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Piotr Ogonowski *

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Gravitational Waves from Alena Tensor

Piotr Ogonowski

Kozminski University, Jagiellonska 57/59, 03-301 Warsaw, Poland; piotrogonowski@kozminski.edu.pl

Abstract: Alena Tensor is a recently discovered class of energy-momentum tensors that proposes a general equivalence of the curved path and the geodesic for the analyzed spacetimes which allows the analysis of physical systems in curvilinear, classical and quantum descriptions. In this paper it is shown that using this approach naturally leads to the existence of gravitational waves, which, next to the classical interpretation, can be also interpreted as vacuum pressure waves with tensor amplitude. It is also shown that Alena Tensor gives decomposition of energy-momentum tensor of the electromagnetic field using two null-vectors, which allows for further analysis of metrics for curved spacetimes Petrov type D (and degenerated ones) with effective cosmological constant. The calculated trace of the metric tensor describing curved spacetime turned out to be invariant. A certain simplification of the analysis of gravitational waves has also been proposed, which may help both in their analysis and in the proof of the validity of the Alena Tensor. The article has been supplemented with the Alena Tensor equations with a positive value of the electromagnetic field tensor invariant (related to cosmological constant) which may help in further analysis of this approach.

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1. Introduction

Gravitational waves are a well-understood and researched issue [1], and it seems that the area of this research will develop dynamically both in theoretical understanding [2] and methods of waves detection [3,4]. The existence of gravitational waves is the key argument for the correctness of the General Relativity, and for this reason it is also a good tool for verifying the correctness of alternative to GR theories [5–7] and the theories of quantum gravity [8].

Alena Tensor is at the beginning of its research journey. It is a recently discovered class of energy-momentum tensors that allows for equivalent description and analysis of physical systems in flat spacetime (with fields and forces) and in curved spacetime (using Einstein Field Equations) proposing the overall equivalence of the curved path and the geodesic. In this method it is assumed that the metric tensor is not a feature of spacetime, but only a method of its mathematical description. In previous publications [9–11] it was already shown that this approach allows for a unified description of a physical system (curvilinear, classical and quantum) ensuring compliance with GR and QM results. Due to this property, the Alena Tensor seems to be a useful tool for studying unification problems, quantum gravity and many other applications in physics.

Many researchers try to reproduce the GR equations in flat spacetime or vice versa [12,13] or include electromagnetism in GR, connecting the spacetime geometry with electromagnetism [14–21]. There are known such approaches on the basis of differential geometry [22,22,23], based on field equations [24,25] as well as promising analyses of spinor fields [26] or helpful approximations for a weak field [27]. For this reason, the Alena Tensor should be viewed as another theory requiring theoretical and experimental verification, and it seems worth checking whether the this approach ensures the existence of gravitational waves and what their interpretation is.

In this paper it will be analyzed the possibility of describing gravitational waves using the Alena Tensor. Due to the fact that research on this approach is a relatively young field, to facilitate the analysis of the article, the next section summarizes the results obtained so far and introduces the necessary notation. Although at the first moment the paradigm shift proposed by this approach may

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seem incomprehensible, the author hopes that the reader will resist the temptation to burn this article and trust the scientific method, which encourages us to calculate and check everything based on the correctness of the results obtained.

2. Short Introduction to Alena Tensor

The following chapter briefly explains the conclusions from the previous publications on Alena Tensor. The author uses the metric signature (+,-,-,-) which provides a positive value of the electromagnetic field tensor invariant. In previous publications it was treated as negative (reversal of the order of terms in the energy-momentum tensor of the electromagnetic field). The following equations remove this inconvenience while maintaining the correctness of the obtained results.

2.1. Transforming a Curved Path into a Geodesic

To understand the Alena Tensor, it is easiest to recreate the reasoning that led to its creation [10] using the example of the electromagnetic field. One may consider the energy-momentum tensor in flat spacetime for a physical system with an electromagnetic field in the following form

$$T^{\alpha\beta} = \varrho\, U^\alpha U^\beta - \frac{1}{\mu_r}\Upsilon^{\alpha\beta} \tag{1}$$

where $T^{\alpha\beta}$ is energy-momentum tensor for a physical system, ϱ is density of matter, U^α is four-velocity, μ_r is relative permeability, $\Upsilon^{\alpha\beta}$ is energy-momentum tensor for the electromagnetic field.

