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Methodology to Determine the Stress Distribution Based on Fatigue Data with Bilinear Behavior and Its P-S-N Field and Testing Plan

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Abstract: In this paper, the method to determine the percentiles of the P-S-N field when it presents bilinear (s_1 *and* s_2) behavior (e.g competence failure mode) is given. The percentiles are determined around the Weibull scale (η) parameter in Cycles that corresponds to the median applied stress of the whole bilinear behavior. The reliability ($R(t)$) index that corresponds to each P-S-N percentile are also determined. As well as the confidence reliability percentile (CL) that corresponds to each P-S-N percentile. The formulation to relate CL with its corresponding normal P-S-N percentile is also given. Numerically, the percentiles of the normal (one, two, and three) sigma levels, as well as those that correspond to the capability Cp and Cpk indices are related with their CL values, and $R(t)$ indices. To validate the Weibull cycle bilinear family, a test plan is designed and numerically performed. In the numerical application we use the 42CrMo4 steel material.

Keywords: fatigue P-S-N field; Weibull/IPL model; bilinear model; reliability percentiles; testing plan; capability indices; six sigma

1. Introduction

Fatigue is one of the principal causes of failure in mechanical and structural elements [1]. When the fatigue element presents different fatigue slopes, determining from both behaviors an equivalent S-N slope is fundamental. Then the equivalent slope is used on the Basquin model to characterize the bilinear fatigue behavior as in ASTM 739 standard [2] and Eurocode 3 [3]. However, the Basquin model is efficient only for linear behavior. Thus, because the Basquin model does not represent multiple failure modes, its use is not efficient to represent the bilinear behavior [4]. In a S-N curve a bilinear behavior represents a transition between two competitive failure modes, or a step of variant stress. In the fatigue frame this transition is called a knee-point [5]. And although its consideration improves the analysis, it does not incorporate the probabilistic behavior [6]. Thus, because fatigue is random, formulating a probabilistic P-S-N curve method is necessary to get a more reliable failure prediction [7].

During the last decades, several fatigue probabilistic models have been proposed. Castillo and Canteli [8] introduced the Weibull fatigue model. In [9], a model for fatigue crack growth was formulated. In [10], a model for fatigue based on the statistical characteristics of the cycles to failure was given. Similar research is found in [11]. Unfortunately, none of them are focused on bilinear behavior. Among research focused on bilinear behavior we have [12]. They mention that to improve fatigue life prediction, a bilinear model with probabilistic focus must be used to model the uncertainties associated with the deterioration process produced by fatigue phenomenon. In [13], an application of the bilinear model to fourteen kinds of alloys was performed. They show that this model provides a better representation of fatigue behavior than the linear model. However, despite their utility, both linear and bilinear models lack probabilistic information.

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Therefore, the novelty of this paper is the method to consider the probabilistic behavior in the bilinear fatigue analysis and to generate its corresponding P-S-N field (see Table 8). The used material for the analysis is 42CrMo4 steel. And since the material S-N curve represents median values, then the analysis was performed based on the median stress of collected data (see Table 1). And because the reliability index is based on the time domain then the Weibull parameters in cycles to failure that corresponds to the median stress value were determined. They are $W(\beta = 4.8032, \eta = 1445.7208 \text{ cycles})$. Consequently, by considering β to be constant, the P-S-N percentiles were determined around η. However, because for each estimated percentile, its corresponding eta (η_i) value exists, and since for each η_i value different $R(t)$ index is determined, then the confidence reliability percentile (CL) that corresponds to the difference between the η_i value and $\eta_i = 1445.7208$ cycles were also determined. Additionally, the formulation to relate the CL value with the P-S-N percentile was also given. Therefore, a summary table that shows the P-S-N percentile, the CL value, the $R(t)$ index and the minimum and maximum η_i values is presented for the one, two, and three sigma levels, as well as those that correspond to the capability Cp and Cpk indices. Finally, the test plan that can be used to validate the Weibull cycle bilinear family is designed and numerically performed to show how the proposed methodology can be implemented.

