, and accessibility within this category.

1. Cost (C1): Should be moderate (fuzzy number 0.5, 0.6, 0.7) because these cities balance quality and affordability. They are neither excessively expensive nor extremely cheap, making them attractive for middle-income professionals and businesses looking for cost-effective but well-equipped locations.
2. Infrastructure (C2): Should be good but not premium (fuzzy number 0.5, 0.6, 0.7). These cities provide high-quality public services, efficient transportation, and modern urban planning, but they may lack the cutting-edge facilities of world-class metropolises.
3. Safety (C3): Should be high but not extreme (fuzzy number 0.5, 0.6, 0.7). These cities offer a safe environment with low to moderate crime rates, ensuring a comfortable living and working atmosphere without reaching the ultra-secure standards of Cluster 2 cities.
4. Accessibility (C4): Should be high but not at the maximum level (fuzzy number 0.5, 0.6, 0.7). These cities have strong regional and international connectivity, including well-developed airports and transport networks, but they do not match the global reach of the top-tier business hubs in Cluster 2.

Table 2. Summary of Optimal Scores for Each Cluster – Cluster Centroids.

	Cost (C1)			Infrastructure (C2)			Safety (C3)			Accessibility (C4)		
Cluster	a	b	c	a	b	c	a	b	c	a	b	c
1	0.9	1	1	0.5	0.6	0.7	0.4	0.5	0.6	0.4	0.5	0.6
2	0.05	0.1	0.12	0.8	0.95	1	0.95	0.95	1	0.7	0.95	1
3	0.5	0.6	0.7	0.5	0.6	0.7	0.5	0.6	0.7	0.5	0.6	0.7

The scores in Table 3 were carefully crafted through a collaborative process involving a diverse panel of experts and analysts. This team combined the expertise of professionals in event planning and logistics with valuable insights from tourist feedback and reviews in reputable travel and tourism journals. By incorporating these perspectives, the evaluation captured not only the practical and logistical aspects of hosting an international event but also the experiences and opinions of travelers. This comprehensive approach ensured that the assessment reflected both operational feasibility and the city's appeal as a vibrant and welcoming destination. For each city and criterion, the experts provided individual scores based on their knowledge, experience, and insights drawn from reliable sources like government reports, tourist feedback, and industry analyses. Naturally, these scores varied, reflecting the diverse perspectives and priorities of the specialists involved.

To ensure fairness and consistency, the final scores were calculated by averaging individual assessments for each criterion and city, then represented as fuzzy numbers. This approach harmonizes differing opinions and minimizes potential bias, resulting in well-rounded and objective scores for a more balanced evaluation.

Table 3. Clustered Data of Cities for Event Hosting Analysis.

	Cost (C1) ($K)			**Infrastructure (C2)**			**Safety (C3)**			**Accessibility (C4)**		
	a	b	c	a	b	c	a	b	c	a	b	c
City A	11	14	16	4	5	6	6	7	8	5	6	7
City B	9	11	14	2	3	4	3	4	5	3	4	5
City C	34	39	45	8	9	10	8	9	10	8	9	10
City D	32	37	43	8	9	10	8	9	10	8	9	10
City E	18	20	23	6	7	8	6	7	8	7	8	9
City F	16	18	20	5	6	7	5	6	7	7	8	9

 doi:10.20944/preprints202502.1684.v1

City G	14	16	18	4	5	6	4	5	6	5	6	7
City H	20	23	25	3	4	5	3	4	5	6	7	8
City I	29	35	41	7	8	9	7	8	9	7	8	9
City J	32	37	43	8	9	10	7	8	9	8	9	10
City K	14	16	18	5	6	7	5	6	7	6	7	8
City L	18	20	23	4	5	6	4	5	6	6	7	8

5. Results and Discussion

In this section, the proposed models are applied, including both the new clustering and normalization methods, along with the TOPSIS method, to the case study. The results are presented step by step, analyzed, and compared with alternative methods.

Table 4 presents the processing of the data from Table 3. Using the fuzzy number of each alternative for each criterion, the corresponding centroid is calculated (columns 2 to 5), using Eq. (2). These centroids are then normalized using the Min-Max method (columns 6 to 9), using Eq.(s) 3 and 4.

The centroids of each alternative will be used to evaluate the distance between these centroids and the optimal scores for each cluster, as presented in Table 2.

Table 4. Normalized Centroids for the Four Considered Criteria Across Selected Cities.

