Article

The Black Hole Mass-Gap and its Relation to Bekenstein-Hawking Entropy Leads to Potential Quantization of Black Holes and a Minimum Gravitational Acceleration in the Universe

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The Black Hole Mass-Gap and its Relation to Bekenstein-Hawking Entropy Leads to Potential Quantization of Black Holes and a Minimum Gravitational Acceleration in the Universe

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Abstract: We demonstrate that black holes likely have an energy or mass gap, $\frac{E_g}{c^2} = m_g$, that is of the order $m_g \approx \frac{m_p^2}{M_{BH}}$. Interestingly, the mass of the black hole divided by the mass gap seems closely related to the Bekenstein-Hawking entropy and thereby potentially leads to a quantization of black holes. Even if mathematically trivial, this could be a potentially important step toward better understanding the potential to quantize black holes. Our focus is mainly on Schwarzschild black holes, but we also briefly discuss Reissner-Nordström black holes.

Keywords: black hole; Bekenstein-Hawking entropy; quantization; Planck mass; Compton wavelength; minimum acceleration in the universe

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1. Expressing the Kilogram Mass of Any Mass from the Compton Formula

The Compton [1] wavelength formula is given by $\lambda = \frac{h}{mc}$. Solving for mass, we obtain:

$$m = \frac{h}{\lambda}\frac{1}{c} \tag{1}$$

Some may believe that this formula applies only to the electron, as electrons are the only particles whose Compton wavelength has been indirectly demonstrated through Compton scattering. However, as early as 1958, Levitt [2] discussed the Compton wavelength in relation to the proton. See also the works of Bohr and Trinhammer [3]. It is likely that only elementary particles have a physical Compton wavelength.

Nevertheless, as demonstrated by Haug [4–6] in multiple papers, any composite mass has an aggregated Compton wavelength that is consistent with the sum of its constituent masses. Since this is a significant point, we briefly revisit it here. For a composite mass, the relationship is as follows:

$$\begin{aligned}
M &= m_1 + m_2 + m_3 + m_n \pm \frac{E_1}{c^2} \pm \frac{E_2}{c^2} \pm \frac{E_3}{c^2} \pm \frac{E_N}{c^2} \\
\frac{h}{\lambda}\frac{1}{c} &= \frac{h}{\lambda_1}\frac{1}{c} + \frac{h}{\lambda_2}\frac{1}{c} + \frac{h}{\lambda_3}\frac{1}{c} + \frac{h}{\lambda_n}\frac{1}{c} \pm \frac{h\frac{c}{\lambda_1}}{c^2} \pm \frac{h\frac{c}{\lambda_2}}{c^2} \pm \frac{h\frac{c}{\lambda_3}}{c^2} \pm \frac{h\frac{c}{\lambda_N}}{c^2} \\
\lambda &= \frac{h}{Mc} = \frac{1}{\sum_i^n \frac{1}{\lambda_i} \pm \sum_j^N \frac{1}{\lambda_j}}
\end{aligned} \tag{2}$$

The Compton wavelength of a composite mass is simply the aggregate of the Compton wavelengths of the elementary particles that constitute the mass, along with the contributions from binding energies. The energy term can be either positive or negative because some energies are binding energies that must be subtracted to determine the aggregate mass. This concept is well-known in nuclear physics.

2. The Schwarzschild Black Hole Energy or Mass Gap

As any mass or mass equivalent can be expressed in the form:

$$m = \frac{E}{c^2} = \frac{h\frac{c}{\lambda}}{c^2} = \frac{h}{\lambda}\frac{1}{c} \tag{3}$$

and the mass is inverse proportional to the wavelength λ then the lowest mass possible in a black hole above zero must be related to the longest possible wavelength inside the black hole, so the mass gap is given by

$$m_g = \frac{E_{min}}{c^2} = \frac{h\frac{c}{\lambda_{max}}}{c^2} = \frac{h}{\lambda_{max}}\frac{1}{c} \tag{4}$$

