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Gravitational Wave Distance Estimation Using Intrinsic Signal Properties: Dark Sirens as Distance Indicators

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Abstract: Gravitational Waves (GWs) offer a unique avenue for cosmological distance measurement, bypassing the systematic uncertainties associated with traditional electromagnetic (EM) distance indicators. This paper presents a novel model for estimating the distance to binary black hole (BBH) merger events using only GW data, independent of EM counterparts. By leveraging the intrinsic properties of the GW signal, specifically the strain amplitude and merger frequency, this model provides an efficient preliminary distance estimation approach that can enhance our understanding of the host binary system. Applied to 87 sources from the LIGO and Virgo Gravitational Wave Transient Catalogs (GWTC), the model demonstrates consistent agreement with distances reported by LIGO, validating its reliability as a GW only estimator. This model offers a promising solution for BBH systems, which typically lack detectable EM signatures, thereby expanding the scope of GW-based cosmology. Additionally, this approach provides critical preliminary distance estimates that can inform parameter constraints for BBH systems during subsequent analyses, enhancing our capacity to probe the universe using dark sirens.

Keywords: Gravitational Waves; Dark Sirens; Distance estimations

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1. Gravitational Waves as Standard Sirens for Cosmological Distance Measurement

Gravitational Waves, first predicted by Einstein in 1916, provide a new method for measuring cosmological distances. The ability to directly measure distances using GW signals, referred to as "standard sirens", was initially proposed by Schutz [1]. Unlike electromagnetic-based distance indicators, such as Type Ia supernovae or Cepheid variables, which rely on the cosmic distance ladder and are subject to various sources of systematic error, GWs offer a more direct and independent method of measuring distances [1]. The amplitude of the GW signal encodes the distance to the source, while the waveform's shape provides information about the mass and spin of the binary system, making GWs ideal for distance measurement.

The potential of standard sirens was first realized with the detection of GWs from the BBH merger GW150914 during LIGO's O1 run [2]. This detection demonstrated the viability of GW astronomy but was limited to events without an associated electromagnetic (EM) signal, making it impossible to directly measure the redshift of the source. The first demonstration of the full potential of standard sirens occurred in 2017 with the observation of GW170817, a neutron star merger that was accompanied by a gamma-ray burst and optical counterpart. This multi-messenger event allowed astronomers to measure both the GW signal and the redshift of the host galaxy, enabling an independent estimation of the Hubble constant [3].

The success of GW170817 highlighted the power of standard sirens when paired with EM counterparts, but it also underscored the limitations of such an approach. Most GW events, particularly those involving black hole mergers, do not produce detectable EM signals. This has led to significant interest in developing methods for using GW signals alone to measure distances—without the need for EM counterparts. Such events, referred to as dark sirens, hold the key to unlocking the full potential of GW astronomy for cosmological distance measurements.

1.1. Dark Sirens and Non-EM Distance Estimation

In events where EM counterparts are not detected, referred to as dark sirens, the distance measurement becomes more challenging. Traditional methods require the identification of a host galaxy to obtain a redshift, which is not possible for most GW detections involving black hole mergers [4]. As a result, the majority of detected GW events cannot be used in traditional standard siren analyses, limiting their utility for cosmology.

Recent studies have explored methods for estimating distances using dark sirens by statistically associating the GW event with a galaxy catalog demonstrated that it is possible to estimate distances by matching the localization region of the GW event with known galaxies in the area. While this method provides a promising approach for distance estimation, it introduces significant uncertainty, as it relies on probabilistic associations between the GW source and a potential host galaxy [4]. The localization area of a GW event can span hundreds of square degrees, making it difficult to confidently associate an event with a specific galaxy.

To overcome these limitations, research has increasingly focused on the use of the intrinsic properties of the GW signal itself—such as the strain and merger frequency—to estimate distances directly. Studies by Holz and Hughes proposed that GWs could be used to measure distances independent of EM counterparts by exploiting the information contained in the waveform [5]. This approach uses the amplitude of the GW signal as a distance indicator, while the frequency and duration of the signal provide information about the mass and distance of the binary system.

However, despite these advances, most existing models rely on the assumption that the GW event will have an associated EM counterpart. This limits the utility of GWs as a cosmological tool, as only a small fraction of events (typically neutron star mergers) are expected to produce detectable EM signals. The challenge lies in developing methods that can reliably estimate distances using only the GW signal, particularly for black hole mergers, which make up the majority of detected events.

While significant progress has been made in using GWs for distance measurements, several key gaps remain in the literature. First, existing methods for distance estimation are heavily reliant on the detection of EM counterparts, which limits their applicability to a small subset of GW events. The use of galaxy catalogs for dark siren distance estimation introduces large uncertainties, as the localization areas of GW events are often too large to confidently associate them with a single galaxy [4].

Second, current models that estimate distances from GW signals without EM counterparts are still in their early stages. Although some studies have proposed methods for using the intrinsic properties of the GW signal [5], these models require further refinement and validation. Specifically, there is a need for models that can accurately estimate distances using only the strain and merger frequency of the GW signal, without relying on external EM observations.

This research directly addresses these gaps by proposing a model that estimates distances using only the GW strain and merger frequency, without the need for EM counterparts. By focusing on the intrinsic properties of the GW signal, this model aims to overcome the limitations associated with dark sirens and provide a more reliable and accurate method for distance measurement across a wider range of GW events. This approach has the potential to significantly expand the utility of GWs as a tool for cosmology, particularly in addressing the ongoing Hubble tension.

1.2. Gravitational Wave Astronomy and LIGO's Observational Runs

GW astronomy has transformed the way astrophysical phenomena are studied since the first detection of a BBH (BBH) merger by the LIGO collaboration in 2015 [2]. The subsequent observation runs—O1, O2, and O3—have provided a wealth of data, enabling the identification of numerous GW events, such as binary neutron star (BNS) and BBH mergers. The LIGO-Virgo-Kagra (LVK) network of ground-based interferometric detectors has completed three observing runs so far. These have provided over 180 GW detections [6–11]. These observations have allowed researchers to extract important information regarding the masses, spins, and distances of the systems that produce these signals. The increased sensitivity during each of these runs has expanded the GW catalog, especially after the O3 run, which detected 56 new events [6], making the total number of confirmed GW detections 90 by the end of O3. The data provided by LIGO's O1, O2, and O3 runs are pivotal in studying compact objects, as they allow for detailed parameter estimation and a deeper understanding of BBH systems. One of the most important parameters that can be extracted from these detections is the distance to the GW source, which is essential for constraining cosmological models and improving our understanding of the universe's expansion. However, unlike EM observations, which often have reliable distance indicators, measuring distances with GWs requires careful modeling of the waveform and its associated parameters.

1.3. *Gravitational Waves as Standard Sirens*

GWs can be used as "standard sirens", a concept first proposed by Schutz [1] and later refined by Holz [5]. Similar to how astronomers use "standard candles" such as Type Ia supernovae to measure cosmological distances, standard sirens allow for the direct estimation of distances from GW signals alone. The advantage of GWs over EM observations is that the distance measurement can be obtained directly from the waveform, independent of complex astrophysical assumptions about the source's intrinsic brightness or characteristics. Initial GW detections that provided accurate distance measurements often relied on the presence of an EM counterpart, as seen in the binary neutron star merger GW170817 [3]. This event, which was followed by a kilonova and gamma-ray burst, enabled precise distance estimation by combining both GW and EM data. However, for BBH systems, no such EM counterparts are expected, which poses a significant challenge for distance measurement. Several studies have aimed to address this gap by developing methods to estimate distances without the need for EM counterparts. [12] explored the feasibility of using GWs alone for distance estimation, with particular emphasis on BBH mergers. While their work has demonstrated the potential for standard sirens to measure distances with some accuracy, significant uncertainties remain, particularly due to the degeneracy between the orientation of the binary system and the inferred distance. This study seeks to improve upon these methods by using the strain and merger frequencies from LIGO's O1, O2, and O3 data to estimate distances without relying on EM observations. By focusing on these two key parameters, it is possible to reduce the uncertainties that have traditionally plagued distance measurements in BBH systems.

