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Article

Qualitative Analysis, and Novel Exact Solitons to the Compound Korteweg-De Vries Burgers Equation

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Abstract: The paper consists on the exact wave results of the (1+1)-dimensional nonlinear compound Korteweg-De Vries, and Burgers (KdVB) equation with a truncated M-fractional derivative. This model represents the generalization of Korteweg-De Vries-modified Korteweg-De Vries and Burgers equations. We obtained the periodic, combo-singular, dark-bright, and other wave results with the use of the extended sinh-Gordon equation expansion (EShGEE), and the modified (G'/G^2)-expansion techniques. The use of effective fractional derivative makes our results much better than the existing results. The gained solutions are useful as well as applicable in the various fields including the mathematical physics, plasma physics, ocean engineering, optics, etc. The obtained solutions are demonstrated by 2-D, 3-D, and Contour plots. The achieved results are fruitful in future research concerned equation. Stability analysis is used to check that the results are precise as well as exact. The modulation instability (MI) is performed to find stable steady-state solutions of the concerned model. At the end, it is suggested that the methods used are easy and reliable.

Keywords: nonlinear fractional compound KdVB model; stability and modulation instability analysis; analytical techniques; new wave solutions

1. Introduction

Non-linear mathematical models are used to depict phenomena that occur naturally. Non-linear partial differential equations are utilized to express a number of different models. Different kinds of fractional derivatives have only been utilized in recent years, including unstable nonlinear Schrödinger equation [1], Gross-Pitaevski model [2], Schrödinger-Hirota equation [3], Cahn-Hilliard equation [4], stochastic concatenation equation [5], Boiti-Leon-Manna-Pempinelli model [6], Estevez-Mansfield-Clarkson model [7] and others.

Two dependable and efficient methods were employed in our study: the modified (G'/G^2)-expansion method and the extended sinh-Gordon equation expansion scheme (EShGEES). The relevant methods are applied to different models. Instantly; the extended sinh-Gordon equation expansion method is applied for the Biswas-Arshed model [8], Kundu-Eckhaus equation [9], generalized nonlinear Schrödinger equation [10], Boussinesq-Burgers model [11], Van der Waals model [12], Westervelt model [13]. The modified (G'/G^2)-expansion technique is utilized for third-order dispersion nonlinear Schrödinger equation [14], Fokas-Lenells equation [15], classical Boussinesq model [16], coupled Drinfel'd-Sokolov-Wilson model [17], Wazwaz Kaur Boussinesq equation [18], and many others.

Our research's primary goal is to utilize the modified (G'/G^2)-expansion scheme and the EShGEE scheme to determine the new exact wave solutions of the compound KdVB equation in the context of truncated M-fractional derivative. To verify the accuracy and precision of the solutions obtained, as well as the stability of the relevant model, stability and modulation instability analyses are conducted.

The paper is divided into various sections. Section 2 presents the model description and mathematical analysis; Section 3 presents the extended sinh-Gordon equation expansion method and wave solutions; Section 4 presents the modified (G'/G^2)-expansion scheme and wave solutions; Section 5 mentions a graphic explanation; Section 6 shows a physical description; Section 7 performs stability analysis; Section 7 modulates instability analysis; and Section 9 concludes.

Truncated M-fractional Derivative

Definition: Suppose $h(t) : [0, \infty) \rightarrow \mathbb{R}$, therefore truncated M-fractional derivative of h of order ϵ [19]

$$D_{M,t}^{\epsilon,\varrho} h(t) = \lim_{\epsilon \rightarrow 0} \frac{h(t E_{\varrho}(\epsilon t^{1-\epsilon})) - h(t)}{\epsilon}, \quad \epsilon \in (0, 1], \quad \varrho > 0,$$

here $E_{\varrho}(\cdot)$ indicates a truncated Mittag-Leffler profile [20]

$$E_{\varrho}(z) = \sum_{m=0}^i \frac{z^m}{\Gamma(\varrho m + 1)}, \quad \varrho > 0 \text{ and } z \in \mathbb{C}.$$

Characteristics: Suppose $a, b \in \mathbb{R}$, and g, h are ϵ -differentiable at a point $t > 0$, from [19]:

$$(a) D_{M,t}^{\epsilon,\varrho}(ag(t) + bh(t)) = aD_{M,t}^{\epsilon,\varrho}g(t) + bD_{M,t}^{\epsilon,\varrho}h(t)$$

$$(b) D_{M,t}^{\epsilon,\varrho}(g(t) \cdot h(t)) = g(t)D_{M,t}^{\epsilon,\varrho}h(t) + h(t)D_{M,t}^{\epsilon,\varrho}g(t)$$

$$(c) D_{M,t}^{\epsilon,\varrho}\left(\frac{g(t)}{h(t)}\right) = \frac{h(t)D_{M,t}^{\epsilon,\varrho}g(t) - g(t)D_{M,t}^{\epsilon,\varrho}h(t)}{(h(t))^2}$$

$$(d) D_{M,t}^{\epsilon,\varrho}(A) = 0, \text{ where } A \text{ is a constant.}$$

$$(e) D_{M,t}^{\epsilon,\varrho}g(t) = \frac{t^{1-\epsilon}}{\Gamma(\varrho + 1)} \frac{dg(t)}{dt}.$$

2. The Model Representation and Mathematical Treatment

Consider a (1+1)-dimensional compound KdVB model is given as [21];

$$h_t + ah h_x + bh^2 h_x + \theta h_{xx} - \tau h_{xxx} = 0. \quad (1)$$

The wave function in this case is denoted by $h = h(x, t)$, where the parameters are a, b, θ , and τ . Eq. The form of the Korteweg-De Vries-modified Korteweg-De Vries and Burgers equations is generalized in Eq.(1). Different wave solutions of Eq.(1) can be obtained by using different schemes, such as the novel (G'/G) -expansion scheme in [21], the (G'/G) -expansion technique in [22], the generalized (G'/G) -expansion scheme in [23], the extended (G'/G) -expansion scheme in [24], singular-kink, singular-periodic, and single soliton solutions in [25].

A (1+1)-dimensional KdVB model in truncated M-fractional derivative is given as;

$$D_{M,t}^{\epsilon,\varrho} h + ah D_{M,x}^{\epsilon,\varrho} h + bh^2 D_{M,x}^{\epsilon,\varrho} h + \theta D_{M,x}^{2\epsilon,\varrho} h - \tau D_{M,x}^{3\epsilon,\varrho} h = 0. \quad (2)$$

Let us apply the following wave transformation:

$$h(x, t) = H(\mathfrak{U}), \quad \mathfrak{U} = \frac{\Gamma(1+\varrho)}{\epsilon} (x^\epsilon - \lambda t^\epsilon). \quad (3)$$

Here, H represents the amplitude where as $\mathfrak{U}$ denotes the argument of the wave profile, which represents the wave's position and time dependence. The parameter λ represents the velocity of soliton.

By applying the Eq.(3) into Eq.(2), we gain the nonlinear ODE given as;

$$3aH^2 + 2bH^3 - 6\tau H'' + 6\theta H' - 6H\lambda = 0. \quad (4)$$

By utilizing Homogenous Balance method in Eq.(4) and balancing the terms H^3 and H'' ; we gain the value of m is 1.

3. Explanation and Implementation of Extended sinh-Gordon Equation Expansion Scheme

3.1. Explanation

There are main stages for the concerned technique.

Stage 1:

Assume a nonlinear partial frctional differential equation:

$$Z(g, D_{M,t}^{\epsilon,\varrho} g^2, g^2 D_{M,x}^{\epsilon,\varrho} g, D_{M,x}^{\epsilon,\varrho} g, \dots) = 0. \quad (5)$$

Here, $g = g(x, t)$ indicates a wave-function.

Considering a given relation:

$$g(x, t) = G(\mathcal{U}), \quad \mathcal{U} = \frac{\Gamma(1+\varrho)}{\epsilon} (x^\epsilon + \kappa t^\epsilon). \quad (6)$$

Inserting Eq. (6) in Eq. (5), yields:

$$H(G, G^2 G', G'', \dots) = 0. \quad (7)$$

Stage 2:

Consider the solution of Eq. (7) is shown below:

$$G(p) = \alpha_0 + \sum_{j=1}^m (\beta_j \sinh(p) + \alpha_j \cosh(p))^j. \quad (8)$$

Here $\alpha_0, \alpha_j, \beta_j$ ($j = 1, 2, 3, \dots, m$) are unknowns. Consider a novel function p of $\mathcal{U}$, satisfies:

$$\frac{df}{d\mathcal{U}} = \sinh(f). \quad (9)$$

Positive integer "m" is calculated with the help of Homogenous balance method. Eq. (9) is obtained by using the following:

$$q_{xt} = \kappa \sinh(v). \quad (10)$$

By [26], one gets the result of Eq. (10) shown as:

$$\sinh p(\mathcal{U}) = \pm \operatorname{csch}(\mathcal{U}) \quad \text{OR} \quad \cosh p(\mathcal{U}) = \pm \operatorname{coth}(\mathcal{U}). \quad (11)$$

And

$$\sinh p(\mathcal{U}) = \pm \operatorname{sech}(\mathcal{U}) \quad \text{OR} \quad \cosh p(\mathcal{U}) = \pm \tanh(\mathcal{U}). \quad (12)$$

$$l^2 = -1.$$

Stage 3:

The system that results from putting Eqs. (8) and Eq. (9) in the Eq. (7) is $p'^k(\mathcal{U}) \sinh^l p(\mathcal{U}) \cosh^m p(\mathcal{U})$ ($k = 0, 1; l = 0, 1; m = 0, 1, 2, \dots$). A system involving α_j and β_j ($j = 1, 2, 3, \dots, m$) and others can be obtained by taking every co-efficient of $p'^k(\mathcal{U}) \sinh^l p(\mathcal{U}) \cosh^m p(\mathcal{U})$ equal to 0.

Stage 4:

Get the answers for the unknown by solving the obtained system. The solutions for Eq. (7) are obtained by using Eqs. (11) and (12), which are displayed as follows:

$$G(\mathcal{U}) = \alpha_0 + \sum_{j=1}^m (\pm \beta_j \operatorname{csch}(\mathcal{U}) \pm \alpha_j \operatorname{coth}(\mathcal{U}))^j. \quad (13)$$

and

$$G(\mathcal{U}) = \alpha_0 + \sum_{j=1}^m (\pm \iota \beta_j \operatorname{sech}(\mathcal{U}) \pm \alpha_j \tanh(\mathcal{U}))^j. \quad (14)$$

By using this scheme, we can gain the sech, csch, tanh and coth consisting solutions.

