A Theory of Gravity Based on Dimensional Perturbations of Objects in Flat Spacetime

William Northcutt [*]

Article

A Theory of Gravity Based on Dimensional Perturbations of Objects in Flat Spacetime

William Northcutt 🄳

Palmyra, Virginia, USA; wnorthcutt@protonmail.com

Abstract: A covariant classical theory of gravity is given assuming absolute flat spacetime and the strong equivalence principle (SEP). It is shown that adherence to these postulates requires that the gravitational field "dimensionally perturbs"—induces fractional length and duration changes—all physical objects at a location universally. Such perturbations are referred to as "gravity shifts," and it is found that all gravitational phenomena may be determined by them. Two classes of observers emerge in "gravity shift theory"—"natural observers" using gravity shifted instruments as is, applicable for all presently available observations, and "absolute observers" that correct for the gravity shifting applied to instruments. Absolute observers accurately measure quantities, including the absolute spacetime metric as it actually is. Natural observers do not accurately measure quantities, but their system of measurement is observationally consistent, yielding a curved "natural metric" to characterize spacetime. When a local gravitational system is surrounded by a "background system" with negligible curvature effects, its gravity shifting induces a diffeomorphism applied to the local system, yielding satisfaction of the SEP for natural observers. Gravity shift theory is the only existing theory other than general relativity that fully satisfies the SEP, which is required for successful predictions of observations. Using the naturally observed inertial form of physical law in free-fall frames, covariant formulation in all coordinates establishes the natural metric as the universally coupled gravitational metric in physical law. The unique field equation determining gravity shifts, and therefore the natural metric, is developed. The resultant bimetric theory is parameterless, complete, and self-consistent. The field equation yields the observed post-Newtonian natural metric and linearizes to the predictive linearized Einstein equation, which, along with SEP satisfaction, results in successful prediction of a wide variety of observed gravitational phenomena. A supplement is provided that extends the range of prediction verification to include low post-Newtonian order radiation cases, and also the strong-field cases consisting of the properties of black and neutron stars plus nearby matter and light.

Keywords: absolute spacetime; flat spacetime; bimetric theory; dimensional perturbations; gravity shifts; diffeomorphism; strong equivalence principle

---

1. Introduction and Summary

It has long been understood that general relativity is fundamentally incompatible with quantum theory due to the single utilized metric specifying both the gravitational field and the spacetime structure, leading to an inconsistent quantization of spacetime when attempting to quantize the field (see Section 2 for discussion of this issue). Accepting this, a novel classical theory of gravity is presented here that is posed in absolute flat spacetime, circumventing spacetime quantization if the gravitational field were later quantized. The provided gravity theory is therefore not fundamentally incompatible with quantum theory, as is understood for all theories of gravity posed in absolute flat spacetime. In addition to formulation in flat spacetime, the strong equivalence principle (SEP) applicable for gravitational systems is assumed to hold due to the extensive observational evidence supporting its validity, including the observations verifying the contained Einstein equivalence principle (EEP) applicable for nongravitational systems only (see Will [1], Chaps. 2 & 8). Both absolute flat spacetime and the SEP are given as *postulates* from which the theory is based.

In order to satisfy the Einstein equivalence principle while adhering to flat spacetime, it is found that on local scales *all physical objects necessarily undergo universal dimensional perturbations that are gravitationally induced, referred to as "gravity shifts"* (shown in Section 3.1 using the Schild argument for observed gravitational redshifting). These local "dimensional perturbations" of objects consist of fractional changes in their lengths, and fractional changes in durations for the physical processes occurring within the objects, or equivalently fractional changes in the rates of the processes. Associated with these purely "dimensional shifts" are additionally shifts in the dynamic properties of objects such as mass. These "dynamic shifts" indeed result from application of the dimensional shifts. As gravity shifts are universal, all physical instruments are gravity shifted as well. As will be shown, measurement of universally gravity shifted objects, as made using gravity shifted instruments, *is the only means by which the equivalence principle may be satisfied assuming flat spacetime.*

Gravity shifts may be considered in terms of local "partner objects" over which the shifting may be approximated as being uniform, with the "unshifted partner" being an object without gravity shifting applied, and the "shifted partner" being the corresponding gravity shifted object. The "material content" of the shifted partner object—meaning its matter and nongravitational fields—is *identical to* the material content of its unshifted partner. Partner objects "share" then the *same* material content, with the only difference between the partners being the dimensional perturbations of the material content of the shifted object relative to the unperturbed unshifted partner with the same content (the resultant shifting in dynamic properties does not alter the makeup of the material content). All gravity shifts may be expressed by the single 1-to-1 linear "partner relation"

$$\mathrm{d}x_S^\alpha = S^\alpha_{\bar\mu}\,\mathrm{d}x_{US}^\mu \tag{1}$$

giving the universal gravity shifting between partner infinitesimal spacetime displacements "tied" to local partner objects, meaning that their endpoints are events that spatially and temporally locate any of the shared material content, such as the "partner events" occurring for a particular shared particle. The rank-2 "shift tensor" $S^\alpha_{\bar\mu}$ is the formal quantity that relates local partner objects, where the "bar" over the second indice indicates conversion from an unshifted partner displacement $\mathrm{d}x_{US}^\mu$, and no bar over the first indice indicates conversion to the corresponding shifted partner displacement $\mathrm{d}x_S^\alpha$. The shifted partner displacements $\mathrm{d}x_S^\alpha$ are the *actual* (i.e., existing) absolute manifold displacements $\mathrm{d}x^\alpha$ due to the gravitational field being present for the actual case, so $\mathrm{d}x_S^\alpha = \mathrm{d}x^\alpha$, whereas the unshifted partner displacements $\mathrm{d}x_{US}^\alpha$ are the *hypothetical* displacements obtained if gravitation were removed in theory. It is found that gravity shifts, as specified by the shift tensor, may be employed to depict the gravitational field and determine all gravitational phenomena. For this reason, the theory given here is referred to as "gravity shift theory," or "GS theory" for short.

The gravity shifts in a system may be determined by using observations and modelling. The effects of gravity shifting on instruments may then be determined and accounted for, yielding "shift-corrected" instruments that *accurately* measure quantities (such as the measurable quantities depicting shifted objects). The *actual values* of quantities are therefore obtained. Use of a shift-corrected instrument is the same as use of its hypothetical unshifted partner, which with the perturbing gravity shifting removed again accurately measures quantities. Measurement with shift-corrected instruments is referred to as "absolute measurement," yielding the class of "absolute observers." Using shift-corrected proper frame clocks and rulers, absolute observers accurately measure the proper intervals $\mathrm{d}s_A$ of the absolute manifold, expressed by $\mathrm{d}s_{A(A)} = \mathrm{d}s_A$, where in general (A) designates absolute measurement using shift-corrected "absolute instruments" (the notation "$\mathrm{d}s$" is generically used to represent temporal, spatial, and null proper intervals, with discernment from specifically spatial intervals $\mathrm{d}s$ made by context). Therefore,

$$\mathrm{d}s_{A(A)}^2 = \mathrm{d}s_A^2 = a_{\mu\nu}\,\mathrm{d}x^\mu\,\mathrm{d}x^\nu, \tag{2}$$

so the absolutely measured metric is indeed the absolute metric $a_{\mu\nu}$, yielding an accurate characterization of the absolute spacetime manifold, which is the reason for the nomenclature "absolute observers."

The class of absolute observers use the absolute inertial frames of the flat spacetime manifold as their "preferred" frames of reference. This is the case since the absolute metric is the Minkowski metric $\eta_{\mu\nu}$ in the global inertial coordinates (ICs) of the absolute inertial frames, yielding absolutely observed geodesic motion, under the zero-valued absolute metric connection, that is *inertial* in the global ICs. An entire "absolute worldview" holds for the class of absolute observers. As an example, absolute observers conceive of gravitation as an *ordinary force* due to absolutely perceived gravitational acceleration of objects relative to their preferred absolute inertial frames.

Measurement made with "raw" gravity shifted instruments that have not been shift-corrected is referred to as "natural measurement," as the instruments are used as is. Exclusive use of such "natural instruments" yields the class of "natural observers." All presently available observations are identified then as having been made by natural observers using raw gravity shifted instruments, as absolute shift-corrected instruments have heretofore not been utilized. Natural instruments, having been perturbed by gravity shifting, will not accurately measure quantities. But under the universality of gravity shifting, natural measurement of the gravity shifted objects present is an *observationally consistent* system of measurement for natural observers with its own properties. Natural observers use gravity shifted instruments to measure local shifted objects for the actual "shifted partner case" when gravitation is present. Whereas when gravitation is removed in theory, the hypothetical "unshifted partner case" is yielded where the unshifted partners of the instruments make the same measurements on the unshifted partners of the objects. As instruments measuring local objects gravity shift under the partner relation (1) the same as the objects, then there is no difference between the shifted and unshifted partner cases except the universal gravity shifting applied to their shared material content consisting of both the instruments and the objects being measured. With the shifted partner case just a dimensionally perturbed version of the unshifted partner case, then *for natural observers, any shifted instrument reading for the shifted partner case, which is the actual case, is the same as the reading from the unshifted partner instrument for the hypothetical unshifted partner case*. This key equivalence for natural measurement of local partner objects is referred to as the "partner equivalence property," or "partner equivalence" for short.

A key example of the partner equivalence property is natural measurement of the partner absolute manifold proper intervals, $ds_S^2 = a_{\mu\nu}\, dx_S^\mu\, dx_S^\nu$ and $ds_{US}^2 = a_{\mu\nu}\, dx_{US}^\mu\, dx_{US}^\nu$, for the partner displacements dx_S^α and dx_{US}^μ tied to local partner objects. The natural proper interval standards consist of raw gravity shifted clocks and rulers utilized as is. Applying partner equivalence, natural measurement of shifted partner proper intervals, ds_S, with these shifted standards, yields values equal to the naturally measured unshifted partner proper intervals, ds_{US}, utilizing the unshifted partners of these standards, as shown in detail later. This property is formally expressed by $ds_{S(N)} = ds_{US(N)}$, where in general (N) designates natural measurement. As a hypothetical unshifted instrument accurately measures quantities due to no perturbing gravity shifting applied to it, then $ds_{US(N)} = ds_{US}$ for natural unshifted proper interval measurement using unshifted proper interval standards. With dx_S^α the actual displacement dx^α (from above), then their absolute manifold proper intervals are the same, so $ds_S = ds_A$. Therefore, $ds_{A(N)} = ds_{S(N)} = ds_{US(N)} = ds_{US}$, resulting in

$$ds_{A(N)}^2 = ds_{US}^2 = a_{\mu\nu}\, dx_{US}^\mu\, dx_{US}^\nu$$

for the natural measurement of actual/shifted proper intervals $ds_A = ds_S$ for the absolute manifold. The 1-to-1 partner relation (1) is *invertible*, yielding the "reverse" partner relation

$$dx_{US}^\mu = S^{\bar\mu}{}_\alpha\, dx_S^\alpha, \tag{3}$$

where $S^{\bar\mu}{}_\alpha$ is the reverse shift tensor satisfying $S^\alpha{}_{\bar\mu} S^{\bar\mu}{}_\beta = \delta^\alpha{}_\beta$. Substitution of the reverse partner relation into above, and applying $dx_S^\alpha = dx^\alpha$, yields

$$ds_{A(N)}^2 = g_{\alpha\beta}\, dx^\alpha\, dx^\beta, \tag{4}$$

where $g_{\alpha\beta}$ is given by the "metric relation"

$$g_{\alpha\beta} = a_{\mu\nu} S^{\bar{\mu}}_{\ \alpha} S^{\bar{\nu}}_{\ \beta}. \tag{5}$$

The quantity $g_{\alpha\beta}$ can be seen to be the "natural metric" determining the natural measurements of the absolute proper intervals $\mathrm{d}s_A$ for actual displacements $\mathrm{d}x^\alpha$. The metric relation is a covariant relation between the absolute and natural metrics in any coordinates, so the shift tensor determines the natural metric given the absolute metric $a_{\mu\nu}$. The reverse partner relation (3) is generally *not an integrable condition*, so it is not the differential form of a diffeomorphism. As a result, the natural metric $g_{\alpha\beta}$ obtained via the metric relation possesses *curvature*. Therefore, *natural observers perceive the absolute flat spacetime manifold to be a curved manifold with the metric* $g_{\alpha\beta}$. Even though natural observers do not accurately measure absolute proper intervals since $\mathrm{d}s_{A(N)} \neq \mathrm{d}s_A$ in general, it can be seen that observational consistency is yielded due to the resultant emergence of the natural metric to determine $\mathrm{d}s_{A(N)}$ and therefore the self-consistent natural characterization of the absolute flat manifold.

A key property to be shown is that *the equivalence principle is satisfied under natural observation*, which as stated above is required, along with universal gravity shifting, for the equivalence principle to hold assuming flat spacetime. As a result, natural observers perceive the local gravitational free-fall frames as being inertial. They may form Cartesian local inertial coordinates to map events in the free-fall frames, with the natural metric given as the Minkowski metric $\eta_{\mu\nu}$ in these coordinates. The natural observers therefore use the free-fall frames as their "preferred" frames of reference. Similar to the arguments used in general relativity, natural observers do not perceive gravitation to be a force due to a lack of perceived gravitational acceleration in their preferred locally inertial free-fall frames. The relative accelerations of the various free-fall frames are equated then with "natural curvature"—i.e., the curvature of the natural metric—in the "natural spacetime" framework. So they equate the perceived natural curvature with gravitation. As can be seen, the "natural observers" in GS theory are equivalent to the *only* "observers" in general relativity, identified using GS theory as *natural observers*. Therefore, *the entire gravitational worldview in general relativity holds for the class of natural observers in gravity shift theory*, referred to as the "natural worldview."

Similar to general relativity, all laws of nongravitational physics may be given by first beginning with free-fall frame natural observations of these laws being identical to their equivalent inertial forms without gravitation present. Covariant formulation in all coordinates establishes the influence of gravitation in the nongravitational physics laws, with all gravitational influence explicitly given by the "gravitational metric" (and its compatible affine connection) that emerges when transforming from the naturally observed Minkowski metric used in the free-fall frames. *The natural metric is therefore the gravitational metric*. Since all nongravitational physics laws may be incorporated into GS theory via this methodology, GS theory is a *complete* theory of gravitation (as per discussed in Will [1], Chap. 2). As in general relativity, the gravitational field may be depicted by the gravitational metric that emerges via use of the equivalence principle, with all matter and the nongravitational fields "universally coupled" to the gravitational and therefore natural metric. An example is exclusive use of the gravitational/natural metric as the field quantity in the "natural" matter stress-energy (SE) tensor $T^{\alpha\beta}$ utilized to depict the energy-momentum density of matter and the nongravitational fields as naturally observed, which is the same as in general relativity due to its "observers" identified as natural observers. As the natural metric is obtained by combining the shift tensor with the absolute metric in the metric relation (5), then within the flat spacetime background, the shift tensor gives the gravitational field as depicted using the natural metric. However, it can be seen that the gravitational/natural metric is a *derived* quantity, whereas the shift tensor is the *fundamental* quantity depicting the gravitational field. The GS theory field equation determines the shift tensor as the field quantity as opposed to the natural metric, as discussed below.

The concept that universal dimensional perturbations of objects posed in a Euclidean (flat) space results in observed curvature, by no means is a new one. Extensive philosophical writings exist on the subject, primarily as part of the Conventionalism school [2], demonstrating how an

underlying Euclidean spatial or spacetime framework can be perceived as being curved due to universal dimensional perturbations of objects that vary as a function of location, which includes the physical instruments utilized to make displacement measurements. The first systematic introduction of this concept is often attributed to Poincaré in his treatise *Science and Hypothesis* [3] (Chap. 4), though earlier Helmholtz as well discussed this concept [4] (Chaps. 1 & 4). After the introduction of general relativity, the dimensional perturbations of objects and instruments, and the resulting induction of apparent curvature, are often equated with gravitation, resulting in the "gravity shift concept" as presented above. This is demonstrated in Reichenbach's book *The Philosophy of Space & Time* [5], where the term "universal forces" is used to refer to gravity shifts. In *Concepts of Space* [6], Jammer provides some history on the debate between the gravity shift concept and the presently utilized "curved geometry" viewpoint for spacetime in general relativity (see pp. 165–174, 207–210, and 221–226), in which Einstein was a participant. Use of the gravity shift concept has led to formal gravity shift based theories being given [7–11], in various stages of development. A well known example is the field theory approach to general relativity, established over decades by a number of particle physicists. The field theory approach utilizes a gravity shift formulation as a key step in its development (see *Feynman Lectures on Gravitation* [12], Lecture 5, for a popular pedagogical rendering), so it may be considered a gravity shift based theory even though the end result is general relativity, which is devoid of either explicit gravity shift or explicit absolute flat spacetime expression.

For the available theories, the defined gravity shifts are not given using a rank-2 tensor based linear transformation relating unshifted and shifted partner objects such as the GS theory partner relation (1), resulting in significantly different formulations. If the EEP is adhered to in flat spacetime, however, then necessarily the particular form of gravity shifts used in GS theory is yielded (as will be shown). For formulation in absolute flat spacetime, the gravity shifting in GS theory is shown to transform according to the global Lorentz transforms. However, the gravity shifting in the available theories does not do so either explicitly or implicitly, or the transform properties are left undetermined, so the given shifting is not explicitly specified as adhering to global Lorentz transforms. The lack of adherence results in a key "breakdown" in their ability to explicitly adhere to the absolute flat spacetime postulate. In addition, only in Broekaert's theory [7] (to the author's knowledge) is the explicit recognition made that universal gravity shifting results in the emergence of both absolute and natural observers as two distinct but related observational classes with their own formulations and worldviews.

The "bimetric" theories, such as Rosen's theory [13], commonly utilize the absolute flat metric $a_{\mu\nu}$ for the spacetime metric, and an additional gravitational metric $g_{\alpha\beta}$ to give gravitational phenomena posed in flat spacetime. Due to the use of a flat spacetime metric combined with a gravitational one, *gravity shift theory may be considered a bimetric theory*. Elements of the formalism in available bimetric theory are also present in GS theory, which may be conveniently utilized. As stated above, adherence to both the EEP and flat spacetime necessarily implies the existence of gravity shifts. However, bimetric theories are developed using absolute and gravitational metrics given *a priori*, as opposed to first constructing gravity shifts in absolute spacetime and then determining the gravitational metric induced by them. A bimetric theory may contain additional gravitational quantities relating the metrics such as the vielbein-based metric relation $g_{\alpha\beta} = a_{AB}E^A_\alpha E^B_\beta$ for dRGT massive gravity [14], which can be seen to be similar to the GS theory metric relation (5). For the available bimetric theories though, the gravitational quantities used in any metric relations or field equations are not identified or utilized as gravity shifts, so explicitly given gravity shifting such as the GS theory partner relation (1) is not provided. This is the case even though some developers recognize gravity shifting being present via use of the metrics. For example, in his seminal paper (first reference in [13]), Rosen recognizes and evaluates the gravity shifting taking place in observed gravitational redshifting using only the metrics, and then reaches the conclusion that the EEP holds as a result of the shifting. Similar to most available gravity shift based theories, the recognition of the distinct absolute and natural observational classes is not made and utilized in the available bimetric theories (to the author's knowledge).

