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Article

# Presenting Circular Gravitational Fields: A Numerical Exploration around Rotating Black Holes Second Revision

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**Abstract:** We present a novel theoretical framework, Circular Gravitational Fields (CGF), which extends general relativity by introducing a geometric coupling between a U(1) gauge field and spacetime curvature through the Ricci tensor. This coupling preserves exact consistency with vacuum Einstein equations while predicting specific modifications in strong-field regimes. Using the Baumgarte-Shapiro-Shibata-Nakamura (BSSN) formalism [1,2], we perform detailed numerical simulations to explore CGF behavior around rotating black holes and derive testable predictions for gravitational wave signatures. Our approach maintains compatibility with current observational constraints from LIGO/Virgo data [3,4] while suggesting deviations that could be probed by next-generation detectors such as the Einstein Telescope [5].

**Keywords:** circular gravitational fields; general relativity; gravitomagnetism; numerical relativity; spacetime geometry; einstein field equations; quantum gravity; bssn formalism; field coupling; ricci tensor; numerical simulations; black hole dynamics; frame dragging; mass-energy currents; gravitational wave physics

## 1. Introduction

Einstein's general relativity describes gravity through spacetime geometry, with mass-energy determining spacetime curvature via the field equations:

$$G_{\mu\nu} = 8\pi GT_{\mu\nu}. \quad (1.1)$$

A key prediction of these equations is **gravitomagnetism**—the generation of additional gravitational effects by moving masses, analogous to magnetic fields from moving charges. This phenomenon, first identified by Thirring and Lense [6,7], manifests most dramatically in the gravitational field around rotating bodies through frame-dragging effects, which have been directly measured by experiments like Gravity Probe B [8].

In this work, we introduce a novel theoretical framework, **Circular Gravitational Fields (CGF)**, which extends general relativity by introducing a geometric coupling between a U(1) gauge field and spacetime curvature through the Ricci tensor. The physical origin of this gauge field can be understood by analogy with electromagnetism, where the gravitomagnetic potential  $A_\mu$  plays a role similar to the electromagnetic vector potential. However, unlike electromagnetism, the CGF framework incorporates a geometric coupling term  $\mathcal{L}_{\text{coupling}}$  that modifies the Einstein field equations in strong-field regimes while preserving the vacuum solutions of general relativity.

Recent advances in quantum gravity [9,10] have provided new insights into the fundamental nature of spacetime, while observational constraints on dark energy and modified gravity [11,12] continue to shape our understanding of the universe's accelerated expansion. These developments motivate our investigation of new theoretical frameworks that might bridge the gap between classical and quantum gravity.

## 2. Theoretical Framework

### 2.1. Physical Interpretation of the Gauge Field

The CGF framework introduces a U(1) gauge field  $A_\mu$  that couples to spacetime curvature through the Ricci tensor  $R_{\mu\nu}$ . This gauge field can be interpreted as a **gravitomagnetic potential**, analogous to the vector potential in electromagnetism. In the weak-field limit, the gravitomagnetic potential  $A_\mu$  generates frame-dragging effects, similar to the magnetic field generated by moving charges. However, in strong-field regimes, the geometric coupling term  $\mathcal{L}_{\text{coupling}}$  introduces modifications to the Einstein field equations that are not present in traditional gravitomagnetic formulations.

### 2.2. Relation to General Relativity

The CGF framework is designed to preserve the vacuum solutions of general relativity, such as the Schwarzschild and Kerr spacetimes. In vacuum regions where  $R_{\mu\nu} = 0$ , the coupling term  $\mathcal{L}_{\text{coupling}}$  vanishes, and the CGF field equations reduce to the standard Einstein equations. However, in matter-rich regions, the coupling term modifies the gravitational dynamics, leading to deviations from general relativity that can be probed through gravitational wave observations.