3.2. Implementation of EShGEE Technique

Eq.(8) takes the given form for $m = 1$:

$$V(\mathcal{U}) = \alpha_0 + \alpha_1 \cosh(f(\mathcal{U})) + \beta_1 \sinh(f(\mathcal{U})). \quad (15)$$

Using Eq.(15) into Eq.(4) along Eq.(9), we gain a system containing $\alpha_0, \alpha_1, \beta_1, \lambda$ and other parameters. With the help of Mathematica tool, one gains the sets:

Set 1:

$$\left\{ \alpha_0 = -\frac{3a}{4b}, \alpha_1 = \frac{2\theta}{a}, \beta_1 = -\frac{2\theta}{a}, \lambda = \frac{4b\theta^2}{3a^2} - \frac{3a^2}{16b}, \tau = \frac{8b\theta^2}{3a^2} \right\}. \quad (16)$$

$$h_1(x, t) = -\frac{3a}{4b} + \frac{2\theta}{a} (\pm \operatorname{coth}(\frac{\Gamma(1+\varrho)}{\epsilon}(x^\epsilon - (\frac{4b\theta^2}{3a^2} - \frac{3a^2}{16b})t^\epsilon)) \mp \operatorname{csch}(\frac{\Gamma(1+\varrho)}{\epsilon}(x^\epsilon - (\frac{4b\theta^2}{3a^2} - \frac{3a^2}{16b})t^\epsilon))). \quad (17)$$

$$h_2(x, t) = -\frac{3a}{4b} + \frac{2\theta}{a} (\pm \tanh(\frac{\Gamma(1+\varrho)}{\epsilon}(x^\epsilon - (\frac{4b\theta^2}{3a^2} - \frac{3a^2}{16b})t^\epsilon)) \mp \iota \operatorname{sech}(\frac{\Gamma(1+\varrho)}{\epsilon}(x^\epsilon - (\frac{4b\theta^2}{3a^2} - \frac{3a^2}{16b})t^\epsilon))). \quad (18)$$

Set 2:

$$\left\{ \alpha_0 = -\frac{3a}{4b}, \alpha_1 = \frac{2\theta}{a}, \beta_1 = \frac{2\theta}{a}, \lambda = \frac{4b\theta^2}{3a^2} - \frac{3a^2}{16b}, \tau = \frac{8b\theta^2}{3a^2} \right\}. \quad (19)$$

$$h_1(x, t) = -\frac{3a}{4b} \pm \frac{2\theta}{a} (\operatorname{coth}(\frac{\Gamma(1+\varrho)}{\epsilon}(x^\epsilon - (\frac{4b\theta^2}{3a^2} - \frac{3a^2}{16b})t^\epsilon)) + \operatorname{csch}(\frac{\Gamma(1+\varrho)}{\epsilon}(x^\epsilon - (\frac{4b\theta^2}{3a^2} - \frac{3a^2}{16b})t^\epsilon))). \quad (20)$$

$$h_2(x, t) = -\frac{3a}{4b} \pm \frac{2\theta}{a} (\tanh(\frac{\Gamma(1+\varrho)}{\epsilon}(x^\epsilon - (\frac{4b\theta^2}{3a^2} - \frac{3a^2}{16b})t^\epsilon)) + \iota \operatorname{sech}(\frac{\Gamma(1+\varrho)}{\epsilon}(x^\epsilon - (\frac{4b\theta^2}{3a^2} - \frac{3a^2}{16b})t^\epsilon))). \quad (21)$$

Set 3:

$$\{\alpha_0 = -\frac{\sqrt{a^2 - 8b\theta} + a}{4b}, \alpha_1 = -\frac{\sqrt{a^2 - 8b\theta} + a}{4b}, \beta_1 = -\frac{\sqrt{a\sqrt{a^2 - 8b\theta} + a^2 - 4b\theta}}{2\sqrt{2}b},$$

$$\lambda = -\frac{a\sqrt{a^2 - 8b\theta} + a^2 + 8b\theta}{12b}, \tau = \frac{a\sqrt{a^2 - 8b\theta} + a^2 - 4b\theta}{12b}\}. \quad (22)$$

$$h_1(x, t) = -\frac{\sqrt{a^2 - 8b\theta} + a}{4b} \mp \left(\frac{(\sqrt{a^2 - 8b\theta} + a)}{4b} \coth\left(\frac{\Gamma(1+q)}{\epsilon}(x^\epsilon + \left(\frac{a\sqrt{a^2 - 8b\theta} + a^2 + 8b\theta}{12b}\right)t^\epsilon)\right) + \frac{\sqrt{a\sqrt{a^2 - 8b\theta} + a^2 - 4b\theta}}{2\sqrt{2}b} \operatorname{csch}\left(\frac{\Gamma(1+q)}{\epsilon}(x^\epsilon + \left(\frac{a\sqrt{a^2 - 8b\theta} + a^2 + 8b\theta}{12b}\right)t^\epsilon)\right) \right). \quad (23)$$

$$h_2(x, t) = -\frac{\sqrt{a^2 - 8b\theta} + a}{4b} \mp \left(\frac{(\sqrt{a\sqrt{a^2 - 8b\theta} + a^2 - 4b\theta})}{2\sqrt{2}b} \operatorname{sech}\left(\frac{\Gamma(1+q)}{\epsilon}(x^\epsilon + \left(\frac{a\sqrt{a^2 - 8b\theta} + a^2 + 8b\theta}{12b}\right)t^\epsilon)\right) + \frac{(\sqrt{a^2 - 8b\theta} + a)}{4b} \tanh\left(\frac{\Gamma(1+q)}{\epsilon}(x^\epsilon + \left(\frac{a\sqrt{a^2 - 8b\theta} + a^2 + 8b\theta}{12b}\right)t^\epsilon)\right) \right). \quad (24)$$

Set 4:

$$\{\alpha_0 = -\frac{\sqrt{a^2 - 8b\theta} + a}{4b}, \alpha_1 = -\frac{\sqrt{a^2 - 8b\theta} + a}{4b}, \beta_1 = \frac{\sqrt{a\sqrt{a^2 - 8b\theta} + a^2 - 4b\theta}}{2\sqrt{2}b},$$

$$\lambda = -\frac{a\sqrt{a^2 - 8b\theta} + a^2 + 8b\theta}{12b}, \tau = \frac{a\sqrt{a^2 - 8b\theta} + a^2 - 4b\theta}{12b}\}. \quad (25)$$

$$h_1(x, t) = -\frac{\sqrt{a^2 - 8b\theta} + a}{4b} \mp \frac{(\sqrt{a^2 - 8b\theta} + a)}{4b} \coth\left(\frac{\Gamma(1+q)}{\epsilon}(x^\epsilon + \left(\frac{a\sqrt{a^2 - 8b\theta} + a^2 + 8b\theta}{12b}\right)t^\epsilon)\right) \pm \frac{\sqrt{a\sqrt{a^2 - 8b\theta} + a^2 - 4b\theta}}{2\sqrt{2}b} \operatorname{csch}\left(\frac{\Gamma(1+q)}{\epsilon}(x^\epsilon + \left(\frac{a\sqrt{a^2 - 8b\theta} + a^2 + 8b\theta}{12b}\right)t^\epsilon)\right). \quad (26)$$

$$h_2(x, t) = -\frac{\sqrt{a^2 - 8b\theta} + a}{4b} \mp \frac{(\sqrt{a^2 - 8b\theta} + a)}{4b} \tanh\left(\frac{\Gamma(1+q)}{\epsilon}(x^\epsilon + \left(\frac{a\sqrt{a^2 - 8b\theta} + a^2 + 8b\theta}{12b}\right)t^\epsilon)\right) \pm \frac{(\sqrt{a\sqrt{a^2 - 8b\theta} + a^2 - 4b\theta})}{2\sqrt{2}b} \operatorname{sech}\left(\frac{\Gamma(1+q)}{\epsilon}(x^\epsilon + \left(\frac{a\sqrt{a^2 - 8b\theta} + a^2 + 8b\theta}{12b}\right)t^\epsilon)\right). \quad (27)$$

Set 5:

$$\{\alpha_0 = \frac{\sqrt{a^2 - 8b\theta} - a}{4b}, \alpha_1 = \frac{\sqrt{a^2 - 8b\theta} - a}{4b}, \beta_1 = -\frac{\sqrt{-a\sqrt{a^2 - 8b\theta} + a^2 - 4b\theta}}{2\sqrt{2}b},$$

$$\lambda = -\frac{-a\sqrt{a^2 - 8b\theta} + a^2 + 8b\theta}{12b}, \tau = \frac{-a\sqrt{a^2 - 8b\theta} + a^2 - 4b\theta}{12b}\}. \quad (28)$$

$$h_1(x, t) = \frac{\sqrt{a^2 - 8b\theta} - a}{4b} \pm \frac{(\sqrt{a^2 - 8b\theta} - a)}{4b} \coth\left(\frac{\Gamma(1 + \varrho)}{\epsilon} \left(x^\epsilon + \left(\frac{-a\sqrt{a^2 - 8b\theta} + a^2 + 8b\theta}{12b}\right)t^\epsilon\right)\right) \\ \mp \frac{\sqrt{-a\sqrt{a^2 - 8b\theta} + a^2 - 4b\theta}}{2\sqrt{2}b} \operatorname{csch}\left(\frac{\Gamma(1 + \varrho)}{\epsilon} \left(x^\epsilon + \left(\frac{-a\sqrt{a^2 - 8b\theta} + a^2 + 8b\theta}{12b}\right)t^\epsilon\right)\right). \quad (29)$$

$$h_2(x, t) = \frac{\sqrt{a^2 - 8b\theta} - a}{4b} \pm \frac{(\sqrt{a^2 - 8b\theta} - a)}{4b} \tanh\left(\frac{\Gamma(1 + \varrho)}{\epsilon} \left(x^\epsilon + \left(\frac{-a\sqrt{a^2 - 8b\theta} + a^2 + 8b\theta}{12b}\right)t^\epsilon\right)\right) \\ \mp \frac{\sqrt{-a\sqrt{a^2 - 8b\theta} + a^2 - 4b\theta}}{2\sqrt{2}b} \operatorname{sech}\left(\frac{\Gamma(1 + \varrho)}{\epsilon} \left(x^\epsilon + \left(\frac{-a\sqrt{a^2 - 8b\theta} + a^2 + 8b\theta}{12b}\right)t^\epsilon\right)\right). \quad (30)$$

Set 6:

$$\{\alpha_0 = \frac{\sqrt{a^2 - 8b\theta} - a}{4b}, \alpha_1 = \frac{\sqrt{a^2 - 8b\theta} - a}{4b}, \beta_1 = \frac{\sqrt{-a\sqrt{a^2 - 8b\theta} + a^2 - 4b\theta}}{2\sqrt{2}b}, \\ \lambda = -\frac{-a\sqrt{a^2 - 8b\theta} + a^2 + 8b\theta}{12b}, \tau = \frac{-a\sqrt{a^2 - 8b\theta} + a^2 - 4b\theta}{12b}\}. \quad (31)$$

$$h_1(x, t) = \frac{\sqrt{a^2 - 8b\theta} - a}{4b} \pm \frac{(\sqrt{a^2 - 8b\theta} - a)}{4b} \coth\left(\frac{\Gamma(1 + \varrho)}{\epsilon} \left(x^\epsilon + \left(\frac{-a\sqrt{a^2 - 8b\theta} + a^2 + 8b\theta}{12b}\right)t^\epsilon\right)\right) \\ \pm \frac{\sqrt{-a\sqrt{a^2 - 8b\theta} + a^2 - 4b\theta}}{2\sqrt{2}b} \operatorname{csch}\left(\frac{\Gamma(1 + \varrho)}{\epsilon} \left(x^\epsilon + \left(\frac{-a\sqrt{a^2 - 8b\theta} + a^2 + 8b\theta}{12b}\right)t^\epsilon\right)\right). \quad (32)$$

$$h_2(x, t) = \frac{\sqrt{a^2 - 8b\theta} - a}{4b} \pm \frac{(\sqrt{a^2 - 8b\theta} - a)}{4b} \tanh\left(\frac{\Gamma(1 + \varrho)}{\epsilon} \left(x^\epsilon + \left(\frac{-a\sqrt{a^2 - 8b\theta} + a^2 + 8b\theta}{12b}\right)t^\epsilon\right)\right) \\ \pm \frac{\sqrt{-a\sqrt{a^2 - 8b\theta} + a^2 - 4b\theta}}{2\sqrt{2}b} \operatorname{sech}\left(\frac{\Gamma(1 + \varrho)}{\epsilon} \left(x^\epsilon + \left(\frac{-a\sqrt{a^2 - 8b\theta} + a^2 + 8b\theta}{12b}\right)t^\epsilon\right)\right). \quad (33)$$