Unconstrained, the 4×4 shift tensor $S^{\alpha}{}_{\bar{\mu}}$ could consist of 16 independent terms with arbitrary values, which would yield gravity shifting consisting of not only spatial and temporal dimensional shifts as discussed above, but additionally spatial rotation and shearing, as well as "temporal and spatial cross shifting." However, it will be shown that the shift tensor $S^{\alpha}{}_{\bar{\mu}}$ at any location is *diagonalizable* in global ICs via Lorentz transformation. Use of the partner relation (1) in such global ICs yields gravity shifting strictly consisting of three spatial dimensional shifts (i.e., changing lengths), each of which is parallel to a spatial coordinate axis for the ICs, and a temporal dimensional shift (i.e., a changing duration) parallel to the IC time coordinate axis, which yields an increase or decrease in the rates of the physical processes for the matter present. All of these dimensional shifts are therefore *orthogonal* to each other in flat spacetime. As a result, the gravity shifting may be depicted by a *geometrically invariant* "shift tetrad" $\vec{S}_{(\alpha)}$ consisting of four vectors parallel to the IC axes giving the four orthogonal directions of the dimensional shifts, with their lengths relative to unity specifying the amount of shifting in terms of fractional increase or decrease (a shift tetrad vector may be reversed in direction and still express the same shift, since a dimensional shift is an expansion or contraction along the given spacetime direction). Transformation out of these global ICs into other coordinates yields a shift tensor $S^{\alpha}{}_{\bar{\mu}}$ that is generally no longer diagonal, resulting in an apparent "mixing" of spatial and temporal shifting in general coordinates. But in actuality, the gravity shifting still consists of the orthogonal spatial and temporal dimensional shifts depicted by the shift tetrad $\vec{S}_{(\alpha)}$, so the shift tetrad depicts the "intrinsic" gravity shifting. When stating above that gravity shifts consist of spatial and temporal dimensional shifts, it was the *intrinsic* shifting that was being referred to. For any system, the intrinsic gravity shifting at all locations may be given by a map of the shift tetrad field, *providing a geometrically invariant complete depiction of the gravitational field*.

In absolute flat spacetime, it is considered impossible that gravity shifting could overlap the spatially distributed matter of an unshifted object on top of itself when shifted, since then an infinite density "matter singularity" would result. It is also considered impossible that events at different times tied to a particle in an unshifted object, such as an atom, could occur at the same time when shifted, since then a "temporal singularity" would be yielded where the frequency of the physical processes for that particle would be infinite. Separate events tied to an unshifted object are barred then from overlapping in the shifted partner. This "overlap restriction" implies that when the shift tensor $S^{\alpha}{}_{\bar{\mu}}$ at a location is diagonalized in global ICs, its diagonal terms giving the dimensional shifts are *always positive* (as shown later). This allows each diagonal term to be given by the *exponential of a real number* of any finite value, where a positive value yields an increasing shift, a negative value yields a decreasing shift, and zero yields unity which is no shifting. In general coordinates, the shift tensor may therefore be given by its "potential form"

$$S^{\alpha}{}_{\bar{\mu}} = \exp(w^{\alpha}{}_{\mu}), \tag{6}$$

where $\exp(w^{\alpha}{}_{\mu})$ is shorthand for the α, μ component of the exponential power series for the "potential tensor" $w^{\beta}{}_{\nu}$. In order that the GS theory field equation has shift tensor solutions that adhere to the overlap restriction, it is assumed that it is the *potential tensor* $w^{\alpha}{}_{\mu}$ that is the field operand as opposed to the shift tensor $S^{\alpha}{}_{\bar{\mu}}$ directly, which is the reason for the nomenclature "potential tensor." Using the metric relation (5), any field equation potential solution will therefore yield the natural metric

$$g_{\alpha\beta} = a_{\mu\nu} \exp(-w^{\mu}{}_{\alpha}) \exp(-w^{\nu}{}_{\beta}). \tag{7}$$

The natural metric is *devoid of event horizons*, since when the potential tensor is diagonalized in global ICs, application of the Minkowski absolute metric $\eta_{\mu\nu}$ in (7) yields a diagonal natural metric with non-zero and finitely large diagonal terms.

The speed of all shifted objects is limited by the shifted light speed c_S, which is *variable* in general. It will be shown that the shifted light speed is $c_S = e^{-2M/R}$ at the surface of a collapsing "black star" with radius R (renaming from "black hole" since there is no event horizon), so the surface collapse

speed is limited by a shifted light speed that becomes infinitesimally small *exponentially* as the black star collapses towards a singularity. Under this exponential "light speed governor," it would take an *infinite* amount of time for a black star to completely collapse to a singularity, thereby preventing singularity formation over the finite age of our universe. The exponential light speed dependence comes about due to the exponential dependence between the potential and the natural metric in (7), where the shifted light speed c_S may be obtained by applying the null condition $ds_{A(N)} = 0$ to the natural metric line element (4). The exponential light speed governor applies for collapsing objects in general, resulting in all collapsing objects remaining finitely large. The exponential potential form (6) for the shift tensor therefore results in *singularities of any kind being barred*, whether they be collapse-based singularities or the gravity shift overlap singularities discussed above.

Gravity shifted objects may not evolve backwards in time, as clearly this would be a causality violation in absolute flat spacetime. The exponential form (6) bars shifted objects evolving backwards in time as obtained from unshifted objects (which always evolve forwards in time), since when the shift tensor is diagonalized in global ICs, via (1) the intrinsic temporal shifting is given by $dt_S = S^0_{\bar{0}} dt_{US}$ where $S^0_{\bar{0}}$ is positive. The "null speed" v_{Null} for the absolute manifold is the speed obtained by applying the null condition $ds_A = 0$ to the absolute spacetime line element (2), yielding $v_{Null} = 1$ in global ICs (using geometrized units). This is the IC speed $c_{US} = 1$ of unshifted light when gravitation is removed in theory, the speed limit applicable in special relativity theory. Shifted object motion faster than the null speed would yield causality violation. With the shifted light speed c_S the shifted object speed limit, the potential solution $w^\alpha{}_\mu$ for the field equation must be such that the resultant shifted light speed does not exceed the manifold null speed, which is the required "energy condition" for the gravitational source matter used in the field equation. This energy condition is evidently satisfied by the use of ordinary source matter. Satisfaction of the energy condition, combined with the barring of shifted objects evolving backwards in time, *prevents causality violations of any kind*.

Due to explicit formulation in absolute flat spacetime, GS theory is compatible with quantum theory, as demonstrated below. Then with the elimination of event horizons, singularities, and causality violations, *all physical law and modelling using gravity shift theory is physically plausible*. In contrast, general relativity has serious plausibility issues since it predicts event horizons and singularities, and since it is fundamentally incompatible with quantum theory due to dual use of the metric $g_{\mu\nu}$ to determine gravitational effects and give the spacetime structure (discussed below).

Consider a finitely large local system posed in an absolute inertial frame of reference with no surrounding background system present, referred to as the "inertial case." Now surround the local system by a background system so that the gravitational field of the background system perturbs the local system, referred to as the "gravitational case." As will be shown, so long as the effects of the background system's *curvature* may be considered negligible for the local system, the gravity shifting of the background system yields a *diffeomorphism* applied to the inertial-case local system to yield its gravitational case. This occurs even though the partner relation (1) giving the background system gravity shifting is generally not a directly integrable condition, so the "morph" (short for "diffeomorphism") formalism does not take the form of the integrated partner relation. Similar to gravity shifts though, the morph expresses a 1-to-1 field relationship between "unmorphed" and "morphed" partner events x_{UM} and x_M tied to the shared matter and fields of the unmorphed inertial-case and morphed gravitational-case partner systems. The existence of the partner event field relationship is established by showing that over a spacetime region where background system curvature may be neglected, all unshifted paths running from a common "shift origin" X to an arbitrary unshifted event, x_{UM}, yield partner gravity shifted paths running from X to a *single* shifted event, x_M. Since the morph is universal, it is also applied to any physical instruments. As a result, for natural observers using raw physical instruments as is, the naturally observed morphed gravitational case is observationally indistinguishable from the naturally observed unmorphed inertial case, yielding *satisfaction of the equivalence principle*. The morph is initially developed utilizing local nongravitational systems so that only EEP satisfaction is yielded. Then the SEP is invoked as a *postulate* to infer that the morph is applicable

for local gravitational systems as well, establishing the morph mechanism as the *means* for yielding SEP satisfaction. Morph utilization will be shown in detail to yield both EEP and SEP satisfaction for natural observers, including the morph-based formulation and use of natural physical law for local nongravitational systems under background system gravitation, and the morph-based application of the natural observer utilized gravitational field equation (the "natural field equation," discussed below) employed to obtain the potential $w^\alpha{}_\mu$ for local gravitational systems under background gravitation.

There are currently no available gravity theories that fully satisfy the SEP other than general relativity. As discussed in Will [1] (Chap. 3), this is due to coupling of either dynamical elements (such as field-determined scalars or tensors) or prior geometric elements (such as an absolute metric) to the gravitational metric $g_{\alpha\beta}$, resulting in SEP violations such as frame or location dependencies. Indeed, under the introduction of auxiliary dynamical or prior geometric fields that arise in theories other than general relativity, it is conjectured that only general relativity *can* satisfy the SEP. However, via the emergence and application of the local diffeomorphism yielded by gravity shifting, *gravity shift theory fully satisfies the strong equivalence principle, even though the dynamical shift tensor and prior geometric absolute metric are coupled to the gravitational metric through the metric relation* (Eq. (5)). The significance of GS theory satisfying the SEP cannot be overemphasized, as SEP satisfaction is mandatory for a theory to successfully predict available observations, which in all cases satisfy the SEP. With the presented GS theory being the only theory other than general relativity to satisfy the SEP, *only gravity shift theory has the potential capability of competing with general relativity as a successfully predictive theory of gravity*.

With the morph-based satisfaction of the equivalence principle established for natural observers, the subject of absolute and natural observation and formulation may be systematically examined. This is found to be a deep subject, so only some basics are provided along with examples. Under EEP satisfaction for natural observers in their preferred infinitesimal free-fall frames of gravitational systems, *natural observers are limited to perceiving matter and the nongravitational fields as universally coupled to the natural metric*. Therefore, natural observers perceive the natural metric $g_{\alpha\beta}$, but they do not perceive the absolute metric $a_{\mu\nu}$, the shift tensor $S^\alpha{}_{\bar\mu}$, or the potential tensor $w^\alpha{}_\mu$. On the other hand, *absolute observers perceive all quantities*. There exists an absolutely and naturally measured value for each naturally measurable physical quantity. A "quantity partner relation" exists relating the absolute and natural values via use of the shift tensor. An example is the above metric relation (5) between the absolute and natural metrics. In addition, every natural form for a physical law has a "partner" absolute form for the law, where the partner laws are related to each other via use of the quantity partner relations for the partner quantities utilized. As a result, partner physical laws are *equivalent*. There are, for instance, equivalent partner natural and absolute gravitational laws of motion, with the natural law simply the usual law of geodesic motion using the natural metric as the gravitational metric, and the equivalent partner absolute law a force-based law of motion where the gravitational field imposes the force (developed below). As will be shown, there are *two* equivalent partner field equations, the "natural field equation" utilized by natural observers to model gravitational systems, and the partner "absolute field equation" utilized by absolute observers. Note that the above mentions of the "field equation" for GS theory refer to the *natural* field equation in particular, with the reasoning for this use explained below.

When a gravitational theory posed in flat spacetime is attempting to satisfy the equivalence principle in some sense, a common problem encountered is the apparent internal contradictions that arise between the contained flat spacetime satisfying formulation and the contained equivalence principle satisfying formulation. Some oft-noted examples are the Schild argument, discussed below, and the conflict between gravitation seen as a force in flat spacetime as opposed to curvature under satisfaction of the equivalence principle. Such conflicts are resolved in GS theory via the recognition and use of the gravity shift mechanism as well as the resultant absolute and natural observational classes that arise due to gravity shifts, yielding the above-discussed equivalencies of what "on the surface" appears to be the contradicting absolute and natural quantities, laws, and concepts that arise when respectively adhering to flat spacetime and to the equivalence principle. Therefore, *gravity shift*

theory is considered to be a self-consistent theory of gravitation, even though it rigidly adheres to both the absolute flat spacetime postulate and the strong equivalence principle.

The general formulation of GS theory may be considered a *physical deduction* based on the absolute flat spacetime and SEP postulates, since there are essentially no additional "free choices" made during its development. The "general formulation" means everything except the natural and absolute field equations and their solutions. This includes the existence and form of the gravity shifts, the resultant bimetric formulation, the existence and form of the morph utilized to satisfy the SEP, and the emergence and formulations of the absolute and natural classes of observers, which include the absolute and natural quantities as well as the partner relations between them. Therefore, *the general formulation of gravity shift theory is uniquely determined from the absolute flat spacetime and SEP postulates.* Concluding, if the absolute flat spacetime and SEP postulates hold, *general gravity shift theory must be the valid general theory of classical gravitation.*

The natural field equation is developed using a Lagrangian-based formulation, where from above, the potential tensor $w^{\alpha}{}_{\mu}$ is utilized as the operand. The most general possible Lagrangian is formed under the assumed requirements, which are the well-accepted assumptions for formulation of Einstein's equation, and that all explicit $w^{\alpha}{}_{\mu}$ use must be linear in order to self-consistently yield SEP satisfaction under morph application (as shown later). For predictive success, the undetermined constants in the resultant Euler-Lagrange form field equation are set to yield the observed post-Newtonian natural metric, and also the observationally predictive linearized Einstein equation in the linearized case. The solution for the constants satisfying these two conditions is unique, resulting in the *unique* natural field equation

$$H^{\alpha\beta}[w; a] = 8\pi\, T^{\alpha\beta}, \tag{8}$$

having started with the most general possible Lagrangian. The quantity $H^{\alpha\beta}[w; a]$ is the "natural field tensor," which uses the potential tensor $w^{\alpha}{}_{\mu}$ as its operand and is dependent on $a_{\mu\nu}$, given in detailed form later. The resultant natural field equation is parameterless, satisfies the SEP under morph application, linearizes to the observationally predictive linearized Einstein equation, and yields the observed post-Newtonian approximation for the natural metric. As is understood, a wide variety of naturally observed gravitational phenomena are successfully predicted from these observational properties.

A Lagrangian-based formulation of the absolute field equation is similarly performed, resulting in the *unique* Euler-Lagrange form

$$H_A^{\alpha\beta}[w; a] = 8\pi\, E_A^{\alpha\beta}, \tag{9}$$

having started with the most general possible Lagrangian based on its starting assumptions (given below). The quantity $H_A^{\alpha\beta}[w; a]$ is the "absolute field tensor," which uses the potential tensor $w^{\alpha}{}_{\mu}$ as its operand and is dependent on $a_{\mu\nu}$, given in detailed form later. The quantity $E_A^{\alpha\beta}$ is the "absolute total SE tensor" for all matter and fields combined, including the gravitational field.

As discussed above, the absolute field equation is the equivalent absolute partner form of the natural field equation, and so is specifically developed to achieve this property. However, for most gravitational systems, the absolute total SE tensor $E_A^{\alpha\beta}$ is not known a priori since it is field dependent. On the other hand, for general systems, the natural matter SE tensor $T^{\alpha\beta}$ *is known* by natural observers. Therefore, the natural field equation is *preferable* to the absolute field equation for determining the field for general systems. The natural field equation is also preferable for natural observer use since it directly predicts naturally observed gravitational phenomena, and is the field equation form for which morph application yields natural observer SEP compliance, as discussed above. These reasons are why the above mentions of the GS theory "field equation" refer to the natural field equation. The absolute field equation is useful though for determining $E_A^{\alpha\beta}$, and also the "absolute field SE tensor" $t_A^{\alpha\beta}$ giving the energy-momentum density of the gravitational field as absolutely conceived and observed (natural observers do not detect its presence, preserving satisfaction of the equivalence principle). This

is accomplished by substituting the potential solution $w^\alpha{}_\mu$ from the natural field equation (8) into the absolute field equation (9).

Summarizing, the "complete" GS theory—comprised of general GS theory and both the natural and absolute field equations—is *uniquely obtained* from the flat spacetime and SEP postulates as well as the additional assumptions made for development of the field equations. Therefore, for both natural and absolute observers, *all observational predictions made using the complete gravity shift theory are uniquely obtained from its postulates and the additional field equation assumptions*. Based on the assumed *physical validity* of the postulates and the natural field equation assumptions, then utilizing the resulting unique natural field equation to predict natural gravitational observations, *the provided complete gravity shift theory is expected to successfully predict all natural observations of classical gravitational phenomena*. As a verification, again a wide variety of naturally observed gravitational phenomena, discussed below, are successfully predicted from the observational properties utilized to develop the natural field equation. The accompanying supplemental material [S1]—a paper titled *Gravity Shift Theory Observational Predictions*, referred to as the "Supplement" for convenience—extends the range of verification to cover *all presently available* natural observations of local systems utilized to test gravitational theories. The available test cases are categorized in Will [1], plus recent gravity wave detections using pulsar timing arrays (PTAs) [15].

The Supplement provides comprehensive "gravity shift post-Minkowskian theory" (or "GS PM theory") at low order. The post-Newtonian (PN) expansion for near-zone systems is given to 1.5PN order, which when truncated to 1PN is found to yield the observed post-Newtonian natural metric utilized to develop the natural field equation, as expected. In addition, the shortwave approximation of gravitational radiation is given to 1.5PN order (using the "PN" designation system where quadrupole radiation is set to "1PN"). As is commonly understood, a "*successful*" prediction is one that "*formally agrees*" with its corresponding observation, meaning that it agrees to within the uncertainty range obtained by combining the specified observation error with any astrophysical modelling uncertainties encountered (such as the uncertainties in the modelling of neutron stars). As is commonly accepted, no prediction made with general relativity (utilizing then Einstein's equation) has been found that formally disagrees with observation, so all predictions are successful to date. In GS theory, it is understood that the successful prediction using general relativity (GR) is *naturally observable* prediction, since again the only "observers" in general relativity are natural observers. In both the near-zone and radiation cases, it is shown that *the GS PM theory 1.5PN expansions yield the same naturally observable predictions as the corresponding 1.5PN expansions in GR PM theory*, including the predictions utilizing radiative energy-momentum balance equations for obtaining near-zone system behavior under 1.5PN radiation losses, such as the secular decay of compact binary orbits. Therefore, *for all natural observations successfully predicted by the GR 1.5PN near-zone and radiation formulation, the corresponding GS theory 1.5PN formulation yields the same successful predictions*.