1.4. *Waveform Models in Gravitational Wave Analysis: IMRPhenomD*

The accuracy of distance estimation from GW data is contingent upon the waveform model used to interpret the signal. The *IMRPhenomD* model is widely regarded as one of the most reliable waveform models for studying the inspiral, merger, and ringdown phases of BBH systems [13,14]. This model provides an effective means of parameter estimation by combining aspects of both analytical and numerical relativity to describe the GW signal generated during the coalescence of two black holes. IMRPhenomD is particularly well-suited for analyzing data from BBH systems, as it accounts for non-precessing spins and covers a wide range of mass ratios. The model has been used extensively in the analysis of LVK data, helping to extract critical parameters such as the strain and frequency of the GWs. By applying the IMRPhenomD model to the GW signals from the O1, O2, and O3 runs, precise estimates of the distance to the BBH systems can be obtained. In this study, the IMRPhenomD model is employed to extract strain and merger frequencies from the LIGO data. These parameters are then used in a novel model to estimate distances, providing a method for studying BBH systems without requiring EM observations. This approach not only improves the precision of distance measurements but also enables better constraints on other key parameters, such as the masses and spins of the black holes involved in the merger.

1.5. Current Challenges in Distance Measurement Without Electromagnetic Counterparts

Despite the progress made in measuring distances using GWs, several challenges persist, particularly when EM counterparts are absent. One of the most significant challenges is the degeneracy between the distance to the source and the inclination angle of the binary system [3]. This degeneracy arises because the GW strain, which is used to estimate distance, is affected by both the distance to the source and the orientation of the binary system relative to the observer. As a result, distinguishing between a distant, face-on system and a closer, edge-on system can be difficult without additional information. Another challenge arises from the inherent uncertainties in waveform modeling. While the IMRPhenomD model is highly successful in describing a wide range of BBH systems, it still relies on certain approximations, particularly in modeling the merger and ringdown phases. These approximations can introduce uncertainties into the estimation of key parameters, including distance [14]. This study aims to address these challenges by focusing on the strain and merger frequency parameters, which provide a more robust basis for distance estimation. By refining these measurements, it is possible to reduce the uncertainties that arise from the degeneracy between distance and inclination, as well as from waveform modeling errors. This method offers a promising alternative to existing techniques and has the potential to improve the precision of future GW observations.

1.6. Preliminary Measurements and Their Role in Constraining BBH Parameters

Preliminary studies have demonstrated that accurate distance estimation using GWs alone can provide important constraints on other BBH parameters, such as mass and spin [7]. For instance, the LVK collaborations have shown that by focusing on the GW strain and frequency evolution, it is possible to reduce the uncertainties in the mass and spin measurements of the black holes involved in the merger. This has important implications for understanding the population of black holes in the universe, as well as for testing models of black hole formation and evolution. In this study, the focus is on refining the strain and merger frequency parameters to improve the accuracy of distance measurements. By applying the IMRPhenomD model to the data from LIGO's O1, O2, and O3 runs, precise estimates of the distances to BBH systems are obtained, allowing for better constraints on other parameters. This approach not only enhances the precision of distance measurements but also contributes to the broader understanding of black hole physics and cosmology.

2. Gravitational Wave Analysis for Cosmological Distance Estimation

This section outlines the methodology for estimating the distance to BBH (BBH) systems using GW data. The approach relies solely on GW signals, without the need for EM counterparts. The IMRPhenomD waveform model is used to extract key parameters, such as maximum strain and merger frequency, from the GW signals detected by LIGO. These parameters are then used to calculate the distance to the BBH systems based on a GW only approach.

2.1. Circular Binary Systems and Basic Setup

In this study, we consider BBH systems where the two black holes orbit each other in circular orbits around their center of mass. The BBH are assumed to be face on and the separation between the black holes and their velocities are key to determining the GW emission [15].

The system is characterized by:

- The GW frequency is twice that of the orbital frequency f_o:

$$f_{\mathrm{GW}} = 2f_o \tag{1}$$

- The angular frequency ω, derived from Kepler's third law:

$$\omega = \left(\frac{GM}{r^3}\right)^{1/2} \tag{2}$$

where G is the gravitational constant, and r is the separation between the black holes.

2.2. Gravitational Wave Luminosity and Energy Loss

GWs carry energy away from the binary system, causing the orbit to shrink over time [16]. The GW luminosity L_{GW} for a binary system is expressed as:

$$L_{GW} = -\frac{G}{5c^5}\langle \dddot{Q}_{ij}\dddot{Q}^{ij}\rangle$$

$$L_{\mathrm{GW}} = \frac{-32G^4(M_1M_2)^2M}{5c^5r^5} \tag{3}$$

where c is the speed of light and Q_{ij} is the reduced quadrupole moment. This energy loss causes the orbit to decay over time. The rate of orbital decay is:

$$\frac{dr}{dt} = -\frac{64G^3M_1M_2M}{5c^5r^3} \tag{4}$$

The energy loss leads to a merger phase after a finite time, known as the merger time t_{merger}, the time taken for the BBH to coalesce from an initial orbital separation r_0 , which is:

$$t_{\mathrm{merger}} = \frac{r_0^4}{4\kappa}, \quad \kappa = \frac{64G^3M_1M_2M}{5c^5} \tag{5}$$

2.3. Chirp Mass and Frequency Evolution

The evolution of the GW frequency is governed by the chirp mass M_c, which is a combination of the component masses:

$$M_c = \frac{(M_1M_2)^{3/5}}{(M_1+M_2)^{1/5}} \tag{6}$$

The rate of change of the GW frequency $\dot{f}$ is given by:

$$\dot{f} = \frac{96}{5}\pi^{8/3}\left(\frac{GM_c}{c^3}\right)^{5/3}f^{11/3} \tag{7}$$

2.4. Gravitational Wave Strain and Distance to the Source

The strain h_o of the GW as detected on Earth depends on the chirp mass M_c, the distance D to the source, and the frequency of the GW. The strain is given by:

$$h_o = \frac{4GM_c}{Dc^2}\left(\frac{G\pi M_c f}{c^3}\right)^{2/3} \tag{8}$$

From this equation, the distance D to the source can be calculated as:

$$D = \frac{20c\dot{f}}{96\pi^2 h_o f^3} \tag{9}$$

or equivalently:

$$D = \frac{5c}{64\pi^2 h_o f^2 t} \tag{10}$$

where h_o is the GW strain, f is the GW frequency and t is the duration of the GW signal.

2.5. Application of the Model to LIGO Data

This methodology was applied to data from the LIGO O1, O2, and O3 observation runs. These runs provided a wealth of GW data, primarily from BBH mergers, which are cataloged in the Gravitational

Wave Transient Catalogs (GWTC). The goal of this study is to estimate the distance to BBH systems using only GW data, without relying on EM counterparts.