Set 7:

$$\{\alpha_0 = \frac{\sqrt{a^2 + 8b\theta} - a}{4b}, \alpha_1 = \frac{a - \sqrt{a^2 + 8b\theta}}{4b}, \beta_1 = -\frac{\sqrt{-a\sqrt{a^2 + 8b\theta} + a^2 + 4b\theta}}{2\sqrt{2}b}, \\ \lambda = \frac{a\sqrt{a^2 + 8b\theta} - a^2 + 8b\theta}{12b}, \tau = \frac{-a\sqrt{a^2 + 8b\theta} + a^2 + 4b\theta}{12b}\}. \quad (34)$$

$$h_1(x, t) = \frac{\sqrt{a^2 + 8b\theta} - a}{4b} \pm \frac{(a - \sqrt{a^2 + 8b\theta})}{4b} \coth\left(\frac{\Gamma(1 + \varrho)}{\epsilon} \left(x^\epsilon - \left(\frac{a\sqrt{a^2 + 8b\theta} - a^2 + 8b\theta}{12b}\right)t^\epsilon\right)\right) \\ \mp \frac{\sqrt{-a\sqrt{a^2 + 8b\theta} + a^2 + 4b\theta}}{2\sqrt{2}b} \operatorname{csch}\left(\frac{\Gamma(1 + \varrho)}{\epsilon} \left(x^\epsilon - \left(\frac{a\sqrt{a^2 + 8b\theta} - a^2 + 8b\theta}{12b}\right)t^\epsilon\right)\right). \quad (35)$$

$$h_2(x, t) = \frac{\sqrt{a^2 + 8b\theta} - a}{4b} \pm \frac{(a - \sqrt{a^2 + 8b\theta})}{4b} \tanh\left(\frac{\Gamma(1 + \varrho)}{\epsilon} \left(x^\epsilon - \left(\frac{a\sqrt{a^2 + 8b\theta} - a^2 + 8b\theta}{12b}\right)t^\epsilon\right)\right) \\ \mp \frac{\iota\sqrt{-a\sqrt{a^2 + 8b\theta} + a^2 + 4b\theta}}{2\sqrt{2}b} \operatorname{sech}\left(\frac{\Gamma(1 + \varrho)}{\epsilon} \left(x^\epsilon - \left(\frac{a\sqrt{a^2 + 8b\theta} - a^2 + 8b\theta}{12b}\right)t^\epsilon\right)\right). \quad (36)$$

Set 8:

$$\{\alpha_0 = \frac{\sqrt{a^2 + 8b\theta} - a}{4b}, \alpha_1 = \frac{a - \sqrt{a^2 + 8b\theta}}{4b}, \beta_1 = \frac{\sqrt{-a\sqrt{a^2 + 8b\theta} + a^2 + 4b\theta}}{2\sqrt{2}b}, \\ \lambda = \frac{a\sqrt{a^2 + 8b\theta} - a^2 + 8b\theta}{12b}, \tau = \frac{-a\sqrt{a^2 + 8b\theta} + a^2 + 4b\theta}{12b}\}. \quad (37)$$

$$h_1(x, t) = \frac{\sqrt{a^2 + 8b\theta} - a}{4b} \pm \frac{(a - \sqrt{a^2 + 8b\theta})}{4b} \coth\left(\frac{\Gamma(1 + \varrho)}{\epsilon} \left(x^\epsilon - \left(\frac{a\sqrt{a^2 + 8b\theta} - a^2 + 8b\theta}{12b}\right)t^\epsilon\right)\right) \\ \pm \frac{\sqrt{-a\sqrt{a^2 + 8b\theta} + a^2 + 4b\theta}}{2\sqrt{2}b} \operatorname{csch}\left(\frac{\Gamma(1 + \varrho)}{\epsilon} \left(x^\epsilon - \left(\frac{a\sqrt{a^2 + 8b\theta} - a^2 + 8b\theta}{12b}\right)t^\epsilon\right)\right). \quad (38)$$

$$h_2(x, t) = \frac{\sqrt{a^2 + 8b\theta} - a}{4b} \pm \frac{(a - \sqrt{a^2 + 8b\theta})}{4b} \tanh\left(\frac{\Gamma(1 + \varrho)}{\epsilon} \left(x^\epsilon - \left(\frac{a\sqrt{a^2 + 8b\theta} - a^2 + 8b\theta}{12b}\right)t^\epsilon\right)\right) \\ \pm \frac{\iota\sqrt{-a\sqrt{a^2 + 8b\theta} + a^2 + 4b\theta}}{2\sqrt{2}b} \operatorname{sech}\left(\frac{\Gamma(1 + \varrho)}{\epsilon} \left(x^\epsilon - \left(\frac{a\sqrt{a^2 + 8b\theta} - a^2 + 8b\theta}{12b}\right)t^\epsilon\right)\right). \quad (39)$$

Set 9:

$$\{\alpha_0 = -\frac{\sqrt{a^2 + 8b\theta} + a}{4b}, \alpha_1 = \frac{\sqrt{a^2 + 8b\theta} + a}{4b}, \beta_1 = -\frac{\sqrt{a\sqrt{a^2 + 8b\theta} + a^2 + 4b\theta}}{2\sqrt{2}b}, \\ \lambda = -\frac{a\sqrt{a^2 + 8b\theta} + a^2 - 8b\theta}{12b}, \tau = \frac{a\sqrt{a^2 + 8b\theta} + a^2 + 4b\theta}{12b}\}. \quad (40)$$

$$h_1(x, t) = -\frac{\sqrt{a^2 + 8b\theta} + a}{4b} \pm \frac{(\sqrt{a^2 + 8b\theta} + a)}{4b} \coth\left(\frac{\Gamma(1 + \varrho)}{\epsilon} \left(x^\epsilon + \left(\frac{a\sqrt{a^2 + 8b\theta} + a^2 - 8b\theta}{12b}\right)t^\epsilon\right)\right) \\ \mp \frac{\sqrt{a\sqrt{a^2 + 8b\theta} + a^2 + 4b\theta}}{2\sqrt{2}b} \operatorname{csch}\left(\frac{\Gamma(1 + \varrho)}{\epsilon} \left(x^\epsilon + \left(\frac{a\sqrt{a^2 + 8b\theta} + a^2 - 8b\theta}{12b}\right)t^\epsilon\right)\right). \quad (41)$$

$$h_2(x, t) = -\frac{\sqrt{a^2 + 8b\theta} + a}{4b} \pm \frac{(\sqrt{a^2 + 8b\theta} + a)}{4b} \tanh\left(\frac{\Gamma(1 + \varrho)}{\epsilon} \left(x^\epsilon + \left(\frac{a\sqrt{a^2 + 8b\theta} + a^2 - 8b\theta}{12b}\right)t^\epsilon\right)\right) \\ \mp \frac{\iota\sqrt{a\sqrt{a^2 + 8b\theta} + a^2 + 4b\theta}}{2\sqrt{2}b} \operatorname{sech}\left(\frac{\Gamma(1 + \varrho)}{\epsilon} \left(x^\epsilon + \left(\frac{a\sqrt{a^2 + 8b\theta} + a^2 - 8b\theta}{12b}\right)t^\epsilon\right)\right). \quad (42)$$

Set 10:

$$\{\alpha_0 = -\frac{\sqrt{a^2 + 8b\theta} + a}{4b}, \alpha_1 = \frac{\sqrt{a^2 + 8b\theta} + a}{4b}, \beta_1 = \frac{\sqrt{a\sqrt{a^2 + 8b\theta} + a^2 + 4b\theta}}{2\sqrt{2}b}, \\ \lambda = -\frac{a\sqrt{a^2 + 8b\theta} + a^2 - 8b\theta}{12b}, \tau = \frac{a\sqrt{a^2 + 8b\theta} + a^2 + 4b\theta}{12b}\}. \quad (43)$$

$$h_1(x, t) = -\frac{\sqrt{a^2 + 8b\theta} + a}{4b} \pm \frac{(\sqrt{a^2 + 8b\theta} + a)}{4b} \coth\left(\frac{\Gamma(1 + \varrho)}{\epsilon} \left(x^\epsilon + \left(\frac{a\sqrt{a^2 + 8b\theta} + a^2 - 8b\theta}{12b}\right)t^\epsilon\right)\right) \\ \pm \frac{\sqrt{a\sqrt{a^2 + 8b\theta} + a^2 + 4b\theta}}{2\sqrt{2}b} \operatorname{csch}\left(\frac{\Gamma(1 + \varrho)}{\epsilon} \left(x^\epsilon + \left(\frac{a\sqrt{a^2 + 8b\theta} + a^2 - 8b\theta}{12b}\right)t^\epsilon\right)\right). \quad (44)$$

$$h_2(x, t) = -\frac{\sqrt{a^2 + 8b\theta} + a}{4b} \pm \frac{(\sqrt{a^2 + 8b\theta} + a)}{4b} \tanh\left(\frac{\Gamma(1 + \varrho)}{\epsilon} \left(x^\epsilon + \left(\frac{a\sqrt{a^2 + 8b\theta} + a^2 - 8b\theta}{12b}\right)t^\epsilon\right)\right) \\ \pm \frac{i\sqrt{a\sqrt{a^2 + 8b\theta} + a^2 + 4b\theta}}{2\sqrt{2}b} \operatorname{sech}\left(\frac{\Gamma(1 + \varrho)}{\epsilon} \left(x^\epsilon + \left(\frac{a\sqrt{a^2 + 8b\theta} + a^2 - 8b\theta}{12b}\right)t^\epsilon\right)\right). \quad (45)$$

Set 11:

$$\left\{ \alpha_0 = -\frac{3a}{4b}, \alpha_1 = \frac{4\theta}{a}, \beta_1 = 0, \lambda = \frac{16b\theta^2}{3a^2} - \frac{3a^2}{16b}, \tau = \frac{8b\theta^2}{3a^2} \right\}. \quad (46)$$

$$h_1(x, t) = -\frac{3a}{4b} \pm \frac{4\theta}{a} \coth\left(\frac{\Gamma(1 + \varrho)}{\epsilon} \left(x^\epsilon - \left(\frac{16b\theta^2}{3a^2} - \frac{3a^2}{16b}\right)t^\epsilon\right)\right). \quad (47)$$

$$h_2(x, t) = -\frac{3a}{4b} \pm \frac{4\theta}{a} \tanh\left(\frac{\Gamma(1 + \varrho)}{\epsilon} \left(x^\epsilon - \left(\frac{16b\theta^2}{3a^2} - \frac{3a^2}{16b}\right)t^\epsilon\right)\right). \quad (48)$$

Set 12:

$$\left\{ \alpha_0 = -\frac{\sqrt{a^2 - 16b\theta} + a}{4b}, \alpha_1 = -\frac{\sqrt{a^2 - 16b\theta} + a}{4b}, \beta_1 = 0, \lambda = -\frac{a\sqrt{a^2 - 16b\theta} + a^2 + 16b\theta}{12b}, \right. \\ \left. \tau = \frac{a\sqrt{a^2 - 16b\theta} + a^2 - 8b\theta}{48b} \right\}. \quad (49)$$

$$h_1(x, t) = -\frac{(\sqrt{a^2 - 16b\theta} + a)}{4b} (1 \pm \coth\left(\frac{\Gamma(1 + \varrho)}{\epsilon} \left(x^\epsilon + \left(\frac{a\sqrt{a^2 - 16b\theta} + a^2 + 16b\theta}{12b}\right)t^\epsilon\right)\right)). \quad (50)$$

$$h_2(x, t) = -\frac{(\sqrt{a^2 - 16b\theta} + a)}{4b} (1 \pm \tanh\left(\frac{\Gamma(1 + \varrho)}{\epsilon} \left(x^\epsilon + \left(\frac{a\sqrt{a^2 - 16b\theta} + a^2 + 16b\theta}{12b}\right)t^\epsilon\right)\right)). \quad (51)$$