The paper is organized as follows. Section 2 presents the general background of the bilinear fatigue and Weibull/IPL analysis. In Section 3, the steps of the proposed method are given. In Section 4, the numerical application is performed. Finally, in Section 5, the conclusions are presented.

2. Bilinear and Weibull/IPL General Background

In fatigue analysis, it is fundamental to understand the models for evaluating the service life of materials. This section offers the fundamentals of the fatigue models, emphasizing the S-N curve and the Basquin equation. We also discussed the bilinear model for fatigue characterization and present the Weibull distribution and the Weibull/IPL model concepts which play a key role in the P-S-N analysis.

2.1. Fatigue Model

When mechanical elements are subject to cycling loading, a fatigue analysis is required. The analysis requires determining the cycles until the material fails. Its graphical representation is the S-N curve, also known Wöhler curve, where the stress (S) is plotted against the logarithm of the cycles to failure (N) [14]. The relationship between applied stress and the number of cycles is given by the Basquin model:

$$\log(S) = A \cdot \log(N) + B \tag{1}$$

Where A is the slope and B is the ordinate to the origin. Its application to the bilinear behavior is as follows.

2.2. Bilinear Model

The bilinear model is an extension of the linear model used in fatigue analysis. In the bilinear behavior, the S-N curve of a material is divided into two regions, because of the change of the slope. For example, region I can be governed by the plastic region, while Region II can be governed by the elastic zone. The point where the two regions intersect is called knee-point [5]. The S-N curve shown in Figure 1, is characterized by a high slope in the first phase of the bilinear model, and a low slope in the second phase.

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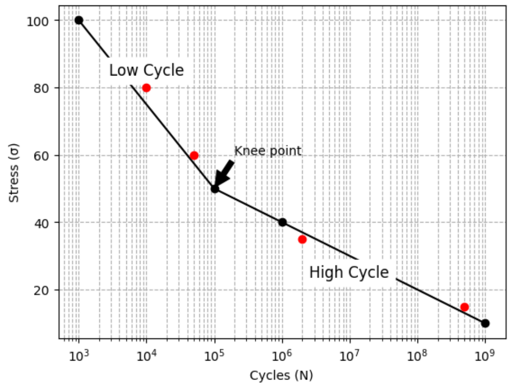

Figure 1. S-N curve with bilinear behavior.

Understanding the characteristics of the S-N curve, particularly the knee-point is crucial for fatigue analysis. The high slope in the first phase indicates a higher rate of damage accumulation, whereas the second phase, with its lower slope, suggests a slower progression of fatigue. Thus, to consider the spread of data, a probability function is necessary. Here we use the Weibull distribution. Its generalities are as follows.

2.3. Weibull Distribution

The Weibull distribution is widely used in reliability analysis [15]. It was proposed by Wallody Weibull [16], and its probability density function (*pdf*) is given by:

$$f(t) = \frac{\beta}{\eta}\left(\frac{t}{\eta}\right)^{\beta-1} \exp\left\{-\left(\frac{t}{\eta}\right)^{\beta}\right\} \tag{2}$$

The corresponding reliability and cumulative failure distribution are:

$$R(t) = \exp\left\{-\left(\frac{t}{\eta}\right)^{\beta}\right\} \tag{3}$$

$$F(t) = 1 - \exp\left\{-\left(\frac{t}{\eta}\right)^{\beta}\right\} \tag{4}$$

Where β is the shape parameter and η is the scale parameter. Furthermore, the Inverse Power Law (IPL) model, which is frequently used to relate the failure times with a non-thermal stresses [17], is defined by:

$$L(S) = \frac{1}{KS^n} \tag{5}$$

Where L represents characteristic life, S represents the stress level, and K and n are parameters to be determined. By setting $\eta = L(S)$ in eq.(2), the Weibull/IPL model is as follows:

$$f(t,S) = \beta KS^n (KS^n t)^{\beta-1} e^{-(KS^n t)^{\beta}} \tag{6}$$

Thus, the Weibull/IPL reliability function is given by:

$$R(T,S) = e^{-(KS^nT)^\beta} \quad (7)$$

The parameters (β, K, and n) of the Weibull/IPL model defined in eq.(6) are estimated by using the maximum likelihood method. With these Weibull/IPL parameters the steps to perform the P-S-N analysis are as follows.