We will assume that the longest possible wavelength is from the central singularity to the surface of the black hole, corresponding to the radius of the black hole. One could argue that it might instead be the circumference of the black hole, but this would only alter the derivation by a factor of 2π. The wavelength of the mass gap of a Schwarzschild [7] black hole must therefore be:

$$\bar{\lambda}_{max} = \frac{2GM_{BH}}{c^2} = R_s \tag{5}$$

This mean the mass gap is :

$$m_g = \frac{\hbar}{R_s}\frac{1}{c} \tag{6}$$

This can be re-written as

$$\begin{aligned} m_g &= \frac{\hbar}{\frac{2GM_{BH}}{c^2}}\frac{1}{c} \\ m_g &= \frac{\hbar^2}{2\frac{G\hbar}{c^3}c^2}\frac{1}{M_{BH}} \end{aligned} \tag{7}$$

and since the Planck [8,9] length is given by $l_p^2 = \frac{G\hbar}{c^3}$ we can re-write this further as:

$$\begin{aligned} m_g &= \left(\frac{\hbar}{l_p}\frac{1}{c}\right)^2 \frac{1}{2M_{BH}} \\ m_g &= \frac{m_p^2}{2M_{BH}} \end{aligned} \tag{8}$$

This means that the mass gap of a Schwarzschild black hole is simply equal to the Planck mass squared divided by twice the black hole's mass. We can also express this as:

$$m_g = \frac{\frac{1}{2}m_p}{f_c} \tag{9}$$

where f_c is the reduced Compton frequency of the black hole mass per Planck time $f_c = \frac{c}{\bar{\lambda}_{BH}}t_p = \frac{l_p}{\bar{\lambda}_{BH}}$.

The mass of the Schwarzschild black hole divided by the mass gap is:

$$\frac{M_{BH}}{m_g} = \frac{M_{BH}}{\frac{\hbar}{R_s}\frac{1}{c}} = \frac{2l_p^2c^3M_{BH}R_s}{2\hbar l_p^2c^2} = \frac{2GM_{BH}R_s}{c^2 2l_p^2} = \frac{R_s^2}{2l_p^2} = \frac{S_{BH}}{2\pi} \tag{10}$$

where $S_{BH} = \frac{A}{4l_p^2} = \frac{4\pi R_s^2}{4l_p^2}$ is the Bekenstein-Hawking entropy [10,11]. This indicates a clear connection between the Bekenstein-Hawking entropy and the black hole mass divided by the black hole mass gap. The black hole mass divided by the mass gap can potentially also be interpreted as the maximum number of microstates in the black hole. Consequently, this ratio, as well as the entropy of the black hole, could be viewed as a form of quantization of the black hole. Naturally, this interpretation requires further investigation.

3. Reissner-Nordström Extremal Solution Mass Gap

In the extremal solution of the Reissner-Nordstr"om [12,13] metric, as well as in the minimal solution of the Haug-Spavieri [14] metric, the event horizon r_h is given by:

$$r_h = \frac{GM_{BH}}{c^2} = \frac{1}{2}R_s \tag{11}$$

This means that the event horizon for such a black hole is half the radius of a Schwarzschild black hole. Consequently, the mass gap for an extremal Reissner-Nordstr"om black hole is:

$$m_g = \frac{\hbar}{r_h}\frac{1}{c} = \frac{\hbar}{\frac{GM_{BH}}{c^2}}\frac{1}{c} = \frac{m_p^2}{M_{BH}} \tag{12}$$

rather than $\frac{m_p^2}{2M_{BH}}$ as in the Schwarzschild metric.