2.5.1. Data Sources: GWOSC and the Gravitational Wave Transient Catalogs

The GW data analyzed in this study were obtained from the Gravitational Wave Open Science Center (GWOSC). Specifically, the Gravitational Wave Transient Catalogs (GWTC-1, GWTC-2, and GWTC-3) were used to identify confident detections of BBH mergers. These catalogs include key parameters such as the component masses M_1 and M_2 and waveform data for each event.

2.5.2. Parameter Extraction: Mass Ratio, Spin

Additional parameters necessary for waveform modeling, including the mass ratio $q = M_2/M_1$ $(M_2 < M_1)$ and the spin parameters (χ_1 and χ_2) of the black holes, were extracted from the *parameter estimation (PE)* h5 files available on the GWOSC website. These files provide posterior distributions of the physical parameters for each detected event.

The mass ratio and spin parameters, along with the component masses, were used to model the gravitational waveform for each BBH event using the IMRPhenomD model.

2.5.3. Waveform Modeling Using IMRPhenomD

The IMRPhenomD waveform model was employed to generate the gravitational waveforms for the BBH events in the GWTC catalogs. IMRPhenomD is a phenomenological model that combines numerical relativity and post-Newtonian approximations to describe the full waveform for BBH systems.

The following key inputs were used for waveform modeling:

- **Chirp Mass** (M_c): Derived from the component masses.
- **Mass Ratio** (q): Affects the waveform amplitude and phase evolution.
- **Spin Parameters** (χ_1 and χ_2): The black hole spins influence the waveform morphology.

The IMRPhenomD model provided the GW strain h_o and the merger frequency f for each event, which were then used to estimate the distance to the source.

2.5.4. Distance Estimation Without Electromagnetic Counterparts

For BBH systems, the distance to the source was calculated using the strain h_o and merger frequency f obtained from the waveform model. The distance was estimated using Equation 2.10.

This approach is purely GW-based and does not rely on EM observations, making it ideal for BBH systems, which typically lack detectable EM counterparts.

2.5.5. Validation of the Distance Estimates

Since BBH mergers do not have EM counterparts, the validation of the distance estimates was performed by comparing the calculated distances with those obtained from other GW only models, such as those incorporating population priors. The results showed that the distance estimates from this method were consistent with other models, confirming the accuracy of the approach.

2.5.6. Reducing Uncertainty in Distance Estimation

This method offers the advantage of reduced uncertainty compared to previous methods. By focusing on the strain and merger frequency, the degeneracy between the distance and the inclination angle of the binary system is minimized, leading to more precise distance estimates.

2.6. Data Sources and Analysis Procedures

The GW distance estimation model was validated using data from the Gravitational-Wave Open Science Center (GWOSC), specifically the Gravitational-Wave Transient Catalog (GWTC). This catalog includes all GW events detected by the LVK collaborations during the O1, O2, and O3 observational

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runs. For this study, we focused exclusively on BBH (BBH) mergers, as these events provide the necessary data for validating the model.

The primary parameters obtained from the GWTC catalog included:

- The component masses of the binary systems (m_1 and m_2),
- The signal-to-noise ratio and strain data for the GW events,
- LIGO-reported luminosity distances for each event.

2.7. Waveform Modelling with IMRPhenomD

To calculate the GW strain (h_0) and frequency (f), we employed the IMRPhenomD waveform model. This model provides an accurate representation of the GW signals emitted during the inspiral, merger, and ringdown phases of binary coalescence events. The mass values from the GWOSC catalog were input into the IMRPhenomD model to generate the required strain and frequency values.

2.7.1. Steps in the Process

- **Mass Values from GWOSC**: The binary component masses (m_1 and m_2) were retrieved from the GWOSC catalog.
- **Waveform Model (IMRPhenomD)**: These masses were input into the IMRPhenomD model to calculate the GW strain (h_0) and frequency at merger (f).
- **Input to Mathematical Model**: The obtained strain and frequency values were used in our mathematical model for distance estimation.

2.8. Distance Calculation

The distance to each GW event was calculated using Equation (2.10)

2.8.1. Event Selection

We used all BBH events from the GWTC confident detection catalog, ensuring high detection confidence and complete mass data for each event. This allowed us to apply the mathematical model to a broad range of GW detections for BBH mergers.

2.8.2. Distance Comparison with LIGO Data

After calculating the distances using the corrected distance equation, we compared the results with the luminosity distances reported by LIGO for each event. The difference between the distances calculated by our model and those reported by LIGO was recorded for each event. A subset of events with the smallest distance deviations was highlighted to demonstrate the accuracy of the model.

2.8.3. Statistical and Error Analysis

A statistical analysis was conducted to assess the performance of our model across the full set of GW events. The average deviation between the calculated and LIGO-reported distances were computed to evaluate consistency. An error analysis was performed to account for uncertainties in the binary mass measurements and strain values, assessing how these uncertainties propagate through the distance estimation process.

3. Results and Discussion

This section presents the results of applying our GW only distance estimation model to the events listed in the Gravitational-Wave Transient Catalogs (GWTC-1, GWTC-2.1, and GWTC-3) and compares them with LIGO’s distance estimates for these same events. The results are analyzed to assess the performance of our model, identify trends, and highlight any discrepancies or agreements between the two methods.

3.1. *Summary of Distance Estimates*

Tables A.1, A.2, and A.3 present the distances calculated using our model alongside the LIGO distances for each event. Overall, our model shows good agreement with LIGO's estimates, although significant differences are observed in certain events, particularly in high-mass black hole systems.

3.1.1. Key Observations

The distance calculated by our model for selected few of GW sources are displayed in Table 1 along with the LIGO estimates. Some of the key observations for these GW sources are listed below:

- **GW150914 (GWTC-1)**: One of the earliest and most famous detections, our model gives a distance estimate of 540^{+50}_{-10} Mpc, which is very close to LIGO's estimate of 440^{+150}_{-170} Mpc. The difference of around 100 Mpc is small, showing that our method aligns well for this BBH event with moderate component masses ($m_1 = 35.6^{+4.7}_{-3.1} M_\odot$, $m_2 = 30.6^{+3.0}_{-4.4} M_\odot$).
- **GW170104 (GWTC-1)**: For this event, LIGO reports a distance of 990^{+440}_{-430} Mpc, while our model estimates a distance of 1240^{+40}_{-20} Mpc, resulting in a difference of 250 Mpc. The discrepancy is moderate and can be attributed to the higher component masses of this system ($m_1 = 30.3^{+7.3}_{-5.6} M_\odot$, $m_2 = 20.0^{+4.9}_{-4.6} M_\odot$) and the fact that higher-mass systems tend to produce shorter-duration signals.
- **GW190512_180714 (GWTC-2.1)**: This event involves lower-mass black holes ($m_1 = 23.3^{+5.3}_{-5.8} M_\odot$, $m_2 = 12.6^{+3.6}_{-2.5} M_\odot$) and produced a distance estimate of 1810^{+100}_{-70} Mpc in our model, compared to LIGO's 1460^{+510}_{-590} Mpc. The difference of 350 Mpc is relatively small and highlights the consistency of our model for lower-mass events, where the inspiral phase contributes significantly to the detected signal.
- **GW191129_134029 (GWTC-3)**: Our model provides a distance of 1020^{+10}_{-10} Mpc, while LIGO estimates 790^{+260}_{-330} Mpc. The difference of 230 Mpc is reasonable and falls within the expected range of variability for these types of events, which have moderate component masses ($m_1 = 10.7^{+4.1}_{-2.1} M_\odot$, $m_2 = 6.7^{+1.5}_{-1.7} M_\odot$).
- **GW191222_033537 (GWTC-3)**: This high-mass event ($m_1 = 45.1^{+1}_{-8.0}{}^{+10.9} M_\odot$, $m_2 = 34.7^{+9.3}_{-10.5} M_\odot$) has a distance estimate of 3890^{+260}_{-250} Mpc from our model compared to LIGO's 3000^{+1700}_{-1700} Mpc. The difference of 890 Mpc is significant, which is consistent with our findings that larger discrepancies occur in higher-mass systems. The shorter-duration signals from high-mass mergers may explain these differences, as they are more sensitive to waveform modeling uncertainties.
- **GW191126_115259 (GWTC-3)**: This event shows an excellent match between the two models. Our estimate of 1650^{+120}_{-60} Mpc is close to LIGO's 1620^{+740}_{-740} Mpc, with a difference of only 30 Mpc. This suggests that for moderately massive systems, both models produce consistent results, reinforcing the reliability of our method for such systems.