3. Steps of the Proposed Method

In this section we present the methodological steps to determine the probabilistic P-S-N field. They are:

Step 0. Collect Experimental fatigue data. Collected data must contain failure times and their corresponding stress.

Step 1. Separate data into two homogeneous sets s_1 and s_2.

Step 2. Determine the set of data that presents the best fit between times and stress: For each set, determine the multiple regression coefficient R^2, and select the data set with the highest R^2 value as the one on which the P-S-N analysis will be performed.

Step 3. For each data set, determine the Weibull/IPL parameters (β, K, and n) defined in eq.(6).

Step 4. Determine the η_i values that correspond to each stress value of the set s_1 and s_2 by using its corresponding β, K, and n parameters in eq.(5).

Step 5. For both sets, determine the reliability $R(t)$ index that corresponds to each observed failure time using the η_i values of step 4 in eq.(7).

Step 6. Using the β, K, and n parameters of s_2, determine the equivalent failure times of set one (say s_1) that corresponds to the same reliability percentile ($R(s_1) = R(s_2)$) in set two (say s_2). They are given by:

$$N_{eq} = \left(-\ln\left(R(t_i)\right)\right)^{1/\beta} \cdot \eta_i \quad (8)$$

Step 7. Form the whole set data for the analysis of the P-S-N field by adding to set s_2 the equivalent failure times determined in step 6, (s_2 is the set with the highest R^2 index).

Step 8. Determine the median stress of the whole stress data.

Step 9. Determine the η value in cycles that correspond to the median stress by using the median stress value in eq.(5) with the Weibull/IPL parameters of set s_2.

Step 10. Based on the reliability indices calculated in step 5 and using eq.(8) determine the predicted failure times. Then using the maximum likelihood method determine the Weibull parameters and the Fisher matrix, and using eq.(9) determine the standard deviation of the eta parameter as:

$$\sigma_\eta = \sqrt{Var(\eta)} \quad (9)$$

Step 11. Use eqs.(10 and 11) to determine the upper and lower limits of η that correspond to a desired P-S-N percentile.

$$\eta_U = \eta \cdot \exp^{\frac{k_\alpha\sqrt{var(\eta)}}{\eta}} \text{ (upper bound)} \quad (10)$$

$$\eta_L = \frac{\eta}{\exp^{\frac{k_\alpha\sqrt{var(\eta)}}{\eta}}} \text{ (lower bound)} \quad (11)$$

Where k_α is the z value of the normal distribution that corresponds to the desired two sizes percentile determined as:

$$\alpha = \frac{1-\delta}{2} \quad (12)$$

Which for one-sided bound is given as:

$$\alpha = 1 - \delta \quad (13)$$

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Where δ is the corresponding confidence level. The numerical application is as follows.

4. Numerical Application

In this section the steps to determine the P-S-N field of fatigue data with bilinear behavior and its corresponding testing plan are presented. The bilinear analysis is as follows.

4.1. Bilinear Numerical Analysis

The application is performed by using the 42CrMo4 steel material. Its mechanical properties are Modulus of elasticity; 210 GPa, Poisson ratio 0.30, and shear modulus 80 GPa. The numerical application aims to demonstrate how the methodology allows us to determine the P-S-N field of 42CrMo4 steel. The methodological steps to perform the bilinear analysis are as follows.

Step 0: Collected bilinear data is given in Table 1 and in Figure 2.

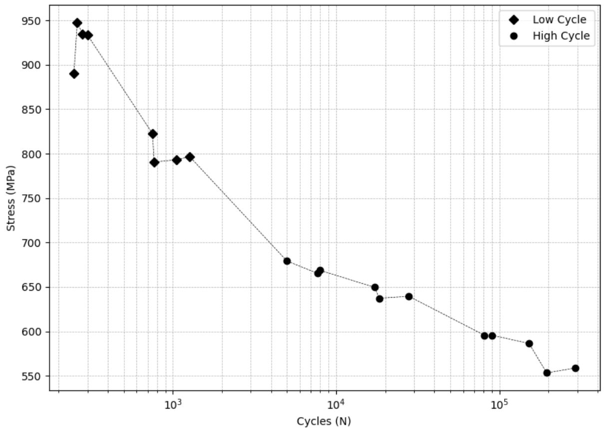

Figure 2. S-N curve of 42CrMo4.