Set 13:

$$\left\{ \alpha_0 = \frac{\sqrt{a^2 - 16b\theta} - a}{4b}, \alpha_1 = \frac{\sqrt{a^2 - 16b\theta} - a}{4b}, \beta_1 = 0, \lambda = -\frac{-a\sqrt{a^2 - 16b\theta} + a^2 + 16b\theta}{12b}, \right. \\ \left. \tau = \frac{-a\sqrt{a^2 - 16b\theta} + a^2 - 8b\theta}{48b} \right\}. \quad (52)$$

$$h_1(x, t) = -\frac{(a - \sqrt{a^2 - 16b\theta})}{4b} (1 \pm \coth\left(\frac{\Gamma(1 + \varrho)}{\epsilon} \left(x^\epsilon + \left(\frac{-a\sqrt{a^2 - 16b\theta} + a^2 + 16b\theta}{12b}\right)t^\epsilon\right)\right)). \quad (53)$$

$$h_2(x, t) = -\frac{(a - \sqrt{a^2 - 16b\theta})}{4b} (1 \pm \tanh\left(\frac{\Gamma(1 + \varrho)}{\epsilon} \left(x^\epsilon + \left(\frac{-a\sqrt{a^2 - 16b\theta} + a^2 + 16b\theta}{12b}\right)t^\epsilon\right)\right)). \quad (54)$$

Set 14:

$$\left\{ \alpha_0 = \frac{\sqrt{a^2 + 16b\theta} - a}{4b}, \alpha_1 = \frac{a - \sqrt{a^2 + 16b\theta}}{4b}, \beta_1 = 0, \lambda = \frac{a\sqrt{a^2 + 16b\theta} - a^2 + 16b\theta}{12b}, \right. \\ \left. \tau = \frac{-a\sqrt{a^2 + 16b\theta} + a^2 + 8b\theta}{48b} \right\}. \quad (55)$$

$$h_1(x, t) = \frac{(a - \sqrt{a^2 + 16b\theta})}{4b} (-1 \pm \coth(\frac{\Gamma(1+\varrho)}{\epsilon} (x^\epsilon - (\frac{a\sqrt{a^2 + 16b\theta} - a^2 + 16b\theta}{12b})t^\epsilon))). \quad (56)$$

$$h_2(x, t) = \frac{(a - \sqrt{a^2 + 16b\theta})}{4b} (-1 \pm \tanh(\frac{\Gamma(1+\varrho)}{\epsilon} (x^\epsilon - (\frac{a\sqrt{a^2 + 16b\theta} - a^2 + 16b\theta}{12b})t^\epsilon))). \quad (57)$$

Set 15:

$$\{\alpha_0 = -\frac{\sqrt{a^2 + 16b\theta} + a}{4b}, \alpha_1 = \frac{\sqrt{a^2 + 16b\theta} + a}{4b}, \beta_1 = 0, \lambda = -\frac{a\sqrt{a^2 + 16b\theta} + a^2 - 16b\theta}{12b}, \tau = \frac{a\sqrt{a^2 + 16b\theta} + a^2 + 8b\theta}{48b}\}. \quad (58)$$

$$h_1(x, t) = \frac{(\sqrt{a^2 + 16b\theta} + a)}{4b} (-1 \pm \coth(\frac{\Gamma(1+\varrho)}{\epsilon} (x^\epsilon + (\frac{a\sqrt{a^2 + 16b\theta} + a^2 - 16b\theta}{12b})t^\epsilon))). \quad (59)$$

$$h_2(x, t) = \frac{(\sqrt{a^2 + 16b\theta} + a)}{4b} (-1 \pm \tanh(\frac{\Gamma(1+\varrho)}{\epsilon} (x^\epsilon + (\frac{a\sqrt{a^2 + 16b\theta} + a^2 - 16b\theta}{12b})t^\epsilon))). \quad (60)$$

4. Description of the Modified (G'/G^2)– Expansion Technique

Here, we will brief fundamental steps of the concerned technique given as [14].

Step 1:

Let us assume the Eqs. (5), (6) and (7).

Step 2:

Assume the result for Eq.(7) given as;

$$Q(\eta) = \sum_{i=0}^m \alpha_i \left(\frac{G'}{G^2}\right)^i, \quad (61)$$

Where $\alpha_i (i = 0, 1, 2, 3, \dots, m)$ are undetermined where $\alpha_j \neq 0$. A new profile $G=G(\eta)$ satisfies a mentioned equation,

$$\left(\frac{G'}{G^2}\right)' = p + q\left(\frac{G'}{G^2}\right)^2, \quad (62)$$

here p and q indicate the constants. We obtain the given solutions to Eq. (62) depends on the values of p:

Case 1: when $pq < 0$, we have

$$\left(\frac{G'}{G^2}\right) = -\frac{\sqrt{|pq|}}{q} + \frac{\sqrt{|pq|}}{2} \left(\frac{C_1 \sinh(\sqrt{pq} \eta) + C_2 \cosh(\sqrt{pq} \eta)}{C_1 \cosh(\sqrt{pq} \eta) + C_2 \sinh(\sqrt{pq} \eta)} \right), \quad (63)$$

Case 2: if $pq > 0$, then

$$\left(\frac{G'}{G^2}\right) = \sqrt{\frac{p}{q}} \left(\frac{C_1 \cos(\sqrt{pq} \eta) + C_2 \sin(\sqrt{pq} \eta)}{C_1 \sin(\sqrt{pq} \eta) - C_2 \cos(\sqrt{pq} \eta)} \right), \quad (64)$$

Case 3: if $p = 0$ and $q \neq 0$, then

$$\left(\frac{G'}{G^2}\right) = -\frac{C_1}{q(C_1\eta + C_2)}. \quad (65)$$

Here C_1 and C_2 are constants.

Step 3:

Putting Eq. (61) into the Eq. (7) along Eq. (62), and summing up coefficients of every order of $(\frac{G'}{G^2})^i$ to 0, then solving the obtained system involving α_i, p, q, v and others.

Step 4:

Eq. (61) of which α_i, v and other parameters that are obtained in the step 3 in the Eq. (7), one can gain the results of Eq. (5).

4.1. Exact Soliton Solutions Through Modified (G'/G^2) – Expansion Technique

For $m = 1$, Eq.(61) changes into:

$$Q(\eta) = \alpha_0 + \alpha_1 \frac{G'(\eta)}{G^2(\eta)}. \quad (66)$$

Here α_0 and α_1 are undetermined. Using Eq. (66) and Eq. (62) in the Eq. (4) and with the use of Mathematica tool, yields the given sets:

Set 1:

$$\left\{ \alpha_0 = -\frac{3a}{4b}, \alpha_1 = \frac{4\theta p}{a}, \lambda = -\frac{16b\theta^2 pq}{3a^2} - \frac{3a^2}{16b}, \tau = \frac{8b\theta^2}{3a^2} \right\}. \quad (67)$$

$$\begin{aligned} h(x, t) = & -\frac{3a}{4b} + \frac{4\theta p}{a} \left(-\frac{\sqrt{|pq|}}{q} + \frac{\sqrt{|pq|}}{2} \left((C_1 \sinh(\sqrt{pq} \frac{\Gamma(1+q)}{\epsilon} (x^\epsilon + (\frac{16b\theta^2 pq}{3a^2} - \frac{3a^2}{16b})t^\epsilon)) \right. \right. \\ & + C_2 \cosh(\sqrt{pq} \frac{\Gamma(1+q)}{\epsilon} (x^\epsilon + (\frac{16b\theta^2 pq}{3a^2} - \frac{3a^2}{16b})t^\epsilon)) / (C_1 \cosh(\sqrt{pq} \frac{\Gamma(1+q)}{\epsilon} (x^\epsilon + (\frac{16b\theta^2 pq}{3a^2} - \frac{3a^2}{16b})t^\epsilon)) \\ & \left. \left. - \frac{3a^2}{16b})t^\epsilon) + C_2 \sinh(\sqrt{pq} \frac{\Gamma(1+q)}{\epsilon} (x^\epsilon + (\frac{16b\theta^2 pq}{3a^2} - \frac{3a^2}{16b})t^\epsilon)) \right) \right). \quad (68) \end{aligned}$$

$$\begin{aligned} h(x, t) = & -\frac{3a}{4b} + \frac{4\theta p}{a} \left(\sqrt{\frac{p}{q}} \left((C_1 \cos(\sqrt{pq} \frac{\Gamma(1+q)}{\epsilon} (x^\epsilon + (\frac{16b\theta^2 pq}{3a^2} - \frac{3a^2}{16b})t^\epsilon)) + \right. \right. \\ & C_2 \sin(\sqrt{pq} \frac{\Gamma(1+q)}{\epsilon} (x^\epsilon + (\frac{16b\theta^2 pq}{3a^2} - \frac{3a^2}{16b})t^\epsilon)) / (C_1 \sin(\sqrt{pq} \frac{\Gamma(1+q)}{\epsilon} (x^\epsilon + (\frac{16b\theta^2 pq}{3a^2} - \frac{3a^2}{16b})t^\epsilon)) \\ & \left. \left. - C_2 \sin(\sqrt{pq} \frac{\Gamma(1+q)}{\epsilon} (x^\epsilon + (\frac{16b\theta^2 pq}{3a^2} - \frac{3a^2}{16b})t^\epsilon)) \right) \right). \quad (69) \end{aligned}$$

Set 2:

$$\begin{aligned} \{ \alpha_0 = & -\frac{a + \sqrt{a^2 - 16ib\theta\sqrt{p}\sqrt{q}}}{4b}, \alpha_1 = \frac{i\sqrt{p}(a + \sqrt{a^2 - 16ib\theta\sqrt{p}\sqrt{q}})}{4b\sqrt{q}}, \\ & \lambda = -\frac{a\sqrt{a^2 - 16ib\theta\sqrt{p}\sqrt{q}} + a^2 + 16ib\theta\sqrt{p}\sqrt{q}}{12b}, \tau = -\frac{a\sqrt{a^2 - 16ib\theta\sqrt{p}\sqrt{q}} + a^2 - 8ib\theta\sqrt{p}\sqrt{q}}{48bpq} \}. \quad (70) \end{aligned}$$

$$\begin{aligned}
h(x,t) = & -\frac{a + \sqrt{a^2 - 16ib\theta\sqrt{p}\sqrt{q}}}{4b} + \frac{i\sqrt{p}(a + \sqrt{a^2 - 16ib\theta\sqrt{p}\sqrt{q}})}{4b\sqrt{q}} \left(-\frac{\sqrt{|pq|}}{q} + \right. \\
& \frac{\sqrt{|pq|}}{2} \left((C_1 \sinh(\sqrt{pq}) \frac{\Gamma(1+q)}{\epsilon} (x^\epsilon + (\frac{a\sqrt{a^2 - 16ib\theta\sqrt{p}\sqrt{q}} + a^2 + 16ib\theta\sqrt{pq}}{12b})t^\epsilon)) + \right. \\
& C_2 \cosh(\sqrt{pq}) \frac{\Gamma(1+q)}{\epsilon} (x^\epsilon + (\frac{a\sqrt{a^2 - 16ib\theta\sqrt{p}\sqrt{q}} + a^2 + 16ib\theta\sqrt{pq}}{12b})t^\epsilon))) / (C_1 \\
& \cosh(\sqrt{pq}) \frac{\Gamma(1+q)}{\epsilon} (x^\epsilon + (\frac{a\sqrt{a^2 - 16ib\theta\sqrt{p}\sqrt{q}} + a^2 + 16ib\theta\sqrt{pq}}{12b})t^\epsilon)) \\
& \left. + C_2 \sinh(\sqrt{pq}) \frac{\Gamma(1+q)}{\epsilon} (x^\epsilon + (\frac{a\sqrt{a^2 - 16ib\theta\sqrt{p}\sqrt{q}} + a^2 + 16ib\theta\sqrt{pq}}{12b})t^\epsilon)) \right) \right). \quad (71)
\end{aligned}$$