This leaves the observed strong-field cases, which involve black and neutron stars. These cases require prediction using either analytical modelling, "high-order" (greater than 1.5PN) post-Minkowskian formulation, or numerical modelling—or a combination of them. Neither high-order PM formulation nor numerical modelling has been attempted. However, using analytical modelling, the Supplement provides approximate predictions for the gross observational properties of black and neutron stars, as well as nearby matter and light when present. Via comparison with the corresponding successful predictions using general relativity, all approximately predicted gross properties are shown to either formally agree with their corresponding observations or, at minimum, approximately agree without formal agreement specified. A key example is the predicted "blackness" of observed black stars, shown by predicting that they are so faint, present instruments cannot detect them. With the aid of the approximately predicted gross properties, key cases of black and neutron star systems are examined demonstrating predictions that formally or approximately agree with their observations, including predictions for detected gravity waves generated by merging compact binaries. To aid in establishing agreement of GS theory predictions with observations for the strong-field cases, use

is made of the 1.5PN agreement of GS theory and GR predictions. As examples under the 1.5PN agreement, the early "1.5PN parts" of detected gravity waves generated by merging compact binaries are successfully predicted by GS theory, and the indirect detections of gravity waves using PTAs are also successfully predicted. Finally, an argument is made concluding with the claim that, *with the exception of the "high-order parts" of detected gravity wave signals generated by merging compact binaries through merger and ringdown (meaning after the early "1.5PN parts" successfully predicted), all presently available observations of black and neutron star systems are indeed successfully predicted by GS theory*, meaning again that the predictions formally agree with their corresponding observations to within the combined observation/modelling uncertainty range for each case. Using analytical modelling, the high-order parts of gravity waves are "grossly" predicted through merger and ringdown, meaning that their general features are predicted. So it can at least be said that no direct contradiction with the detected waves is apparent.

As shown in the Supplement, all available natural observations of local systems utilized to test gravitational theories, as categorized in Will [1] plus the PTA gravity wave detections [15], are covered by combining the provided strong-field predictions with predictions made using the 1.5PN post-Minkowskian formulation. As a result, *all available local system test cases are successfully predicted using gravity shift theory with the exception of the high-order parts of detected gravity waves*, which at least are shown to be grossly predicted using the present analytical modelling. This result provides extensive verification supporting the above conclusion that the given complete GS theory is expected to successfully predict all natural observations of classical gravitational phenomena.

The cosmological natural metric must take the form of the Robertson-Walker (RW) metric due to adherence to the cosmological principle as naturally observed, which is a well-established conclusion obtained using GR theory and also applicable for GS theory. Natural adherence to the cosmological principle results in absolutely observed adherence as well. Application of the cosmological principle assuming an absolute flat spacetime background *necessarily* implies that *the absolutely observed cosmography of the universe is given by the Milne cosmology*. Proof of this key finding may be obtained by assuming an absolute flat spacetime background, as well as an absolute observer interpretation, for the cosmography development in Milne's book *Relativity, Gravitation, and World Structure* [8]. A key property of the Milne cosmology can be established by applying the spherical symmetry of the surrounding universe as absolutely observed from comoving matter, with then no directional gravitational force applied as measured in any instantaneously comoving absolute inertial frame, yielding a *fixed velocity* for any comoving matter in all inertial frames (adapting the proof given by [8], Sec. 91). The naturally observed cosmology, and therefore the natural RW metric, is dependent on the gravity shifting present on cosmological scales. Starting with the flat RW metric $a_{\mu\nu}$ for the absolute Milne cosmology, the natural RW metric $g_{\alpha\beta}$ may be obtained via application of the cosmological shift tensor $S^{\alpha}_{\bar{\mu}}$ in the metric relation (5). Natural cosmological modelling and prediction is not further covered in this paper (or the Supplement), so the cosmic $S^{\alpha}_{\bar{\mu}}$ value is not predicted, leaving the natural RW metric in its general unspecified form. However, it is assumed that application of the natural RW metric accounts for cosmological effects in the naturally observed properties of distant local systems, similar to when using GR theory.

2. Postulates

Classical gravity shift theory is based on two postulates:

- spacetime is absolute and flat,
- the strong equivalence principle holds.

The SEP postulate is satisfied for general relativity due to exclusive use of the gravitational metric $g_{\mu\nu}$ to depict the gravitational field for all formulation, including Einstein's field equation. But GS theory has in addition the first postulate assuming that spacetime is absolute and flat. The commonly utilized covariant formulation is assumed, but this is not elevated to the level of a postulate since in modern gravitational theory it is understood that virtually any theory may be put into convenient covariant

form (such as the covariant formulation of Newton's theory as given in Misner, Thorne, and Wheeler (MTW) [16], Chap. 12).

Spacetime is assumed to be absolute based on available arguments showing the incompatibility of quantum theory with the notion of a matter-dependent gravitational field that is equated with the spacetime structure. What may be considered the most powerful of these are the "circularity arguments" that arise based on the dual role that the gravitational metric $g_{\mu\nu}$ plays as both the quantity that determines gravitational effects and the quantity that gives the spacetime structure. Quantization of the gravitational effects then leads to quantization of the spacetime structure. As discussed by Wald [17] (Chap. 14), the difficulties caused by the dual role of the metric are (quoting) "perhaps best illustrated" by the simple example involving the commutator of the quantized metric operator $\widehat{g}_{ab}$ for spacelike separated events. Then $[\widehat{g}_{ab}(x), \widehat{g}_{cd}(x')] = 0$ must hold for x and x' spacelike separated (assuming gravitons are integer-spin quanta, such as the expected spin-2 gravitons). This equation must hold independently of the value of the metric, but it is unknown if x and x' are spacelike separated until the metric is known, yielding a circularity in the use of the commutator that cannot be resolved. As Wald points out, one cannot assume that classical general relativity applies for gravity in order to resolve these difficulties, since the indeterminate source matter state generating the field would yield acausal discontinuous changes in the spacetime structure when the source state "collapses" upon measurement. Such "duality-based" incompatibilities are taken as *unavoidable* here. In order to circumvent these difficulties, it is concluded that the spacetime structure is indeed independent from the gravitational field, yielding an "absolute" spacetime structure that is not dependent on matter as the field is. This is a commonly made assumption proposed in gravitational theory in order to resolve the fundamental incompatibilities with quantum theory that arise when the gravitational field and the spacetime structure are equated.

Of course the spacetime structure is treated as classical for the classical theory being considered here. Even if GS theory were quantized at a future time, however, *the spacetime structure is considered to be classical at all scales and precisions*, since the spacetime structure is absolute. With all of the successful quantum formulations of the nongravitational fields posed in classical flat spacetime, it is assumed that the classical spacetime structure is indeed *flat*. In particular, the spacetime utilized for the quantized nongravitational fields is a *four-dimensional Riemannian manifold with a Lorentz signature metric that has no curvature*, which is then the specification of the absolute classical flat spacetime manifold assumed for GS theory. The usual qualifiers accompanying this specification are assumed. For instance, included in this specification is the acceptance that the affine connection for a physical manifold is the unique (torsion-free) metric compatible one, with the Riemann curvature tensor obtained then from the compatible affine connection. It is also of course assumed that the manifold is topologically Euclidean, being then simply connected and open.

As is well understood, covariant formulation in absolute flat spacetime is the same as in general relativity, with simply the designation made that the selected curvature for the manifold is globally zero. Formulation may be given then as covariant tensor calculus using the available coordinates of the flat manifold, or equivalently in coordinate independent geometric form with all tensor quantities the invariant geometric objects of the manifold. In order to adhere to the absolute spacetime postulate, all geometric objects used to depict gravitation, as well as physical objects under gravitation, are subject to the same rules as any geometric objects in the flat manifold. The sign convention employed throughout this paper is the "Landau Lifshitz Spacelike Convention" (the same convention as used in MTW).

From the summary, the metric for the absolute manifold is specified by $a_{\mu\nu}$, with the absolute manifold proper intervals given by

$$ds_A^2 = a_{\mu\nu}\, dx^\mu\, dx^\nu \tag{10}$$

in any available coordinates for the manifold. Formalizing the above specification of the absolute spacetime postulate, all of the properties of the absolute classical flat spacetime manifold may be obtained by setting the "absolute curvature tensor" to zero at all locations, expressed by

$$R^{\alpha}{}_{\beta\mu\nu}^{A} = \frac{\partial A^{\alpha}_{\beta\nu}}{\partial x^{\mu}} - \frac{\partial A^{\alpha}_{\beta\mu}}{\partial x^{\nu}} + A^{\alpha}_{\sigma\mu} A^{\sigma}_{\beta\nu} - A^{\alpha}_{\sigma\nu} A^{\sigma}_{\beta\mu} = 0 \tag{11}$$

in any available manifold coordinates, where the compatible affine connection for the absolute metric is provided by the "absolute Christoffel symbol"

$$A^{\alpha}_{\mu\nu} = \frac{1}{2} a^{\alpha\sigma} (a_{\sigma\nu,\mu} + a_{\mu\sigma,\nu} - a_{\mu\nu,\sigma}). \tag{12}$$

The rest of the properties of the absolute (classical) flat manifold follow from this specification, as is commonly understood. The absolute curvature tensor expression (11) acts as a "field equation" for the absolute metric, yielding the Minkowski metric $\eta_{\mu\nu}$ as a constant global solution, with the coordinates utilized for this solution identified as the Cartesian global ICs of an absolute inertial frame of reference due to geodesic motion under the zero-valued absolute connection $A^{\alpha}_{\mu\nu}$ being inertial in these coordinates. The Minkowski metric solution is preserved under global Lorentz transforms, yielding the infinite set of absolute inertial frames as specified using their respective global ICs. The events of the absolute flat manifold therefore adhere to global Lorentz transforms. Viewed geometrically, all tensors are invariant geometric objects under the global Lorentz symmetry group, so the entire formulation of physical law in flat spacetime may be said to exhibit "global Lorentz invariance." Adherence to global Lorentz invariance as a symmetry for all formulation implies that *the principle of special relativity holds*. As understood in modern terms, the "principle of special relativity" is a symmetry property of the flat spacetime manifold, with the behavior of any (closed) system set up in one absolute inertial frame being identical to the behavior if the setup is duplicated in any other inertial frame. It is emphasized here that even when considering gravitation, adherence to global Lorentz invariance—and therefore the principle of special relativity—is maintained. Any available coordinate system for the absolute flat manifold *must* be obtainable via coordinate transform from the global ICs of an absolute inertial frame, with covariant formulation in any such coordinate system yielded from global IC given covariant formulation under the transform.

The SEP is postulated due to the extensive observational evidence supporting its validity, including the observations verifying the contained EEP for nongravitational systems [1] (Chaps. 2 & 8), as discussed in the summary. There are a variety of statements of the "equivalence principle" found in the literature. For GS theory, an "observational form" of the equivalence principle is given: *Within an arbitrary gravitational field, the behavior of all phenomena as observed in any gravitational free-fall frame of reference, of sufficiently small spacetime extent, is indistinguishable from behavior without the field present as observed in an absolute inertial frame of reference*. This property may be applied to matter and the nongravitational fields for EEP compliance, and additionally to local gravitational systems for SEP compliance by identifying the "gravitational field" in the condition with the field provided by a "background system." This form of the equivalence principle is consistent with commonly used forms in the literature. It makes explicit, though, that the *observational process* is involved in the equivalence of the gravitational and inertial cases, meaning that the two cases are equivalent as measured by observers using their instrumentation. This form then does not exclude the possibility that the instruments themselves are affected by the gravitational field, allowing for the observational process utilizing such instruments to participate in satisfaction of the equivalence. This form may be applied to conclude that *the local laws of physics applicable in special relativity theory (i.e., in absolute inertial frames without background gravitation present) are also observationally applicable in the local gravitational free-fall frames*, which is a commonly stated form of the EEP (though the "observation" qualification is typically not stated). Similar to general relativity, a corollary to the free-fall frame version of the equivalence principle is the indistinguishability of phenomena as observed in a local frame within the field that is not in free

fall, from as observed in an accelerating and/or rotating frame without gravitation present. As is well understood, the equivalence principle only holds if the effects of the background system's curvature may be neglected for a local system. The "smallness" qualification for the spacetime extent is provided in equivalence principle statements for the express purpose of nullifying the effects of background system curvature on the local system of interest, with it understood that the smaller the local system, the smaller the effects of the background curvature.

The equivalence principle is of course a statement about the gravitational field, which is expected to be subject to quantization. The equivalence principle is assumed to hold for the *classical* conditions assumed here, but is not expected to hold at the smaller scales or higher precisions that would be applicable for a quantized formulation of gravity shift theory.

3. Gravity Shifts and the Natural Metric

3.1. The Existence of Gravity Shifts

Using the postulates of GS theory, consider the "Schild argument" evaluating light transmitted at the bottom of a tower and measured by a receiver at the top (see Schutz [18], Chap. 5, for background). For the purposes of this discussion, the rotation of the Earth and its revolution around the Sun may be ignored. Assuming that spacetime is flat, an absolute inertial frame stationary with respect to the Earth is utilized to evaluate the physical properties for this case, with a global IC system used in order to map events. The front and rear of a wave train of light with a given number of wavelengths will respectively take the same amount of IC time to climb the tower. This equality of transit time will hold even if the speed of light varies along the length of the tower, as the gravitational conditions are static for light traveling up the tower at all times. So the duration it takes for the wave train to be received at the top is the same as it takes to be transmitted at the bottom. As the number of wavelengths in the wave train is fixed, the frequency of the light at the top is the same as at the bottom. In flat spacetime then, *the global IC given frequency of light cannot change as it travels through a static gravitational field*. But in reality, the light at the top is actually measured by the receiver to have a frequency that is slower than the frequency emitted by the transmitter. This has been shown in the Pound-Rebka experiment using gamma ray emission and absorption for a particular nuclear transition as the transmitter and receiver (respectively). The observed frequency change is in agreement with the equivalence principle, with the equivalent nongravitational case being acceleration of the entire apparatus at 1g relative to an absolute inertial frame. The contradiction between the observed frequency change, and the lack of allowed change assuming flat spacetime, may be construed to indicate that spacetime is indeed curved, as in general relativity.

There is another possibility, however. To consider this possibility, note that both the transmitter and receiver have "operating frequencies" that are the frequencies of their respective nuclear transitions. The operating frequency of the transmitter is also the frequency of the transmitted light. The operating frequency of the receiver is the "reference standard" to which the frequency of the received light is compared in order to measure it. If it is accepted that spacetime is indeed flat, then again the Schild argument establishes that there is no gravitational frequency shifting while the light travels up the tower. Therefore, to explain the observed frequency shift assuming flat spacetime, *it must be the case that the operating frequencies of the transmitter and receiver have been perturbed by the gravitational field, with the degree of perturbation differing between the bottom and the top of the tower*. The difference in the operating frequencies of these instruments yields the measured shift in the frequency of the light, since with the frequency of the light arriving at the receiver the same as the transmitter frequency, the shift in frequency as measured by the receiver is due to the relative difference between its operating frequency and the transmitter frequency. The relative difference between the instrument operating frequencies is such as to yield a receiver-measured frequency shift in agreement with the equivalence principle. Notable is that all of the measured change is due then to the difference in operating frequencies of the instruments with no contribution made from light frequency shifting while in flight, which is

"opposite" the assumption made in general relativity that the measured change is entirely due to frequency shifting of the light while in flight.

The established perturbation in the operating frequencies of the instruments in this case—based on assuming flat spacetime—is an example of how the gravitational field will "dimensionally perturb" objects. Here, the dimensional perturbation is "temporal" as it takes the form of a change in operating frequencies. Dimensional perturbations of objects induced by the gravitational field are referred to as "gravity shifts," as previously coined. This example demonstrates how satisfaction of the equivalence principle relies on natural measurement made with "raw" gravity shifted instruments used "as is"—i.e., without corrections made to compensate for the dimensional perturbations applied to the instruments. This single example indeed *proves* that *satisfaction of the equivalence principle in flat spacetime requires the existence of gravity shifts and the use of natural observation*, as there is evidently no other possibility available if both the equivalence principle and flat spacetime are accepted. Note that the entirety of the apparatus used here consists of matter and nongravitational fields, so that satisfaction of the equivalence principle in this case is specifically satisfaction of the EEP. Assuming then flat spacetime, only satisfaction of the EEP is required to establish the necessary existence of gravity shifts, as opposed to additionally requiring the SEP applicable for local gravitational systems.

The operating frequency of an instrument (when it has one) is the "characteristic frequency" of a physical process occurring within it. In the case at hand, the gravitational field has modified the rates of the physical processes that establish the transmitter and receiver operating frequencies, nuclear transitions for the Pound-Rebka experiment. With the rates of the processes changed in the instruments measuring light frequency shifting, it would be expected that the field would change the rates in other instruments. Consider the corresponding case where two atomic clocks are synchronized, placed at the top and bottom of the light experiment tower, allowed to run for a while, and then their times are compared. Actual experiments with atomic clocks yield a variation in clock times with height that agrees with the equivalence principle. So the clock times agree with the equivalence principle for the tower case here, with the equivalent nongravitational case being acceleration of the entire apparatus at 1g relative to an absolute inertial frame. From the time difference, the fractional difference in the clocks' operating frequencies may be determined, and is the same as the measured fractional frequency change for the light (ratioing the lower clock frequency over the upper frequency), as expected since the equivalence principle applies in both cases. Assuming flat spacetime, it is concluded that the rates of the physical processes that establish these clocks' operating frequencies, in this case atomic resonances, have been modified by the same degree at each location as for the processes in the light apparatus. Satisfaction of the equivalence principle in this case is due to natural measurement—i.e., the clock readings—using gravity shifted clocks, offering further proof that gravity shifts and natural observation are required to satisfy the equivalence principle in flat spacetime (particularly the EEP here).

To establish that the temporal gravity shifting at a location is indeed universal, the equivalence principle may be invoked. If the gravitational field (gravity) shifted the rates of the processes in various objects differently, this difference would be perceptible by natural observers, contrary to the equivalent nongravitational case in which no shifting occurs. For instance, clocks of different constructions would be naturally observed to keep time differently right next to each other. Therefore, *the gravitational field universally shifts the rates of the physical processes in all objects at a location to the same fractional degree*. Natural observation is again required in order to satisfy the equivalence principle as used here, with the rates of all processes when measured using gravity shifted clocks, of any construction, the same as the rates in the equivalent nongravitational case.

Above, the dimensional perturbation examined for objects is the "temporal" one, with the rates of all processes modified, or equivalently their durations (temporal lengths) modified. Similarly, there exists the possibility that the field will also dimensionally perturb objects "spatially." The "spatial" analog to the temporal dimensional perturbation is a change in the *lengths* of objects, again referred to as "gravity shifts." This possibility is left open and worked with in the subsequent development,

with then the resultant GS theory utilizing spatial gravity shifts in addition to the temporal shifts. The *predictive success* of the resultant GS theory is invoked to establish the existence of spatial gravity shifts. As in the temporal case, the equivalence principle requires that *the gravitational field universally shifts the lengths of all objects at a location to the same fractional degree*. Otherwise, if the gravitational field shifted the lengths of various objects differently, this difference would be perceptible by natural observers, contrary to the equivalent nongravitational case in which no shifting occurs. For instance, length standards of different constructions would be naturally observed to be different right next to each other. Note that the length perturbations, quantified as a universally applied fractional change in length, *could vary by direction*, which is assumed as a possibility in subsequent development. But so long as the directionally dependent length perturbations are universally applied to all objects, a local natural observer using gravity shifted length standards, of any construction, will not be able to tell the difference between this case and the equivalent nongravitational case.

As will be generally shown, and as demonstrated in the examples above, the existence of universal gravity shifts *allows* systems to be posed in absolute flat spacetime and yet still yield observation, specifically natural observation, that adheres to the equivalence principle. Turning this around, adherence to the equivalence principle in a flat spacetime framework *requires* the existence of universal gravity shifts and the use of natural observation, as proven using the temporal examples above. The gravitational theory presented here is referred to as "gravity shift theory" (as coined above) since gravity shifts may be used to depict the gravitational field and determine all gravitational phenomena.