Step 1. The two bilinear behavior groups s_1 (First eight data), and s_2 (last eleven data) are in Table 1. This data was published in [18].

Table 1. Fatigue test of 42CrMo4 steel.

	s_1								s_2										
Cycles (N)	248	260	280	300	750	770	1050	1270	5000	7700	7950	17200	18400	27900	81000	90000	152000	195000	290000
Stress (S) MPa	890.4	947.5	934.8	933.7	822.8	790.6	793.1	796.6	679.2	665.3	668.8	649.6	637.3	639.5	595.7	595.7	586.4	553.3	558.7

Step 2. Because the R^2 index for s_1 is 0.8083, and for s_2 is 0.8280, we select the s_2 set as the base set to perform the P-S-N analysis.

Step 3. From eq.(6), by using the maximum likelihood method, the estimated parameters of the Weibull/IPL model for each data set are given in Table 2.

Table 2. Weibull/IPL parameters for data sets are.

	β	K	n
s_1	6.2428	4.74E-28	8.3738
s_2	4.8032	3.43E-61	20.0032

Step 4. By using the (β, K, and n) parameters of step 3 in eq.(5), the η_i values that correspond to each stress value of Table 1, are given in Table 3:

Table 3. η values in MPa for each stress.

	s_1													s_2					
η_i	29.11	8.40	11	11.26	141.24	313.88	294.67	269.83	6548.75	9903.54	8916.79	15968.12	23405.74	21846.65	90311.39	90311.39	123719.22	395533.92	325694.34
Stress (S) MPa	890.4	947.5	934.8	933.7	822.8	790.6	793.1	796.6	679.2	665.3	668.8	649.6	637.3	639.5	595.7	595.7	586.4	553.3	558.7

Step 5. By using the η_i values of Table 3 in eq.(3), The reliability indices of the failure times of both data sets are given in Table 4.

Table 4. Reliability of each failure times.

					s_1										s_2				
Cycles (N)	248	260	280	300	750	770	1050	1270	5000	7700	7950	17200	18400	27900	81000	90000	152000	195000	290000
R(t)	0.9643	0.2838	0.3724	0.2396	0.5560	0.9177	0.4957	0.0552	0.7606	0.7419	0.5620	0.2396	0.7299	0.0393	0.5527	0.3740	0.0680	0.9671	0.5640

Step 6. The equivalent failure cycles N_{eq} of s_1 that should be incorporated to the s_2 sets are given in Table 5.

Table 5. Equivalent Failure Times to use in S_2.

					s_1			
R(t)	0.9643	0.2838	0.3724	0.2396	0.5560	0.9177	0.4957	0.0552
N_{eq}	14.60	8.81	10.97	12.13	126.41	188.26	273.73	336.71

Step 7. The whole set of data that incorporate the equivalent failure times of step 6 into set s_2 are given in Table 6.

Table 6. Grouped s_1 and s_2 data set.

					s_1										s_2				
$N_{grouped}$	14.60	8.81	10.97	12.13	126.41	188.26	273.73	336.71	5000	7700	7950	17200	18400	27900	81000	90000	152000	195000	290000
Stress (S)	890.4	947.5	934.8	933.7	822.8	790.6	793.1	796.6	679.2	665.3	668.8	649.6	637.3	639.5	595.7	595.7	586.4	553.3	558.7

Step 8. The median stress of the whole set is $S = 732.4806$ MPa.

Step 9. From eq.(5), The η value that corresponds to the median stress value in cycles is $\eta = 1445.7208$ cycles .

Step 10. Using the reliability indices from Table 4 in eq.(3), the predicted failure times are given in Table 7.