$$\begin{aligned}
h(x,t) = & -\frac{a + \sqrt{a^2 - 16ib\theta\sqrt{p}\sqrt{q}}}{4b} + \frac{i\sqrt{p}(a + \sqrt{a^2 - 16ib\theta\sqrt{p}\sqrt{q}})}{4b\sqrt{q}} \left(\sqrt{\frac{p}{q}} \right. \\
& \times \left((C_1 \cos(\sqrt{pq}) \frac{\Gamma(1+q)}{\epsilon} (x^\epsilon + (\frac{a\sqrt{a^2 - 16ib\theta\sqrt{p}\sqrt{q}} + a^2 + 16ib\theta\sqrt{pq}}{12b})t^\epsilon)) + \right. \\
& C_2 \sin(\sqrt{pq}) \frac{\Gamma(1+q)}{\epsilon} (x^\epsilon + (\frac{a\sqrt{a^2 - 16ib\theta\sqrt{p}\sqrt{q}} + a^2 + 16ib\theta\sqrt{pq}}{12b})t^\epsilon))) / (C_1 \\
& \sin(\sqrt{pq}) \frac{\Gamma(1+q)}{\epsilon} (x^\epsilon + (\frac{a\sqrt{a^2 - 16ib\theta\sqrt{p}\sqrt{q}} + a^2 + 16ib\theta\sqrt{pq}}{12b})t^\epsilon)) - C_2 \\
& \left. \sin(\sqrt{pq}) \frac{\Gamma(1+q)}{\epsilon} (x^\epsilon + (\frac{a\sqrt{a^2 - 16ib\theta\sqrt{p}\sqrt{q}} + a^2 + 16ib\theta\sqrt{pq}}{12b})t^\epsilon)) \right) \right). \quad (72)
\end{aligned}$$

Set 3:

$$\begin{aligned}
\{\alpha_0 = & \frac{-a + \sqrt{a^2 - 16ib\theta\sqrt{p}\sqrt{q}}}{4b}, \alpha_1 = -\frac{i\sqrt{p}(-a + \sqrt{a^2 - 16ib\theta\sqrt{p}\sqrt{q}})}{4b\sqrt{q}}, \\
\lambda = & -\frac{-a\sqrt{a^2 - 16ib\theta\sqrt{p}\sqrt{q}} + a^2 + 16ib\theta\sqrt{pq}}{12b}, \tau = \frac{a\sqrt{a^2 - 16ib\theta\sqrt{p}\sqrt{q}} - a^2 + 8ib\theta\sqrt{p}\sqrt{q}}{48bpq}\}. \quad (73)
\end{aligned}$$

$$\begin{aligned}
h(x,t) = & \frac{-a + \sqrt{a^2 - 16ib\theta\sqrt{p}\sqrt{q}}}{4b} - \frac{i\sqrt{p}(-a + \sqrt{a^2 - 16ib\theta\sqrt{p}\sqrt{q}})}{4b\sqrt{q}} \left(-\frac{\sqrt{|pq|}}{q} + \right. \\
& \frac{\sqrt{|pq|}}{2} \left((C_1 \sinh(\sqrt{pq}) \frac{\Gamma(1+q)}{\epsilon} (x^\epsilon + (\frac{-a\sqrt{a^2 - 16ib\theta\sqrt{p}\sqrt{q}} + a^2 + 16ib\theta\sqrt{pq}}{12b})t^\epsilon)) + \right. \\
& C_2 \cosh(\sqrt{pq}) \frac{\Gamma(1+q)}{\epsilon} (x^\epsilon + (\frac{-a\sqrt{a^2 - 16ib\theta\sqrt{p}\sqrt{q}} + a^2 + 16ib\theta\sqrt{pq}}{12b})t^\epsilon))) / (C_1 \\
& \cosh(\sqrt{pq}) \frac{\Gamma(1+q)}{\epsilon} (x^\epsilon + (\frac{-a\sqrt{a^2 - 16ib\theta\sqrt{p}\sqrt{q}} + a^2 + 16ib\theta\sqrt{pq}}{12b})t^\epsilon)) + \\
& \left. C_2 \sinh(\sqrt{pq}) \frac{\Gamma(1+q)}{\epsilon} (x^\epsilon + (\frac{-a\sqrt{a^2 - 16ib\theta\sqrt{p}\sqrt{q}} + a^2 + 16ib\theta\sqrt{pq}}{12b})t^\epsilon)) \right) \right). \quad (74)
\end{aligned}$$

$$\begin{aligned}
h(x,t) = & \frac{-a + \sqrt{a^2 - 16ib\theta\sqrt{p}\sqrt{q}}}{4b} - \frac{i\sqrt{p}(-a + \sqrt{a^2 - 16ib\theta\sqrt{p}\sqrt{q}})}{4b\sqrt{q}} \left(\sqrt{\frac{p}{q}}\right. \\
& \times ((C_1 \cos(\sqrt{pq}) \frac{\Gamma(1+q)}{\epsilon} (x^\epsilon + (\frac{-a\sqrt{a^2 - 16ib\theta\sqrt{pq}} + a^2 + 16ib\theta\sqrt{pq}}{12b})t^\epsilon)) + \\
& C_2 \sin(\sqrt{pq}) \frac{\Gamma(1+q)}{\epsilon} (x^\epsilon + (\frac{-a\sqrt{a^2 - 16ib\theta\sqrt{pq}} + a^2 + 16ib\theta\sqrt{pq}}{12b})t^\epsilon))) / (C_1 \\
& \sin(\sqrt{pq}) \frac{\Gamma(1+q)}{\epsilon} (x^\epsilon + (\frac{-a\sqrt{a^2 - 16ib\theta\sqrt{pq}} + a^2 + 16ib\theta\sqrt{pq}}{12b})t^\epsilon)) - \\
& C_2 \sin(\sqrt{pq}) \frac{\Gamma(1+q)}{\epsilon} (x^\epsilon + (\frac{-a\sqrt{a^2 - 16ib\theta\sqrt{pq}} + a^2 + 16ib\theta\sqrt{pq}}{12b})t^\epsilon)))). \quad (75)
\end{aligned}$$

Set 4:

$$\begin{aligned}
\{ \alpha_0 = -\frac{a + \sqrt{a^2 + 16ib\theta\sqrt{p}\sqrt{q}}}{4b}, \alpha_1 = -\frac{i\sqrt{p}(a + \sqrt{a^2 + 16ib\theta\sqrt{p}\sqrt{q}})}{4b\sqrt{q}}, \\
\lambda = -\frac{a\sqrt{a^2 + 16ib\theta\sqrt{pq}} + a^2 - 16ib\theta\sqrt{pq}}{12b}, \tau = -\frac{a\sqrt{a^2 + 16ib\theta\sqrt{pq}} + a^2 + 8ib\theta\sqrt{pq}}{48bpq} \}. \quad (76)
\end{aligned}$$

$$\begin{aligned}
h(x,t) = & -\frac{a + \sqrt{a^2 + 16ib\theta\sqrt{p}\sqrt{q}}}{4b} - \frac{i\sqrt{p}(a + \sqrt{a^2 + 16ib\theta\sqrt{p}\sqrt{q}})}{4b\sqrt{q}} \left(-\frac{\sqrt{|pq|}}{q} + \right. \\
& \frac{\sqrt{|pq|}}{2} ((C_1 \sinh(\sqrt{pq}) \frac{\Gamma(1+q)}{\epsilon} (x^\epsilon + (\frac{a\sqrt{a^2 + 16ib\theta\sqrt{pq}} + a^2 - 16ib\theta\sqrt{pq}}{12b})t^\epsilon)) + \\
& C_2 \cosh(\sqrt{pq}) \frac{\Gamma(1+q)}{\epsilon} (x^\epsilon + (\frac{a\sqrt{a^2 + 16ib\theta\sqrt{pq}} + a^2 - 16ib\theta\sqrt{pq}}{12b})t^\epsilon))) / (C_1 \\
& \cosh(\sqrt{pq}) \frac{\Gamma(1+q)}{\epsilon} (x^\epsilon + (\frac{a\sqrt{a^2 + 16ib\theta\sqrt{pq}} + a^2 - 16ib\theta\sqrt{pq}}{12b})t^\epsilon)) + \\
& C_2 \sinh(\sqrt{pq}) \frac{\Gamma(1+q)}{\epsilon} (x^\epsilon + (\frac{a\sqrt{a^2 + 16ib\theta\sqrt{pq}} + a^2 - 16ib\theta\sqrt{pq}}{12b})t^\epsilon)))). \quad (77)
\end{aligned}$$

$$\begin{aligned}
h(x,t) = & -\frac{a + \sqrt{a^2 + 16ib\theta\sqrt{p}\sqrt{q}}}{4b} - \frac{i\sqrt{p}(a + \sqrt{a^2 + 16ib\theta\sqrt{p}\sqrt{q}})}{4b\sqrt{q}} \left(\sqrt{\frac{p}{q}}\right. \\
& \times ((C_1 \cos(\sqrt{pq}) \frac{\Gamma(1+q)}{\epsilon} (x^\epsilon + (\frac{a\sqrt{a^2 + 16ib\theta\sqrt{pq}} + a^2 - 16ib\theta\sqrt{pq}}{12b})t^\epsilon)) + \\
& C_2 \sin(\sqrt{pq}) \frac{\Gamma(1+q)}{\epsilon} (x^\epsilon + (\frac{a\sqrt{a^2 + 16ib\theta\sqrt{pq}} + a^2 - 16ib\theta\sqrt{pq}}{12b})t^\epsilon))) / (C_1 \\
& \sin(\sqrt{pq}) \frac{\Gamma(1+q)}{\epsilon} (x^\epsilon + (\frac{a\sqrt{a^2 + 16ib\theta\sqrt{pq}} + a^2 - 16ib\theta\sqrt{pq}}{12b})t^\epsilon)) - \\
& C_2 \sin(\sqrt{pq}) \frac{\Gamma(1+q)}{\epsilon} (x^\epsilon + (\frac{a\sqrt{a^2 + 16ib\theta\sqrt{pq}} + a^2 - 16ib\theta\sqrt{pq}}{12b})t^\epsilon)))). \quad (78)
\end{aligned}$$