The density of four-forces acting in a physical system can be considered as a four-divergence. One may therefore denote the four-force densities occurring in the system:

- $f^\beta \equiv \partial_\alpha \varrho\, U^\alpha U^\beta$ is the density of the total four-force acting on matter
- $\frac{1}{\mu_r} f^\beta_{em} + f^\beta_{gr} \equiv \partial_\alpha \frac{1}{\mu_r}\Upsilon^{\alpha\beta}$ are forces due to the field, where
- f^β_{em} is the density of the electromagnetic four-force
- $f^\beta_{gr} = \Upsilon^{\alpha\beta}\partial_\alpha \frac{1}{\mu_r}$ was shown in [9] as related to the presence of gravity in the system.

One may assume that the forces balance, which will provide a vanishing four-divergence of the energy-momentum tensor for the entire system

$$0 = \partial_\alpha T^{\alpha\beta} = f^\beta - \frac{1}{\mu_r} f^\beta_{em} - f^\beta_{gr} \tag{2}$$

It may be noticed, that if one wanted to use $T^{\alpha\beta}$ for a curvilinear description, which would describe the same physical system but curvilinearly, then in curved spacetime the forces due to the field can be replaced with help of Christoffel symbols of the second kind. This means, that the entire field term can simply disappear from the equation in curved spacetime, because instead of a field and the forces associated with it, there will be corresponding curvature.

This would mean, that in curved spacetime $\frac{1}{\mu_r}\Upsilon^{\alpha\beta} = 0 \;\rightarrow\; T^{\alpha\beta} = \varrho\, U^\alpha U^\beta$. As shown in [10], a minor amendment to continuum mechanics provides this property. Assuming ϱ_o as rest mass density and $\varrho U^\alpha \equiv \varrho_o \gamma U^\alpha$ one gets mass density taking into account motion and Lorentz contraction of the volume and provides

$$\partial_\alpha \varrho U^\alpha = 0 \;\rightarrow\; U^\alpha{}_{,\alpha} = -\frac{d\gamma}{dt} \;\rightarrow\; U^\alpha{}_{;\alpha} = 0\,;\; U^\alpha U^\beta{}_{;\alpha} = 0\,;\; \frac{D\, U^\beta}{D\,\tau} = 0\,;\; \left(\varrho\, U^\alpha U^\beta\right)_{;\alpha} = 0 \tag{3}$$

One may thus generalize $\Upsilon^{\alpha\beta}$ making the following substitution

$$\Upsilon^{\alpha\beta} \equiv \Lambda_\rho \left(\frac{4}{\Bbbk}\Bbbk^{\alpha\beta} - g^{\alpha\beta}\right) = \frac{1}{\mu_o} F^{\alpha\delta}\, g_{\delta\gamma}\, F^{\beta\gamma} - \Lambda_\rho g^{\alpha\beta} \tag{4}$$

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where $F^{\alpha\delta}$ is electromagnetic field strength tensor, μ_o is vacuum magnetic permeability, $g^{\alpha\beta}$ is metric tensor with the help of which the spacetime is considered, and

- $\Lambda_\rho = \frac{1}{4\mu_o} F^{\alpha\mu}\, g_{\mu\gamma}\, F^{\beta\gamma} g_{\alpha\beta}$ is invariant of the electromagnetic field tensor,
- $\mathbb{k} = g_{\mu\nu}\, \mathbb{k}^{\mu\nu}$ is trace of $\mathbb{k}^{\alpha\beta}$,
- $\mathbb{k}^{\alpha\beta} \equiv 2\, \frac{F^{\alpha\delta}\, g_{\delta\gamma}\, F^{\beta\gamma}}{\sqrt{F^{\alpha\delta}\, g_{\delta\gamma}\, F^{\beta\gamma}\, g_{\mu\beta}\, F_{\alpha\eta}\, g^{\eta\xi}\, F^{\mu}_{\ \xi}}}$ is a metric tensor of a spacetime for which $\Upsilon^{\alpha\beta}$ vanishes.