3.2. Establishing the Partner Relation and Shift Tensor

As gravity shifts are the dimensional perturbations of objects, then any gravity shifted object may be considered to be the result of applying gravity shifting to it from an original "unshifted" state. Indeed, in the above discussion, the notion of dimensional perturbations being applied to objects presupposes that these objects were originally in an unshifted state and then gravity shifting was applied. A (gravity) shifted object and its corresponding unshifted counterpart are referred to as being "partners" (from the summary), with again the "unshifted partner" being an object without gravity shifting applied, and the "shifted partner" being the corresponding gravity shifted object. Gravity shifts are *defined* then in terms of dimensional relationships between partner objects. Under this definition, the "material content" of the shifted partner object—meaning its matter and nongravitational fields—is *identical to* the material content of its unshifted partner. Partner objects "share" then the *same* material content, with the only difference between the partners being the dimensional perturbations of the material content of the shifted object relative to the unperturbed unshifted partner with the same content. As shown below, gravity shifting alters the dynamic properties of objects but does not alter the makeup of their material content. *The objects actually present in a gravitational field are all gravity shifted objects, whereas the unshifted partner objects are the hypothetical objects obtained if gravitation were removed in theory*.

The events locating the material content of objects provide both the spatial and temporal locations of their matter and nongravitational field contributors. For example, the events locating an array of atoms, idealized as point particles, each consist of an event giving both the spatial location of an atom and a particular time in its evolution. For a nongravitational field example, events may be used to locate the beginning and end of a cycle of the electromagnetic field for light. Events locating the material content of objects are referred to as being "tied" to the material content. Since gravity shifting consists of dimensional perturbations of the material content of shifted objects relative to their unshifted partners, then for each "unshifted partner event" x_{US} tied to a given material component of an unshifted partner object, a corresponding "shifted partner event" x_S is yielded that is tied to the same component of the shifted partner object, in each case giving the spacetime location of the shared material component. For the "atom array" example, the unshifted event x_{US} tied to an atom in the unshifted partner, giving the atom's spatial location and evolution time in the unshifted array, yields the shifted partner event x_S tied to the same atom in the shifted partner, now giving the atom's spatial location and evolution time in the shifted array. Under the dimensional perturbations of the

shared material content of partner objects, the spacetime location of an event x_{US} tied to the material content of an unshifted object, in general differs from the location of the partner event x_S tied to the same material content in the shifted partner object. As will become evident below, this perturbation in the locations of events tied to the shared material content of partner objects, universally applied for partner objects of all constructions, *provides a convenient and powerful means of depicting gravity shifts.*

As it is the shifted objects that are actually present in a gravitational field, the shifted partner events x_S tied to the shifted objects are the *actual* events x of the absolute spacetime manifold. This identification is formally expressed by

$$x_S = x, \qquad x_S^\alpha = x^\alpha, \tag{13}$$

where the right-hand formula gives their coordinate location equality. On the other hand, the unshifted partner events, x_{US}, tied to the unshifted partner objects, are the *hypothetical* events obtained if gravitation were removed in theory. For convenience going forward, the substitution of x for x_S and vice versa is automatically assumed for formulations or discussion, unless substitution is explicitly stated for clarity.

Variations in gravity shifting with location and over time will yield a complex relationship between *extended* partner objects, as discussed below. However, if the partner objects are small enough spatially, and the temporal extents for their evaluation are short enough, the gravity shifting over their spacetime extents may be approximated as being *uniform*, meaning both spatially and temporally constant. Such partner objects are referred to as being "local," with the uniform gravity shifting over their spacetime extents referred to as "uniform-scale" gravity shifting. The ability to treat partner objects as being "local" depends on the context, but it is always possible to use partner objects that are small enough such that the gravity shifting over their spacetime extents may be approximated as being uniform.

To obtain a formal expression for uniform-scale gravity shifting, consider infinitesimal *spacetime displacements* $d\vec{x}$ "tied" to the shared material content of local partner objects, meaning that the two events $x(\text{head})$ and $x(\text{tail})$ at the "head" and "tail" of each displacement, referred to as the "displacement event pair," are both tied to the shared material content. Then an infinitesimal unshifted partner displacement, $d\vec{x}_{US} = x_{US}(\text{head}) - x_{US}(\text{tail})$, tied to the material content of the unshifted partner object, yields a shifted partner displacement, $d\vec{x}_S = x_S(\text{head}) - x_S(\text{tail})$, tied to the same material content of the shifted partner object, where $x_{US}(\text{head})$ and $x_S(\text{head})$ are partner events, and $x_{US}(\text{tail})$ and $x_S(\text{tail})$ are partner events. It can be seen that uniform-scale gravity shifting is linear, since for example, doubling the span of an unshifted displacement $d\vec{x}_{US}$ yields a doubling of the resultant span of the shifted partner displacement $d\vec{x}_S$. Therefore, the formal relation between the infinitesimal partner displacements depicting uniform-scale gravity shifting, applicable for local objects, *must be a linear transformation*. Writing down the *most general possible* linear transformation (utilizing the component forms dx_{US}^μ and dx_S^α of the partner displacement vectors) yields

$$dx_S^\alpha = S^\alpha_{\bar{\mu}} \, dx_{US}^\mu \tag{14}$$

depicting the most general possible uniform-scale gravity shifting for local partner objects, establishing the "partner relation" (1) stated in the summary. The quantity $S^\alpha_{\bar{\mu}}$ providing the linear transformation implementing the gravity shifting is referred to as the "shift tensor," since it will be shown to be a rank-2 *tensor*. Though the partner relation has the appearance of a differential coordinate transform, both partner displacements are given in the *same coordinates*, so a "bar" is conveniently used over the second index in $S^\alpha_{\bar{\mu}}$ to designate conversion from unshifted displacements, and no bar is used over the first index to designate conversion to shifted partner displacements. In deep space far from gravitation, the shift tensor $S^\alpha_{\bar{\mu}}$ becomes the identity tensor $\delta^\alpha_{\bar{\mu}}$, expressing the condition of no gravity shifting.

As stated above, gravity shifts may be employed to depict the gravitational field and determine all gravitational phenomena. This is demonstrated by showing that the shift tensor $S^{\alpha}_{\bar{\mu}}$ utilized in the partner relation (14), which formally depicts the gravity shifting, may indeed be so employed as found throughout all subsequent formulation. Based on this demonstration, this capability is taken as a "given" in further discussion.

Since the shifted partner events, x_S, tied to the shifted objects present in a gravitational system, are the absolute manifold events x that exist in actuality as per (13), then

$$\mathrm{d}x^{\alpha}_{S} = \mathrm{d}x^{\alpha} \tag{15}$$

for the shifted displacements $\mathrm{d}x^{\alpha}_{S}$ tied to local shifted objects. On the other hand, the unshifted partner displacements, $\mathrm{d}x^{\mu}_{US}$, tied to the local unshifted partner objects, are the hypothetical displacements obtained when gravitation is removed in theory. As shown below, the partner relation (14) is generally not an integrable condition (as discussed in the summary), so the local unshifted object displacements $\mathrm{d}x^{\mu}_{US}$ in the partner relation cannot in general be integrated over *extended* regions to yield hypothetical unshifted events x_{US} with locations x^{μ}_{US}.

Using the partner relation in global ICs, if the unshifted partner vector displacement $\mathrm{d}\vec{x}_{US}$ runs parallel to a spatial axis (designated by n), the spatial gravity shifting along the same spatial axis direction is given by $\mathrm{d}x^{n}_{S} = S^{n}_{\bar{n}} \, \mathrm{d}x^{n}_{US}$ (no sum), where $S^{n}_{\bar{n}} > 1$ yields expansion, $S^{n}_{\bar{n}} < 1$ yields contraction, and $S^{n}_{\bar{n}} = 1$ yields no shifting. If $\mathrm{d}\vec{x}_{US}$ runs parallel to the time axis, the temporal gravity shifting along the time axis direction is given by $\mathrm{d}x^{0}_{S} = S^{0}_{\bar{0}} \, \mathrm{d}x^{0}_{US}$. The condition $S^{0}_{\bar{0}} > 1$ yields "temporal expansion" where the time increases between events occurring in the material content, resulting in the physical processes slowing down. The condition $S^{0}_{\bar{0}} < 1$ yields "temporal contraction" resulting in the physical processes speeding up, and $S^{0}_{\bar{0}} = 1$ yields no temporal shifting so the rates of the physical processes do not change. For the above redshifting and corresponding atomic clock examples, the light transmitter and receiver, as well as the clocks, may be treated as the shifted partners of local partner objects over which the shifting may be approximated as being uniform at their locations of use. Using $\mathrm{d}\vec{x}_{US}$ running parallel to the utilized IC time axis in these examples to depict their unshifted partner temporal displacements, the temporal gravity shifting of the physical processes in the transmitter, receiver, and clocks may therefore be expressed by $\mathrm{d}x^{0}_{S} = S^{0}_{\bar{0}} \, \mathrm{d}x^{0}_{US}$ at their locations. Due to naturally measured adherence to the equivalence principle at each location, both the light source nuclear resonance and the clock atomic resonance at the bottom of the tower are shifted by the same $S^{0}_{\bar{0}}(\text{bottom})$, and both the light receiver resonance and the clock resonance at the top of the tower are shifted by the same $S^{0}_{\bar{0}}(\text{top})$. The naturally measured redshifting (using frequency) and clock rate ratio (bottom over top) are both equal to $S^{0}_{\bar{0}}(\text{top})/S^{0}_{\bar{0}}(\text{bottom})$, which is less than unity and equal to the same ratio obtained if the tower accelerated at 1g relative to an absolute inertial frame of reference without gravitation present, satisfying the EEP.

As discussed above, the equivalence principle requires that the gravitational field universally shifts the lengths of all local objects at a location to the same fractional degree in each direction, and universally shifts the rates of their processes to the same fractional degree, since otherwise differences between the lengths and/or rates would be naturally perceived, differing from the equivalent inertial case in violation of the equivalence principle. Prior to constraining the shift tensor $S^{\alpha}_{\bar{\mu}}$ (as done below), it can be seen that universal shifting holds for all possible forms of gravity shifting applied to all local objects at a location, as required to satisfy the equivalence principle for natural observers. Therefore, the most general possible uniform-scale gravity shifting, as given by the partner relation (14) with an unconstrained shift tensor, *is universally applicable for all possible local objects placed at any location*.

The *local* unshifted partner event field, x_{US}, tied to local unshifted objects, may be generated by running the unshifted partner displacements $\mathrm{d}\vec{x}_{US}$ from their tails $x_{US}(\text{tail})$ set at a single "unshifted shift origin" X_{US}, so $x_{US} = X_{US} + \mathrm{d}\vec{x}_{US}$. Similarly, the *local* shifted partner event field, x_S, tied to the local shifted partner objects, may be generated by running the shifted partner displacements $\mathrm{d}\vec{x}_S$ from their tails $x_S(\text{tail})$ set at a single "shifted shift origin" X_S, so $x_S = X_S + \mathrm{d}\vec{x}_S$. The utilized partner

displacements $d\vec{x}_{US}$ and $d\vec{x}_S$ are obtained from the partner relation (14) applied on the uniform scale, so a *fixed* shift tensor is used to generate the partner displacements and therefore the local partner event fields x_{US} and x_S. This method yields a depiction of gravity shifting on the uniform scale using the local partner event fields x_{US} and x_S tied to the shared material content of local partner objects.

However, the partner relation is specifically a relation between event *displacements* as opposed to being a relation between events themselves. As a result, the relative separation $X_S - X_{US}$ between the partner shift origins is left *unspecified* by the partner relation, yielding an unspecified overall relative separation between the local partner event fields x_{US} and x_S. Therefore, *under a given shift tensor, the relationship between all local partner objects is indeterminate by a single overall spacetime translation that is the separation $X_S - X_{US}$ of the partner shift origins.* As previously stated, gravity shifts may be employed to determine all gravitational phenomena in GS theory. With the partner relation (14) the formal expression of gravity shifting, then since the relative separation of local partner objects is not specified by the partner relation, their relative separation plays no part in determining gravitational phenomena. This is made clear under the above-given recognition that all gravitational phenomena may be quantified by only the shift tensor $S^\alpha{}_{\bar{\mu}}$ in the partner relation, which has the same value regardless of the relative separation of local partner objects utilized. Therefore, when evaluating local partner objects for a given shift tensor, *the overall separation of the partner objects has no physical significance, so the indeterminacy of their overall separation is of no consequence.* If desired, *any arbitrary separation $X_S - X_{US}$ may be specified* for the two shift origins without impacting the physics, yielding a specific hypothetical local unshifted event field x_{US} corresponding to the known shifted event field x_S tied to local shifted objects present in actuality. The hypothetical unshifted shift origin X_{US} may be placed then at the *same location* as the actual shifted shift origin X_S, yielding local partner event fields mapped by running partner displacements from a "common shift origin" $X \equiv X_{US} = X_S$. As the shift tensor utilized is fixed over the uniform scale utilized, its fixed value may be set to the shift tensor value $S^\alpha{}_{\bar{\mu}}|_X$ at the common shift origin X. Substituting the component form of $d\vec{x}_{US} = x_{US} - X$ and $d\vec{x}_S = x_S - X$ into the partner relation (14), with $S^\alpha{}_{\bar{\mu}}|_X$ as the fixed shift tensor, yields the local "event partner relation"

$$(x_S^\alpha - X^\alpha) = S^\alpha{}_{\bar{\mu}}|_X (x_{US}^\mu - X^\mu) \quad \text{(infinitesimal)}, \tag{16}$$

applicable in any coordinates. Technically, the event partner relation is generally applicable over *infinitesimally sized* spacetime regions, as indicated, since it gives *the displacement-based partner relation* (14) *in equivalent event-based form* with $x_{US} - X$ and $x_S - X$ the infinitesimal partner displacements $d\vec{x}_{US}$ and $d\vec{x}_S$. Therefore, the event partner relation (16) *is not obtained by integrating the displacement partner relation* (14), which is indeed not generally integrable (as shown below). Any location of interest x in an actual shifted system may be used as a common shift origin $X = x$, with the local partner event fields mapped over the surrounding infinitesimally sized region with then uniform shifting. The use of a common shift origin provides a convenient means to compare partner events, displacements, and objects. This is the practice followed unless otherwise specified, so *partner displacements and infinitesimally sized local partner objects are considered to reside at the same location $x = X$.*

Gravity shifts may be "directly" characterized by the dimensional perturbations of objects themselves, referred to as the "object form" of depicting shifts. The object form is less general than the "event form" developed above, since specific objects are utilized. Due though to the universality of gravity shifts, the shifting of specific "test objects" may be used to represent the shifts for all objects. Examples are the use of "rigid" metal rulers to represent spatial shifts of objects, and "fixed resonance" clocks to represent temporal shifts, which only dimensionally change then under application of gravity shifts (assuming imposed nongravitational forces are not too severe). The representative test objects may be used to generate the field of partner displacements and/or events, which may in turn be utilized to determine the shifting for all objects, establishing an *equivalency* between the object and event forms of depicting gravity shifts. The object form may be made more general by using generic "test matter" as opposed to specific test objects, quantifying gravity shifts in terms of, for instance,

fractional length and duration changes for any matter. In the development presented here, the event form and object form of depicting gravity shifts are used interchangeably.

Consider gravity shifting occurring within "extended" objects over which the gravity shifting is not uniform. Now an extended object under gravitation will in general have gravitational stresses and strains introduced. Removing gravitation over the entire extended object removes its gravity shifting as well. The relation between an extended object and its unshifted counterpart will in general then be nonlinear and complex due to the introduction of stresses and strains when gravitation is introduced. However, *given* the stressed and strained state of matter in an extended object under gravitation, if the gravity shifting were removed for *only* an infinitesimally sized portion of that object, that portion would unshift the same as any "isolated" (i.e., not embedded within an extended object) infinitesimal object at the same location. This must be the case due to the universality of gravity shifting yielded under uniform-scale application of the equivalence principle. Therefore, *for infinitesimal portions of extended objects, the partner relation* (14) *holds if the "isolated unshifting condition" is assumed, and similarly the equivalent local event partner relation* (16) *holds*. For all extended objects, gravity shifting of infinitesimal portions is evaluated under the isolated unshifting condition unless otherwise specified. This practice yields *universal applicability of the partner relation and the local event partner relation for both local and extended objects*. Along with this, the identification of shifted partner displacements and events with actual displacements and events, as expressed by (15) and (13), is also *universally applicable for both local and extended objects*.

"Gravity shifts" are limited to the possible relations that may occur between partner objects as given by the partner relation (14) (with the local event partner relation (16) the partner relation in event form), as this relation is the most general expression that may be formed consistent with the given characterization of gravity shifts in terms of linear transformations between partner displacements tied to partner objects. In the absence of any criteria limiting its components, however, the 4×4 shift tensor $S^{\alpha}_{\bar{\mu}}$ at each location consists of 16 components that could have any values. Therefore, without additional criteria being imposed, any possible relations between partners when employing arbitrary shift tensors are considered to be candidates of "gravity shifts." Gravity shifts could include then more than just fractional changes in lengths and durations, such as spatial rotation or shearing, as well as "temporal and spatial cross shifting." As shown below though, the absolute flat spacetime and equivalence principle postulates may be employed to obtain various physical criteria limiting gravity shifting, resulting in a shift tensor that is highly constrained. Indeed, as discussed in the summary, at any location the shift tensor $S^{\alpha}_{\bar{\mu}}$ may be *diagonalized* in global ICs using Lorentz transforms, resulting in "intrinsic" gravity shifting consisting of purely spatial and temporal dimensional shifts, i.e., just fractional changes in lengths and durations.

3.3. Transformational Properties of Gravity Shifting

In GS theory the flat spacetime structure is "absolute," meaning that the spacetime structure is unaffected by the material content (as discussed above). Therefore, *the flat manifold of events exists a priori, unaffected by material content and therefore gravitation*. Since all of the events used to characterize gravity shifted objects are events of the absolute manifold, then their coordinate locations transform according to the flat spacetime transformational properties. When using the partner relation (14) (or the equivalent local event partner relation (16)), the shifted partner events x_S tied to the material content of shifted objects are part of the inventory of events used to characterize shifted objects, so they are events of the absolute manifold as any other events are, transforming then under the flat spacetime transforms. When gravitation is removed in theory, unshifted objects remain, and again the events used to characterize them are events of the absolute manifold. The unshifted partner events x_{US} tied to the material content of the unshifted objects are also then events of the absolute manifold, again transforming under the flat spacetime transformations. Therefore, when transforming between absolute inertial frames, the global IC locations of events tied to partner objects transform according to the global Lorentz transforms. As a result, the global IC given partner event displacements

tied to partner objects, formed by using event pairs tied to the objects at the heads and tails of the displacements, transform according to the global Lorentz transforms as well, or formally

$$\mathrm{d}\check{x}'^{\beta}_{S} = \Lambda^{\beta'}{}_{\alpha}\,\mathrm{d}\check{x}^{\alpha}_{S}, \qquad \mathrm{d}\check{x}'^{\nu}_{US} = \Lambda^{\nu'}{}_{\mu}\,\mathrm{d}\check{x}^{\mu}_{US}, \tag{17}$$

where the "check" indicates global IC use for clarity. These transforms may be inverted and substituted into the partner relation $\mathrm{d}\check{x}^{\alpha}_{S} = \check{S}^{\alpha}{}_{\bar{\mu}}\,\mathrm{d}\check{x}^{\mu}_{US}$, as given in the original "unprimed" global ICs, to obtain the corresponding partner relation $\mathrm{d}\check{x}'^{\beta}_{S} = \check{S}'^{\beta}{}_{\bar{\nu}}\,\mathrm{d}\check{x}'^{\nu}_{US}$ in any "primed" global ICs, where

$$\check{S}'^{\beta}{}_{\bar{\nu}} = \Lambda^{\beta'}{}_{\alpha}\check{S}^{\alpha}{}_{\bar{\mu}}\Lambda^{\mu}{}_{\nu'}. \tag{18}$$

The gravity shifts, as given by the partner relation and shift tensor, therefore adhere to the global Lorentz transforms since the events characterizing gravity shifts adhere to the global Lorentz transforms.