Set 5:

$$\{\alpha_0 = \frac{-a + \sqrt{a^2 + 16ib\theta\sqrt{p}\sqrt{q}}}{4b}, \alpha_1 = \frac{i\sqrt{p}(-a + \sqrt{a^2 + 16ib\theta\sqrt{p}\sqrt{q}})}{4b\sqrt{q}},$$

$$\lambda = \frac{a\sqrt{a^2 + 16ib\theta\sqrt{p}\sqrt{q}} - a^2 + 16ib\theta\sqrt{p}\sqrt{q}}{12b}, \tau = -\frac{-a\sqrt{a^2 + 16ib\theta\sqrt{p}\sqrt{q}} + a^2 + 8ib\theta\sqrt{p}\sqrt{q}}{48bpq}\}. \quad (79)$$

$$h(x, t) = \frac{-a + \sqrt{a^2 + 16ib\theta\sqrt{p}\sqrt{q}}}{4b} + \frac{i\sqrt{p}(-a + \sqrt{a^2 + 16ib\theta\sqrt{p}\sqrt{q}})}{4b\sqrt{q}} \left(-\frac{\sqrt{|pq|}}{q} + \frac{\sqrt{|pq|}}{2} \left((C_1 \sinh(\sqrt{pq}) \frac{\Gamma(1+q)}{\epsilon} (x^\epsilon - (\frac{a\sqrt{a^2 + 16ib\theta\sqrt{p}\sqrt{q}} - a^2 + 16ib\theta\sqrt{p}\sqrt{q}}{12b})t^\epsilon))\right.\right.$$

$$+ C_2 \cosh(\sqrt{pq}) \frac{\Gamma(1+q)}{\epsilon} (x^\epsilon - (\frac{a\sqrt{a^2 + 16ib\theta\sqrt{p}\sqrt{q}} - a^2 + 16ib\theta\sqrt{p}\sqrt{q}}{12b})t^\epsilon))) / (C_1 \cosh(\sqrt{pq}) \frac{\Gamma(1+q)}{\epsilon} (x^\epsilon - (\frac{a\sqrt{a^2 + 16ib\theta\sqrt{p}\sqrt{q}} - a^2 + 16ib\theta\sqrt{p}\sqrt{q}}{12b})t^\epsilon)) +$$

$$C_2 \sinh(\sqrt{pq}) \frac{\Gamma(1+q)}{\epsilon} (x^\epsilon - (\frac{a\sqrt{a^2 + 16ib\theta\sqrt{p}\sqrt{q}} - a^2 + 16ib\theta\sqrt{p}\sqrt{q}}{12b})t^\epsilon)))). \quad (80)$$

$$h(x, t) = \frac{-a + \sqrt{a^2 + 16ib\theta\sqrt{p}\sqrt{q}}}{4b} + \frac{i\sqrt{p}(-a + \sqrt{a^2 + 16ib\theta\sqrt{p}\sqrt{q}})}{4b\sqrt{q}} \left(\sqrt{\frac{p}{q}} \times ((C_1 \cos(\sqrt{pq}) \frac{\Gamma(1+q)}{\epsilon} (x^\epsilon - (\frac{a\sqrt{a^2 + 16ib\theta\sqrt{p}\sqrt{q}} - a^2 + 16ib\theta\sqrt{p}\sqrt{q}}{12b})t^\epsilon)) + \right.$$

$$C_2 \sin(\sqrt{pq}) \frac{\Gamma(1+q)}{\epsilon} (x^\epsilon - (\frac{a\sqrt{a^2 + 16ib\theta\sqrt{p}\sqrt{q}} - a^2 + 16ib\theta\sqrt{p}\sqrt{q}}{12b})t^\epsilon))) / (C_1 \sin(\sqrt{pq}) \frac{\Gamma(1+q)}{\epsilon} (x^\epsilon - (\frac{a\sqrt{a^2 + 16ib\theta\sqrt{p}\sqrt{q}} - a^2 + 16ib\theta\sqrt{p}\sqrt{q}}{12b})t^\epsilon)) -$$

$$C_2 \sin(\sqrt{pq}) \frac{\Gamma(1+q)}{\epsilon} (x^\epsilon - (\frac{a\sqrt{a^2 + 16ib\theta\sqrt{p}\sqrt{q}} - a^2 + 16ib\theta\sqrt{p}\sqrt{q}}{12b})t^\epsilon)))). \quad (81)$$

5. Graphically Explanation

Here, we used 2-D, 3-D, and contour plots to display the obtained solutions. The 2-D graphs are displayed for various ϵ .

6. Physically Explanation

Now we will give the dynamical behaviour of the solutions obtained for the compound KdVB equation with fractional derivative.

Figure 1: denotes a combo singular solitons for; $a = -0.5, b = 1, \theta = -0.2$, and $q = 1$. Fig(a) shows the 2D plot for $-12 < x < 12$ when $\epsilon = 1$, blue line at value of t is 0, orange line when the t is 5, while green line when the t is 10. Fig(b) shows a 2D graph for $-12 < x < 12$ at $0 < t < 10$, while red line when $\epsilon = 0.5$, black line when $\epsilon = 0.7$, while blue line if $\epsilon = 1$. Fig(c) indicates the 3D graph if $\epsilon = 0.6$ when $0 < t < 10$. Fig(d) represents the Contour graph when $\epsilon = 0.6$ for $0 < t < 10$.

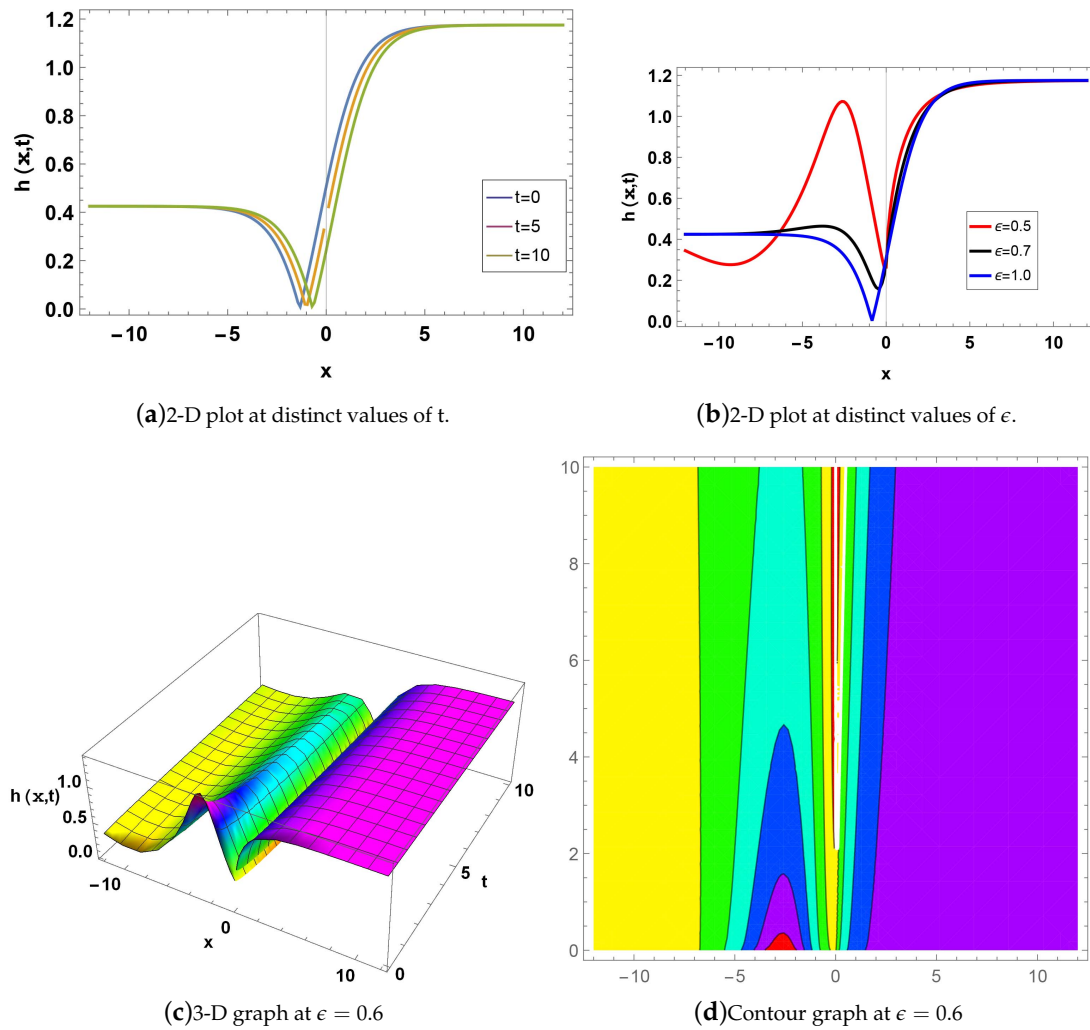


Figure 1. Graph of $h(x, t)$ is given in Eq.(17).

Figure 2: denotes a dark-bright soliton for; $a = 0.3, b = 2, \theta = 0.2$, and $\varrho = 1$. Fig(a) shows the 2D plot for $-12 < x < 12$ when $\epsilon = 1$, blue curve at value of t is 0, orange line when the t is 5, while green line when the t is 10. Fig(b) shows a 2D graph for $-12 < x < 12$ at $0 < t < 10$, while red line when $\epsilon = 0.5$, black line when $\epsilon = 0.7$, while blue line if $\epsilon = 1$. Fig(c) indicates the 3D graph if $\epsilon = 0.6$ when $0 < t < 10$. Fig(d) represents the Contour graph when $\epsilon = 0.6$ for $0 < t < 10$.

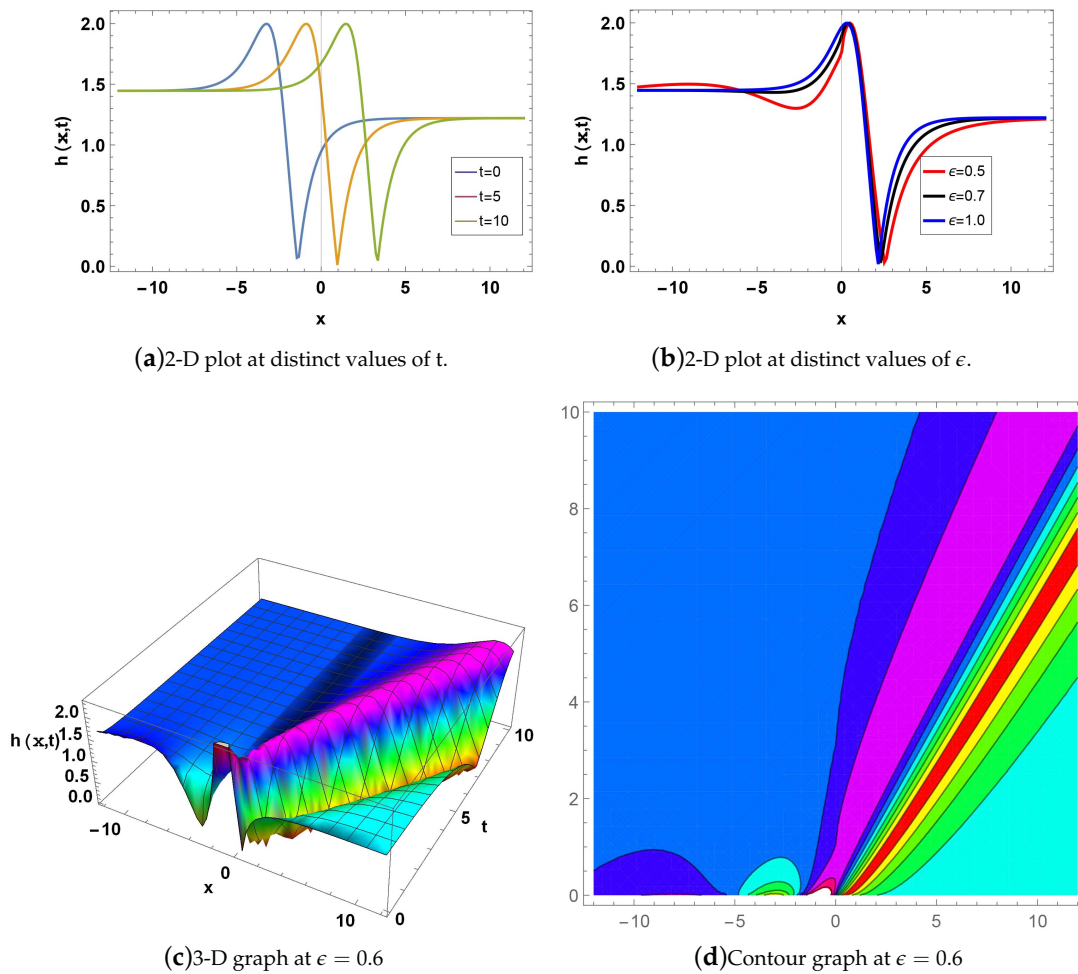


Figure 2. Graph of $h(x,t)$ is given in Eq.(18).