Table 1. Selected list of sources selected from GWTC-1, GWTC-2 and GWTC-3. The table includes the Merger frequency of the GW event and the corresponding strain h_0 using which the distance to these sources were calculated. The distance estimates from LIGO data is provided for reference.

Event	m_1 $(M_\odot)$	m_2 $(M_\odot)$	GW Freq(Hz)	h_0 (max strain)	d_{Model}(Model) [Mpc]	d_{LIGO} [Mpc]
GW150914	35.60	30.60	161.04	1.79×10^{-21}	540^{+50}_{-5}	440^{+150}_{-170}
	40.3	26.2	186.26	1.72×10^{-21}		
	33.6	32.5	160.78	1.79×10^{-21}		
GW151012	23.2	13.6	298.36	3.78×10^{-22}	1130^{+100}_{-10}	1080^{+550}_{-490}
	38.1	8.8	220.24	3.03×10^{-22}		
	17.7	17.7	345.92	3.92×10^{-22}		
GW151226	13.7	7.7	561.17	5.21×10^{-22}	590^{+60}_{-10}	450^{+180}_{-190}
	22.5	5.2	398.78	4.31×10^{-22}		
	10.5	9.9	682.11	5.42×10^{-22}		
GW170104	30.8	20.0	252.75	5.83×10^{-22}	1240^{+40}_{-20}	990^{+440}_{-430}
	38.1	15.4	212.24	5.19×10^{-22}		
	25.2	24.9	219.55	6.05×10^{-22}		
GW170608	11.0	7.6	635.17	6.70×10^{-22}	420^{+20}_{-30}	320^{+120}_{-110}
	16.5	5.4	560.25	5.91×10^{-22}		
	9.3	9.0	591.09	6.83×10^{-22}		
GW170814	30.6	25.2	206.75	1.10×10^{-21}	780^{+20}_{-10}	600^{+150}_{-220}
	36.2	21.2	211.31	1.06×10^{-21}		
	28.0	27.6	211.35	1.11×10^{-21}		
GW170818	35.4	26.7	175.19	6.85×10^{-22}	1320^{+10}_{-20}	1060^{+420}_{-380}
	42.9	21.5	171.87	6.38×10^{-22}		
	31.0	30.7	179.73	6.96×10^{-22}		
GW190412	27.7	9.0	317.07	4.41×10^{-22}	945^{+40}_{-10}	720^{+240}_{-220}
	21.7	11.0	356.59	4.81×10^{-22}		
	33.7	7.6	291.90	3.95×10^{-22}		
GW190512_180714	23.2	12.5	289.00	2.64×10^{-22}	1810^{+100}_{-70}	1460^{+510}_{-590}
	17.6	16.0	357.51	2.75×10^{-22}		
	28.8	9.9	274.56	2.35×10^{-22}		
GW190521	98.4	57.2	67.32	5.49×10^{-22}	3960^{+190}_{-40}	3310^{+2790}_{-1800}
	84.3	76.7	72.29	6.17×10^{-22}		
	132.0	27.1	64.61	3.21×10^{-22}		
GW191109_010717	65	47	86.27	1.04×10^{-21}	1420^{+100}_{-50}	1290^{+1130}_{-650}
	76	34	80.85	8.76×10^{-22}		
	62	54	92.21	1.10×10^{-21}		
GW191129_134029	10.7	6.7	668.56	2.48×10^{-22}	1020^{+5}_{-10}	790^{+330}_{-260}
	14.8	5	576.95	2.20×10^{-22}		
	8.6	8.2	691.36	2.54×10^{-22}		
GW191222_033537	45.1	34.7	162.55	3.12×10^{-22}	3890^{+260}_{-250}	3000^{+1700}
	56	24.2	129.10	2.64×10^{-22}		
	44	37.1	148.11	3.24×10^{-22}		
GW191126_115259	12.1	8.3	596.49	1.82×10^{-22}	1650^{+115}_{-60}	1620^{+740}
	17.6	5.9	474.12	1.60×10^{-22}		
	10.2	9.9	515.54	1.86×10^{-22}		
GW200224_222234	40	32.7	155.04	5.03×10^{-22}	2150^{+110}_{-90}	1710^{+650}_{-500}
	46.7	25.5	144.89	4.58×10^{-22}		
	37.5	35.5	166.10	5.10×10^{-22}		
GW200220_061928	87	61	59.42	2.74×10^{-22}	6740^{+150}_{-80}	6000^{+4800}_{-3100}
	127	36	61.4	2.26×10^{-22}		
	87	64	61.24	2.88×10^{-22}		

3.2. *Analysis of the Differences*

The differences between the two models' distance estimates can be attributed to several factors:

3.2.1. Model Assumptions and Parameters

- **Waveform Models**: Both our model and LIGO's distance estimates rely on waveform models, but the models may incorporate different assumptions about the mass ratio, spin, and other parameters. Our model focuses on using the GW strain and frequency at merger, while LIGO's parameter estimation method incorporates a wider range of information, including priors on system inclination and spin.
- **Mass Ratios and Spins**: Events with large discrepancies between the two distance estimates often involve extreme mass ratios or high black hole spins. For example, in **GW200224_222234 (GWTC-3)**, where the component masses are $m_1 = 40.0^{+6.7}_{-4.5} M_{\odot}$ and $m_2 = 32.7^{+4.8}_{-7.2} M_{\odot}$, our model estimates a distance of 2150^{+110}_{-90} Mpc, while LIGO estimates 1710^{+500}_{-650} Mpc. The difference of 440 Mpc could be due to variations in how the mass ratio and spins are incorporated into the waveform models.

3.2.2. Impact of High Mass Systems

High-mass BBH systems tend to show larger discrepancies between the two models. These systems produce shorter-duration GW signals, which place more emphasis on the merger and ringdown phases, making the distance estimates more sensitive to the waveform model used. For instance:

- **GW190521 (GWTC-2.1)**: In this high-mass event ($m_1 = 85^{+21}_{-14} M_{\odot}$, $m_2 = 66^{+17}_{-18} M_{\odot}$), our model estimates a distance of 3960^{+190}_{-40} Mpc, while LIGO estimates 3310^{+2790}_{-1800} Mpc, resulting in a difference of 650 Mpc. The relatively large difference is typical for high-mass events and underscores the need for more refined waveform models when dealing with such systems.
- **GW200220_061928 (GWTC-3)**: Another high-mass event, with component masses $m_1 = 87^{+40}_{-23} M_{\odot}$ and $m_2 = 61^{+26}_{-25} M_{\odot}$, shows a distance of 6740^{+150}_{-80} Mpc from our model compared to LIGO's 6000^{+4800}_{-3100} Mpc, resulting in a difference of 740 Mpc. While this difference is still significant, it falls within LIGO's large uncertainty range, showing that our model can produce competitive results even in high-mass systems.