　　As discussed in the summary, a number of gravity shift based theories have been given (at various stages of completion). These formulations may initially pose gravity shifts in an absolute inertial frame, but for every available theory the event locations under the specified gravity shifting do not adhere to the global Lorentz transforms (to the author's knowledge), either stated explicitly or implicitly inherent in the formulation such as the "field theory" approach to general relativity [12], or the transform properties are left undetermined. So the given shifting is not explicitly specified as adhering to global Lorentz transforms. *The lack of adherence to the global Lorentz transforms for the gravity shifting specified subsequently "breaks down" the ability of a gravity shift based theory to explicitly preserve a flat, and therefore absolute, spacetime structure.* As gravity shifts are a *material* property, then the lack of adherence to the global Lorentz transforms yields a materially dependent spacetime structure, thereby reintroducing fundamental incompatibilities with quantum theory (see the Postulates section 2 for background). It is imperative then that this adherence be satisfied if it is desired to explicitly preserve a flat, and therefore absolute, spacetime structure. For the specified gravity shifts in GS theory, the adherence to the global Lorentz transforms is approached (above) by recognizing that for an absolute flat spacetime structure, the flat manifold of events exists a priori, unaffected by material content. Then since a characterization of gravity shifts is given *based* on events—i.e., partner event displacements tied to local partner objects—adherence of the specified gravity shifts to the global Lorentz transforms is established via the materially independent adherence of the depicting events. It can be seen that the characterization of gravity shifts in terms of events is "very powerful" since it is ideally suited for establishing that gravity shifts must adhere to the global Lorentz transforms, explicitly maintaining adherence to the absolute flat spacetime postulate.

　　Since partner displacements are again displacements running between events of the absolute flat manifold, then partner displacements transform between arbitrary available flat manifold coordinates according to

$$\mathrm{d}x'^{\beta}_{S} = L^{\beta'}{}_{\alpha}\,\mathrm{d}x^{\alpha}_{S}, \qquad \mathrm{d}x'^{\nu}_{US} = L^{\nu'}{}_{\mu}\,\mathrm{d}x^{\mu}_{US}. \tag{19}$$

These transforms may be inverted and substituted into the partner relation $\mathrm{d}x^{\alpha}_{S} = S^{\alpha}{}_{\bar{\mu}}\,\mathrm{d}x^{\mu}_{US}$, as given in any "unprimed" coordinates, to obtain $\mathrm{d}x'^{\beta}_{S} = S'^{\beta}{}_{\bar{\nu}}\,\mathrm{d}x'^{\nu}_{US}$ in any "primed" coordinates, where

$$S'^{\beta}{}_{\bar{\nu}} = L^{\beta'}{}_{\alpha}S^{\alpha}{}_{\bar{\mu}}L^{\mu}{}_{\nu'}. \tag{20}$$

It can be seen that *the partner relation is manifestly covariant*, where the so-called "shift tensor" is indeed a *tensor quantity* since $S^{\alpha}{}_{\bar{\mu}}$ transforms as a mixed rank-2 tensor.

　　With the shift tensor established as a tensor quantity in flat spacetime, its indices may be raised and lowered using the absolute metric in the usual fashion. For example, $S_{\alpha\bar{\mu}} = a_{\alpha\beta}S^{\beta}{}_{\bar{\mu}}$ and $S^{\alpha\bar{\mu}} = a^{\mu\nu}S^{\alpha}{}_{\bar{\nu}}$. Then $S^{\alpha}{}_{\bar{\mu}}$, $S_{\alpha\bar{\mu}}$, $S^{\alpha\bar{\mu}}$, and $S_{\alpha}{}^{\bar{\mu}}$ are equivalent forms of the shift tensor. Taking the determinant of (20) on both sides, then $|S'^{\beta}{}_{\bar{\nu}}| = |L||S^{\alpha}{}_{\bar{\mu}}||L^{-1}|$. Since $|L||L^{-1}| = 1$, then $|S'^{\beta}{}_{\bar{\nu}}| = |S^{\alpha}{}_{\bar{\mu}}|$. The determinant of the shift tensor $S^{\alpha}{}_{\bar{\mu}}$ is therefore *invariant* under all coordinate transforms, as is generally the case

for mixed rank-2 tensors. As with all scalars formed from tensors, the trace $S^\alpha_{\bar\alpha}$ of the shift tensor is a *scalar invariant*.

3.4. The 1-to-1 Requirement

If two events tied to *separate* material components of an unshifted object (at arbitrary times) were then coincident (both spatially and temporally) in the shifted partner, the material components would "spatially overlap." Due to the linearity of gravity shifting on the uniform scale, in addition all unshifted material tied to the "event line" running between the two events would also spatially overlap in the shifted partner, resulting in an *infinite density* "matter singularity" considered physically impossible in flat spacetime. Note that in general relativity, matter can "spatially overlap" on itself at *spacetime* singularities to yield infinite densities, such as predicted at the centers of black holes. Infinite curvature at the spacetime singularities provides an explanation as to how this matter overlap can occur, allowing the matter overlap via "compaction" of the spacetime structure containing the matter. But with spacetime assumed to be flat in GS theory, spacetime singularities do not occur, so the infinite densities that would occur under spatial overlap are considered to be nonphysical. The predicted singularities in black holes are examples of "spacetime pathologies" in general relativity that allow matter conditions considered physically impossible in flat spacetime. On the temporal front, if two events tied to separate occurrences for a process in a *single* material component of an unshifted object, such as an atom, were then coincident in its shifted partner, the physical process would run at *infinitely fast* rates. Such a "temporal singularity" is considered to be impossible in flat spacetime due to the absence of any spacetime pathologies allowing for infinitely fast rates. Summarizing, in GS theory, adherence to the absolute flat spacetime postulate bars the existence of "spacetime pathologies," so separate events tied to unshifted objects cannot be coincident in the shifted partners, as this would result in physically impossible matter and/or temporal singularities. The barring of shifted event overlap is referred to as the "overlap restriction."

The tensor partner relation (14) may be given in matrix form by

$$\{\mathrm{d}x_S\} = \{S\}\{\mathrm{d}x_{US}\},\tag{21}$$

where $\{\mathrm{d}x_S\} \equiv \{\mathrm{d}x_S^\alpha\}$ and $\{\mathrm{d}x_{US}\} \equiv \{\mathrm{d}x_{US}^\mu\}$ are the column vectors for the partner vector displacements, and $\{S\} \equiv \{S^\alpha_{\bar\mu}\}$ is the 4×4 square matrix for the shift tensor. The formal overlap restriction barring shifted event overlap is given in matrix form by

$$\{S\}\{\mathrm{d}x_{US}\} = \{\mathrm{d}x_S\} \neq \{0\} \quad \text{if } \{\mathrm{d}x_{US}\} \neq \{0\}.\tag{22}$$

From linear algebra, this condition is satisfied if and only if the square shift matrix $\{S\}$ has an *inverse* $\{S^{-1}\}$, yielding the "reverse" shift tensor $S^{\bar\mu}_\alpha$ satisfying the conditions

$$S^{\bar\mu}_\alpha S^\alpha_{\bar\nu} = \delta^\mu_\nu, \qquad S^\alpha_{\bar\mu} S^{\bar\mu}_\beta = \delta^\alpha_\beta.\tag{23}$$

Of course, *the invertibility of the shift tensor $S^\alpha_{\bar\mu}$ limits what values the shift tensor components may have.*

Applying $S^{\bar\mu}_\alpha$ to both sides of the partner relation (14), then

$$\mathrm{d}x_{US}^\mu = S^{\bar\mu}_\alpha \, \mathrm{d}x_S^\alpha,\tag{24}$$

which is the "reverse" partner relation giving unshifted partner displacements by applying the reverse shift tensor to shifted partner displacements (establishing (3)). The partner relation (14) is optionally referred to as the "forward" partner relation for distinguishing it from the reverse partner relation. Since both partner displacements are given in the same coordinates for both the forward and reverse partner relations, the "bar" is consistently used to indicate the unshifted displacement indice for both the "forward" shift tensor $S^\alpha_{\bar\mu}$ and the reverse shift tensor $S^{\bar\mu}_\alpha$ used in their respective partner relations, with no bar indicating the shifted displacement indice for both. Similar to the forward partner relation

and shift tensor, the partner displacement transforms (19) may be substituted into the reverse partner relation to show that it is manifestly covariant, as well as to show that $S^{\bar{\mu}}{}_{\alpha}$ is indeed a tensor quantity. Applying $S^{\bar{\mu}}{}_{\alpha}|_X$ to both sides of the forward event partner relation (16) yields the local reverse event partner relation

$$(x^{\mu}_{US} - X^{\mu}) = S^{\bar{\mu}}{}_{\alpha}|_X (x^{\alpha}_S - X^{\alpha}) \quad \text{(infinitesimal)}, \tag{25}$$

giving *the displacement-based reverse partner relation* (24) *in equivalent event-based form* with $x_{US} - X$ and $x_S - X$ the infinitesimal partner displacements $d\vec{x}_{US}$ and $d\vec{x}_S$. Similar to the forward case, the reverse event partner relation (25) is *not obtained by integrating the reverse displacement partner relation* (24), which is indeed not generally integrable (as shown below).

    From linear algebra, a square matrix based linear transformation is 1-to-1 if and only if it is invertible. Therefore, *the partner relation* (14) *is a 1-to-1 relation between partner displacements*. Now the events tied to the material content of partner objects form a *continuum* for each partner since the material content does (such as the continuous classical electromagnetic field always present in theory). Since the events at the endpoints of partner displacements—i.e., the displacement event pairs—are taken from the continuum of events tied to the material content, the partner displacements in the partner relation therefore each form a continuum. As the partner relation is a relation between partner displacements in common coordinates used for both, then the partner relation (14) is formally a *homeomorphism* (i.e., a 1-to-1 continuous relation in common coordinates) between partner displacements. As a result, the equivalent local event partner relation (16) is also a homeomorphism, *yielding gravity shifting as a 1-to-1 "morphing" between the shared material content of partner objects tied to the local partner event fields*.

    With the partner relation (14) being a 1-to-1 linear transformation between spaces of the same dimension, then it is also *onto*. Indeed, for a given *fixed* shift tensor obtained by running the shift tensor at a location over all of spacetime, the domain of all possible unshifted displacement 4-vectors, which runs over all spacetime, yields the range consisting of all possible shifted displacement 4-vectors covering all spacetime, and vice versa. Therefore, *for a fixed shift tensor running over all of spacetime, the 1-to-1 relation of partner event fields, as obtained from the event partner relation* (16), *spans all of spacetime for both fields*. With shift tensors for actual systems typically varying with location though, then of course the application of fixed shift tensor based shifting is limited to the uniform scale.

    Using the forward and reverse forms of the partner relation, if a displacement is the zero vector, its partner is zero as well. Therefore, with the partner relation being 1-to-1, a nonzero displacement cannot yield a zero vector partner. So separate events tied to either a shifted or unshifted object cannot overlap in its partner (as established above for forward shifting, here including reverse shifting).

    Under the usual exclusion of discontinuity of physical effects in classical physics, *the shift tensor components are considered to be spacetime differentiable to all orders*. Therefore, each forward and reverse shift tensor component is *continuous* with respect to spacetime location, which is a key physical limitation.

    In order that the partner relation in either forward or reverse form is invertible, then from linear algebra the determinants of the forward and reverse shift tensor matrices satisfy $|S| \neq 0$ and finite and equivalently $|S^{-1}| \neq 0$ and finite. Due to continuity of the shift tensor components, then the forward and reverse shift tensor determinants are continuous as well. In deep space, the forward and reverse shift tensors are given by the identity matrix $\{I\}$ with unity determinant. Now since the shift tensor in either form can never have a zero determinant, then it can never be negative at any location, as continuity out to any deep-space location would imply that at some location the determinant would have to be zero in order to yield the deep-space value of unity. Therefore, since the shift tensor in either form must have a finite determinant, the forward and reverse shift tensors are limited by the constraints

$$0 < |S| < \infty, \qquad 0 < |S^{-1}| < \infty. \tag{26}$$

Either of these equivalent constraints may be used as the *single* condition required to yield an invertible 1-to-1 partner relation at all locations that is consistent with the deep-space "no shift" condition. It has

been shown that the determinant of the shift tensor is invariant under coordinate transforms, so the conditions (26) hold in any coordinates.

3.5. The Natural Metric, and Absolute and Natural Measurement of Absolute Proper Intervals

For establishing the properties of gravity shifts, use of the natural metric is helpful. Fundamental natural metric formulation and characterization is provided here, which includes use of the absolute metric, as well as evaluation of absolute and natural measurement of absolute manifold proper intervals ds_A to help characterize the natural metric. Absolute and natural measurement cannot be systematically evaluated until morph-based establishment of the SEP and EEP, but absolute measurement of absolute proper intervals is straightforward conceptually, and as discussed in the summary, the partner equivalence property may be employed to evaluate natural measurement of absolute proper intervals.

The absolute metric line element $ds_A^2 = a_{\mu\nu} \, dx^\mu \, dx^\nu$ (10) gives the absolute manifold proper interval ds_A obtained from an actual displacement dx^α. From the summary, the class of absolute observers uses absolute instruments that have been corrected for the gravity shifting that has been applied to them, where the use of a shift-corrected instrument is the same as use of its hypothetical unshifted partner, which is not perturbed by gravity shifting, yielding accurate measurements of quantities. Using shift-corrected clocks and rulers, then *absolute observers accurately measure the absolute manifold proper intervals* ds_A, formally expressed by

$$ds_{A(A)} = ds_A \tag{27}$$

for temporal, spatial, and null absolute intervals. Therefore,

$$ds_{A(A)}^2 = ds_A^2 = a_{\mu\nu} \, dx^\mu \, dx^\nu, \tag{28}$$

which implies that *absolute observers accurately measure the absolute manifold metric* $a_{\mu\nu}$. This is the reason for the nomenclature "absolute observers" (as discussed in the summary).

Using (15) to substitute dx_S^α for dx^α in the absolute metric line element (10) yields the equivalent "shifted line element"

$$ds_S^2 = a_{\alpha\beta} \, dx_S^\alpha \, dx_S^\beta = ds_A^2, \tag{29}$$

where the "shifted" proper interval ds_S is the absolute manifold proper interval for the shifted partner displacement dx_S^α. Note the equality of ds_S and ds_A obtained from this derivation, with the shifted and actual displacement intervals used interchangeably going forward similar to the displacements dx_S^α and dx^α. With ds_S and ds_A the same, their absolute measurements $ds_{S(A)}$ and $ds_{A(A)}$ are the same, yielding

$$ds_{S(A)} = ds_{A(A)} = ds_A = ds_S \tag{30}$$

via (27). So *absolute observers accurately measure the shifted proper intervals* $ds_S = ds_A$.

The "unshifted" proper interval ds_{US} is the absolute manifold proper interval for the unshifted partner displacement dx_{US}^α, given by the "unshifted line element"

$$ds_{US}^2 = a_{\mu\nu} \, dx_{US}^\mu \, dx_{US}^\nu. \tag{31}$$

This is the proper interval yielded for the unshifted partner displacement dx_{US}^μ corresponding to the shifted partner displacement dx_S^α, so the unshifted proper interval ds_{US} is the unshifted partner of the shifted proper interval ds_S. There are therefore *two* absolute proper intervals of interest for a shifted/actual event displacement $dx_S^\alpha = dx^\alpha$, the shifted/actual proper interval $ds_S = ds_A$, and the hypothetical unshifted partner proper interval ds_{US}. Note that in general, the proper frames are different for partner displacements, so the measurement of $ds_S = ds_A$, as made by proper-frame absolute clocks and rulers, is generally in a different proper frame than the hypothetical measurement of ds_{US} made by the same standards.

As discussed in the summary, the class of natural observers uses raw shifted instruments that have not then been shift-corrected. So the natural proper interval standards consist of raw gravity shifted clocks and rulers used as is. To evaluate the natural measurement of absolute proper intervals ds_A, infinitesimally sized evaluation regions are utilized so that the shifting may be approximated as being uniform, with the uniform-scale gravity shifting given by the partner relation (14). The utilized shifted clocks and rulers are infinitesimally sized as well then (in theory), and may therefore be treated as the shifted partners of local partner objects under the uniform-scale gravity shifting. In the summary, the generally applicable partner equivalence property was invoked to imply that the natural measurement of shifted partner proper intervals, ds_S, with these shifted standards, yields values equal to the naturally measured unshifted partner proper intervals ds_{US} utilizing the unshifted partners of these standards, stated by $ds_{S(N)} = ds_{US(N)}$. This is proven here directly. Consider first natural measurement of shifted proper intervals ds_S when gravitation is removed, yielding the hypothetical unshifted partner case. Natural measurement of a spatial unshifted proper interval may be obtained by running between two "tickmarks" for an unshifted proper frame ruler, and natural measurement of a temporal unshifted proper interval may be obtained by running between two "clock ticks" for an unshifted proper frame clock, yielding the naturally measured unshifted proper interval values $ds_{US(N)}$. When gravitation is reintroduced to obtain the actual gravity shifted case, all objects present shift the same under the partner relation (14), including the proper interval standards. So the shifted partner case is just a dimensionally perturbed version of the unshifted partner case, yielding then the same utilized tickmarks for the rulers and clocks ticks for the clocks. As a result, natural observers, using the raw shifted partner proper interval standards, will again read the same hypothetical unshifted partner proper interval values $ds_{US(N)}$ off the interval standards. However, in actuality the displacements being spanned by the shifted proper interval standards are the shifted partner displacements dx_S^α, which have absolute manifold proper intervals ds_S, given by (29), that are the shifted partners of the unshifted partner intervals ds_{US}. So natural measurement of the shifted partner intervals, utilizing the shifted proper interval standards, yields values $ds_{S(N)}$ equal to $ds_{US(N)}$, completing the proof. As a hypothetical unshifted instrument accurately measures quantities due to no gravity shifting perturbing it, then $ds_{US(N)} = ds_{US}$ for natural unshifted interval measurement using unshifted proper interval standards. Combining these equalities and utilizing $ds_S = ds_A$, then

$$ds_N \equiv ds_{A(N)} = ds_{S(N)} = ds_{US(N)} = ds_{US}, \tag{32}$$

stating that *natural observers obtain the hypothetical unshifted partner proper interval values ds_{US} when measuring the shifted/actual absolute manifold proper intervals $ds_S = ds_A$ with raw gravity shifted clocks and rulers used as is*. Added to this equality is the definition of ds_N, which is convenient shorthand for $ds_{A(N)}$. This example demonstrates how universal gravity shifting, applied to both the instruments and the measured objects, yields adherence to the partner equivalence property. The above proof is applicable for the natural measurement of local objects, as well as for the natural measurement of infinitesimal regions of extended objects under the isolated unshifting condition, so (32) is *universally applicable*.