Tensor $\mathbb{k}^{\alpha\beta}$ may be calculated in flat spacetime as $\mathbb{k}^{\alpha\beta} = 2\, \frac{F^{\alpha\gamma} F^{\beta}_{\ \gamma}}{\sqrt{F^{\alpha\gamma}\, F^{\mu}_{\ \gamma}\, F_{\alpha\nu}\, F_{\mu}^{\ \nu}}}$ and may be treated as fixed, since the value of $\mathbb{k}^{\alpha\beta}$ is independent of the $g^{\alpha\beta}$ adopted for analysis. In this way one obtains a generalized description of the tensor $\Upsilon^{\alpha\beta}$, which has the following properties:

- in flat spacetime $\Upsilon^{\alpha\beta}$ is the usual, classical energy-momentum tensor of the electromagnetic field
- its trace vanishes in any spacetime, regardless of the considered metric tensor $g^{\alpha\beta}$
- in spacetime for which $g^{\alpha\beta} = \mathbb{k}^{\alpha\beta}$ the entire tensor $\Upsilon^{\alpha\beta}$ vanishes
- $\mathbb{k}^{\alpha\beta}\mathbb{k}_{\alpha\beta} = 4$ which is expected property of the metric tensor (it was already shown in [10] that $\mathbb{k}^{\alpha\beta}$ indeed is a metric tensor)

In the above manner one obtains the Alena Tensor $T^{\alpha\beta}$ in form of

$$T^{\alpha\beta} = \varrho\, U^\alpha U^\beta - \frac{1}{\mu_r}\Lambda_\rho \left(\frac{4}{\mathbb{k}}\, \mathbb{k}^{\alpha\beta} - g^{\alpha\beta} \right) \tag{5}$$

with the yet unknown $\frac{1}{\mu_r}$ for which in curved spacetime ($g^{\alpha\beta} = \mathbb{k}^{\alpha\beta}$) the energy-momentum tensor of the field $\Upsilon^{\alpha\beta}$ vanishes.

The reasoning carried out above for electromagnetism is universal and allows to consider the Alena Tensor also for energy-momentum tensors associated with other fields. This leads to obtaining an energy-momentum tensor $T^{\alpha\beta}$ for the system that can be considered both in flat spacetime and in curved spacetime.

2.2. *Connection with Continuum Mechanics, GR and QFT/QM*

To make the Alena Tensor consistent with Continuum Mechanics in flat spacetime, it is enough to adopt the substitution $\frac{1}{\mu_r} \equiv \frac{-p}{\Lambda_\rho}$ where p is the negative pressure in the system and it is equal to $p \equiv \varrho c^2 - \Lambda_\rho$ where c is the speed of light in a vacuum. Such substitution yields

$$\varrho\, U^\alpha U^\beta - T^{\alpha\beta} = p\, \eta^{\alpha\beta} - c^2 \varrho\, \frac{4}{\mathbb{k}}\, \mathbb{k}^{\alpha\beta} + \Lambda_\rho \frac{4}{\mathbb{k}}\, \mathbb{k}^{\alpha\beta} \tag{6}$$

where $\eta^{\alpha\beta}$ is the metric tensor of flat Minkowski spacetime. Introducing deviatoric stress tensor $\Pi^{\alpha\beta} \equiv -c^2 \varrho\, \frac{4}{\mathbb{k}}\, \mathbb{k}^{\alpha\beta}$ one obtains relativistic equivalence of Cauchy momentum equation (convective form) in which only f_{em} appears as a body force

$$f^\alpha = \partial^\alpha\, p + \partial_\beta\, \Pi^{\alpha\beta} + f^\alpha_{em} \tag{7}$$

The above substitution also provides a connection to General Relativity in curved spacetime. For this purpose, one may introduce the following tensors, which can be analyzed in both flat and curved spacetime

$$R^{\alpha\beta} \equiv 2\varrho\, U^\alpha U^\beta - p\, g^{\alpha\beta} \quad ; \quad R \equiv R^{\alpha\beta}\, g_{\alpha\beta} = 2\Lambda_\rho - 2p \quad ; \quad G^{\alpha\beta} \equiv R^{\alpha\beta} - \frac{1}{2} R\, \frac{4}{\mathbb{k}}\, \mathbb{k}^{\alpha\beta} \tag{8}$$