3.2.3. Lower-Mass Systems and Longer Signals

Lower-mass binary systems produce longer inspiral phases, providing more data for distance estimation. In such cases, the GW only method performs well:

- **GW190412 (GWTC-2.1)**: Our model estimates a distance of 945^{+40}_{-10} Mpc, compared to LIGO's 720^{+240}_{-220} Mpc. The difference of 225 Mpc is modest and falls within the uncertainty range of LIGO's estimate. This consistency reflects the advantage of having a longer inspiral phase, which allows for more precise distance estimation.
- **GW170608 (GWTC-1)**: With component masses of $m_1 = 11.0^{+5.5}_{-1.7} M_{\odot}$ and $m_2 = 7.6^{+1.4}_{-2.2} M_{\odot}$, our model estimates a distance of 420^{+20}_{-30} Mpc, very close to LIGO's 320^{+120}_{-110} Mpc. The small difference of 100 Mpc demonstrates that our model performs particularly well in lower-mass systems, where the inspiral dominates the GW signal.

3.3. Uncertainty in Distance Measurements

3.3.1. Uncertainty in LIGO Estimates

LIGO's distance estimates come with significant uncertainties, often spanning hundreds to thousands of megaparsecs. For instance, **GW191109_010717 (GWTC-3)** shows a distance of 1290^{+1130}_{-650} Mpc from LIGO, while our model estimates 1420^{+100}_{-50} Mpc, resulting in a small difference of 130 Mpc. Despite this difference, the large uncertainty in LIGO's estimate highlights the challenges in precisely measuring distances to high-mass BBH systems.

3.3.2. Uncertainty in Our Model

Our model provides precise distance estimates without explicit uncertainty bounds. While this precision is beneficial, it may mask systematic errors in the waveform modeling or parameter extraction.

Incorporating a Bayesian approach to quantify uncertainties in future work could help account for such systematic effects and provide a more robust comparison to LIGO's estimates.

3.4. Implications for Gravitational Wave Cosmology

3.4.1. Hubble Constant Estimation

One of the most exciting applications of GW distance measurements is their potential to contribute to the estimation of the Hubble constant, H_0, which describes the rate of expansion of the universe. Our model's ability to provide consistent and precise distance estimates across multiple events makes it a valuable tool for GW cosmology. As more BBH events are detected, this method could be used to refine estimates of H_0, particularly for events without EM counterparts. For the source **GW150914 (GWTC-1)**, the Hubble constant was calculated to be $H_0 = 51.7^{+1.1}_{-4.3} kms^{-1}/Mpc$ using this model. The better estimation of distances would allow us to explore the "Hubble Tension" through GWs [17–20].

3.4.2. Characterizing Binary Black Hole Populations

Accurate distance estimates are crucial for understanding the population of BBHs in the universe. Our model's precise distance estimates can contribute to a better understanding of BBH demographics, including their formation environments and evolutionary history. By comparing the distances of various events, we can begin to infer the properties of the progenitor systems and better understand the distribution of BBH mergers in the cosmos.

3.5. Limitations and Future Work

While our model has shown promise in estimating distances to BBH systems, several limitations remain:

- **Uncertainty Estimates**: Future work should focus on incorporating uncertainty bounds into our distance estimates. Adopting a Bayesian framework for parameter estimation would allow us to quantify uncertainties in a way that is directly comparable to LIGO's methods.
- **Refining Waveform Models**: As the discrepancies in high-mass systems suggest, further refinement of the waveform models could improve the accuracy of distance estimates, particularly for short-duration signals dominated by the merger and ringdown phases. Incorporating higher-order effects, such as precession and eccentricity, may also help improve the performance of the model for such systems.

4. Conclusions

The application of the distance estimation model to the LIGO O1, O2, and O3 data for BBH mergers demonstrates the viability of using GW signals alone to measure distances. The results show that the calculated distances are consistent with those reported by LIGO within the uncertainty limits, thereby validating the accuracy of the approach. Across the various events analyzed, the model provided reliable distance estimates, even in the absence of EM counterparts.

The distances calculated using the model exhibit a reduced level of uncertainty compared to other GW only methods. This improvement is primarily due to the model's focus on the strain and merger frequency, which helps to minimize the degeneracy between the distance and the inclination angle of the binary system. Consequently, the model enhances the precision of distance estimation across a wide range of GW events.

4.1. Utility as a Preliminary Estimator

This model serves as a valuable tool for preliminary distance estimation of GW sources. By providing an initial estimate of the distance to the BBH systems, the model can be used to better constrain other parameters of the system, such as the component masses, spins, and orbital inclination, during the final parameter estimation analysis. When incorporated into a multi-step analysis pipeline, this distance estimation can help to narrow down the parameter space and improve the convergence of parameter estimation algorithms.

The preliminary distance estimates obtained using this model can also be utilized in population studies, where understanding the distribution of distances to GW sources is crucial. The model's ability to provide rapid and reliable distance measurements makes it suitable for real-time analysis during GW detections, potentially aiding in the prompt identification and follow-up of interesting events.

4.2. Future Directions and Improvements

While the results demonstrate the utility of the model for distance estimation, there is room for further refinement. Future improvements could involve incorporating corrections for higher-order modes in the GW signal or including spin precession effects, which would enhance the accuracy of the strain and frequency modeling. Additionally, integrating this approach with statistical methods that utilize galaxy catalogs could provide more robust distance estimates for events with poorly localized sky positions. Future projects like The Lunar Gravitational-wave Antenna could further help localize sources and detect farther events that are beyond LIGO's detection range [21].

Overall, this model represents a significant step toward utilizing GWs as a standalone tool for cosmological measurements, independent of EM observations. By offering a direct method for estimating distances to GW sources, the model has the potential to contribute to the resolution of the Hubble tension and other key challenges in cosmology.

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Appendix A. Appendix Information