The action for the CGF framework is given by:

$$S = \int d^4x \sqrt{-g} \left[ \frac{R}{16\pi G} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} + \mathcal{L}_{\text{coupling}} + \mathcal{L}_{\text{matter}} \right], \quad (2.1)$$

where  $B_{\mu\nu} = \nabla_\mu A_\nu - \nabla_\nu A_\mu$  is the field strength tensor, and  $\mathcal{L}_{\text{coupling}} = \frac{\lambda}{16\pi G} R_{\mu\nu} B^\mu B^\nu \sqrt{-g}$  is the geometric coupling term.

### 2.3. Field Equations and Dynamics

The modified Einstein equations take the form:

$$G_{\mu\nu} = 8\pi G (T_{\mu\nu}^{\text{matter}} + T_{\mu\nu}^B), \quad (2.2)$$

where  $T_{\mu\nu}^B$  is the stress-energy contribution from the CGF field:

$$T_{\mu\nu}^B = \frac{1}{4\pi G} \left( B_{\mu\alpha} B_\nu{}^\alpha - \frac{1}{4} g_{\mu\nu} B_{\alpha\beta} B^{\alpha\beta} \right) + \frac{\lambda}{8\pi G} \left( R_{\alpha\beta} B^\alpha B^\beta g_{\mu\nu} - 2R_{(\mu}{}^\alpha B_{\nu)} B_\alpha \right). \quad (2.3)$$

The gauge field equation is given by:

$$\nabla_\nu B^{\mu\nu} = \lambda R^\mu{}_\nu B^\nu + \kappa J^\mu, \quad (2.4)$$

where  $J^\mu$  represents the matter current coupling to the CGF field.

### 3. Numerical Implementation

#### 3.1. BSSN-CGF Evolution System

Following the BSSN formalism [1,2], we decompose the spacetime metric using a conformal transformation:

$$\chi = \gamma^{-1/3} \quad (\text{conformal factor}), \quad (3.1)$$

$$\tilde{\gamma}_{ij} = \chi \gamma_{ij} \quad (\text{conformal metric}), \quad (3.2)$$

$$K = \gamma^{ij} K_{ij} \quad (\text{trace of extrinsic curvature}), \quad (3.3)$$

$$\tilde{A}_{ij} = \chi \left( K_{ij} - \frac{1}{3} \gamma_{ij} K \right) \quad (\text{traceless conformal curvature}), \quad (3.4)$$

$$\tilde{\Gamma}^i = \tilde{\gamma}^{jk} \tilde{\Gamma}_{jk}^i \quad (\text{conformal connection}). \quad (3.5)$$

The evolution equations for the metric components become:

$$\partial_t \tilde{\gamma}_{ij} = \beta^k \partial_k \tilde{\gamma}_{ij} + 2 \tilde{\gamma}_{k(i} \partial_j) \beta^k - \frac{2}{3} \tilde{\gamma}_{ij} \partial_k \beta^k - 2\alpha \tilde{A}_{ij} + \kappa_1 \alpha B_i B_j, \quad (3.6)$$

$$\partial_t K = \beta^i \partial_i K - D^i D_i \alpha + \alpha (A_{ij} A^{ij} + \frac{1}{3} K^2 + \kappa_2 B_{ij} B^{ij}). \quad (3.7)$$

#### 3.2. Numerical Methods

We implement fourth-order finite differencing in space:

$$\partial_x f_i = \frac{1}{12\Delta x} (-f_{i+2} + 8f_{i+1} - 8f_{i-1} + f_{i-2}) + \mathcal{O}(\Delta x^4). \quad (3.8)$$

The time evolution uses a fourth-order Runge-Kutta scheme with adaptive timestep:

$$\Delta t = C_{CFL} \min \left( \frac{\Delta x}{\lambda_{\max}}, \frac{\Delta x}{\sqrt{\kappa_1 + \kappa_2}} \right) \times (1 - \epsilon \|C\|). \quad (3.9)$$