Above allows to rewrite Alena Tensor as

$$G^{\alpha\beta} + \Lambda_\rho\, g^{\alpha\beta} = 2\, T^{\alpha\beta} + \varrho c^2 \left(g^{\alpha\beta} - \frac{4}{\mathbb{k}}\, \mathbb{k}^{\alpha\beta} \right) \tag{9}$$

Analyzing the above equation in curved spacetime ($g^{\alpha\beta} = \Bbbk^{\alpha\beta}$), one obtains simplifications

$$G^{\alpha\beta} + \Lambda_\rho \, g^{\alpha\beta} = 2\, T^{\alpha\beta} \quad ; \quad G^{\alpha\beta} = R^{\alpha\beta} - \frac{1}{2} R \, g^{\alpha\beta} \tag{10}$$

thus above can be interpreted as the main equation of General Relativity up to the constant $\frac{4\pi G}{c^4}$ where $G^{\alpha\beta}$ and $R^{\alpha\beta}$ can be interpreted in curved spacetime, respectively, as Einstein curvature tensor and Ricci tensor both with an accuracy of $\frac{4\pi G}{c^4}$ constant.

Analyzing the $G^{\alpha\beta}$ tensor in flat spacetime ($g^{\alpha\beta} = \eta^{\alpha\beta}$) one can also see that it is related to the non-body forces seen in the description of the Cauchy momentum equation

$$\partial_\beta G^{\alpha\beta} = \partial^\alpha \, p + \partial_\beta \, \Pi^{\alpha\beta} = f^\alpha_{gr} + f^\alpha_{rr} \tag{11}$$

which means that in the Alena Tensor analysis method gravity is not a body force, and as shown in [9] in above

- $f^\alpha_{rr} = \left(\frac{1}{\mu_r} - 1\right) f^\alpha_{em}$ is the density of the radiation reaction four-force
- $f^\alpha_{gr} = \varrho \left(\frac{d\phi}{d\tau} U^\alpha - c^2 \partial^\alpha \phi \right)$ is density of the four-force related to gravity, where
- $\phi = -ln(\mu_r)$ is related to the effective potential in the system with gravity.

  It can be calculated that f^α_{gr} vanishes in two cases:
- $\vec{u} = \vec{u}_{ff} \equiv -c \, \frac{\nabla \phi}{\partial^0 \phi}$ - which turns out to be the case of free fall
- $\partial^\alpha \phi = 0$ which occurs in the case of circular orbits

Neglecting the electromagnetic force and the radiation reaction force, using the above equation one can reproduce the motion of bodies in the effective potential obtained from the solutions of General Relativity. Such a description has already been done for the Schwarzschild metric [9] for

$$\phi + c_o \equiv \sqrt{\frac{E^2}{m^2 c^4} - \left(\frac{1}{c} \frac{dr}{d\tau} \right)^2} = \sqrt{\left(1 - \frac{r_s}{r}\right)\left(1 + \frac{L^2}{r^2}\right)} \tag{12}$$

where c_o is a certain constant. The solutions obtained in this way enforce the existence of gravitational waves due to time-varying ϕ (except for free fall and circular orbits).

In the above description, gravity itself is not a force, because the above description is based on an effective potential. However, one can see a similarity to Newton's classical equations for the stationary case with a stationary observer, for which f^α_{gr} can be approximated by Newton's gravitational force with the opposite sign. Thus for stationary observer f^α_{gr} represents a force that must exist to keep a stationary observer suspended above the source of gravity in fixed place.

The description of gravity obtained in this way is surprisingly consistent with current knowledge, despite the fact that gravity itself in this description is not a force, and the force f^α_{gr} is not a body force.

The Alena Tensor constructed in presented way according to [9,11] may be simplified in flat spacetime to

$$T^{\alpha\beta} = \Lambda_\rho \eta^{\alpha\beta} - \frac{1}{\mu_o} F^{\alpha\gamma} \partial^\beta A_\gamma \quad ; \quad \mathcal{L} = T^{00} = -\Lambda_\rho = -\frac{1}{4\mu_o} F^{\alpha\beta} F_{\alpha\beta} \tag{13}$$

which allows its analysis in classical field theory and quantum theories. Obtained canonical four-momentum $H^\alpha \equiv -\frac{1}{c} \int T^{\alpha 0} \, d^3x$ provides $0 = H^\alpha{}_{,\alpha} = H^\alpha H_\alpha$ and

$$\partial^\alpha H^\mu X_\mu = H^\alpha = \mu_r P^\alpha + q\mathcal{E}^\alpha \quad ; \quad -L = \frac{mc^2}{\gamma} - W_{pv} \tag{14}$$

where P^α is four-momentum, $W_{pv} = -\int p \, d^3x$ is pressure-volume work, and where $q\mathcal{E}^\alpha$ and $-\mu_r P^\alpha$ are in fact two gauges of electromagnetic four-potential. In above $(\mu_r - 1)P^\alpha$ is responsible for the force