	(C1) Centroid	(C2) Centroid	(C3) Centroid	(C4) Centroid	(C1) Centroid Normalized	(C1) Centroid Normalized	(C1) Centroid Normalized	(C1) Centroid Normalized
City A	14	5	7	6	0,92	0,33	0,6	0,4
City B	11	3	4	4	1,00	0,00	0	0
City C	39	9	9	9	0,00	1,00	1	1
City D	37	9	9	9	0,08	1,00	1	1
City E	20	7	7	8	0,68	0,67	0,6	0,8
City F	18	6	6	8	0,76	0,50	0,4	0,8
City G	16	5	5	6	0,84	0,33	0,2	0,4
City H	23	4	4	7	0,60	0,17	0	0,6
City I	35	8	8	8	0,16	0,83	0,8	0,8
City J	37	9	8	9	0,08	1,00	0,8	1
City K	16	6	6	7	0,84	0,50	0,4	0,6
City L	20	5	5	7	0,68	0,33	0,2	0,6

Table 5 presents these distances, which are evaluated using the Euclidean norm, as defined by Eq. (5). As we can see, the distance of each alternative to the optimal value within each cluster varies. To determine the cluster to which each alternative belongs, we consider the minimum distance, as represented by Eq. (6). In the last column of Table 5, we can identify the minimum distance obtained for each alternative, and in bold, the assigned cluster for each alternative.

Table 5. Evaluation of Distances to the Centroids of Each Cluster.

	(C1) distance	(C2) distance	(C3) distance	min
City A	**0,31**	1,18	0,46	0,31
City B	**0,93**	1,84	1,11	0,93
City C	1,26	**0,17**	0,92	0,17
City D	1,20	**0,15**	0,87	0,15
City E	0,43	0,74	**0,23**	0,23

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City F	0,39	0,98	**0,34**	0,34
City G	**0,43**	1,31	0,57	0,43
City H	0,76	1,36	**0,74**	0,74
City I	0,94	**0,22**	0,57	0,22
City J	1,13	**0,22**	0,79	0,22
City K	**0,21**	1,07	0,33	0,21
City L	0,50	1,16	**0,49**	0,49

Table 6 presents the results aggregated by cluster, showing a grouping consistent with the definition of each cluster. Cluster 1, Cost-Effective Cities with Moderate Infrastructure, includes alternatives with the lowest costs, while ratings for other criteria remain moderate, confirming that the alternatives are classified accordingly. Cluster 2, High-Investment Cities with World-Class Infrastructure, consists of alternatives that align with the cluster definition, exhibiting the highest costs as well as the highest ratings for the criteria. Similarly, Cluster 3, Balanced Cities with a Mix of Features, follows its definition, with alternatives displaying intermediate costs between Cluster 1 and Cluster 2, while the majority of the criteria ratings fall between those of the first two clusters.

Table 6. Result of the Distribution of Alternatives Using the Proposed Clustering Method.

	Cluster	(C1) Centroid	(C2) Centroid	(C3) Centroid	(C4) Centroid
City A	1	14	5	7	6
City B	1	11	3	4	4
City G	1	16	5	5	6
City K	1	16	6	6	7
City C	2	39	9	9	9
City D	2	37	9	9	9
City I	2	35	8	8	8
City J	2	37	9	8	9
City E	3	20	7	7	8
City F	3	18	6	6	8
City H	3	23	4	4	7
City L	3	20	5	5	7

Based on these results, we can conclude that the proposed model produces outcomes consistent with expectations. This means that analyzing the distribution of alternatives across the different clusters confirms that the results are logical and align with the expected distribution of alternatives within each cluster.

Table 7 compares the proposed clustering method with the Fuzzy K-Means approach, showing that the results are almost identical, except for one key difference: City K is assigned to Cluster 1 in the proposed method, whereas Fuzzy K-Means places it in Cluster 3. While this might seem like a small discrepancy, it actually highlights an important distinction in how each method interprets distances and cluster assignments. Overall, the consistency in results suggests that the proposed method is a strong alternative to traditional fuzzy clustering techniques.

Table 7. Comparison between the proposed clustering method and the Fuzzy K-Means method.