Table 7. The predicted Failure Times.

					s_1										s_2				
N_{pre}	725.206	1516.815	1442.048	1557.252	1293.946	867.110	1342.984	1804.107	1103.814	1124.048	1288.972	1557.254	1136.528	1846.307	1296.663	1440.737	1776.197	712.747	1287.278
R(t)	0.9643	0.2838	0.3724	0.2396	0.5560	0.9177	0.4957	0.0552	0.7606	0.7419	0.5620	0.2396	0.7299	0.0393	0.5527	0.3740	0.0680	0.9671	0.5640

By using the maximum likelihood method and the predicted data, the estimated Weibull parameters are $\beta = 4.8032$, $\eta = 1445.7208$ cycles, and from the corresponding Fisher matrix given as:

$$\begin{pmatrix} 0.769 & 19.8171 \\ 19.8171 & 5278.8121 \end{pmatrix}$$

The standard deviation of η is $\sigma_\eta = \sqrt{5278.8121} = 72.6554$ cycles.

Step 11. With $\eta = 1445.7208$ cycles in eqs.(9 and 10), the upper and lower limits of its P-S-N field are given in Table 8.

Table 8. Upper and lower limits of η for severals percentiles.

Percentile	50%	55%	1σ	80%	90%	95%	2σ	97%	99%	3σ
ηL	1445.7208	1437.8689	1411.6007	1393.9324	1367.6036	1346.2362	1327.9943	1332.5387	1307.0509	1257.0788
ηU	1445.7208	1453.6180	1480.6681	1499.4358	1528.3025	1552.5597	1573.8863	1568.5189	1599.1054	1662.6740

The percentiles given in Table 8 represent the P-S-N field around the η value (1445.7208 cycles) that corresponds to the analyzed median stress (732.4806 MPa). Here we use the median stress because the S-N curve represents median values, but any other desired stress value can be used. Therefore, the Weibull distribution that represents the median stress of the bilinear behavior in cycles is $W(4.8072, 1445.7208$ cycles), and its field percentiles are those given in Table 8. Now, the testing plan that we can use to determine the reliability index that the addressed Weibull distribution represents is as follows.

4.2. Test Plan of the Addressed Weibull Distribution

Here we numerically perform the vibration test plan given in appendix C of the GMW3172 norm. The steps to perform the mentioned test plan for the addressed bilinear Weibull distribution are as follows.

Step 1. Determine the six test plan elements. They are 1) Required reliability $R(t)$. 2) Required confidence level CL for the desired $R(t)$ index. 3) The sample size n to be tested. 4) The testing time t. 5) The Weibull (β and η) parameters. And 6) The testing conditions.

Step 2. Determine the required sample size n that corresponds to the addressed η value. Based on Piña-Monarrez [19], it is given by:

$$n = \frac{-1}{\ln\big(R(t)\big)} \tag{14}$$

Step 3. Determine the required testing time. It is the time that corresponds to the desired reliability $R(t)$ index. From [20], based on the n value of eq.(14) and the bilinear Weibull parameters of step 1, it is given by:

$$t = \frac{\eta}{n^{1/\beta}} \tag{15}$$

Step 4. Determine the n_2 value that corresponds to the required $R(t)$ and CL indices of step 1 as:

$$n_2 = \frac{\ln\,(1 - CL)}{\ln\big(R(t)\big)} \tag{16}$$

Step 5. By using the n_2 value, determine the corresponding upper Weibull eta η_U parameter as:

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$$\eta_U = n_2^{1/\beta} t \tag{17}$$

Step 6. From Piña-Monarrez [19], determine the lower η_L parameter as:

$$\eta_L = \frac{\eta^2}{\eta_U} \tag{18}$$

Step 7. By using the testing time t of step 3 and η_U from step 5, determine the corresponding reliability index.

Step 8. Determine the P-S-N percentile that corresponds to the required CL value of step 1 as.

$$k_\alpha = -\ln\left(\left(\frac{\eta_L}{\eta_U}\right)^{1/2}\right) * \left(\frac{\eta}{\sigma_\eta}\right) \tag{19}$$

Where σ_η was determined in eq.(9). The numerical application is presented as follows.