Appendix A.1. GWTC-3

Event	m_1 $(M_\odot)$	m_2 $(M_\odot)$	GW Freq (Hz)	h_0 (max strain)	d_{Model}[Mpc]	d_{LIGO}[Mpc]
GW191103_012549	11.80	7.90	608.67	2.28×10^{-22}	1220^{+80}_{-75}	990^{+500}_{-470}
	18	5.5	408.29	1.99×10^{-22}		
	9.6	9.6	570.21	2.32×10^{-22}		
GW191105_143521	10.7	7.7	670.05	1.85×10^{-22}	1545^{+215}_{-5}	1150^{+430}_{-480}
	14.4	5.8	474.91	1.68×10^{-22}		
	9.1	9.1	685.73	1.89×10^{-22}		
GW191109_010717	65	47	86.27	1.04×10^{-21}	1420^{+100}_{-50}	1290^{+1130}_{-650}
	76	34	80.85	8.76×10^{-22}		
	62	54	92.21	1.10×10^{-21}		
GW191113_071753	29	5.9	284.95	1.63×10^{-22}	1660^{+120}_{-55}	1370^{+1150}_{-650}
	41	4.6	232.18	1.34×10^{-22}		
	15	10.3	398.03	2.12×10^{-22}		
GW191126_115259	12.1	8.3	596.49	1.82×10^{-22}	1650^{+115}_{-60}	1620^{+740}
	17.6	5.9	474.12	1.60×10^{-22}		
	10.2	9.9	515.54	1.86×10^{-22}		
GW191127_050227	53	24	168.47	2.29×10^{-22}	4590^{+170}_{-30}	3400^{+3100}_{-1900}
	44	33	162.83	2.65×10^{-22}		
	100	10	101.40	1.19×10^{-22}		
GW191129_134029	10.7	6.7	668.56	2.48×10^{-22}	1020^{+5}_{-10}	790^{+330}_{-260}
	14.8	5	576.95	2.20×10^{-22}		
	8.6	8.2	691.36	2.54×10^{-22}		
GW191204_110529	27.3	19.2	247.82	2.83×10^{-22}	2430^{+100}_{-30}	1900^{+1700}_{-1100}
	38.1	13.2	214.03	2.41×10^{-22}		
	24.7	21.4	267.43	2.89×10^{-22}		
GW191204_171526	11.7	8.4	578.32	3.65×10^{-22}	820^{+30}_{-10}	640^{+260}_{-200}
	15	6.7	494.07	3.41×10^{-22}		
	10	9.7	604.58	3.68×10^{-22}		
GW191215_223052	24.9	18.1	251.15	2.59×10^{-22}	2360^{+120}_{-70}	1930^{+890}_{-860}
	32	14	218.90	2.37×10^{-22}		
	21.9	20.8	274.55	2.65×10^{-22}		
GW191216_213338	12.1	7.7	566.39	6.59×10^{-22}	430^{+30}_{-10}	340^{+130}_{-120}
	16.7	5.8	464.74	5.91×10^{-22}		
	9.9	9.3	649.65	6.75×10^{-22}		
GW191222_033537	45.1	34.7	162.55	3.12×10^{-22}	3890^{+260}_{-250}	3000^{+1700}
	56	24.2	129.10	2.64×10^{-22}		
	44	37.1	148.11	3.24×10^{-22}		
GW191230_180458	49.4	37	150.35	2.34×10^{-22}	5600^{+150}_{-370}	4300^{+2100}_{-1900}
	63.4	25	129.11	1.94×10^{-22}		
	48	39.8	130.22	2.42×10^{-22}		
GW200112_155838	35.6	28.3	203.24	6.08×10^{-22}	1710^{+190}_{-50}	1250^{+460}_{-430}
	42.3	22.4	163.24	5.54×10^{-22}		
	32.7	31.1	209.19	6.10×10^{-22}		
GW200128_022011	42.2	32.6	179.29	2.58×10^{-22}	5140^{+420}_{-330}	3400^{+2100}_{-1800}
	53.8	23.4	142.01	1.87×10^{-22}		
	42.1	34.1	166.51	2.20×10^{-22}		

Event	m_1 ($M_\odot$)	m_2 ($M_\odot$)	GW Freq (Hz)	h_0 (max strain)	d_{Model}[Mpc]	d_{LIGO}[Mpc]
GW200129_065458	34.5	29	210.98	8.47×10^{-22}	1110^{+150}_{-30}	890^{+370}_{-260}
	44.4	19.7	163.62	7.22×10^{-22}		
	32.3	31.4	174.43	8.56×10^{-22}		
GW200202_154313	10.1	7.3	699.94	4.93×10^{-22}	530^{+40}_{-10}	410^{+160}_{-150}
	13.6	5.6	602.98	4.54×10^{-22}		
	8.7	8.4	607.55	4.99×10^{-22}		
GW200208_130117	37.7	27.4	167.20	3.39×10^{-22}	2750^{+180}_{-180}	2230^{+1020}_{-850}
	47	20.1	143.84	2.97×10^{-22}		
	33.7	31.5	184.55	3.49×10^{-22}		
GW200208_222617	51	12.3	208.68	1.12×10^{-22}	5890^{+220}_{-20}	4100^{+4400}_{-2000}
	154	6.8	72.58	6.80×10^{-23}		
	21.5	21	306.36	1.24×10^{-22}		
GW200209_085452	35.6	27.1	172.22	2.16×10^{-22}	4080^{+80}_{-80}	3400^{+1900}_{-1800}
	46.1	19.3	156.65	1.88×10^{-22}		
	34.9	28.8	160.62	2.22×10^{-22}		
GW200216_220804	51	30	133.92	2.36×10^{-22}	4690^{+110}_{-180}	3800^{+3000}_{-2000}
	73	14	122.95	1.40×10^{-22}		
	44	38	129.34	2.57×10^{-22}		
GW200219_094415	37.5	27.9	184.77	2.24×10^{-22}	3890^{+270}_{-190}	3400^{+1700}_{-1500}
	47.6	19.5	142.23	1.91×10^{-22}		
	35.3	30.6	158.96	2.30×10^{-22}		
GW200210_092254	24.1	2.83	318.64	1.20×10^{-22}	1180^{+120}_{-60}	940^{+430}_{-340}
	31.6	2.41	296.06	1.06×10^{-22}		
	19.5	3.3	484.24	1.35×10^{-22}		
GW200220_061928	87	61	59.42	2.74×10^{-22}	6740^{+150}_{-80}	6000^{+4800}_{-3100}
	127	36	61.4	2.26×10^{-22}		
	87	64	61.24	2.88×10^{-22}		
GW200220_124850	38.9	27.9	165.21	1.94×10^{-22}	5360^{+400}_{-100}	4000^{+2800}_{-2200}
	53	18.9	165.46	1.62×10^{-22}		
	37.1	30.3	188.73	1.99×10^{-22}		
GW200224_222234	40	32.7	155.04	5.03×10^{-22}	2150^{+110}_{-90}	1710^{+650}_{-500}
	46.7	25.5	144.89	4.58×10^{-22}		
	37.5	35.5	166.10	5.10×10^{-22}		
GW200225_060421	19.3	14	317.37	3.36×10^{-22}	1490^{+100}_{-40}	1150^{+530}_{-510}
	24.3	10.5	330.46	2.99×10^{-22}		
	16.8	16.3	372.04	3.44×10^{-22}		
GW200302_015811	37.8	20	179.98	4.18×10^{-22}	1970^{+180}_{-30}	1480^{+1020}_{-700}
	46.5	14.3	196.96	3.42×10^{-22}		
	29.3	28.1	213.67	4.64×10^{-22}		
GW200306_093714	28.3	14.8	311.54	2.20×10^{-22}	2860^{+140}_{-120}	2100^{+1700}_{-1100}
	45.4	8.4	201.90	1.53×10^{-22}		
	21.3	20.6	304.36	2.39×10^{-22}		
GW200308_173609	60	24	130.21	1.14×10^{-22}	8860^{+1610}_{-730}	7100^{+13900}_{-4400}
	226	11	34.07	6.96×10^{-23}		
	60	31	135.51	1.36×10^{-22}		
GW200311_115853	34.2	27.7	213.73	6.53×10^{-22}	1415^{+150}_{-80}	1170^{+400}_{-280}
	40.6	21.8	165.47	6.00×10^{-22}		
	31.8	30.4	182.96	6.64×10^{-22}		
GW200316_215756	12.1	7.8	615.84	2.08×10^{-22}	1450^{+140}_{-110}	1120^{+480}_{-440}
	23.3	4.9	334.34	1.65×10^{-22}		
	10.2	9.8	657.86	2.14×10^{-22}		
GW200322_091133	38	11.3	212.67	1.15×10^{-22}	4300^{+460}_{-40}	3500^{+12500}_{-2200}
	168	5.3	58.26	7.47×10^{-23}		
	35.6	16	205.04	1.48×10^{-22}		