	Proposed Method	Fuzzy K-Means
City A	1	1

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City B	1	1
City G	1	1
City K	**1**	**3**
City C	2	2
City D	2	2
City I	2	2
City J	2	2
City E	3	3
City F	3	3
City H	3	3
City L	3	3

More importantly, assigning City K to Cluster 1 actually makes more sense. The city has a low cost, which aligns well with the characteristics of Cluster 1. Furthermore, its cost is identical to that of City G, which was classified as part of Cluster 1 by the Fuzzy K-Means method. The only differences between City K and City G are minor variations (just one point) in the other criteria, making them highly comparable. Given this, it seems much more reasonable for City K to belong to Cluster 1 rather than Cluster 3. This suggests that the proposed method provides a more accurate classification.

Another key advantage of the proposed method is its simplicity and efficiency compared to Fuzzy K-Means, which relies on multiple iterative calculations and a more complex optimization process. The proposed approach follows a more direct and intuitive process, assigning each alternative to the closest centroid without requiring repeated recalculations. In contrast, Fuzzy K-Means involves constant re-evaluation of centroids, which increases computational effort—especially for larger datasets. Additionally, Fuzzy K-Means uses a soft clustering approach, meaning alternatives can partially belong to multiple clusters, rather than being assigned definitively.

On the other hand, the proposed method is deterministic, meaning each alternative is assigned to a single cluster without ambiguity. It also significantly reduces computational effort by eliminating the need for iterative adjustments. Its ease of implementation makes it a practical choice, particularly in scenarios where speed and efficiency are priorities. Given that the overall clustering results are nearly identical and that the proposed method classifies City K in a more logical way, it appears to be not only simpler but also more accurate and reliable than Fuzzy K-Means.

In the next step, the normalization of the rankings obtained for each alternative within each cluster is presented using Logarithmic Normalization (Table 8) and Min-Max Normalization (Table 9).

Table 8. Logarithmic Normalization results.

City	Cluster	C1	C2	C3	C4
City A	1	0,96	0,92	1,00	0,94
City B	1	0,88	0,71	0,77	0,77
City G	1	1,00	0,92	0,86	0,94
City K	1	1,00	1,00	0,94	1,00
City C	2	1,00	1,00	1,00	1,00
City D	2	0,99	1,00	1,00	1,00
City I	2	0,97	0,95	0,95	0,95
City J	2	0,99	1,00	0,95	1,00
City E	3	0,96	1,00	1,00	1,00
City F	3	0,93	0,94	0,94	1,00
City H	3	1,00	0,77	0,77	0,95
City L	3	0,96	0,86	0,86	0,95

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Table 9. Min-Max Normalization results.

City	Cluster	C1	C2	C3	C4
City A	1	0,60	0,67	1,00	0,67
City B	1	0,00	0,00	0,00	0,00
City G	1	1,00	0,67	0,33	0,67
City K	1	1,00	1,00	0,67	1,00
City C	2	1,00	1,00	1,00	1,00
City D	2	0,50	1,00	1,00	1,00
City I	2	0,00	0,00	0,00	0,00
City J	2	0,50	1,00	0,00	1,00
City E	3	0,40	1,00	1,00	1,00
City F	3	0,00	0,67	0,67	1,00
City H	3	1,00	0,00	0,00	0,00
City L	3	0,40	0,33	0,33	0,00

Analyzing the results from Tables 8 and 9, Logarithmic Normalization proves advantageous over Min-Max Normalization, particularly in how it distributes values across the clusters. In Cluster 2, where C1 (cost) has significantly higher values compared to other clusters, Min-Max Normalization stretches these differences, making the cost variation between cities appear more pronounced. In contrast, Logarithmic Normalization compresses the scale, reducing the gap between alternatives while maintaining their relative ranking. This effect is also observed in Cluster 3, where the differences in C1 (cost) and C2 (infrastructure) are more evenly distributed in the logarithmic transformation, preventing extreme values from overshadowing smaller differences. As a result, Logarithmic Normalization provides a more balanced representation, ensuring that alternatives within each cluster are not disproportionately influenced by a single high value, leading to a more stable and interpretable ranking system.

The next step is the application of the TOPSIS method to the normalized tables (Tables 8 and 9), considering the weights for each criterion and each cluster, as presented in Table 10. These weights are derived from the optimal values within each cluster and are essentially obtained by normalizing these values using the Min-Max method.