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associated with gravity and radiation reaction force. It was also shown that cannonical four-momentum H^{μ} may be expressed as

$$H^{\mu} = P^{\mu} + W^{\mu} = -\frac{\gamma L}{c^2} U^{\mu} + \mathbb{S}^{\mu} \tag{15}$$

where $\mathbb{S}^{\mu}$ due to its property $\mathbb{S}^{\mu} U_{\mu} = 0$, seems to be some description of rotation or spin, and where W^{μ} describes the transport of energy due to the field.

The quantum picture obtained from the Alena Tensor [9,11] for the system with electromagnetic field leads to the conclusion that gravity and the radiation reaction force have always been present in Quantum Mechanics and Quantum Field Theory. This conclusion follows from the fact that the quantum equations obtained from the Alena Tensor for the system with electromagnetic field [9] are actually the three main quantum equations currently used:

- simplified Dirac equation for QED:
  $\mathcal{L}_{QED} = \frac{1}{4\mu_o} F^{\alpha\beta} F_{\alpha\beta} = \frac{1}{2\mu_o} F^{0\gamma} \partial^0 A_{\gamma} = \frac{1}{2} \bar{\Psi} \left(i\hbar c \not{D} - mc^2 \right) \Psi$
- Klein-Gordon equation,
- equivalent of the Schrödinger equation: $ic\hbar\, \partial^0 \psi = -\frac{\hbar^2}{m\left(\gamma + \frac{1}{\gamma}\right)} \nabla^2 \psi + cq\hat{A}^0 \psi$

where A^{α} and $\hat{A}^{\alpha}$ are two gauges of electromagnetic four-potential, and where the last equation in the limit of small energies (Lorentz factor $\gamma \approx 1$) turns into the classical Schrödinger equation considered for charged particles.

The above results make the Alena Tensor a useful tool for the analysis of physical systems with fields, allowing modeling phenomena in flat spacetime, curved spacetime, and in the quantum image.

3. Results

One may consider a flat spacetime with an electromagnetic field, described in a way provided by Alena Tensor using notation introduced in section 2. Completing the definition of the first invariant of the electromagnetic field tensor Λ_{ρ}, one may define the second invariant $I_{\perp}$ by electric $\vec{E}$ and magnetic $\vec{B}$ fields as

$$I_{\perp} \equiv \frac{1}{c\mu_o} \vec{E}\vec{B} \tag{16}$$

It can then be seen, after some transformations, that

$$F^{\alpha\gamma} F^{\mu}{}_{\gamma} F_{\alpha\nu} F_{\mu}{}^{\nu} = 4\mu_o^2 \left(2\Lambda_{\rho}^2 + I_{\perp}^2 \right) \tag{17}$$

Therefore the metric tensor for curved spacetime $\Bbbk^{\alpha\beta}$ and its trace $\Bbbk$ (mentioned in introduction) calculated in flat spacetime are

$$\Bbbk^{\alpha\beta} = \frac{\frac{1}{\mu_o} F^{\alpha\gamma} F^{\beta}{}_{\gamma}}{\sqrt{2\Lambda_{\rho}^2 + I_{\perp}^2}} \quad ; \quad \frac{4}{\Bbbk} = \sqrt{2 + \frac{I_{\perp}^2}{\Lambda_{\rho}^2}} \tag{18}$$

The above simplifies further for $I_{\perp} \to 0$, but the key conclusion is that the trace $\Bbbk$ is invariant.