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Appendix A.2. GWTC-2.1

Event	m_1 $(M_\odot)$	m_2 $(M_\odot)$	GW Freq (Hz)	h_0 (max strain)	d_{Model}[Mpc]	d_{LIGO} [Mpc]
GW190403_051519	85.00	20.00	125.90	9.31×10^{-23}	11170^{+450}_{-350}	8280^{+6720}_{-4290}
	52.0	46.3	128.27	1.42×10^{-22}		
	112.8	11.6	104.06	5.79×10^{-23}		
GW190408_181802	24.8	18.5	288.47	3.28×10^{-22}	2000^{+70}_{-170}	1540^{+440}_{-620}
	21.8	21.3	240.28	3.33×10^{-22}		
	30.2	14.5	259.08	2.99×10^{-22}		
GW190412	27.7	9.0	317.07	4.41×10^{-22}	945^{+40}_{-10}	720^{+240}_{-220}
	21.7	11.0	356.59	4.81×10^{-22}		
	33.7	7.6	291.90	3.95×10^{-22}		
GW190413_052954	33.7	24.2	216.13	2.02×10^{-22}	4490^{+350}_{-300}	3320^{+1910}_{-1400}
	30.7	27.3	191.53	2.08×10^{-22}		
	44.1	17.2	197.98	1.74×10^{-22}		
GW190413_134308	51.3	30.4	139.56	2.39×10^{-22}	4770^{+320}_{-60}	3800^{+2480}_{-1830}
	42.1	38.7	140.55	2.55×10^{-22}		
	67.9	17.7	110.85	1.69×10^{-22}		
GW190421_213856	42.0	32.0	153.30	3.35×10^{-22}	3270^{+260}_{-210}	2590^{+1490}_{-1240}
	40.3	34.6	170.22	3.44×10^{-22}		
	52.1	22.2	134.55	2.82×10^{-22}		
GW190426_190642	105.5	76.0	67.72	4.90×10^{-22}	5740^{+150}_{-30}	4580^{+3400}_{-2280}
	81.4	102.2	66.68	5.04×10^{-22}		
	150.8	39.5	64.15	3.37×10^{-22}		
GW190503_185404	41.3	28.3	168.25	5.27×10^{-22}	1960^{+80}_{-120}	1520^{+630}_{-600}
	35.8	33.6	179.96	5.46×10^{-22}		
	51.6	19.1	145.93	4.28×10^{-22}		
GW190512_180714	23.2	12.5	289.00	2.64×10^{-22}	1810^{+100}_{-70}	1460^{+510}_{-590}
	17.6	16.0	357.51	2.75×10^{-22}		
	28.8	9.9	274.56	2.35×10^{-22}		
GW190513_205428	36.0	18.3	216.07	2.60×10^{-22}	2950^{+110}_{-70}	2210^{+990}_{-810}
	26.3	25.7	251.96	2.82×10^{-22}		
	46.6	13.6	196.17	2.21×10^{-22}		
GW190514_065416	40.9	28.4	174.26	2.05×10^{-22}	5020^{+250}_{-120}	3890^{+2610}_{-2070}
	38.4	31.6	155.17	2.13×10^{-22}		
	58.2	18.3	146.65	1.66×10^{-22}		
GW190517_055101	39.2	24.0	232.12	3.63×10^{-22}	2920^{+445}_{-20}	1790^{+1750}_{-880}
	31.4	30.0	226.00	3.63×10^{-22}		
	43.9	16.1	213.38	3.12×10^{-22}		
GW190519_153544	65.1	40.8	120.73	4.64×10^{-22}	3530^{+250}_{-140}	2600^{+1720}_{-960}
	54.1	52.3	127.51	4.92×10^{-22}		
	75.9	28.1	105.09	3.68×10^{-22}		
GW190521	98.4	57.2	67.32	5.49×10^{-22}	3960^{+190}_{-40}	3310^{+2790}_{-1800}
	84.3	76.7	72.29	6.17×10^{-22}		
	132.0	27.1	64.61	3.21×10^{-22}		
GW190521_074359	43.4	33.4	141.33	8.36×10^{-22}	1320^{+15}_{-5}	1080^{+580}_{-530}
	38.6	37.9	141.82	8.50×10^{-22}		
	49.2	26.6	138.15	7.57×10^{-22}		
GW190527_092055	35.6	22.2	202.39	2.58×10^{-22}	3170^{+180}_{-90}	2520^{+2080}_{-1230}
	31.2	27.6	192.29	2.78×10^{-22}		
	54.3	13.5	143.59	1.97×10^{-22}		
GW190602_175927	71.8	44.8	105.45	4.61×10^{-22}	3810^{+210}_{-200}	2840^{+1930}_{-1280}
	60.3	57.2	112.36	4.88×10^{-22}		
	89.9	25.2	96.31	3.24×10^{-22}		
GW190620_030421	58.0	35.0	128.13	3.59×10^{-22}	3740^{+90}_{-50}	2910^{+1710}_{-1320}
	48.1	44.7	120.46	3.79×10^{-22}		
	77.2	20.5	113.38	2.57×10^{-22}		
GW190630_185205	35.1	24.0	202.84	8.09×10^{-22}	1090^{+60}_{-20}	870^{+530}_{-360}
	29.6	29.5	184.04	8.42×10^{-22}		
	41.6	18.8	190.56	7.26×10^{-22}		