Combining (32) with (31) yields

$$ds_{A(N)}^2 = ds_{US}^2 = a_{\mu\nu}\, dx_{US}^\mu\, dx_{US}^\nu \tag{33}$$

for the natural measurement of shifted/actual proper intervals $ds_S = ds_A$ for the absolute manifold. Using the reverse partner relation (24) as well as $dx_S^\alpha = dx^\alpha$ and $ds_N \equiv ds_{A(N)}$, then (33) becomes

$$ds_{A(N)}^2 = ds_N^2 = g_{\alpha\beta}\, dx^\alpha\, dx^\beta, \tag{34}$$

where $g_{\alpha\beta}$ is the natural metric given by the covariant metric relation

$$g_{\alpha\beta} = a_{\mu\nu} S^{\bar\mu}{}_\alpha S^{\bar\nu}{}_\beta. \tag{35}$$

The above formulations, proven in detail here, were given in the summary. Again, the natural metric gives the naturally measured absolute proper intervals for actual displacements $\mathrm{d}x^\alpha$, having then the values $\mathrm{d}s_{A(N)} = \mathrm{d}s_N$. Natural observers therefore perceive the absolute flat spacetime manifold to have the metric $g_{\alpha\beta}$, which yields a naturally perceived curved manifold due to the reverse partner relation (24) not being an integrable condition in general (as shown below). An examination of (35) shows that $g_{\alpha\beta}$ is a *symmetric* tensor, as expected for a metric.

The natural proper intervals (34) dictate the timelike, spacelike, and null "interval categories" for natural observation of the shifted/actual partner displacements $\mathrm{d}x_S^\alpha = \mathrm{d}x^\alpha$. Similarly, the absolute proper intervals (31) dictate the interval categories for unshifted displacements $\mathrm{d}x_{US}^\mu$. Via (32), *the timelike, spacelike, or null natural interval category for a naturally observed shifted/actual displacement,* $\mathrm{d}x_S^\alpha = \mathrm{d}x^\alpha$, *is the same as the absolute interval category for its unshifted partner displacement* $\mathrm{d}x_{US}^\mu$.

From the metric relation (35), the inverse natural metric may be given by the "inverse metric relation"

$$g^{\alpha\beta} = S^\alpha_{\ \bar\mu} S^\beta_{\ \bar\nu} a^{\mu\nu}. \tag{36}$$

This can be seen by forming $g^{\alpha\beta} g_{\alpha\mu}$, substituting (35) and (36) for $g_{\alpha\mu}$ and $g^{\alpha\beta}$, and using the shift tensor inverse property (23) as well as the absolute metric inverse property $a^{\alpha\beta} a_{\alpha\mu} = \delta^\alpha_{\ \mu}$ to obtain the delta tensor $\delta^\alpha_{\ \mu}$. An examination of (36) shows that $g^{\alpha\beta}$ is a *symmetric* tensor, as expected for a metric inverse.

As demonstrated above, the equivalence principle holds for natural observers, which is subsequently proven in detail. So the natural metric $g_{\alpha\beta}$ has the Minkowski value $\eta_{\alpha\beta}$ in the free-fall frames due to naturally perceived inertial behavior. The natural metric therefore has Lorentz signature $(-,+,+,+)$ just as the absolute metric does. Again, the Minkowski metric valued natural metric is the metric exclusively utilized in all nongravitational physics laws when given in their inertial forms as naturally observed in free-fall frames, resulting in the natural metric acting as the "gravitational metric" when covariant formulation of the nongravitational laws applicable in all coordinates is made. Gravity shift theory falls then into the category of a "bimetric" theory, utilizing the absolute metric $a_{\mu\nu}$ to characterize the absolute flat spacetime manifold, and the additional natural/gravitational metric $g_{\alpha\beta}$ to characterize gravitation posed in the flat manifold. But as discussed in the summary, unlike available bimetric theories where the gravitational metric is given a priori, in GS theory the natural metric is a *derived* quantity via the metric relation (35), whereas the shift tensor is considered the *fundamental* quantity depicting the gravitational field.

In any coordinates, the metric relation (35) becomes $\{g\} = \{S^{-1}\}^T \{a\} \{S^{-1}\}$ in matrix form, yielding

$$|g| = |a||S^{-1}|^2. \tag{37}$$

With $0 < |S^{-1}| < \infty$ (26) in any coordinates, and with $|a| = |\eta| = -1$ in global ICs, then

$$-\infty < |g| < 0, \qquad -\infty < |g^{-1}| < 0 \tag{38}$$

in global ICs. Use was made of $|g||g^{-1}| = 1$ to obtain the range of the inverse natural metric determinant. Using $\{g'\} = \{L^{-1}\}^T \{g\} \{L^{-1}\}$ to transform the natural metric matrix between coordinates yields $|g'| = |g||L^{-1}|^2$, so the sign of the metric determinant will not reverse itself under any coordinate transforms. Therefore, (38) holds in *any coordinates* except at coordinate singularities. Similarly $|a'| = |a||L^{-1}|^2$ for the absolute metric under coordinate transforms, so

$$-\infty < |a| < 0, \qquad -\infty < |a^{-1}| < 0 \tag{39}$$

holds in *any coordinates* except at coordinate singularities. At coordinate singularities, both $|a|$ and $|g|$ are zero or infinite since $|L^{-1}|$ will be so.

3.6. The Speed, Temporal, and Null Constraints

As unshifted light is yielded when gravity shifting is removed, then in the global ICs of absolute inertial frames, the unshifted light speed $c_{US} = 1$ (in geometrized units) is the Lorentz invariant speed of light without gravitation present, obtainable by setting the proper interval ds_{US} to zero in the unshifted line element (31) with $a_{\mu\nu} = \eta_{\mu\nu}$. When gravitation is reintroduced, then as will be shown, application of the partner relation to hypothetical unshifted light yields actual shifted light that does not move at the Lorentz invariant unshifted light speed (in general). With spacetime postulated to be absolute and flat, however, the global IC locations of the absolute manifold of events obey the global Lorentz transforms, even when gravitation is present. Therefore, there exists a Lorentz invariant "null speed" $v_{Null} = 1$ for the absolute manifold even when gravitation is present, meaning the IC speed of particles that yields a zero-valued absolute proper interval ds_A for the absolute manifold line element (10) with $a_{\mu\nu} = \eta_{\mu\nu}$. Therefore,

$$v_{Null} = c_{US} = 1, \tag{40}$$

expressing the IC equality of the invariant null and unshifted light speeds in any inertial frame. The "null cone" of the absolute manifold is therefore identical to the unshifted light cone, acting as a Lorentz invariant cone in all global ICs separating timelike and spacelike event paths, even in the presence of gravitation.

Since all of the events x_S tied to shifted objects present under gravitation are indeed events of the absolute flat manifold, *the usual spacetime limitations of special relativity (theory) hold for gravity shifted objects*. If, for instance, a shifted object moved faster than the null speed in one inertial frame, its event path would be spacelike for the absolute manifold, so there would exist an inertial frame where it would be found to move at *infinite* speed. Infinite propagation speeds are of course considered an impossibility in flat spacetime. Therefore, the following "speed constraint" holds (stated in the summary): *The speed of all gravity shifted objects in any absolute inertial frame must not exceed the Lorentz invariant null speed $v_{Null} = 1$, which is the speed of unshifted light.* Adherence to the speed constraint in one inertial frame implies adherence in all since the IC locations of events obey the global Lorentz transforms. Speed constraint adherence bars backwards causal temporal evolution in some frames due to faster than null speed travel in other frames.

Now an unshifted particle moving at a given "unshifted velocity" will yield a shifted particle moving at a partner "shifted velocity." To use the partner relation to relate these "partner velocities," partner displacements may be selected that give the infinitesimal movement of the particles in infinitesimal time. For each partner particle, the two events comprising a displacement event pair are tied then to different occurrences in the material processes of the particle as it moves, such as different occurrences in the internal processes of a moving atom, or different occurrences in the cycling of electromagnetic light. Using such displacement event pairs, the IC partner 3-velocities are given by (with Latin indices indicating the spatial directions)

$$v_{US}^n = \frac{dx_{US}^n}{dt_{US}}, \qquad v_S^n = \frac{dx_S^n}{dt_S}. \tag{41}$$

The partner relation gives $dx_S^n = S^n{}_{\bar{\mu}}\, dx_{US}^{\mu}$ and $dt_S = S^0{}_{\bar{\nu}}\, dx_{US}^{\nu}$ in global ICs, which when substituted into the right of (41) yields the "velocity relation"

$$v_S^n = \frac{S^n{}_{\bar{0}} + S^n{}_{\bar{k}} v_{US}^k}{S^0{}_{\bar{0}} + S^0{}_{\bar{l}} v_{US}^l}. \tag{42}$$

Expressing the IC 3-velocity by $\vec{v} = v\,\hat{r}$ for both partners (where v is the speed and $\hat{r}$ is the unit direction vector), the velocity relation is also given by

$$v_S^n = \frac{S^n{}_{\bar{0}} + v_{US} S^n{}_{\bar{k}} r_{US}^k}{S^0{}_{\bar{0}} + v_{US} S^0{}_{\bar{l}} r_{US}^l}, \tag{43}$$

yielding

$$v_S = \sqrt{v_S^{x2} + v_S^{y2} + v_S^{z2}}, \qquad r_S^n = v_S^n / v_S. \tag{44}$$

It can be seen that in general, the partner velocities are not the same in either speed or direction (in deep space, the identity shift tensor yields equal partner velocities, as expected). The velocity relation may be given in the "reverse" direction as well, with

$$v_{US}^n = \frac{S^{\bar{n}}{}_0 + v_S S^{\bar{n}}{}_k r_S^k}{S^{\bar{0}}{}_0 + v_S S^{\bar{0}}{}_l r_S^l}, \tag{45}$$

yielding

$$v_{US} = \sqrt{v_{US}^{x}{}^2 + v_{US}^{y}{}^2 + v_{US}^{z}{}^2}, \qquad r_{US}^n = v_{US}^n / v_{US}. \tag{46}$$

The "reverse" velocity relation (45) may be derived from the ("forward") velocity relation (43) and vice versa, so they are equivalent.

As in the shifted object case, the speed of unshifted objects is limited by the null speed v_{Null} equal to the unshifted light speed c_{US}, as would be the case for all objects in flat spacetime when no gravitation is present. So unshifted particles can move at all possible velocities that have a speed not exceeding the unshifted light speed. Using these as the available set of all unshifted partner velocities in the velocity relation, the set of all possible shifted particle velocities are obtained for a given inertial frame. The following "partner form" of the speed constraint results: *The velocity relation* (43) *at each location, applied to all possible unshifted partner velocities bounded by the unshifted light speed, must yield the corresponding set of shifted partner velocities such that none exceed the unshifted light speed equal to the manifold null speed.* This is formally given by (in the ICs of any absolute inertial frame)

$$v_S(v_{US}, \hat{r}_{US}) \le c_{US} = v_{Null} \tag{47}$$

over the entire range $0 \le v_{US} \le c_{US}$ of unshifted partner speeds, and over the entire 4π steradian range of unshifted partner directions $\hat{r}_{US}$, where $v_S(v_{US}, \hat{r}_{US})$ is obtained by substituting the velocity relation (43) into (44). Satisfaction of this condition in one inertial frame guarantees satisfaction in all, as all speeds are less than or equal to the invariant null speed $v_{Null} = c_{US} = 1$ of the absolute manifold. The partner form of the speed constraint *limits the shift tensor components* to insure that the speed constraint is satisfied.

Consider the relation between the IC-given temporal components of the partner displacements used to quantify the velocities of partner particles. With $dt_S = S^0{}_{\bar{v}} dx_{US}^v$, then $dt_S = dt_{US}(S^0{}_{\bar{0}} + S^0{}_{\bar{n}} v_{US}^n)$, yielding the "temporal partner relation"

$$dt_S = dt_{US}(S^0{}_{\bar{0}} + v_{US} S^0{}_{\bar{n}} r_{US}^n), \tag{48}$$

where the right-hand side is a function of the shift tensor and the unshifted velocity $\vec{v}_{US}$. The possibility arises that the temporal partners dt_S and dt_{US} could have opposite signs at a location for some unshifted velocities, referred to as a "temporal inversion." In this case, an unshifted partner, with the expected causal evolution forward in time ($dt_{US} > 0$), would yield a shifted partner with a causal evolution *backwards* in time ($dt_S < 0$). *Gravity shifted objects may not evolve backwards in time*, as clearly this would be a causality violation in absolute flat spacetime (from the summary). Therefore, gravity shifting must adhere to the following "temporal constraint": *Temporal inversions cannot occur between partner particles.*

Application of the temporal constraint to the temporal partner relation, (48), yields the equivalent "shift (tensor) form" of the temporal constraint given by (in the ICs of any absolute inertial frame)

$$S^0{}_{\bar{0}} + v_{US} S^0{}_{\bar{n}} r_{US}^n > 0 \tag{49}$$

applicable for all possible v_{US} and $\hat{r}_{US}$, thereby *limiting the shift tensor*. Since v_{US} can be zero, the shift form temporal constraint implies that

$$S^0{}_{\bar{0}} > 0 \tag{50}$$

in any inertial frame. In addition, since v_{US} can be as large as unity, then

$$|S^0{}_{\bar{n}} r^n_{US}| < S^0{}_{\bar{0}} \quad \text{for all } \hat{r}_{US} \tag{51}$$

in any inertial frame. Note that the left-hand side of (49) is the denominator in the velocity relation (43), which is always positive then.

The causal spacetime evolution of a particle adheres to the "null constraint" if *its evolution from an event falls within or on the Lorentz invariant forward null cone of the absolute manifold (the forward unshifted light cone)*. In a given inertial frame, adherence to both the speed constraint and temporal constraint establishes that the evolution of shifted particles adheres to the null constraint, with indeed the null constraint and the combined speed/temporal constraints being equivalent. The null constraint of course holds for the unshifted partners when gravitation is removed. Shifted partner adherence to the null constraint in one inertial frame implies adherence in all, since timelike or null motion in one inertial frame yields timelike or null motion in all. Under the equivalence of the null constraint and the combined speed/temporal constraints, then if the speed and temporal constraints both hold in one inertial frame, they both hold in all. The speed and temporal constraints combined *bars shifted particle backwards causal temporal evolution of any kind*, either by temporal inversions or by speeds exceeding the manifold null speed, which would yield backwards evolution in some absolute inertial frames.

3.7. The Symmetry of the Shift Tensor

The "native" (defined) form of the shift tensor $S^\alpha{}_{\bar{\mu}}$ is a *mixed form* rank-2 tensor. To evaluate its symmetry, one of its indices must be raised or lowered by a metric to put it into "pure (raised or lowered indice) form" $S^{\alpha\bar{\mu}}$ or $S_{\alpha\bar{\mu}}$. Either the absolute or natural metric may be utilized for this purpose, but in general the symmetry property of a native mixed rank-2 tensor is dependent on the metric utilized to put it in pure form. When discussing whether or not the shift tensor is symmetric, use of it in pure form is assumed. The shift tensor (in pure form) could contain both symmetric and antisymmetric parts. A symmetric shift tensor is diagonalizable as shown below, which would yield a significant simplification in the types of gravity shifting that could occur. Gravity shifting would be greatly simplified then if it can be shown that the shift tensor is symmetric, as will be done here.

Applying $S^\beta{}_{\bar{\sigma}}$ on both sides of the metric relation (35) yields

$$S_{\alpha\bar{\sigma}} = S_{\underline{\bar{\sigma}}\alpha}. \tag{52}$$

The technique has been employed in (52) where indices raised/lowered from their native positions by the absolute metric are *underscored*, whereas no underscoring is used for indices raised/lowered by the natural metric or left in their native positions. For example, $S_{\underline{\bar{\sigma}}\alpha} = a_{\mu\sigma} S^{\bar{\mu}}{}_\alpha$ and $S_{\alpha\bar{\sigma}} = g_{\alpha\beta} S^\beta{}_{\bar{\sigma}}$. This "absolute (metric) underscoring" is not required in formulations where only the absolute metric is being utilized (and clearly identified as such), so absolute underscoring is typically used in "mixed-metric" cases where both the absolute and natural metrics are employed. Utilizing (52), if the absolute metric lowered pure form $S_{\underline{\bar{\sigma}}\alpha}$ is symmetric (i.e., $S_{\underline{\bar{\sigma}}\alpha} = S_{\underline{\bar{\alpha}}\sigma}$), then the natural metric lowered pure form $S_{\alpha\bar{\sigma}}$ is symmetric (i.e., $S_{\alpha\bar{\sigma}} = S_{\sigma\bar{\alpha}}$), and vice versa. This would result in both the raised and lowered pure forms of both the forward and reverse shift tensors being symmetric utilizing either the absolute or natural metrics to raise/lower the indices. Therefore, to establish the symmetry for all possible pure forms of the shift tensor using either metric, symmetry need be established for only a single instance of a pure form forward or reverse shift tensor using either metric to form it.

No constraints have been found that would "directly" constrain the shift tensor to being symmetric. The shift tensor is often "paired" with itself when evaluating cases, allowing for a degeneracy and therefore nonsymmetric shift tensors, such as if the metric relation (35) were used to attempt to

establish symmetry based on a symmetric natural metric. At present, the only approach found to establish shift tensor symmetry is via its coupling to its gravitational source as given by a symmetric SE tensor, which may be considered an "indirect" constraint. Application of this constraint is as follows.

As established above, in GS theory the shift tensor is considered the fundamental quantity depicting the gravitational field. This assumption infers that at the fundamental level, *the gravitational field generated by the gravitational source is the shift tensor field*. In modern gravitational theory, the "charge" for the gravitational source consists of its energy-momentum components, typically taking the form of a symmetric rank-2 SE tensor for *tensor* field theories. With the generated gravitational field in GS theory being the shift tensor field, it is similarly assumed that the gravitational source charge takes the form of a symmetric rank-2 SE tensor. This is reflected in the general forms (8) and (9) of both the natural and absolute field equations, which are *equivalent* expressions of the shift tensor field being generated by a symmetric SE tensor charge.

As the fundamental shift tensor field is generated by its source, then it is *coupled* to its source as given by a symmetric SE tensor charge. The absolute and natural metrics are symmetric, so at the fundamental level, the coupling of the shift tensor field to a symmetric SE tensor source charge, with both in the presence of these symmetric metrics (or at the very least the symmetric absolute metric), *results in a symmetric shift tensor when given in pure form*. Either the absolute or natural metric may be used to obtain the symmetric pure form shift tensor from its native mixed form, since as shown above the natural metric obtained pure form is symmetric if the absolute metric obtained pure form is symmetric, and vice versa. The formal statements of the lowered forward and reverse shift tensor symmetries are

$$S_{\underline{\alpha}\bar{\sigma}} = S_{\underline{\sigma}\bar{\alpha}}, \qquad S_{\bar{\underline{\alpha}}\sigma} = S_{\bar{\underline{\sigma}}\alpha}, \qquad S_{\alpha\bar{\sigma}} = S_{\sigma\bar{\alpha}}, \qquad S_{\bar{\alpha}\sigma} = S_{\bar{\sigma}\alpha}, \tag{53}$$

with the raised forms readily obtained from these.