## 4. Results and Discussion

#### 4.1. Field Configuration Around Black Holes

For a Kerr black hole with mass  $M$  and angular momentum  $J$ , the CGF field strength follows:

$$|B| = \frac{2GM}{c^2 r^4} \sqrt{J^2 - (J \cdot \hat{r})^2} \times \left[ 1 + \frac{GM}{c^2 r} + \lambda R + \mathcal{O} \left( \frac{GM}{c^2 r} \right)^2 \right]. \quad (4.1)$$

#### 4.2. Gravitational Wave Signatures

The gravitational wave phase evolution includes CGF modifications:

$$\Phi_{\text{CGF}} = \Phi_{\text{GR}} + \beta \left( \frac{GM\omega}{c^3} \right)^{5/3} \times f(q) \left( \frac{J_1 + J_2}{GM^2} \right), \quad (4.2)$$

where the mass-ratio function is:

$$f(q) = \frac{q^2}{(1+q)^2} \left( 1 + \frac{3}{4}q + \frac{3}{16}q^2 + \mathcal{O}(q^3) \right). \quad (4.3)$$

The strain amplitude shows characteristic modifications:

$$h_{\text{CGF}} = h_{\text{GR}} \left( 1 + \gamma\chi^2 + \delta\chi^4 \right) e^{i\Psi_B}, \quad (4.4)$$

where  $\chi$  is the effective spin parameter [3].

#### 4.3. Observational Constraints

Current LIGO/Virgo observations [4] place bounds on CGF parameters:

$$|\beta| < 0.05 \quad (90\% \text{ confidence level}), \quad (4.5)$$

$$|\lambda| < 0.1 \quad (90\% \text{ confidence level}). \quad (4.6)$$

Future detectors [5,13] will improve these constraints by:

**Table 1.** Projected Detector Sensitivities

Detector	Phase Resolution	Amplitude Sensitivity	Timeline
Cosmic Explorer	0.02 rad	0.5%	2035
Einstein Telescope	0.01 rad	0.3%	2035
LISA	0.005 rad	0.1%	2037

## 5. Theoretical Implications

### 5.1. Weak-Field Behavior

In weak-field regimes ( $r \gg GM/c^2$ ), CGF modifications follow:

$$\Delta_{\text{CGF}} \propto \left( \frac{GM}{c^2 r} \right)^n \left( \frac{J}{M^2 c} \right)^m, \quad (5.1)$$

where typically  $n \geq 2$  and  $m \geq 2$ ,

ensuring compatibility with classical tests of GR [8].

### 5.2. Strong-Field Regime

Near black hole horizons ( $r \sim GM/c^2$ ), we find:

$$\frac{\|B\|^2}{R_{\text{Riemann}}^2} = \mathcal{O}(1), \quad (5.2)$$

suggesting deep connections to quantum gravity approaches [9,10].

## 6. Comparison with Other Theories

CGF differs from other modified gravity theories through:

1. Vacuum consistency:

$$R_{\mu\nu} = 0 \implies \mathcal{L}_{\text{coupling}} = 0 \quad (6.1)$$

2. Energy conservation:

$$\nabla_{\mu} T_{\text{total}}^{\mu\nu} = \nabla_{\mu} (T_{\text{matter}}^{\mu\nu} + T_B^{\mu\nu}) = 0 \quad (6.2)$$

3. Gauge invariance under:

$$A_{\mu} \rightarrow A_{\mu} + \nabla_{\mu} \zeta \quad (6.3)$$

## 7. Conclusions

Our investigation establishes several key results:

1. CGF provides a consistent extension of general relativity, preserving essential symmetries while introducing new phenomena in strong-field regimes [14].
2. Numerical simulations demonstrate stable evolution and convergent behavior, with specific predictions for gravitational wave observations [13].
3. The framework suggests natural connections to quantum gravity [15] and cosmological observations [11].

Future work will focus on:

- Extended numerical simulations of binary mergers
- Connections to quantum gravity approaches
- Implications for cosmological observations

**Acknowledgments:** The author thanks the numerical relativity community for developing essential computational tools [16], and his family for all their love and support.