Table 10. Criterion Weights for Each Cluster.

cluster	(C1) Cluster Centroid	(C2) Cluster Centroid	(C3) Cluster Centroid	(C4) Cluster Centroid	Weight C1	Weight C2	Weight C3	Weight C4
1	1,0	0,6	0,5	0,5	0,38	0,23	0,19	0,19
2	0,1	0,9	1,0	0,9	0,03	0,32	0,34	0,31
3	0,6	0,6	0,6	0,6	0,25	0,25	0,25	0,25

Applying the TOPSIS method to the logarithmic normalization, we obtain the results presented in Table 11, where each alternative is ranked within its respective cluster. Table 12 presents the TOPSIS results using Min-Max normalization, allowing for a comparative analysis between the two normalization approaches.

Table 11. TOPSIS Results Using Logarithmic Normalization.

City	Cluster	C1	C2	C3	C4	D+	D-	TOPSIS Score
City A	1	0,36	0,21	0,19	0,18	0,03	0,08	0,74
City B	1	0,33	0,16	0,15	0,15	0,10	0,00	0,00

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City G	1	0,38	0,21	0,16	0,18	0,03	0,08	0,69
City K	1	0,38	0,23	0,18	0,19	0,01	0,10	**0,89**
City C	2	0,03	0,32	0,34	0,31	0,00	0,03	**1,00**
City D	2	0,03	0,32	0,34	0,31	0,00	0,03	0,98
City I	2	0,03	0,31	0,32	0,30	0,03	0,00	0,00
City J	2	0,03	0,32	0,32	0,31	0,02	0,02	0,57
City E	3	0,24	0,25	0,25	0,25	0,01	0,08	**0,89**
City F	3	0,23	0,23	0,23	0,25	0,03	0,06	0,67
City H	3	0,25	0,19	0,19	0,24	0,08	0,02	0,18
City L	3	0,24	0,22	0,22	0,24	0,05	0,03	0,38

Table 11. TOPSIS Results Using Min-Max Normalization.

City	Cluster	C1	C2	C3	C4	D+	D-	TOPSIS Score
City A	1	0,23	0,15	0,19	0,13	0,18	0,36	0,66
City B	1	0,00	0,00	0,00	0,00	0,52	0,00	0,00
City G	1	0,38	0,15	0,06	0,13	0,16	0,43	0,73
City K	1	0,38	0,23	0,13	0,19	0,06	0,50	**0,89**
City C	2	0,03	0,32	0,34	0,31	0,00	0,56	**1,00**
City D	2	0,02	0,32	0,34	0,31	0,02	0,56	0,97
City I	2	0,00	0,00	0,00	0,00	0,56	0,00	0,00
City J	2	0,02	0,32	0,00	0,31	0,34	0,45	0,57
City E	3	0,10	0,25	0,25	0,25	0,15	0,44	**0,75**
City F	3	0,00	0,17	0,17	0,25	0,28	0,34	0,55
City H	3	0,25	0,00	0,00	0,00	0,43	0,25	0,37
City L	3	0,10	0,08	0,08	0,00	0,37	0,15	0,29

The application of the TOPSIS method to both Min-Max and Logarithmic Normalization identified the top-ranked cities within each cluster. The results show that City K in Cluster 1 and City C in Cluster 2 were consistently ranked highest across both normalization methods, while City E in Cluster 3 showed some variation in its final score depending on the normalization approach.

In Cluster 1, City K emerged as the best-performing alternative, achieving a TOPSIS score of approximately 0.887 in both normalizations. This suggests that City K possesses a well-balanced combination of cost, infrastructure, safety, and accessibility, making it the most favorable option within its group. The stability of its ranking across both normalizations indicates that it is consistently aligned with the ideal conditions of this cluster.

For Cluster 2, City C received a perfect TOPSIS score of 1.000 under both normalization methods. This result confirms that City C perfectly matches the ideal profile for this cluster, making it the most suitable representative. The fact that its ranking remains unchanged, regardless of the normalization technique, suggests that City C is a dominant alternative within its cluster, excelling across all weighted criteria.

In Cluster 3, City E ranked highest, but unlike the other clusters, its TOPSIS score varied between the two normalization methods. Under Min-Max Normalization, it scored 0.747, while in Logarithmic Normalization, its score increased to 0.886. This difference suggests that logarithmic normalization had a stronger impact in this cluster, likely smoothing variations in extreme values and reducing the influence of outliers. As a result, City E appeared closer to the ideal best under Logarithmic Normalization than under Min-Max.