Based on the above, one may reverse the reasoning presented in introduction and consider the field as a manifestation of a propagating perturbation of the curvature of spacetime (which in flat spacetime is just interpreted as a field). For this purpose, one may define a certain perturbation $h^{\alpha\beta}$ of the metric tensor $\Bbbk^{\alpha\beta}$ that describes the deviation from flat spacetime, and also define its trace h as

$$h^{\alpha\beta} \equiv \Bbbk^{\alpha\beta} - \eta^{\alpha\beta} \quad ; \quad h = h^{\alpha\beta}\eta_{\alpha\beta} = \Bbbk - 4 \tag{19}$$

The stress-energy tensor of the electromagnetic field in flat spacetime can be thus represented as follows

$$\frac{\Bbbk}{4\Lambda_{\rho}} \Upsilon^{\alpha\beta} = h^{\alpha\beta} - \frac{h}{4}\eta^{\alpha\beta} \tag{20}$$

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As one can see in the above, considering gravitational waves in the Alena Tensor is natural and does not require classical linearization. Traces h and $\Bbbk$ are invariants, thus $0 = \Box h = \Box \Bbbk$ and condition $0 = \Box \Bbbk^{\alpha\beta} = \Box h^{\alpha\beta}$ can be reduced to Maxwell's equations in vacuum. This would mean that gravitational waves in Alena Tensor approach are de facto a propagating disturbance of the energy-momentum tensor for the field (in the case analyzed, the electromagnetic field energy-momentum tensor).

Denoting the pressure amplitude $\mathcal{P}_o$ and $\bar{h}^{\alpha\beta}$ one obtains

$$\mathcal{P}_o \equiv \frac{4\Lambda_\rho}{\Bbbk} \quad ; \quad \bar{h}^{\alpha\beta} \equiv h^{\alpha\beta} - \frac{h}{4}\eta^{\alpha\beta} \quad \rightarrow \quad \Upsilon^{\alpha\beta} = \mathcal{P}_o\, \bar{h}^{\alpha\beta} \tag{21}$$

which shows that the energy-momentum tensor of the field may be also interpreted as propagating vacuum pressure waves with tensor amplitude.

To provide an analysis of the above equation for gravitational waves and the analysis of the resulting classes of metrics, a representation using null-vectors will be useful. Therefore, in the next few steps it will be shown that Alena Tensor allows representing the energy-momentum tensor of the electromagnetic field with the use of two null-vectors.

At first step one may recall equation (15) and define new four-vector B^μ as

$$B^\mu \equiv -\frac{\gamma L}{c^2} U^\mu - \mathbb{S}^\mu \tag{22}$$

Since it is know from previous publications, that $H^\mu H_\mu = 0$ and $U^\mu \mathbb{S}_\mu = 0$, therefore (15) and (22) also yield $B^\mu B_\mu = 0$. This property therefore allows to represent four-velocity using two null-vectors H^μ and B^μ as follows

$$H^\alpha + B^\alpha = -\frac{2\gamma L}{c^2} U^\alpha \quad \rightarrow \quad H^\alpha B_\alpha = \frac{2\gamma^2 L^2}{c^2} \tag{23}$$

thus

$$H^\alpha B^\beta + B^\alpha H^\beta = \frac{2H^\mu B_\mu}{c^2}\, U^\alpha U^\beta - \left(H^\alpha H^\beta + B^\alpha B^\beta\right) \tag{24}$$

Next, one may define auxiliary parameter α as

$$\alpha \equiv \frac{B^0}{H^0} + \frac{2H^\mu B_\mu}{H^0 mc\gamma} \tag{25}$$

and subtract the linear combination of H^α and B^α from both sides

$$H^\alpha B^\beta + B^\alpha H^\beta - \alpha H^\alpha H^\beta - \frac{H^0}{B^0} B^\alpha B^\beta = \\ = \frac{2H^\mu B_\mu}{c^2} U^\alpha U^\beta - \left([1+\alpha] H^\alpha H^\beta + \left[1 + \frac{H^0}{B^0}\right] B^\alpha B^\beta\right) \tag{26}$$

Next, one may recall from [9] coefficients related to the electromagnetic field

- relative permeability $\mu_r = \frac{\Lambda_\rho}{-p} = \frac{cH^0}{W_{pv}}$
- volume magnetic susceptibility $\chi = \mu_r - 1 = \frac{\varrho c^2}{-p}$
- relative permittivity $\varepsilon_r = \frac{1}{\mu_r} = \frac{-p}{\Lambda_\rho} = \frac{W_{pv}}{cH^0}$
- electric susceptibility $\chi_e = \varepsilon_r - 1 = -\frac{\varrho c^2}{\Lambda_\rho} = -\frac{mc\gamma}{H^0} = -\chi\varepsilon_r$