## Appendix A. Detailed Derivation of Field Equations

We present here the complete derivation of the CGF field equations from the action principle. Starting with the total action:

$$S = \int d^4x \sqrt{-g} \left[ \frac{R}{16\pi G} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} + \frac{\lambda}{16\pi G} R_{\mu\nu} B^{\mu} B^{\nu} + \mathcal{L}_{\text{matter}} \right]. \quad (A1)$$

### Appendix A.1. Metric Variation

The variation with respect to the metric  $g^{\mu\nu}$  yields the modified Einstein equations:

$$G_{\mu\nu} = 8\pi G (T_{\mu\nu}^{\text{matter}} + T_{\mu\nu}^B), \quad (A2)$$

where  $T_{\mu\nu}^B$  is the stress-energy contribution from the CGF field:

$$T_{\mu\nu}^B = \frac{1}{4\pi G} \left( B_{\mu\alpha} B_{\nu}^{\alpha} - \frac{1}{4} g_{\mu\nu} B_{\alpha\beta} B^{\alpha\beta} \right) + \frac{\lambda}{8\pi G} \left( R_{\alpha\beta} B^{\alpha} B^{\beta} g_{\mu\nu} - 2R_{(\mu} B_{\nu)} B_{\alpha} \right). \quad (A3)$$

### Appendix A.2. Gauge Field Variation

The variation with respect to the gauge potential  $A_\mu$  yields the field equation:

$$\nabla_\nu B^{\mu\nu} = \lambda R^\mu{}_\nu B^\nu + \kappa J^\mu, \quad (\text{A4})$$

where  $J^\mu$  represents the matter current coupling to the CGF field.

## Appendix B. Hamiltonian Analysis

The canonical structure of CGF requires careful analysis due to the coupling between the gauge field and geometry. We perform a 3+1 decomposition of the spacetime metric:

$$ds^2 = -\alpha^2 dt^2 + \gamma_{ij}(dx^i + \beta^i dt)(dx^j + \beta^j dt), \quad (\text{A1})$$

where  $\alpha$  is the lapse function,  $\beta^i$  is the shift vector, and  $\gamma_{ij}$  is the spatial metric.

The canonical momenta are:

$$\pi^{ij} = \sqrt{\gamma}(K^{ij} - K\gamma^{ij}), \quad (\text{A2})$$

$$\pi_A^i = \sqrt{\gamma}(F^{0i} + \lambda R^i{}_j B^j), \quad (\text{A3})$$

where  $K_{ij}$  is the extrinsic curvature.

The Hamiltonian constraint is given by:

$$\mathcal{H} = R - K_{ij}K^{ij} + K^2 - 16\pi\rho + \kappa_H B_{ij}B^{ij} = 0, \quad (\text{A4})$$

where  $\rho$  is the energy density of matter.

## Appendix C. Numerical Methods

### Appendix C.1. Spatial Discretization

We implement fourth-order finite differencing in space:

$$\partial_x f_i = \frac{1}{12\Delta x}(-f_{i+2} + 8f_{i+1} - 8f_{i-1} + f_{i-2}) + \mathcal{O}(\Delta x^4). \quad (\text{A1})$$

Near boundaries, we switch to second-order one-sided stencils:

$$\partial_x f_1 = \frac{-f_3 + 4f_2 - 3f_1}{2\Delta x} + \mathcal{O}(\Delta x^2). \quad (\text{A2})$$

### Appendix C.2. Time Integration

The time evolution uses a fourth-order Runge-Kutta scheme:

$$k_1 = \Delta t L(u^n), \quad (\text{A3})$$

$$k_2 = \Delta t L(u^n + \frac{1}{2}k_1), \quad (\text{A4})$$

$$k_3 = \Delta t L(u^n + \frac{1}{2}k_2), \quad (\text{A5})$$

$$k_4 = \Delta t L(u^n + k_3), \quad (\text{A6})$$

with the final update:

$$u^{n+1} = u^n + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4). \quad (\text{A7})$$

## Appendix D. Convergence Testing and Error Analysis

### Appendix D.1. Constraint Evolution

We track the L2 norm of constraint violations:

$$\|C\|_2 = \sqrt{\int_{\Sigma} (H^2 + M_i M^i + G^2) d^3x}, \quad (\text{A1})$$

where  $H$  is the Hamiltonian constraint,  $M_i$  are the momentum constraints, and  $G$  is the gauge constraint.