Figure 3: represents a singular soliton for; $a = 0.65, b = -0.45, \theta = 0.3$, and $q = 1$. Fig(a) shows the 2D plot for $-8 < x < 8$ when $\epsilon = 1$, blue curve at value of t is 0, orange line when the t is 5, while green line when the t is 10. Fig(b) shows a 2D graph for $-8 < x < 8$ at $0 < t < 10$, while red line when $\epsilon = 0.5$, black line when $\epsilon = 0.7$, while blue line if $\epsilon = 1$. Fig(c) indicates the 3D graph if $\epsilon = 0.6$ when $0 < t < 10$. Fig(d) represents the Contour graph when $\epsilon = 0.6$ for $0 < t < 10$.

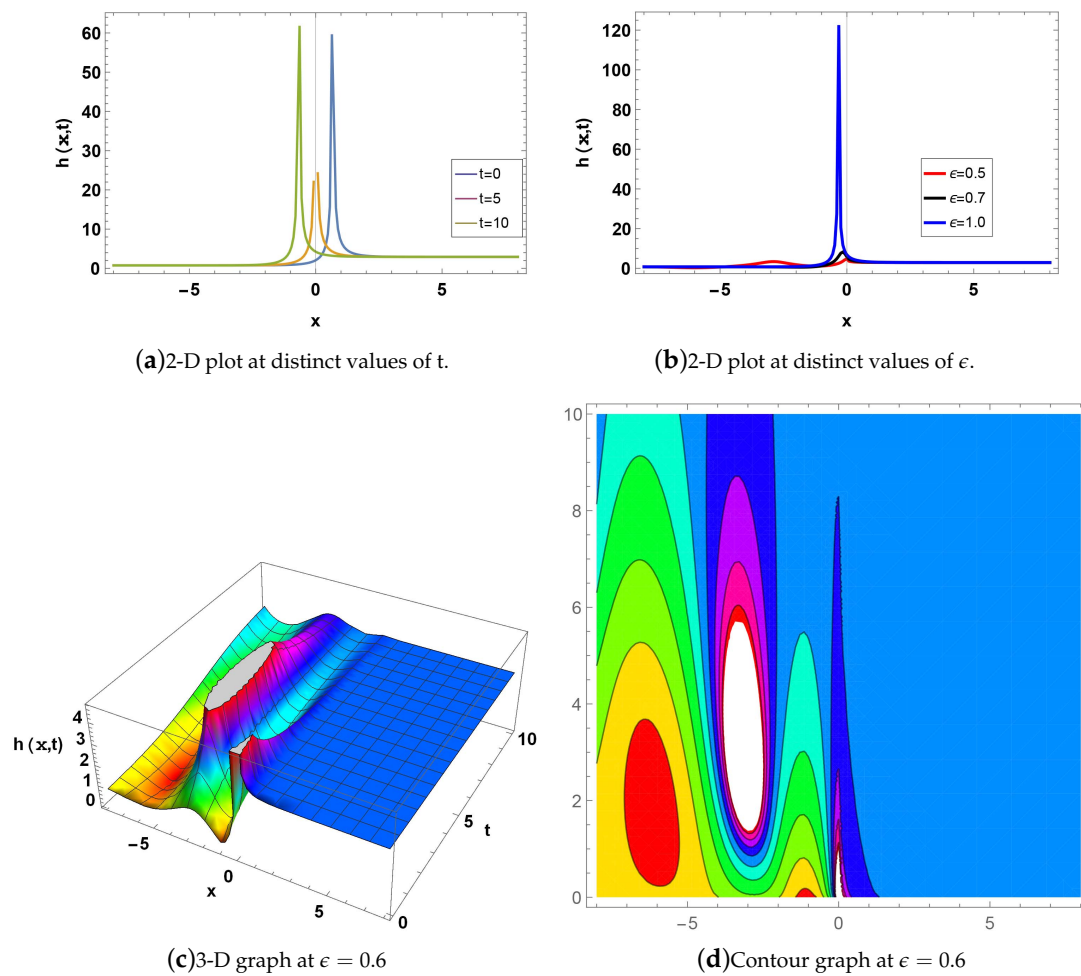


Figure 3. Graph of $h(x,t)$ is given in Eq.(47).

Figure 4: denotes a dark soliton for; $a = 0.55, b = 0.45, \theta = -0.2$, and $q = 1$. Fig(a) shows the 2D plot for $-9 < x < 9$ when $\epsilon = 1$, blue curve at value of t is 0, orange line when the t is 5, while green line when the t is 10. Fig(b) shows a 2D graph for $-9 < x < 9$ at $0 < t < 10$, while red line when $\epsilon = 0.5$, black line when $\epsilon = 0.7$, while blue line if $\epsilon = 1$. Fig(c) indicates the 3D graph if $\epsilon = 0.6$ when $0 < t < 10$. Fig(d) represents the Contour graph when $\epsilon = 0.6$ for $0 < t < 10$.

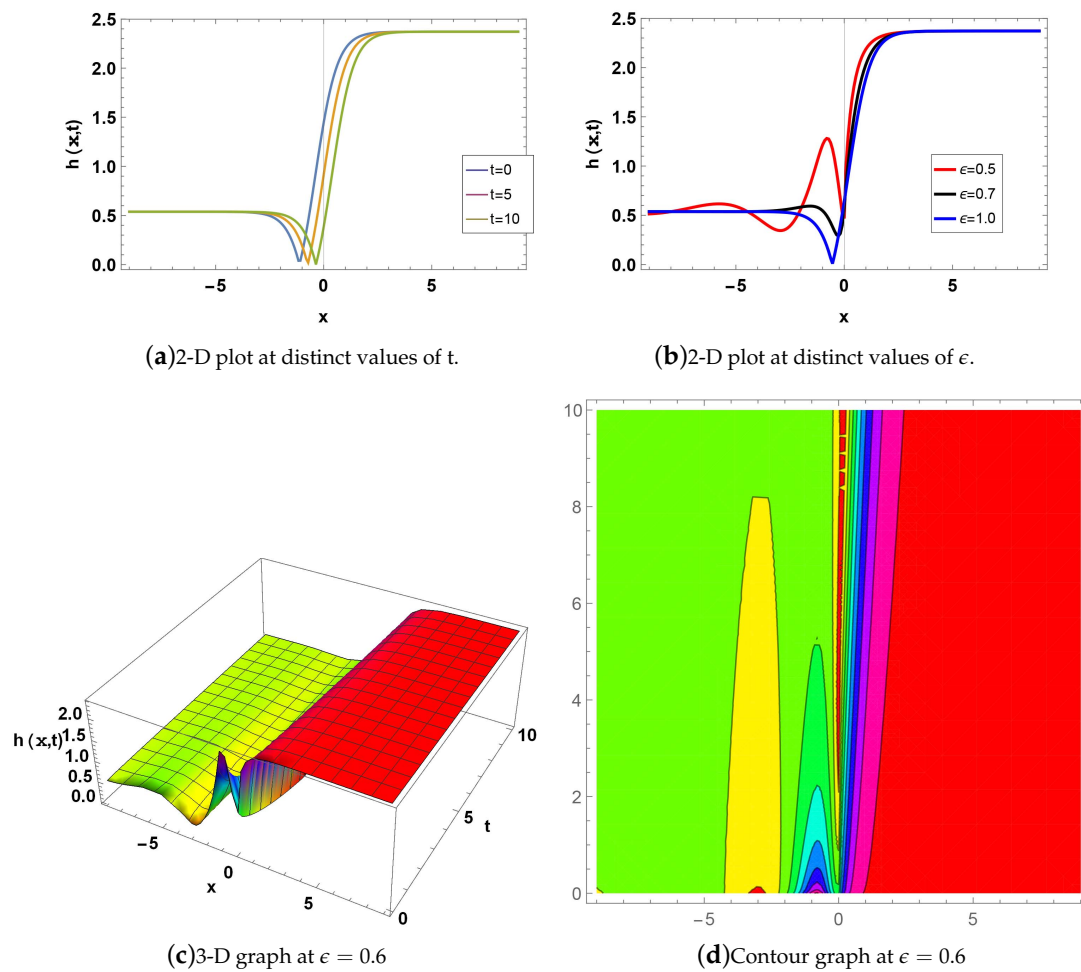


Figure 4. Graph of $h(x,t)$ is given in Eq.(48).

Figure 5: shows a combo kink soliton for; $a = 1, b = 0.9, \theta = -0.2, p = 0.5, q = -0.6, c_1 = 0.5, c_2 = -0.4$ and $q = 1$. Fig(a) shows the 2D plot for $-3 < x < 3$ when $\epsilon = 1$, blue curve at value of t is 0, orange line when the t is 5, while green line when the t is 10. Fig(b) shows a 2D graph for $-3 < x < 3$ at $0 < t < 10$, while red line when $\epsilon = 0.5$, black line when $\epsilon = 0.7$, while blue line if $\epsilon = 1$. Fig(c) indicates the 3D graph if $\epsilon = 0.6$ when $0 < t < 10$. Fig(d) represents the Contour graph when $\epsilon = 0.6$ for $0 < t < 10$.

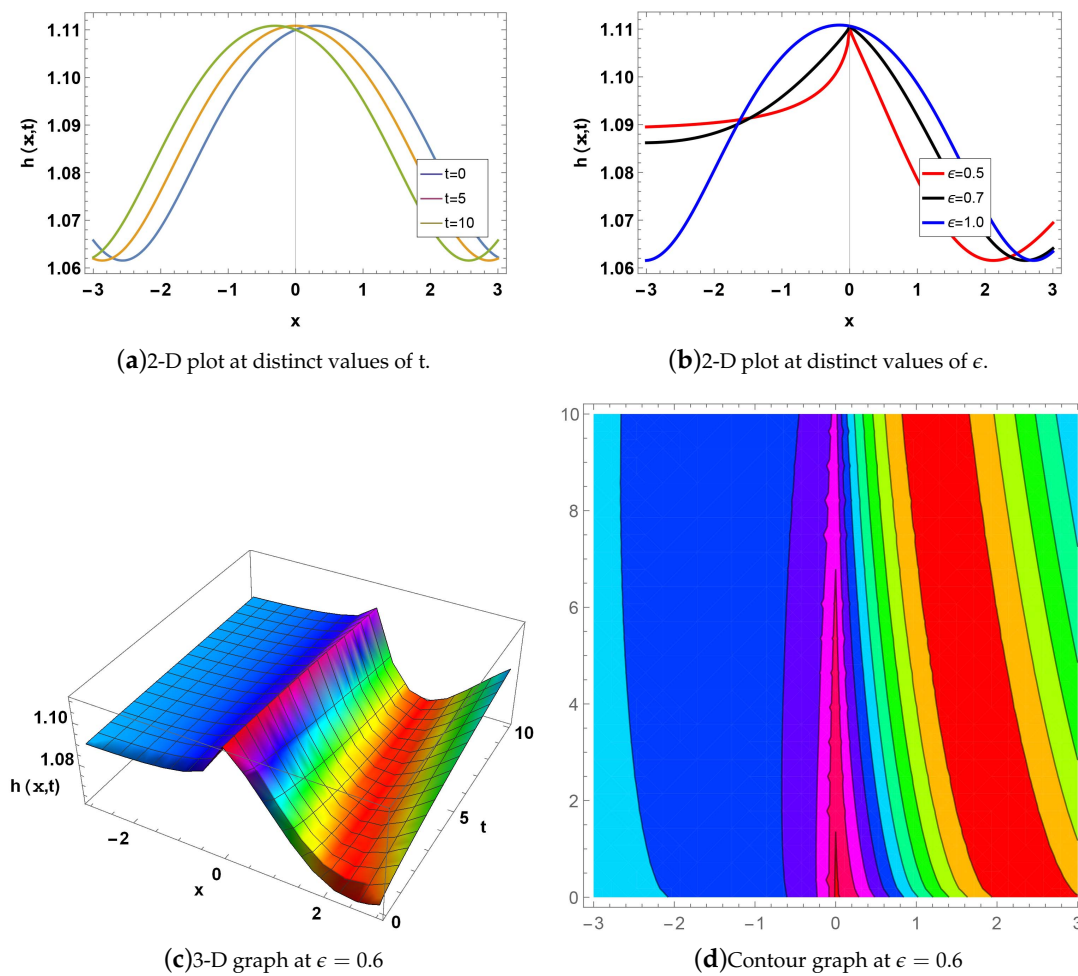


Figure 5. Graph of $h(x,t)$ is given in Eq.(68).