4. The Black Hole Universe Mass Gap

In the critical Friedmann [15] universe, it is well known that the critical mass (equivalent) is given by:

$$M_c = \frac{c^2 R_H}{2G} \tag{13}$$

Solving for the Hubble radius R_H, this yields:

$$R_H = \frac{2GM_c}{c^2} \tag{14}$$

This is the same equation for the radius as the Schwarzschild radius. This similarity is one of several reasons why multiple researchers (see [16,17]) have suggested that the universe could potentially be a gigantic black hole. While this idea is far less popular than the Λ-CDM model, there are still researchers who, even in recent times, continue to discuss the possibility of the universe being a black hole (see [18–21]).

Assuming for simplicity that the universe is a Schwarzschild black hole, then according to our analysis in previous sections, it must have a mass gap in the Hubble sphere black hole of:

$$m_g = \frac{m_p^2}{M_c} = \frac{\hbar}{R_H}\frac{1}{c} = \frac{\hbar H_0}{c^2} \approx 2.54 \times 10^{-69}\ kg \tag{15}$$

when using a Hubble constant of 66.9km/s/Mpc. Naturally, there is some uncertainty in this estimated black hole universe mass gap due to the uncertainty in H_0. However, the primary purpose here is to highlight the equation for the mass gap in the universe under the hypothetical assumption that the Hubble sphere is a Schwarzschild black hole.

The mass gap give a gravitational acceleration over the Planck length distance in the critical Friedmann universe is:

$$g = \frac{m_g}{l_p^2} = \frac{G\frac{m_p^2}{M_c}}{l_p^2} = \frac{c^2\bar{\lambda}_c}{l_p^2} \approx 13 \times 10^{-10}\ m^2/s \tag{16}$$

where $\bar{\lambda}_c = \frac{\hbar}{M_c c}$, see Haug [4,5] how any composite mass also have a composite Compton wavelength. Further in a (extremal) Reissiner-Nordström black hole universe (see [22]) it will be:

$$g = \frac{m_g}{l_p^2} = \frac{G\frac{m_p^2}{M_u}}{l_p^2} = \frac{c^2\bar{\lambda}_u}{l_p^2} \approx 6.5 \times 10^{-10}\ m^2/s \tag{17}$$

where $M_u = \frac{c^3}{GH_0} = 2M_c$ in the RN extremal universe is exactly twice the critical Friedmann mass. Further $\bar{\lambda}_u = \frac{\hbar}{M_u c}$. We can call this the gravitational acceleration gap; it is the smallest possible gravitational acceleration one can likely observe above zero.

Be aware that the extremal Reissner-Nordström universe is more realistic than a Schwarzschild universe, as there is an equilibrium in such a black hole that prevents the universe from collapsing into a singularity (see the paper just referred to). Although much more can be said here, we will soon publish more papers on this topic. This is essentially the missing minimum acceleration needed to explain galaxy rotation curves and likely also the Pioneer anomaly.

Interesting the mass-gap gravitational acceleration over the Planck length distance is identical to a hypothetical gravitational acceleration of the universe mass over the distance of the Hubble radius, that is we have:

$$g_{min} = \frac{Gm_g}{l_p^2} = \frac{GM_u}{R_H^2} = \frac{c^2\bar{\lambda}_u}{l_p^2} \approx 6.5 \times 10^{-10}\ m^2/s \tag{18}$$

This might be related to a different way of understanding how the electrostatic force and gravitational force achieve perfect equilibrium in an extremal RN black hole, as pointed out, for example, by Zee [23] and others. However, this is a topic that requires further investigation, particularly in connection with the recently proposed approach to unifying gravity with quantum mechanics [24].

5. Conclusion

We have demonstrated that black holes likely have a mass gap of the order: $m_g = \frac{m_p^2}{M_{BH}}$, and that this is closely related to the Bekenstein-Hawking entropy. From this perspective, Bekenstein-Hawking entropy can be interpreted as the number of possible microstates in the black hole, providing a type of quantization of black holes. This connection should be further investigated to explore whether it could lead to new and deeper insights into black hole physics.

The mass gap in a black hole universe appears to potentially explain the minimum observed galaxy rotation. It seems this could be related to a gravitational acceleration gap, which is linked to the mass gap in the Hubble sphere.

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