4.3. Numerical Application of the Test Plan

Step 1, 2 and 3. The required test elements are, $R(t) = 0.97$, $CL = 0.75$. ($\beta = 4.8032$ and $\eta = 1445.7208$ cycles), $n = 32.8307$. and $t = 698.8798$ cycles.

Step 4. The required simple with $CL = 0.75$ is $n_2 = 45.5131$.

Step 5. Using $n_2 = 45.5131$ in eq.(17), the upper bound of the Weibull parameter is $\eta_U = 1547.4550$ cycles.

Step 6. The lower bound of the Weibull parameter $\eta_L = 1350.6750$ cycles.

Step 7. The reliability index corresponding to η_U is $R(t) = 0.9783$.

Step 8. The selected percentiles mentioned in section 1 were determined using $\sigma_\eta = 72.6554$ in eqs.(17 and 18). Results are given in Table 9.

Table 9. Results of the test plan.

	Percentile	k_α	ηU	ηL	CL	R(t)
1σ	0.6827	0.4752623	1480.6669	1411.5996	0.6742	0.9732
Cpk	0.9082	1.3297520	1545.6359	1352.2646	0.7480	0.9781
Cp	0.9525	1.6695926	1572.2603	1329.3656	0.7761	0.9798
2σ	0.9545	1.6901461	1573.8852	1327.9932	0.7777	0.9799
3σ	0.9973	2.7821505	1662.6728	1257.0776	0.8588	0.9846
Test plan	0.0880	-1.3531742	1547.4550	1350.6750	0.7500	0.9783

Based on the designed test plan, it must be performed with a confidence level $CL = 0.75$ and a reliability $R(t) = 0.97$. We must run 46 pieces for a testing time of $t = 698.8798$ cycles each, and if none of them fail, we conclude that the analyzed element has a minimum reliability of $R(t) = 0.97$. However, as observed in Table 9, since η_U is greater than η, the expected reliability is $R(t) = 0.9783$. The general conclusions are as follows.

5. Conclusions

1. In practice the bilinear behavior is mainly generated by two competitive failure modes. Although here we are focused on the bilinear behavior of the S-N curve, any bilinear behavior can be analyzed by using the proposed methodology.

2. By applying the methodology, the bilinear behavior can be represented by a Weibull family fitted for a desired stress value. Here because the focus is on the S-N curve, which represents median stresses, we select the median observed stress as the basis of the analysis, but any desired stress value can be used. The median stress value is $S = 732.4806$ MPa.

3. The percentiles estimation was performed on the Weibull scale parameter that represents the selected median stress, because we are interested in the reliability index of the analyzed behavior.

Notice the shape parameter remains constant. In cycles the addressed Weibull parameters are $\beta = 4.8032$, $\eta = 1445.7208$ Cycles.

4. In general, the P-S-N field of the analysis is given in Table 8, and the percentiles of practical interest are given in Table 9.

5. In the performed test plan of section 4.2., notice the *R(t)* and *CL* indices, both determine the required sample size n_2, and n_2 in eq.(17) completely determines the upper eta value. And because for any $CL > 0.632120$ $(1 - e^{-1})$, $\eta_U > \eta$, then a confidence interval always exists, implying in any plan test analysis the given methodology can be used.

6. From the test plan data given in the last row of Table 9, notice although the desired reliability was 0.97, because $\eta_U > \eta$ the final reliability is 0.9783. Also notice by the normal and Weibull relationship given in eq.(19) the normal failure percentile that corresponds to R(t)=0.9783 is 0.0880.

7. It is very important to notice because the normal and Weibull percentiles defined in eq.(19), depend on the Weibull β value (see eqs.(15 and 17)) then an accurate estimation of β is critical in the analysis. Thus, because in step 10 of sec.3 we determine the β value that represents the bilinear behavior, the proposed methodology is efficient and dynamic to be used in the analysis.

8. As a future research, we recommend determining the sensitive behavior of addressed Weibull family to more accurately determine the expected behavior of β and the variance of η.

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