3.8. Gravity Shifted Light

An illustrative and helpful example of gravity shifting is gravity shifted light. Utilizing the global ICs of an absolute inertial frame, for an unshifted light partner with speed $c_{US} = 1$ and velocity $\vec{c}_{US} = c_{US}\,\hat{r}_{US}$, the velocity of the shifted light partner is

$$c_S^n = \frac{S^n{}_{\bar{0}} + S^n{}_{\bar{k}}c_{US}^k}{S^0{}_{\bar{0}} + S^0{}_{\bar{l}}c_{US}^l} \tag{54}$$

as given by the velocity relation (42), with speed c_S and direction $\hat{r}_S$ as per (44). Under the variability of the shift tensor in (54), then *the speed c_S of light in a gravitational field is variable*, generally being a function of both propagation direction and location. In deep space, where the shift tensor $S^\alpha{}_{\bar{\mu}}$ becomes the delta tensor $\delta^\alpha{}_{\bar{\mu}}$, the shifted light speed becomes the fixed unshifted light speed. Note that the variability of shifted light speed was accounted for when applying the Schild argument to establish the existence of gravity shifts. The global IC shifted light velocity c_S^n is the velocity of light as measured by absolute observers in their preferred absolute inertial frames, yielding the variable speed c_S as absolutely observed. Under the equivalence principle applicable for natural observers (proven below), *natural measurement of shifted light yields the perception of inertial unshifted light in the local ICs of their preferred free-fall frames*. The naturally observed shifted light speed is therefore the fixed unshifted light speed $c_{US} = 1$ in free-fall frame ICs.

All possible unshifted light motion at a location may be generated by sweeping over the 4π steradians of direction $\hat{r}_{US}$ in which an unshifted light partner can propagate. This motion may be depicted by a Lorentz invariant spherical velocity surface in the three spatial IC velocity dimensions (a "velocity map"), centered on zero velocity for each absolute inertial frame. All possible shifted light motion at a location may be generated by using (54) and again sweeping over the 4π steradians of direction $\hat{r}_{US}$ in which an unshifted light partner can propagate. For a given inertial frame and ICs, the

resulting velocity surface is an *ellipsoid*. To show this, the equivalent reverse velocity relation (45) may be applied to light to yield

$$c_{US}^n = \frac{S^{\bar{n}}{}_0 + S^{\bar{n}}{}_k c_S^k}{S^{\bar{0}}{}_0 + S^{\bar{0}}{}_l c_S^l}. \tag{55}$$

Substituting (55) in (46) to obtain c_{US}, and setting $c_{US} = 1$, the natural metric based "light equation"

$$g_{00} + 2g_{0n}c_S^n + g_{nm}c_S^n c_S^m = 0 \tag{56}$$

is yielded for the shifted light velocity, having utilized $a_{\mu\nu} = \eta_{\mu\nu}$ in global ICs as well as the metric relation (35) to obtain this result. This is as expected, since utilizing the natural metric line element, (34), in the ICs of the free-fall frames, yields $d\tau_N = 0$ for shifted light motion naturally perceived as inertial unshifted light motion with speed $c_{US} = 1$. Expressing (34) in global ICs with $d\tau_N = 0$ yields the light equation (56). The light equation has the general form of an ellipsoid formula, yielding an ellipsoid for the shifted light velocity map. With the natural metric $g_{\alpha\beta}$ generally having nonzero g_{0n} in global ICs, and a spatial subspace metric g_{nm} that is not diagonal, the velocity ellipsoid is generally not centered on zero velocity, and its three axes are not typically aligned with the velocity map IC axes. The use of arbitrary shift tensors yields natural metric values in the light equation that may result in "pathological" c_S^n solutions that are infinite, complex valued, or yield shifted light overlapping itself. But if the shift tensor adheres to the above physical constraints, specifically having a nonzero determinant (as per (26)) to prevent overlap, as well as yielding shifted light velocities that are finite and real valued as consistent with the speed constraint (47), then the pathological solutions are eliminated, leaving only the "ordinary" ellipsoid solutions. Additional application of the speed limit $c_S \leq c_{US}$ obtained from the speed constraint, (47), yields a shifted light velocity ellipsoid that does not extend beyond the unshifted light velocity sphere. This property holds in any absolute inertial frame and global ICs utilized, so the ellipsoidal shape of the velocity map is an invariant property under the global Lorentz transforms.

With the shifted light velocity map an ellipsoid, light in a general gravitational field behaves as if it were *propagating in an anisotropic crystal* at each location due to different reduced propagation speeds in different directions, with then its velocity ellipsoid not exceeding the "vacuum" unshifted light velocity sphere in any inertial frame. The wavefront for the shifted light at a location is parallel to the velocity ellipsoid surface for its given propagation direction, which is generally not perpendicular to the propagation direction. But under the equivalence principle, when naturally measured in the free-fall frames, the wavefront is perceived as being perpendicular since inertial unshifted light is perceived with its spherical velocity surface.

The global IC velocity sphere for unshifted light may be used to generate the corresponding Lorentz invariant unshifted light cone. Following common practice, the "cone" reference comes from pictorially mapping all possible unshifted light propagation in spacetime diagrams using any two IC spatial dimensions and the time dimension, yielding the invariant conical shape (a "hypercone" is formed in 4-spacetime). Similarly, the shifted light velocity ellipsoid yields a shifted light "cone" that is actually elliptical in spatial cross section, where in general the light cone is "tilted" relative to the IC time axis. In general gravitational systems, the parameters depicting the velocity ellipsoid will vary with location, including variation in its "center velocity," axis lengths, and axis orientations, as mapped in a common global IC system. A corresponding variation is yielded in the tilt, axis lengths, and axis orientations of the elliptical shifted light cone (in any 2+1 dimensions). As the null constraint must hold for shifted light as for any shifted object, then the shifted light cone at any location cannot exceed the absolute manifold forward null cone, which is the unshifted light cone. Under the global Lorentz boosts, the tilt of the shifted light cone changes from one inertial frame to the next, whereas the invariant null cone maintains no tilt. Note that in some inertial frames, the shifted light cone will not contain the IC time axis (if its cone is smaller than the null cone in any inertial frame), so the shifted light will not propagate in all directions.

Having been shifted, the evolution of all matter and fields is bounded by the shifted light cone. This includes the gravitational field itself as given by the shift tensor field, which will be shown to have the same propagation speed as gravity shifted light. As a result, *the causal connectedness of a gravitational system is bounded by its shifted light cone at each location, which cannot exceed the Lorentz invariant forward null cone of the absolute manifold. Closed timelike curves are therefore barred for gravitational systems.*

The unshifted/null light cone is the boundary between timelike and spacelike unshifted displacements dx^μ_{US} as dictated by their absolute proper intervals (31). The shifted partner displacements dx^α_S for the timelike unshifted displacements are contained within the shifted light cone, whereas the shifted partner displacements for the spacelike unshifted displacements fall outside the shifted light cone. Via (32), *the shifted light cone is the boundary separating natural measurement of shifted/actual partner displacements* $dx^\alpha_S = dx^\alpha$ *being timelike or spacelike*, as dictated by their natural proper intervals (34). Natural measurement of shifted displacements along the shifted light cone yields a null natural interval $d\tau_N = 0$, as expected since $d\tau_N = d\tau_{US}$ and the unshifted partner displacements along the unshifted light cone yield $d\tau_{US} = 0$. This result is consistent with $d\tau_N = 0$ obtained above for shifted light motion via derivation of the light equation (56) or use of the equivalence principle. Note that shifted/actual displacements $dx^\alpha_S = dx^\alpha$ that fall between the shifted and null cones are naturally measured as being spacelike, but via (28) are absolutely measured as being *timelike* since they fall within the null cone, demonstrating that natural and absolute displacement measurement may be starkly different in character. But if a shifted/actual displacement $dx^\alpha_S = dx^\alpha$ is naturally measured as timelike or null ($d\tau_N \geq 0$), it is absolutely measured as being timelike or null ($d\tau_A \geq 0$) since it falls within or on the null cone.

3.9. Intrinsic Gravity Shifting

Consider diagonalization of the absolute metric lowered pure form $S_{\beta\bar{\mu}}$ of the shift tensor as given in the global ICs of absolute inertial frames (absolute underscoring is not used here since only the absolute metric is being utilized). As shown here, since $S_{\beta\bar{\mu}}$ is *symmetric*, it may be diagonalized at any location using global Lorentz transforms, yielding a diagonalized $S^\alpha_{\bar{\mu}}$.

The "spatial subspace" of $S_{\beta\bar{\mu}}$, consisting of its space-space components $S_{n\bar{m}} = S_{m\bar{n}}$, may be diagonalized using the *rotations* provided by the (global) Lorentz transforms. This leaves only the time-space components $S_{0\bar{n}} = S_{n\bar{0}}$, which may zeroed out using the *Lorentz boost*s provided by the Lorentz transforms. Note that as is typically the case, if a symmetric tensor is already spatially diagonalized, application of a Lorentz boost to zero out the time-space components will reintroduce off-diagonal space-space components, so an additional rotation is applied to spatially rediagonalize. Alternately, a Lorentz boost may be applied first to zero out the time-space components of a symmetric tensor that has not been spatially diagonalized, and then a single rotation is applied to diagonalize the spatial subspace. In either case, the end result is a fully diagonalized $S_{\beta\bar{\mu}}$. Since $a^{\alpha\beta}$ is the diagonal $\eta^{\alpha\beta}$ in global ICs, then when the absolute metric is used to raise the first indice in the diagonalized $S_{\beta\bar{\mu}}$ to obtain its native mixed form $S^\alpha_{\bar{\mu}}$, the mixed form is also fully diagonal, which is the desired result.

Now the spatial subspace of a symmetric rank-2 tensor, in global ICs, may be readily diagonalized by a rotation regardless of the values of its space-space components, with this property applicable then for the symmetric $S_{\beta\bar{\mu}}$ with arbitrary space-space component values. However, if the time-space components of a symmetric tensor are too large relative to its other components, application of Lorentz boosts $\Lambda^{\alpha'}{}_\mu(\vec{v}_B)$ will not be able to zero them out since the values of the Lorentz boost components are limited due to the speed for the boost velocity $\vec{v}_B$ being less than the absolute manifold null speed $v_{Null} = 1$. For the shift tensor though, the values of its time-space components $S_{0\bar{n}} = S_{n\bar{0}}$ are limited by the speed constraint (47). Under this constraint, a Lorentz boost will indeed be able to zero out its time-space components, shown by utilizing gravity shifted light as follows. As shown above, the IC velocity map for shifted light is an ellipsoid in any absolute inertial frame, with the ellipsoid not extending beyond the unshifted light spherical velocity surface. Therefore, with the unshifted light speed c_{US} equal to the null speed v_{Null}, a single global Lorentz boost may be applied to *center* the velocity ellipsoid in a particular inertial frame. In addition, a rotation may be applied to align the IC

axes with the three ellipsoid axes. This infers that the symmetric $S_{\beta\bar{\mu}}$ must have been diagonalized in the process, since then raising its first index by the diagonal $a^{\alpha\beta} = \eta^{\alpha\beta}$ yields a diagonal $S^{\alpha}{}_{\bar{\mu}}$, which when used in the shifted light velocity equation (54) yields the centered and aligned velocity ellipsoid. Therefore, the time-space components $S_{0\bar{n}} = S_{n\bar{0}}$ are limited by the speed constraint (47) to the extent that a Lorentz boost will zero them. This limitation was implicitly imposed above in order to obtain a shifted light velocity ellipsoid limited by the unshifted light spherical velocity surface with speed $c_{US} = v_{Null}$, allowing the ellipsoid to be centered under a Lorentz boost, and with it, $S_{0\bar{n}} = S_{n\bar{0}}$ is zeroed.

Any global ICs that diagonalize $S^{\alpha}{}_{\bar{\mu}}$ at a location is referred to as an "eigensystem," since as shown below, the IC's basis vectors $\vec{e}_{(\tilde{\alpha})}$ are eigenvectors of the shift tensor as given in any coordinates. A tilde is used to indicate quantities given in an eigensystem. An inventory of diagonal quantities in an IC eigensystem is the shift tensor $\tilde{S}^{\alpha}{}_{\bar{\mu}}$, its inverse $\tilde{S}^{\bar{\mu}}{}_{\alpha}$, the absolute metric $\tilde{a}_{\mu\nu} = \eta_{\mu\nu}$ and its inverse $\tilde{a}^{\mu\nu} = \eta^{\mu\nu}$, as well as the natural metric $\tilde{g}_{\alpha\beta}$ and its inverse $\tilde{g}^{\alpha\beta}$ as obtained via the forward and inverse metric relations (35) and (36). There can be more than one eigensystem at a location, such as in deep space, where all global ICs in all absolute inertial frames are eigensystems since the shift tensor is the diagonal identity tensor $\delta^{\alpha}{}_{\bar{\mu}}$ for all. But there always exists at least one IC eigensystem, since the symmetric shift tensor is always diagonalizable under global Lorentz transforms. As generally discussed in the summary, applying the partner relation in an IC eigensystem, the gravity shifting is depicted by *strictly dimensional shifts* consisting of three spatial dimensional shifts each given by $d\tilde{x}^n_S = \tilde{S}^n{}_{\bar{n}} \, d\tilde{x}^n_{US}$ depicting length change parallel to the n direction IC spatial coordinate axis for the eigensystem, and a temporal dimensional shift given by $d\tilde{x}^0_S = \tilde{S}^0{}_{\bar{0}} \, d\tilde{x}^0_{US}$ depicting duration change parallel to the IC time coordinate axis, yielding a change in the rates of the physical processes for the matter present. The specified dimensional shifts are *orthogonal* to each other in flat spacetime since they run parallel to the orthogonal IC eigensystem axes. Transformation out of an eigensystem into other coordinates yields a shift tensor that is generally no longer diagonal, resulting in an apparent "mixing" of temporal and spatial shifting in general coordinates. But is actuality, gravity shifting may be considered to "intrinsically" consist of the orthogonal dimensional shifts as given in an eigensystem, with the apparent mixing in general coordinates an artifact of general coordinate expression of the intrinsic dimensional shifting.

A means of depicting the intrinsic shifting is use of the shift tetrad $\vec{S}_{(\alpha)}$ discussed in the summary. The shift tetrad at a location may be constructed with the aid of an eigensystem. The four tetrad vectors run parallel to its IC axes, with the magnitude for each given by the scalar "shift factor"

$$S_{(\alpha)} \equiv \tilde{S}^{\alpha}{}_{\tilde{\alpha}} \text{ (no sum)}, \tag{57}$$

yielding

$$\vec{S}_{(\alpha)} \equiv S_{(\alpha)} \vec{e}_{(\tilde{\alpha})} \tag{58}$$

defining the shift tetrad vectors, having utilized the unity-length coordinate basis vectors $\vec{e}_{(\tilde{\alpha})}$ for the IC eigensystem. Each shift tetrad vector $\vec{S}_{(\alpha)}$ depicts a dimensional shift along the spacetime direction of the vector, with the length of the tetrad vector, given by the shift factor $S_{(\alpha)}$, quantifying the shifting in terms of fractional change relative to unity ($S_{(\alpha)} = 1$ indicates no change). Since the eigensystem basis vectors $\vec{e}_{(\tilde{\alpha})}$ are orthogonal to each other, the shift tetrad vectors are orthogonal as well. Note that a shift tetrad vector may be reversed in direction and still express the same shift, since a dimensional shift is an expansion or contraction along its spacetime direction. As can be seen, the shift tetrad $\vec{S}_{(\alpha)}$ depicts the intrinsic gravity shifting consisting of the orthogonal spatial and temporal dimensional shifts along the four IC axis directions for an eigensystem. As the shift tetrad consists of vectors, it provides a *geometrically invariant* expression of intrinsic gravity shifting. For any system, the intrinsic gravity shifting at all locations may be given by a map of the shift tetrad $\vec{S}_{(\alpha)}$ (utilizing any convenient coordinate system). Since the gravity shifting providing the field is *completely specified* by its intrinsic shifting, then for any system, the shift tetrad map *provides a geometrically invariant complete depiction*

of the gravitational field. As can be seen, the shift tetrad map is a convenient and powerful means of completely depicting the gravitational field.

The reason why a global IC system diagonalizing the shift tensor $S^\alpha{}_{\bar\mu}$ is referred to as an "eigensystem," is that its four coordinate basis vectors $\vec{e}_{(\tilde\alpha)}$ are *eigenvectors* of the shift tensor as given in any coordinates, with the diagonal terms of the eigensystem shift tensor—i.e., the shift factors $S_{(\alpha)}$ defined by (57)—being their respective *eigenvalues*. This is formally expressed in matrix and column vector form by the "shift eigenvector equation"

$$\{S\}\{\vec{e}_{(\tilde\alpha)}\} = S_{(\alpha)}\{\vec{e}_{(\tilde\alpha)}\}, \tag{59}$$

applicable for $S^\alpha{}_{\bar\mu}$ and the eigensystem basis vectors $\vec{e}_{(\tilde\alpha)}$ given in any coordinates. The validity of (59) may be shown by first evaluating it in an eigensystem, so that $\{S\}$ is a diagonal matrix, and so that each $\{\vec{e}_{(\tilde\alpha)}\}$ is a column vector with unity for its α component and zero for the other components. Then (59) is obtained as a covariant expression by applying arbitrary coordinate transformation to the eigensystem $\{S\}$ and $\{\vec{e}_{(\tilde\alpha)}\}$. The shift eigenvector equation may be interpreted as shifting each eigensystem basis vector $\vec{e}_{(\tilde\alpha)}$ to yield its shifted partner $\vec{S}_{(\alpha)} = S_{(\alpha)}\vec{e}_{(\tilde\alpha)}$ (using (58)) in the same direction, with vectors not parallel to an $\vec{e}_{(\tilde\alpha)}$ generally not shifting along their original directions. Being proportional to $\vec{e}_{(\tilde\alpha)}$, the shift tetrad vectors $\vec{S}_{(\alpha)}$ are eigenvectors of the shift tensor, with their eigenvalues the shift factors $S_{(\alpha)}$. Using the eigenvector equation, the "shift secular equation"

$$|S_{(\alpha)}\mathrm{I} - S| = 0 \tag{60}$$

is obtained, which yields the same four shift factor roots $S_{(\alpha)}$ regardless of the coordinates used. These roots may then be utilized in the eigenvector equation (59), in any coordinates, to determine the unity magnitude eigenvectors $\vec{e}_{(\tilde\alpha)}$, which when multiplied by their respective $S_{(\alpha)}$ yields the shift tetrad $\vec{S}_{(\alpha)}$. Using this "eigenvector method," the shift tetrad $\vec{S}_{(\alpha)}$ may be determined from the shift tensor $S^\alpha{}_{\bar\mu}$ in any coordinates, providing a convenient means of obtaining a map of the intrinsic shifting in any coordinates. Note that when there is more than one global IC system at a location that diagonalizes the shift tensor, there will be degeneracy in the eigenvector equation, resulting in the multiple eigensystems being yielded, as well as possible multiple secular equation solutions $S_{(\alpha)}$ with the same values. For instance, in deep space, the identity shift tensor $\delta^\alpha{}_{\bar\mu}$ yields four $S_{(\alpha)} = 1$ secular equation solutions, and yields the basis vectors $\vec{e}_{(\tilde\alpha)}$ for all global ICs as eigenvector equation solutions. There is therefore degeneracy in the shift tetrad $\vec{S}_{(\alpha)}$ when there is eigensystem degeneracy, but the specified intrinsic shifting is *unique* under this degeneracy since the multiple shift tetrads specify the same intrinsic shifting.