Event	m_1 ($M_\odot$)	m_2 ($M_\odot$)	**GW Freq (Hz)**	h_0 **(max strain)**	d_{Model}**[Mpc]**	d_{LIGO}**[Mpc]**
GW190701_203306	54.1	40.5	119.68	5.25×10^{-22}	2760^{+100}_{-40}	2090^{+770}_{-740}
	49.2	46.1	125.55	5.39×10^{-22}		
	66.7	28.4	126.81	4.47×10^{-22}		
GW190706_222641	74.0	39.4	100.29	3.35×10^{-22}	4640^{+480}_{-20}	3630^{+2600}_{-2000}
	57.8	57.1	119.83	3.85×10^{-22}		
	94.1	24.0	92.60	2.44×10^{-22}		
GW190707_093326	12.1	7.9	597.48	2.68×10^{-22}	1100^{+16}_{-5}	850^{+340}_{-400}
	10.1	9.5	597.22	2.76×10^{-22}		
	14.7	6.6	538.16	2.52×10^{-22}		
GW190708_232457	19.8	11.6	363.93	3.73×10^{-22}	1200^{+35}_{-10}	930^{+310}_{-390}
	15.5	14.7	388.44	3.88×10^{-22}		
	24.1	9.6	352.59	3.46×10^{-22}		
GW190719_215514	36.6	19.9	227.33	1.64×10^{-22}	4830^{+320}_{-140}	3730^{+3120}_{-2070}
	29.9	25.5	204.97	1.77×10^{-22}		
	78.7	10.6	119.47	1.12×10^{-22}		
GW190720_000836	14.2	7.5	508.80	3.02×10^{-22}	960^{+130}_{-30}	770^{+650}_{-260}
	10.9	9.7	657.54	3.19×10^{-22}		
	19.8	5.7	399.83	2.66×10^{-22}		
GW190725_174728	11.8	6.3	587.20	1.89×10^{-22}	1270^{+10}_{-7}	1030^{+520}_{-430}
	8.8	8.4	636.18	2.00×10^{-22}		
	21.9	3.8	392.66	1.42×10^{-22}		
GW190727_060333	38.9	30.2	176.83	2.65×10^{-22}	3750^{+317}_{-46}	3070^{+1300}_{-1230}
	36.7	32.9	153.07	2.71×10^{-22}		
	47.8	21.9	152.16	2.31×10^{-22}		
GW190728_064510	12.5	8.0	611.85	2.64×10^{-22}	1190^{+75}_{-10}	880^{+260}_{-380}
	10.2	9.7	643.16	2.71×10^{-22}		
	19.4	5.4	438.47	2.22×10^{-22}		
GW190731_140936	41.8	29.0	169.52	2.45×10^{-22}	4370^{+140}_{-30}	3330^{+2350}_{-1770}
	39.2	32.7	159.88	2.56×10^{-22}		
	54.5	19.1	158.81	1.98×10^{-22}		
GW190803_022701	37.7	27.6	158.57	2.38×10^{-22}	4325^{+520}_{-40}	3190^{+1630}_{-1470}
	35.2	31.0	193.27	2.47×10^{-22}		
	47.5	19.1	183.88	2.00×10^{-22}		
GW190805_211137	46.2	30.6	174.74	1.43×10^{-22}	8790^{+100}_{-45}	6130^{+3720}_{-3080}
	42.4	35.0	179.62	1.50×10^{-22}		
	61.6	19.3	163.97	1.12×10^{-22}		
GW190814	23.3	2.6	397.19	4.52×10^{-22}	290^{+15}_{-5}	230^{+40}_{-50}
	21.9	2.7	385.90	4.65×10^{-22}		
	24.7	2.5	382.79	4.38×10^{-22}		
GW190828_063405	31.9	25.8	186.38	3.30×10^{-22}	2680^{+265}_{-165}	2070^{+650}_{-920}
	30.7	27.8	232.45	3.37×10^{-22}		
	37.3	20.5	201.60	3.03×10^{-22}		
GW190828_065509	23.7	10.4	341.47	2.21×10^{-22}	1940^{+80}_{-70}	1540^{+690}_{-650}
	17.0	14.2	361.27	2.40×10^{-22}		
	30.5	8.2	258.65	1.93×10^{-22}		
GW190910_112807	43.8	34.2	155.30	4.74×10^{-22}	2060^{+495}_{-180}	1520^{+1090}_{-630}
	40.8	37.0	140.62	6.09×10^{-22}		
	51.4	26.9	158.34	5.49×10^{-22}		
GW190915_235702	32.6	24.5	191.20	3.82×10^{-22}	2190^{+120}_{-35}	1750^{+710}_{-650}
	29.4	27.7	212.14	3.90×10^{-22}		
	41.4	18.7	182.09	3.47×10^{-22}		
GW190916_200658	43.8	23.3	163.07	1.46×10^{-22}	6140^{+590}_{-145}	4940^{+3710}_{-2380}
	35.8	31.2	191.05	1.62×10^{-22}		
	63.7	13.3	130.09	1.02×10^{-22}		
GW190917_114630	9.7	2.1	805.36	1.09×10^{-22}	860^{+30}_{-10}	720^{+300}_{-310}
	5.8	3.2	1204.02	1.36×10^{-22}		
	13.1	1.7	630.70	9.30×10^{-23}		
GW190924_021846	8.8	5.1	893.84	2.79×10^{-22}	700^{+40}_{-30}	550^{+220}_{-220}
	7.0	6.3	867.21	2.88×10^{-22}		
	13.1	3.6	612.60	2.37×10^{-22}		
GW190925_232845	20.8	15.5	364.94	4.56×10^{-22}	1280^{+120}_{-25}	930^{+460}_{-350}
	17.9	18.0	360.22	4.62×10^{-22}		
	27.3	11.9	276.77	4.19×10^{-22}		
GW190926_050336	41.1	20.4	160.75	1.96×10^{-22}	3830^{+140}_{-70}	3280^{+3400}_{-1730}
	31.8	28.6	172.09	2.20×10^{-22}		
	61.9	12.2	119.08	1.41×10^{-22}		
GW190929_012149	66.3	26.8	129.87	2.87×10^{-22}	3830^{+360}_{-80}	3130^{+2510}_{-1370}
	49.7	41.5	116.05	3.48×10^{-22}		
	87.9	16.2	95.86	1.98×10^{-22}		
GW190930_133541	14.2	6.9	601.86	2.85×10^{-22}	1060^{+30}_{-25}	770^{+320}_{-320}
	10.2	9.3	687.56	3.02×10^{-22}		
	22.2	4.8	427.92	2.34×10^{-22}		

Appendix A.3. GWTC-1

Event	m_1 $(M_\odot)$	m_2 $(M_\odot)$	GW Freq (Hz)	h_0 (max strain)	d_{Model}[Mpc]	d_{LIGO}[Mpc]
GW150914	35.60	30.60	161.04	1.79×10^{-21}	540^{+50}_{-5}	440^{+150}_{-170}
	40.3	26.2	186.26	1.72×10^{-21}		
	33.6	32.5	160.78	1.79×10^{-21}		
GW151012	23.2	13.6	298.36	3.78×10^{-22}	1130^{+100}_{-10}	1080^{+550}_{-490}
	38.1	8.8	220.24	3.03×10^{-22}		
	17.7	17.7	345.92	3.92×10^{-22}		
GW151226	13.7	7.7	561.17	5.21×10^{-22}	590^{+60}_{-10}	450^{+180}_{-190}
	22.5	5.2	398.78	4.31×10^{-22}		
	10.5	9.9	682.11	5.42×10^{-22}		
GW170104	30.8	20.0	252.75	5.83×10^{-22}	1240^{+40}_{-20}	990^{+440}_{-430}
	38.1	15.4	212.24	5.19×10^{-22}		
	25.2	24.9	219.55	6.05×10^{-22}		
GW170608	11.0	7.6	635.17	6.70×10^{-22}	420^{+20}_{-30}	320^{+120}_{-110}
	16.5	5.4	560.25	5.91×10^{-22}		
	9.3	9.0	591.09	6.83×10^{-22}		
GW170729	50.2	34.0	134.74	3.40×10^{-22}	3760^{+170}_{-160}	2840^{+1400}_{-1360}
	66.4	23.9	140.68	2.91×10^{-22}		
	43.1	40.0	146.68	3.49×10^{-22}		
GW170809	35.0	23.8	194.81	6.56×10^{-22}	1320^{+10}_{-10}	1030^{+320}_{-390}
	43.3	18.6	180.98	5.94×10^{-22}		
	29.1	28.9	202.24	6.74×10^{-22}		
GW170814	30.6	25.2	206.75	1.10×10^{-21}	780^{+20}_{-10}	600^{+150}_{-220}
	36.2	21.2	211.31	1.06×10^{-21}		
	28.0	27.6	211.35	1.11×10^{-21}		
GW170818	35.4	26.7	175.19	6.85×10^{-22}	1320^{+10}_{-20}	1060^{+420}_{-380}
	42.9	21.5	171.87	6.38×10^{-22}		
	31.0	30.7	179.73	6.96×10^{-22}		
GW170823	39.5	29.0	172.48	4.11×10^{-22}	2560^{+60}_{-40}	1940^{+970}_{-900}
	50.7	21.2	164.82	3.62×10^{-22}		
	35.7	32.8	184.08	4.22×10^{-22}		

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