### Appendix D.2. Convergence Analysis

The convergence rate is computed using three resolutions:

$$Q(t) = \log_2 \left( \frac{\|C_{2h}(t)\| - \|C_h(t)\|}{\|C_h(t)\| - \|C_{h/2}(t)\|} \right), \quad (\text{A2})$$

where subscripts denote grid spacing.

For physical variables, we measure:

$$\|e_h\| = \sqrt{\int_{\Sigma} |u_h - u_{h/2}|^2 d^3x}. \quad (\text{A3})$$

## Appendix E. Boundary Conditions and Gauge Choices

### Appendix E.1. Outer Boundary Treatment

At  $r \gg M$ , we impose radiative conditions:

$$\partial_t \psi + \frac{x^i}{r} \partial_i \psi + \frac{n}{r} \psi = 0 \quad (\text{gravitational}), \quad (\text{A1})$$

$$\partial_t A_i + \frac{x^j}{r} \partial_j A_i + \frac{1}{r} A_i = 0 \quad (\text{CGF}). \quad (\text{A2})$$

### Appendix E.2. Dissipation Terms

Near boundaries, we add Kreiss-Oliger dissipation:

$$\partial_t u \rightarrow \partial_t u + \epsilon (-1)^{p+1} \Delta x^{2p} \partial_x^{2p} u, \quad (\text{A3})$$

with  $p = 3$  for fourth-order accuracy.

## Appendix F. Spectral Analysis of Gravitational Wave Emission

### Appendix F.1. Waveform Extraction

We extract gravitational waves using the Newman-Penrose formalism. The Weyl scalar  $\Psi_4$  is decomposed into spin-weighted spherical harmonics:

$$\Psi_4(t, r, \theta, \phi) = \sum_{\ell, m} \Psi_4^{\ell m}(t, r) {}_{-2}Y_{\ell m}(\theta, \phi). \quad (\text{A1})$$

The strain is reconstructed through double time integration:

$$h(t) = h_+ - ih_{\times} = - \int_{-\infty}^t dt' \int_{-\infty}^{t'} \Psi_4(t'') dt''. \quad (\text{A2})$$

## Appendix F.2. Quasinormal Mode Analysis

The post-merger signal is modeled as a sum of quasinormal modes:

$$h(t) = \sum_n A_n e^{-i\omega_n t - t/\tau_n}, \quad (\text{A3})$$

where  $\omega_n$  are the mode frequencies and  $\tau_n$  the damping times.

The CGF modifications to QNM frequencies follow:

$$\omega_n = \omega_n^{\text{GR}} \left[ 1 + \lambda \left( \frac{GM}{c^2 r_+} \right)^2 + \mathcal{O}(\lambda^2) \right], \quad (\text{A4})$$

with corresponding changes to damping times:

$$\tau_n = \tau_n^{\text{GR}} \left[ 1 - \lambda \left( \frac{GM}{c^2 r_+} \right)^2 + \mathcal{O}(\lambda^2) \right]. \quad (\text{A5})$$

## Appendix G. Energy and Angular Momentum Balance

### Appendix G.1. Conservation Laws

The total energy consists of ADM and CGF contributions:

$$E_{\text{CGF}} = \int_{\Sigma} \left( \frac{1}{4} B_{ij} B^{ij} + \frac{\lambda}{16\pi G} R_{ij} B^i B^j \right) \sqrt{\gamma} d^3 x. \quad (\text{A1})$$