Overall, the consistency of City K and City C as top-ranked alternatives reinforces the robustness of the methodology and confirms that the chosen criteria effectively distinguish the best-performing cities within each cluster. However, the variation in City E's score highlights how normalization can influence ranking intensity, particularly in datasets where differences between values are more pronounced.

The results show that Logarithmic and Min-Max normalizations produced nearly identical outcomes in the TOPSIS analysis, indicating that when there are no significant outliers, logarithmic normalization performs just as well as the Min-Max method. However, in the presence of extreme values, logarithmic normalization proves to be more effective, as it reduces the impact of outliers and prevents criteria with very high values from distorting the distance calculations in TOPSIS.

Thus, it is observed that for datasets without outliers, logarithmic normalization performs just as well as Min-Max normalization, with the added advantage that when outliers are present, logarithmic normalization delivers better performance. If the goal is to ensure that normalization has a meaningful effect only in cases where data variation is large, logarithmic normalization is preferable due to its ability to smooth extreme values. However, when the data is naturally well-distributed, Min-Max normalization remains a valid option, as it preserves the original proportions without information loss.

6. Conclusions

This study introduced two methodological advancements aimed at enhancing the TOPSIS decision-making framework: Clustering Using Fuzzy Numbers and Centroid-Based Distance Allocation and Logarithmic Normalization in TOPSIS. These innovations address key challenges in clustering and normalization, ensuring a more flexible, robust, and accurate approach to multi-criteria decision-making.

By integrating fuzzy numbers into the clustering process, the proposed method moves beyond rigid classifications, allowing alternatives to be assigned to clusters with a degree of membership rather than a strict, binary assignment. This enhances the ability to capture uncertainty and overlapping characteristics in data, making the clustering process more reflective of real-world conditions. Similarly, the introduction of logarithmic normalization overcomes the limitations of traditional methods like Min-Max and Z-score, which often fail to handle highly skewed distributions and extreme outliers effectively. By preserving relative differences while reducing the influence of dominant criteria, this approach ensures a more balanced ranking system within TOPSIS.

The findings from the case study demonstrate that these methodologies significantly improve the accuracy and stability of decision-making outcomes. The fuzzy clustering method enables a more realistic classification of alternatives, while logarithmic normalization enhances the comparability of criteria without introducing unnecessary complexity. A key strength of both approaches is that, despite their effectiveness, they remain computationally simple and easy to implement, making them accessible for a wide range of applications.

While this study has shown promising results with the proposed Clustering Using Fuzzy Numbers and Centroid-Based Distance Allocation and Logarithmic Normalization in TOPSIS, there are several opportunities to further explore and validate these methods in different contexts. One important next step is to evaluate how these techniques perform in other Multi-Criteria Decision-Making (MCDM) models, such as VIKOR, PROMETHEE, and AHP. Since each decision-making framework has unique characteristics, testing the proposed clustering and normalization approaches in different models would provide deeper insights into their adaptability and effectiveness.

Additionally, applying these methods to a wider range of real-world case studies would help assess their practicality and impact in various industries. Future research could explore applications in finance, environmental management, industrial optimization, healthcare, and risk assessment, where decision-making often involves complex, high-uncertainty data. Understanding how well these methods generalize across different sectors would be key to establishing their broader relevance.

Another valuable direction would be to compare the proposed methods with other clustering and normalization techniques. By benchmarking their performance against alternative fuzzy clustering approaches and different normalization strategies, researchers could measure improvements in classification accuracy, ranking stability, and computational efficiency.

Author Contributions: Conceptualization, V.A. and A.A.; methodology, V.A.; software, V.A.; validation, V.A. and A.A.; formal analysis, V.A.; investigation, V.A.; resources, V.A.; data curation, V.A.; writing—original draft preparation, V.A.; writing—review and editing, A.A.; visualization, A.A.; supervision, V.A.; project administration, V.A.; funding acquisition, A.A. All authors have read and agreed to the published version of the manuscript.

Funding: The authors gratefully acknowledge the support from FCT–Fundação para a Ciência e Tecnologia (Portuguese Foundation for Science and Technology), through IDMEC, under LAETA, project UIDB/50022/2020.

Institutional Review Board Statement: Not applicable.

Informed Consent Statement: Not applicable.

Data Availability Statement: The original contributions presented in the study are included in the article, further inquiries can be directed to the corresponding authors.

Conflicts of Interest: The authors declare no conflicts of interest.

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