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and notice, that one obtains Alena Tensor $T^{\alpha\beta}$ as

$$\frac{\mu_r}{\Lambda_\rho}T^{\alpha\beta} = \frac{\chi}{2H^\mu B_\mu}\left(H^\alpha B^\beta + B^\alpha H^\beta - \alpha H^\alpha H^\beta - \frac{H^0}{B^0}B^\alpha B^\beta\right) =$$
$$= \frac{\chi}{c^2}U^\alpha U^\beta - \frac{\chi}{2H^\mu B_\mu}\left([1+\alpha]H^\alpha H^\beta + \left[1+\frac{H^0}{B^0}\right]B^\alpha B^\beta\right) \quad (27)$$

where electromagnetic stress-energy tensor is equal to

$$\frac{1}{\Lambda_\rho}\Upsilon^{\alpha\beta} = \frac{\chi}{2H^\mu B_\mu}\left([1+\alpha]H^\alpha H^\beta + \left[1+\frac{H^0}{B^0}\right]B^\alpha B^\beta\right) \quad (28)$$

and where $T^{0\beta}$ actually simplifies to

$$\frac{\mu_r}{\Lambda_\rho}T^{0\beta} = -\frac{\mu_r}{H^0}H^\beta \quad (29)$$

Since it is known from existing literature [28], that invariants of electromagnetic stress-energy tensor are

$$\Upsilon^{\alpha\beta}\Upsilon_{\alpha\beta} = 4\left(\Lambda_\rho^2 + I_\perp^2\right) = 4\left(\Upsilon^{0\beta}\Upsilon_{0\beta}\right) \quad (30)$$

therefore from (28) one obtains simplifications

$$B^0 = \frac{H^\mu B_\mu}{4H^0} = \frac{\gamma^2 L^2}{2c^2 H^0} \rightarrow \alpha = \frac{B^0}{H^0}\left(1 - \frac{8}{\chi e}\right) \rightarrow \frac{1}{H^0} + \frac{1}{B^0} = -\frac{4c}{L} \quad (31)$$

and by defining a useful variable φ one gets

$$\gamma \equiv \frac{1}{\sqrt{2}}\cosh(\varphi) \quad \rightarrow \quad e^{2\varphi} = \frac{B^0}{H^0} \quad (32)$$

Finally, defining as below, referring to (18)

$$e^\theta \sinh(\theta) \equiv \frac{-L}{mc^2\gamma} \quad ; \quad I^2 \equiv 1 + \frac{I_\perp^2}{\Lambda_\rho^2} = \left(\frac{4}{\mathbb{k}}\right)^2 - 1 \quad (33)$$

then calculating with the use of W_{pv} from (14)

$$I^2 = \chi^2\gamma^2\left(1 - \frac{2L}{mc^2\gamma}\right) = \chi^2\gamma^2 e^{2\theta} \quad \rightarrow \quad \frac{-\gamma L}{W_{pv}} = I\sinh(\theta) \quad (34)$$

and expressing $\mu_r = \frac{cH^0}{W_{pv}} = e^{-\phi}$ as before in introduction, one gets

$$I = \sqrt{2}\frac{e^{\varphi-\phi}}{\sinh(\theta)} \quad ; \quad \chi\gamma^2 = I^2\frac{e^{-2\theta}}{\chi} \quad (35)$$

which can be further used to model and simplify the description of the electromagnetic field. From the perspective of describing gravitational waves, other elements of the description are crucial, which we will discuss next.