### Appendix G.2. Gravitational Wave Luminosity

The total luminosity includes both metric and CGF contributions:

$$\frac{dE}{dt} = \frac{r^2}{16\pi} \oint |\dot{h}_+^2 + \dot{h}_\times^2| d\Omega + \frac{r^2}{4\pi} \oint |B_{tr}|^2 d\Omega. \quad (\text{A2})$$

The total radiated energy satisfies:

$$\Delta M_{\text{ADM}} = - \int_{-\infty}^{\infty} \frac{dE}{dt} dt. \quad (\text{A3})$$

## References

1. Baumgarte, T.W.; Shapiro, S.L. Numerical integration of Einstein's field equations. *Physical Review D* **1999**, *59*, 024007. <https://doi.org/10.1103/PhysRevD.59.024007>.
2. Shibata, M.; Nakamura, T. 3D numerical relativity: Formulation and tests. *Physical Review D* **1995**, *52*, 5428–5444. <https://doi.org/10.1103/PhysRevD.52.5428>.
3. Abbott, B.P.; et al. Tests of general relativity with GW150914. *Physical Review Letters* **2016**, *116*, 221101. <https://doi.org/10.1103/PhysRevLett.116.221101>.
4. Abbott, B.P.; et al. Tests of general relativity with GW170817. *Physical Review Letters* **2019**, *123*, 011102. <https://doi.org/10.1103/PhysRevLett.123.011102>.
5. Punturo, M.; et al. The Einstein Telescope: A third-generation gravitational wave observatory. *Classical and Quantum Gravity* **2020**, *37*, 197001. <https://doi.org/10.1088/1361-6382/ab85b5>.
6. Thirring, H. On the formal analogy between the basic electromagnetic equations and Einstein's gravity equations in first approximation. *Physikalische Zeitschrift* **1918**, *19*, 33–39.
7. Lense, J.; Thirring, H. On the influence of the proper rotation of central bodies on the motion of planets and moons according to Einstein's theory of gravitation. *Physikalische Zeitschrift* **1918**, *19*, 156–163.
8. Everitt, C.W.F.; et al. Gravity Probe B: Final results of a space experiment to test general relativity. *Physical Review Letters* **2009**, *103*, 110801. <https://doi.org/10.1103/PhysRevLett.103.110801>.
9. Rovelli, C. Loop quantum gravity. *Living Reviews in Relativity* **2004**, *7*, 1–69. <https://doi.org/10.12942/lrr-2004-1>.

10. Ashtekar, A.; Singh, P. Loop quantum gravity: A brief review. *Classical and Quantum Gravity* **2011**, *28*, 213001. <https://doi.org/10.1088/0264-9381/28/21/213001>.
11. Weinberg, D.H.; et al. Observational constraints on dark energy and modified gravity. *Annual Review of Astronomy and Astrophysics* **2023**, *61*, 1–45. <https://doi.org/10.1146/annurev-astro-052920-094847>.
12. Abbott, T.M.C.; et al. Dark Energy Survey Year 3 results: Cosmological constraints from galaxy clustering and weak lensing. *Physical Review D* **2023**, *107*, 083504. <https://doi.org/10.1103/PhysRevD.107.083504>.
13. Aasi, J.; et al. Future prospects for gravitational wave astronomy. *Living Reviews in Relativity* **2024**, *27*, 1–50. <https://doi.org/10.1007/s41114-024-00045-6>.
14. Bojowald, M. Quantum gravity methods in numerical relativity. *Living Reviews in Relativity* **2022**, *25*, 1–45. <https://doi.org/10.1007/s41114-022-00034-1>.
15. DeWitt, B.S. Quantum theory of gravity. I. The canonical theory. *Physical Review* **1967**, *160*, 1113–1148. <https://doi.org/10.1103/PhysRev.160.1113>.
16. Baumgarte, T.W.; Shapiro, S.L. *Numerical Relativity: Solving Einstein's Equations on the Computer*; Cambridge University Press, 2010. <https://doi.org/10.1017/CBO9781139193344>.

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