Figure 6: shows a periodic soliton for; $a = 0.15, b = 0.45, \theta = 0.1, p = 0.5, q = -0.3, c_1 = 0.5, c_2 = 1$, and $q = 1$. Fig(a) shows the 2D plot for $-7 < x < 7$ when $\epsilon = 1$, blue curve at value of t is 0, orange line when the t is 5, while green line when the t is 10. Fig(b) shows a 2D graph for $-7 < x < 7$ at $0 < t < 10$, while red line when $\epsilon = 0.5$, black line when $\epsilon = 0.7$, while blue line if $\epsilon = 1$. Fig(c) indicates the 3D graph if $\epsilon = 0.6$ when $0 < t < 10$. Fig(d) represents the Contour graph when $\epsilon = 0.6$ for $0 < t < 10$.

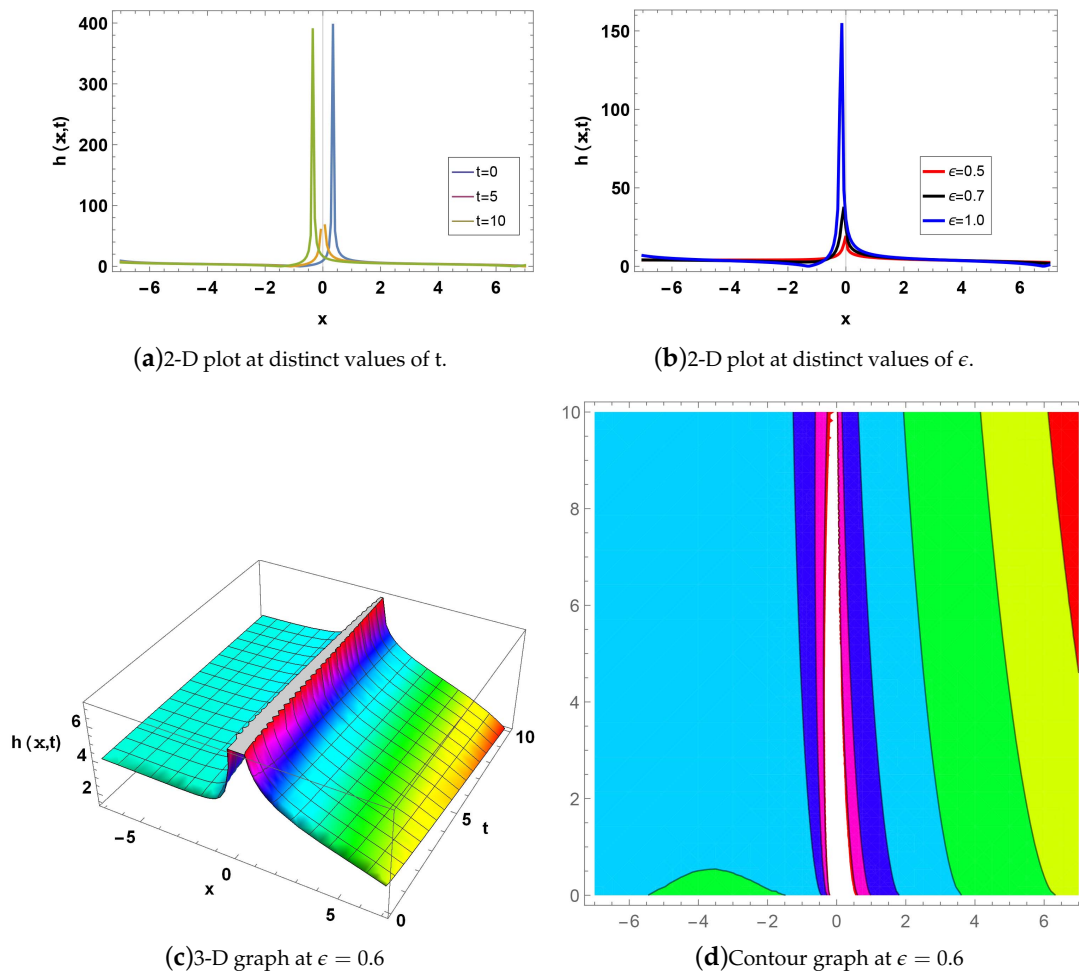


Figure 6. Graph for $h(x, t)$ is given in Eq.(69).

7. Stability Analysis

We will conduct the crucial analysis for the relevant equation in this section. Many equations probably use the stability analysis [27,28]. The Hamiltonian transformation is taken into consideration for the stability analysis of Eq.(1).

$$\mathcal{S} = \frac{1}{2} \int_{-\infty}^{\infty} h^2 dx \quad (82)$$

Here, $\mathcal{S}$ indicates a factor of momentum, and $h(x, t)$ represents the possibility power. An important criterion for the stable-solution is shown as follows;

$$\frac{\partial \mathcal{S}}{\partial \lambda} > 0, \quad (83)$$

here λ shows the soliton velocity, putting Eq.(48) in Eq.(82) yields;

$$\mathcal{S} = \frac{1}{2} \int_{-6}^6 \left(-\frac{3a}{4b} + \frac{4\theta}{a} \tanh\left(x - \left(\frac{16b\theta^2}{3a^2} - \frac{3a^2}{16b}\right)t\right) \right)^2 dx, \quad (84)$$

by using the criterion given in Eq.(83), we get

$$\begin{aligned}
& (24a^2b\theta \cosh(6 - t(\frac{16b\theta^2}{3a^2} - \frac{3a^2}{16b})) \operatorname{sech}(t(\frac{16b\theta^2}{3a^2} - \frac{3a^2}{16b}) + 6) (t \sinh(t(\frac{16b\theta^2}{3a^2} - \frac{3a^2}{16b}) + 6) \\
& \operatorname{sech}(6 - t(\frac{16b\theta^2}{3a^2} - \frac{3a^2}{16b})) + t \cosh(t(\frac{16b\theta^2}{3a^2} - \frac{3a^2}{16b}) + 6) \tanh(6 - t(\frac{16b\theta^2}{3a^2} - \frac{3a^2}{16b})) \operatorname{sech}(6 - t(\frac{16b\theta^2}{3a^2} \\
& - \frac{3a^2}{16b}))) - 64b^2\theta^2(t \operatorname{sech}^2(t(\frac{16b\theta^2}{3a^2} - \frac{3a^2}{16b}) + 6) - t \operatorname{sech}^2(6 - t(\frac{16b\theta^2}{3a^2} - \frac{3a^2}{16b})))) / (8a^2b^2) > 0. \quad (85)
\end{aligned}$$

Hence, Eq.(1) shows that, if the criterion is met, the model is stable.

8. MI Analysis

Consider the given relation for steady state solution of compound KdVB equation [29]:

$$h(x, t) = (H(x, t) + \sqrt{\delta})e^{i\delta t}. \quad (86)$$

Here δ represents the normalizing power of optical. Using Eq.(86) in Eq.(1). By linearizing, we get

$$\theta H_{xx} - \tau H_{xxx} + H_t + \delta t H + \delta^{3/2} t = 0. \quad (87)$$

Let us assume the results of Eq.(87) given as;

$$H(x, t) = A_1 e^{i(px - qt)} + A_2 e^{-i(px - qt)}. \quad (88)$$

here q shows the frequency while p denotes the perturbation normalized wave number. Using the Eq.(88) in Eq.(87). By collecting the co-efficients of $e^{i(px - qt)}$ and $e^{-i(px - qt)}$, we get the dispersion relation .

$$-\delta^2 + p^6 \tau^2 + \theta^2 p^4 - 2p^3 q \tau + q^2 = 0. \quad (89)$$

Finding for q , the dispersion relation derived from Eq.(89) provides

$$q = p^3 \tau \pm \sqrt{\delta^2 - \theta^2 p^4}. \quad (90)$$

The stable steady state solutions are displayed by an attained dispersion relation. In the event that q is not real, the steadily increasing perturbation will make the solution unstable. If q is real, then small perturbations cause the steady state to transform into the stable state. The prerequisite for an unstable solution is;

$$\delta^2 - \theta^2 p^4 < 0. \quad (91)$$

MI gain spectrum $G(p)$ is obtained;

$$G(p) = 2\operatorname{Im}(q) = \pm \sqrt{\delta^2 - \theta^2 p^4}. \quad (92)$$

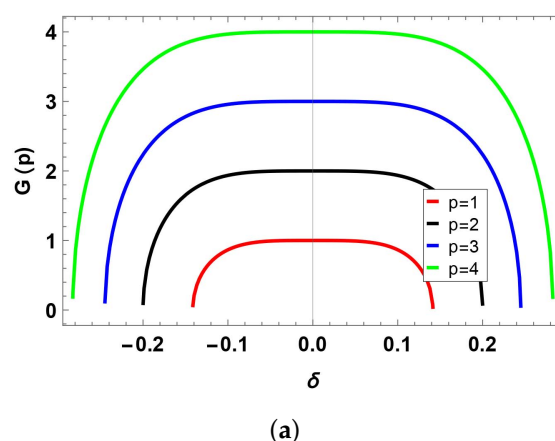


Figure 7. MI gain spectrum for distinct values of p .

9. Conclusion

We have successfully obtained new wave solutions for the non-linear compound KdVB equation along TMFD in this paper. We use the modified (G'/G^2) -expansion methods and the EShGEE to obtain the kink, periodic, dark-bright, singular, and many other soliton solutions. Mathematica software is used to get and check the results. Using the Mathematica tool, the obtained results are displayed as contour, 2-dimensional, and 3-dimensional graphs. The outcomes aid in the development of a relevant equation. The solutions are also helpful in a variety of engineering and applied science fields. Ultimately, this study shows that the employed techniques are not only straightforward and easy to understand, but also effective in solving the unique non-linear partial fractional differential equations (NLPFDEs).

Moreover, a stability analysis and the MI of the governing model have been conducted to check the stability and precision of the achieved results. The methods utilised are not only straightforward but also exceptionally effective in solving non-linear partial differential equations (PDEs). Furthermore, these techniques prove to be valuable for addressing higher-order NLPDEs and larger systems of equations. The findings presented here offer substantial insights and potential applications across various scientific and engineering areas. The obtained solutions are beneficial in a variety of fields, including fluid dynamics, plasma physics, optical fibers, and telecommunications. At the end, it is concluded that the used techniques are reliable and provide the useful results. Now a days, exact solution, Since soliton-like solutions have emerged as a unique area of study in nonlinear science, they have gained significant attention. Because of the unique characteristics of soliton, soliton theory has become more significant. Following interaction and stability, the soliton retains its shape and velocity.

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