To simplify the analysis, one may first normalize four-vectors H^μ and B^μ as follows

$$h^\mu \equiv \frac{1}{H^0}H^\mu \quad ; \quad b^\mu \equiv \frac{1}{B^0}B^\mu \quad (36)$$

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One may now rewrite the electromagnetic field tensor from (28) to obtain (after few calculations using previously derived relationships) its following representation

$$\Upsilon^{\alpha\beta} = \mu_r \Lambda_\rho \left(1 + \frac{1}{2e^\theta \sinh{(\theta)}}\right) h^\alpha h^\beta + \Lambda_\rho \left(\chi \gamma^2 - \frac{\mu_r}{2e^\theta \sinh{(\theta)}}\right) b^\alpha b^\beta \tag{37}$$

As shown in [9] element $\mu_r \Lambda_\rho$ is responsible for electric field energy caried by light, where $\Lambda_\rho \chi \gamma^2$ was shown as describing energy density of magnetic moment linked to matter in motion. The element $\frac{1}{2e^\theta \sinh{(\theta)}}$ is a new term and since it does not actually carry energy (as seen below denoted as $s^{\alpha\beta}$) but only momentum, it can in principle be associated with some description of spin field effects or, potentially, polarization of gravitational waves. This element may be expressed with help of (35) as

$$\Lambda_\rho \, s^{\alpha\beta} \equiv \frac{e^{-(\theta+\varphi)}}{2\sqrt{2}} \Lambda_\rho \, I \left(h^\alpha h^\beta - b^\alpha b^\beta\right) \tag{38}$$

Finally, substituting (37) into (20) using (35) one gets

$$h^{\alpha\beta} = \frac{\Bbbk}{4}\left(e^{-\phi} h^\alpha h^\beta + I^2 \frac{e^{-2\theta}}{\chi} b^\alpha b^\beta + s^{\alpha\beta}\right) + \frac{h}{4}\eta^{\alpha\beta} \tag{39}$$

As can be seen in the above result, the described system is Petrov type D [29], although to be sure, the Weyl tensor should be calculated. However, since type D can degenerate to type N (or with perturbation - go to type II), this means that the Alena Tensor allows for the existence of gravitational waves. It is also worth noting that the obtained term $\frac{h}{4}\eta^{\alpha\beta}$ can in principle be interpreted as a vacuum energy contribution (effective cosmological constant) as in [30,31] playing the role of a metric scaling factor, as e.g. described in [32]

Additionally, one may invoke the scalar field ϕ associated with the presence of matter, where $e^\phi = \frac{1}{\mu_r}$. It is known from 2.2 that e^ϕ is responsible for the presence of sources and in their absence $\mu_r = 1$. Therefore, interpreting whole $\Upsilon^{\alpha\beta}$ as the wave amplitude tensor one would get representation $\frac{1}{\mu_r}\Upsilon^{\alpha\beta} = \mathcal{P}_o \, \bar{h}^{\alpha\beta} e^\phi$ which allows to search for e^ϕ as a certain wave function.

This approach allows for two simplifications related to the analysis of gravitational waves. Considering the force f^α_{gr} responsible for effects related to gravity as shown in (12) and extracting the acceleration $\mathcal{A}^\alpha$ from it, one gets

$$\varrho \mathcal{A}^\alpha \equiv f^\alpha_{gr} = \varrho\left(\frac{d\phi}{d\tau} U^\alpha - c^2 \partial^\alpha \phi\right) \quad \rightarrow \quad c^2 \Box \phi = \gamma^2 \frac{d^2\phi}{dt^2} - \partial_\alpha \mathcal{A}^\alpha \tag{40}$$

since according amendment from [10] $\partial_\alpha \gamma U^\alpha = 0$.

As shown in [9], ϕ is directly related to the effective potential in gravitational systems which can be calculated from the GR equations. This would allow searching for propagating changes of the effective potential itself ($\Box \phi = 0$) similarly as was postulated in [33]. It would significantly simplify both the calculations and perhaps the methods of detecting gravitational waves.

The second simplification results from the possibility of analyzing only the Poynting four-vector $\Upsilon^{\alpha 0}$ as $\frac{1}{\mu_r}\Upsilon^{\alpha 0} = \mathcal{P}_o \, \bar{h}^{\alpha 0} e^\phi$ which might also help simplify the calculations and look for experimental proof of correctness for the Alena Tensor approach.

4. Conclusions and Discussion

As shown in the above article, the Alena Tensor ensures the existence of gravitational waves and allows their physical interpretation. The obtained decomposition of the electromagnetic field stress-energy tensor (39) allows for further analysis of metrics for curved spacetime and the possibility of describing gravitational waves.

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It remains an open question whether the Alena Tensor is a correct way to describe physical systems, but this paper shows that it exhibits many properties that are expected from such a description, including the existence of gravitational waves.

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