Consider a path run from any location of interest in a gravitational system out to deep space, where an IC eigensystem at each point is utilized so the shift tensor has the eigensystem value $\tilde{S}^\alpha{}_{\bar\mu}$. Under the positive determinant range (26) for the shift tensor in any coordinates, then all of the diagonal terms of $\tilde{S}^\alpha{}_{\bar\mu}$ must be nonzero along the entire path. Since all of the diagonal terms are positive for the identity eigensystem shift tensor $\delta^\alpha{}_{\bar\mu}$ in deep space, then under continuity of the shift tensor components, all of the $\tilde{S}^\alpha{}_{\bar\mu}$ diagonal terms must be positive at all path locations, including the start of the path at the location of interest. Therefore, IC eigensystem shift tensors $\tilde{S}^\alpha{}_{\bar\mu}$ *must have strictly positive diagonal terms at all locations*, formally expressed by the "shift factor range" (utilizing (57))

$$0 < (\tilde{S}^\alpha{}_{\bar\alpha} = S_{(\alpha)}) < \infty \quad \text{for each } \alpha. \tag{61}$$

Included in the shift factor range is the infinite upper limit imposed by the infinite upper limit of (26). Adherence to the shift factor range establishes that the shift tensor matrix $\{S\}$ is *positive stable* since all

of its eigenvalues are positive. Using the velocity relation (42) as well as (44), application of the speed constraint (47) in an eigensystem yields the "shift factor speed constraint"

$$(\tilde{S}^n_{\bar{n}} = S_{(n)}) \le (\tilde{S}^0_{\bar{0}} = S_{(0)}) \quad \text{for each } n. \tag{62}$$

Formation and use of (62) relies on the shift factors being positive.

Specification of gravity shifting as intrinsic shifting consisting of orthogonal dimensional shifts in flat spacetime, combined with the shift factor range and speed constraints (61) and (62), provides a clear and compact means of specification such that all previously given shift tensor and gravity shifting constraints are satisfied, as shown here. Gravity shifting intrinsically consisting of orthogonal dimensional shifts in flat spacetime implies the existence of an IC eigensystem where the shift tensor $\tilde{S}^\alpha_{\bar{\mu}}$ is diagonal, yielding a diagonal absolute (Minkowski) metric lowered $\tilde{S}_{\beta\bar{\mu}}$, and therefore a symmetric $S_{\beta\bar{\mu}}$ in any coordinates. Using the shift factor range (61) and the invariance of the shift tensor determinant, the shift tensor determinant range (26) is automatically satisfied for the forward shift tensor $S^\alpha_{\bar{\mu}}$, guaranteeing invertibility of the shift tensor with (26) holding for the reverse shift tensor $S^{\bar{\mu}}_\alpha$, and therefore satisfaction of the 1-to-1 requirement for gravity shifting and the prevention of shifted and unshifted partner event overlap. With $\tilde{S}^0_{\bar{n}} = 0$ in the eigensystem, use of (61) to obtain $\tilde{S}^0_{\bar{0}} > 0$ yields satisfaction of the shift form temporal constraint (49). With both the speed and temporal constraints satisfied in the eigensystem, then as per above (Sec. 3.6) the null constraint is satisfied, yielding adherence to the speed, temporal, and null constraints in any absolute inertial frame (with the speed and temporal constraints given in shift form by (47) and (49), additionally yielding (50) and (51) from the temporal constraint), completing the proof.

When the partner relation (14) was first formed giving the most general possible gravity shifting that could occur, the initially unconstrained 4×4 shift tensor $S^\alpha_{\bar{\mu}}$ consisted of 16 arbitrary components. Via application of the absolute flat spacetime postulate as well as the equivalence principle applicable for natural observers, the shift tensor has been shown to be constrained as follows: the shift tensor is symmetric in pure form; the shift tensor $S^\alpha_{\bar{\mu}}$ may be diagonalized in global ICs using Lorentz transforms; when diagonalized in global ICs, the diagonal terms $\tilde{S}^\alpha_{\bar{\alpha}} = S_{(\alpha)}$ satisfy the shift factor range and speed constraints (61) and (62). As shown, use of the shift tensor with these properties in the partner relation, (14), yields intrinsic shifting consisting of orthogonal dimensional shifts, three spatial and one temporal as geometrically depicted by the shift tetrad $\vec{S}_{(\alpha)}$, with their respective shift factors $S_{(\alpha)}$ quantifying the fractional length or duration changes again subject to the shift factor range and speed constraints. All previously established constraints for the shift tensor and gravity shifting, derived via application of the absolute flat spacetime and equivalence principle postulates, have been shown to be obtainable from the intrinsic shifting and its constraints. *All constraints required for physical validity are considered to be covered above*, completing the effort to constrain the shift tensor $S^\alpha_{\bar{\mu}}$ and the uniform-scale gravity shifting as given by the partner relation (14). Summarizing, application of the absolute flat spacetime and equivalence principle postulates to gravity shifting initially given using a shift tensor, $S^\alpha_{\bar{\mu}}$, with 16 arbitrary components, yields the "simple" orthogonal spatial and temporal dimensional shifts as specified above, severely constraining the types of gravity shifting that may occur.

3.10. The Potential Tensor

For the potential tensor development, some background on tensor exponentials is helpful. The exponential power series of a square matrix is defined by

$$\exp\{B\} \equiv \sum_{n=0}^{\infty} \frac{1}{n!} \{B\}^n. \tag{63}$$

The equivalent tensor exponential power series, applicable for mixed rank-2 tensors, is defined by

$$\exp(B^\alpha{}_\mu) \equiv \delta^\alpha{}_\mu + B^\alpha{}_\mu + \frac{1}{2!}B^\alpha{}_\nu B^\nu{}_\mu + \frac{1}{3!}B^\alpha{}_\nu B^\nu{}_\sigma B^\sigma{}_\mu + \ldots, \tag{64}$$

where each α, μ component for the "shorthand" expression, $\exp(B^\alpha{}_\mu)$, on the left, stands for the α, μ component of the tensor expansion on the right. As can be seen, a single α, μ component for $\exp(B^\alpha{}_\mu)$ generally involves all of the $B^\alpha{}_\mu$ components in the expansion. Application of arbitrary coordinate transformation to $\exp(B^\alpha{}_\mu)$ yields

$$L^{\beta'}{}_\alpha \exp(B^\alpha{}_\mu)L^\mu{}_{\nu'} = \exp(L^{\beta'}{}_\alpha B^\alpha{}_\mu L^\mu{}_{\nu'}). \tag{65}$$

From matrix analysis [19] (Chap. 6),

$$e^{\{B\}}e^{\{C\}} = e^{\{C\}}e^{\{B\}} = e^{\{B+C\}} \quad \text{if } \{B\} \text{ and } \{C\} \text{ commute,} \tag{66}$$

applicable for square matrices. Also from matrix analysis, $\exp\{B\}$ may be given as a polynomial of $\{B\}$ with an order not exceeding one less than the dimension of $\{B\}$, avoiding the need to explicitly work with its infinite exponential series. The resultant $\exp\{B\}$ matrix is a closed analytic form containing scalar exponentials of the *eigenvalues* λ for $\{B\}$ as the only exponentials present, which are obtainable from its characteristic equation $|\lambda I - B| = 0$.

   With each diagonal term $\tilde{S}^\alpha{}_{\tilde\alpha} = S_{(\alpha)}$ for the shift tensor in an IC eigensystem being positive as per (61), then it may be given by

$$\tilde{S}^\alpha{}_{\tilde\alpha} = S_{(\alpha)} = \exp(w_{(\alpha)}), \tag{67}$$

the *exponential of a real number* $w_{(\alpha)}$ *of any value* (prior to applying the speed constraint (62)), as discussed in the summary. Again, a positive value for $w_{(\alpha)}$ yields an increasing dimensional shift along the α IC axis direction for a diagonal term, a negative value yields a decreasing shift, and $w_{(\alpha)} = 0$ yields unity, which is no shifting. The property (67) yields the matrix form $\{\tilde{S}\} = \exp\{\tilde{w}\}$, where $\exp\{\tilde{w}\}$ is the exponential power series (63) of the diagonal matrix $\{\tilde{w}\}$. In tensor form this becomes $\tilde{S}^\alpha{}_{\bar\mu} = \exp(\tilde{w}^\alpha{}_\mu)$, where $\exp(\tilde{w}^\alpha{}_\mu)$ is the exponential power series (64) of the diagonal quantity $\tilde{w}^\alpha{}_\mu$. Application of arbitrary coordinate transformation to $\tilde{S}^\alpha{}_{\bar\mu} = \exp(\tilde{w}^\alpha{}_\mu)$, and utilizing (65), results in the covariant "potential form" of the shift tensor

$$S^\alpha{}_{\bar\mu} = \exp(w^\alpha{}_\mu) \tag{68}$$

in any coordinates, with $\exp(w^\alpha{}_\mu)$ the exponential power series of the "potential tensor" $w^\alpha{}_\mu$ (establishing (6)). The quantity $w^\alpha{}_\mu$ is indeed a *tensor*, since it transforms as a tensor under coordinate transform of (68) due to (65) holding.

   Since by its definition $\tilde{w}^\alpha{}_\mu$ is diagonal for the diagonal $\tilde{S}^\alpha{}_{\bar\mu}$, then when the shift tensor $S^\alpha{}_{\bar\mu}$ is diagonalized via transform to an IC eigensystem, the potential tensor $w^\alpha{}_\mu$ is also diagonalized, and vice versa. With the absolute and natural metrics $\tilde{a}_{\mu\nu} = \eta_{\mu\nu}$ and $\tilde{g}_{\alpha\beta}$ (and their inverses $\tilde{a}^{\mu\nu} = \eta^{\mu\nu}$ and $\tilde{g}^{\alpha\beta}$) also diagonal in an IC eigensystem (from above), then raising/lowering the indices of $\tilde{w}^\alpha{}_\mu$ by either metric yields a diagonal potential in pure indice form (such as $\tilde{w}_{\alpha\beta}$), resulting in the potential $w^\alpha{}_\mu$ being a *symmetric tensor* in any coordinates when given in pure form using either metric to raise/lower its indices (similar to the shift tensor $S^\alpha{}_{\bar\mu}$).

   As can be seen, the quantities $w_{(\alpha)}$ used in (67), referred to as the "potential factors," are given by

$$w_{(\alpha)} = \tilde{w}^\alpha{}_{\tilde\alpha} \quad \text{(no sum)}, \tag{69}$$

the diagonal values of an eigensystem potential tensor. Similar to (59) holding for the shift tensor, the "potential eigenvector equation"

$$\{w\}\{\vec{e}_{(\tilde\alpha)}\} = w_{(\alpha)}\{\vec{e}_{(\tilde\alpha)}\} \tag{70}$$

holds in any coordinates, where again use is made of the eigensystem basis vectors $\vec{e}_{(\tilde{\alpha})}$. So similar to the shift tensor, the IC eigensystem coordinate basis vectors $\vec{e}_{(\tilde{\alpha})}$ are eigenvectors of the potential tensor $w^{\alpha}{}_{\mu}$ as given in any coordinates, with the diagonal terms of the eigensystem potential tensor—i.e., the potential factors $w_{(\alpha)}$—being their respective eigenvalues. An IC eigensystem for the shift tensor $S^{\alpha}{}_{\bar{\mu}}$ is also then the same eigensystem for the potential $w^{\alpha}{}_{\mu}$, and vice versa. Similar to (60) for the shift factors, the potential factors $w_{(\alpha)}$ are the roots of the "potential characteristic equation"

$$| \, w_{(\alpha)} \mathrm{I} - w \, | = 0. \tag{71}$$

Utilizing from above, the potential form $S^{\alpha}{}_{\bar{\mu}} = \exp(w^{\alpha}{}_{\mu})$ of the shift tensor may be given as a $w^{\alpha}{}_{\mu}$ polynomial not exceeding third order, a closed analytic form, with the only exponentials present consisting of the scalar exponentials of the $w^{\alpha}{}_{\mu}$ eigenvalues $w_{(\alpha)}$ as given by its characteristic equation (71). This property may be readily seen, using any coordinates, by transforming the shift tensor $S^{\alpha}{}_{\bar{\mu}}$ and potential $w^{\alpha}{}_{\mu}$ into an eigensystem, forming the diagonal $\tilde{S}^{\alpha}{}_{\bar{\mu}} = \exp(\tilde{w}^{\alpha}{}_{\mu})$ where $\tilde{S}^{\alpha}{}_{\tilde{\alpha}} = \exp(w_{(\alpha)})$ via (67), and then transforming back to the original coordinates.

The potential form of the reverse shift tensor is

$$S^{\bar{\mu}}{}_{\alpha} = \exp(-w^{\mu}{}_{\alpha}). \tag{72}$$

To see that the forward and reverse potential forms are inverses of each other, they may be given in their matrix forms

$$\{S\} = \exp\{w\}, \qquad \{S^{-1}\} = \exp\{-w\}, \tag{73}$$

with $\exp\{w\} \exp\{-w\} = \exp\{0\} = \{I\}$ via application of (66) where $\{w\}$ and $\{-w\}$ commute. In tensor form this becomes $\exp(w^{\alpha}{}_{\mu}) \exp(-w^{\mu}{}_{\beta}) = \delta^{\alpha}{}_{\beta}$, completing the proof. Note that the inverse of the potential tensor $w^{\alpha}{}_{\mu}$ is its *arithmetic* inverse $-w^{\alpha}{}_{\mu}$, as opposed to the inverse of the shift tensor $S^{\alpha}{}_{\bar{\mu}}$ being its *multiplicative* inverse $S^{\bar{\mu}}{}_{\alpha}$. It may be readily shown (via an IC eigensystem) that in any coordinates, the determinants of the forward and reverse shift tensors are

$$|S| = \mathrm{e}^{w}, \qquad |S^{-1}| = \mathrm{e}^{-w}, \tag{74}$$

where w is the trace $w^{\alpha}{}_{\alpha}$. So for any finite real-valued $w^{\alpha}{}_{\mu}$, the ranges (26) are satisfied.

As stated in the summary, the ability to express the shift tensor $S^{\alpha}{}_{\bar{\mu}}$ as the exponential of the potential tensor $w^{\alpha}{}_{\mu}$ is a result of the overlap restriction placed on gravity shifting, which bars forbidden matter and temporal singularities from occurring in absolute flat spacetime. To prove this, recall that (Sec. 3.4) the overlap restriction results in the shift tensor determinant $|S|$ being nonzero, which in turn implies that when diagonalized in an IC eigensystem, the shift tensor $\tilde{S}^{\alpha}{}_{\bar{\mu}}$ has positive diagonal terms as specified by the shift factor range (61), yielding the potential form (68). Again, in order that the shift tensor solutions of the utilized field equation adhere to the overlap restriction, it is assumed that the *potential tensor $w^{\alpha}{}_{\mu}$* is the field operand as opposed to the shift tensor $S^{\alpha}{}_{\bar{\mu}}$ directly, which is the reason for the nomenclature "potential tensor." This is reflected in the potential tensor being the operand in the general forms (8) and (9) of the equivalent natural and absolute field equations. Substituting (72) in the metric relation (35), any field equation solution will therefore yield the "potential form" natural metric

$$g_{\alpha\beta} = a_{\mu\nu} \exp(-w^{\mu}{}_{\alpha}) \exp(-w^{\nu}{}_{\beta}) \tag{75}$$

(establishing (7)). The natural metric is *devoid of event horizons*, since in an IC eigensystem at a location, application of the Minkowski absolute metric $\eta_{\mu\nu}$ in (75) yields a diagonal natural metric with non-zero and finitely large diagonal terms.

3.11. The Squared Shift Tensor

Using absolute metric underscoring, the metric relation (35) may be given by $g_{\alpha\beta} = S_{\underline{\bar{\nu}}\alpha} S^{\bar{\nu}}{}_\beta$. Applying the reverse shift tensor symmetry in (53) yields $g_{\alpha\beta} = S_{\underline{\bar{\alpha}}\nu} S^{\bar{\nu}}{}_\beta$, or $g_{\alpha\beta} = a_{\alpha\mu} S^{\bar{\mu}}{}_\nu S^{\bar{\nu}}{}_\beta$. So the metric relation may be given in the "squared form"

$$g_{\alpha\beta} = a_{\alpha\mu} F^{\bar{\mu}}{}_\beta, \tag{76}$$

where the "reverse squared shift tensor" is defined by

$$F^{\bar{\mu}}{}_\beta \equiv S^{\bar{\mu}}{}_\nu S^{\bar{\nu}}{}_\beta. \tag{77}$$

Similarly, the inverse metric relation (36) may be given in the squared form

$$g^{\alpha\beta} = F^{\alpha}{}_{\bar{\nu}} a^{\nu\beta}, \tag{78}$$

where the "forward squared shift tensor" is defined by

$$F^{\alpha}{}_{\bar{\nu}} \equiv S^{\alpha}{}_{\bar{\mu}} S^{\mu}{}_{\bar{\nu}}. \tag{79}$$

Utilizing (77), (79), and (23), then

$$F^{\bar{\mu}}{}_\alpha F^{\alpha}{}_{\bar{\nu}} = \delta^\mu{}_\nu, \qquad F^{\alpha}{}_{\bar{\mu}} F^{\bar{\mu}}{}_\beta = \delta^\alpha{}_\beta, \tag{80}$$

so the forward and reverse squared shift tensors are *inverses* of each other, similar to the forward and reverse shift tensors.

Note that (76) states that lowering $F^{\bar{\mu}}{}_\beta$ by the absolute metric yields $F_{\underline{\bar{\alpha}}\beta}$ equal to the natural metric $g_{\alpha\beta}$, and similarly (78) states that raising $F^{\alpha}{}_{\bar{\nu}}$ yields $F^{\alpha\underline{\bar{\beta}}}$ equal to the inverse natural metric $g^{\alpha\beta}$. In an IC eigensystem, $\tilde{F}^{\alpha}{}_{\bar{\nu}}$ is diagonal since $\tilde{S}^{\alpha}{}_{\bar{\nu}}$ is. With the absolute and natural metrics $\tilde{a}_{\mu\nu} = \eta_{\mu\nu}$ and $\tilde{g}_{\alpha\beta}$ (and their inverses $\tilde{a}^{\mu\nu} = \eta^{\mu\nu}$ and $\tilde{g}^{\alpha\beta}$) also diagonal in an IC eigensystem (from above), then raising/lowering the indices of $\tilde{F}^{\alpha}{}_{\bar{\nu}}$ by either metric yields a diagonal pure indice form (such as $F_{\underline{\bar{\alpha}}\beta}$), resulting in the squared shift tensor $F^{\alpha}{}_{\bar{\nu}}$ being a *symmetric tensor* in any coordinates when given in pure form using either metric to raise/lower its indices (similar to the shift tensor $S^{\alpha}{}_{\bar{\mu}}$). Using a similar argument, the reverse squared shift tensor $F^{\bar{\mu}}{}_\beta$ is also a *symmetric tensor* in any coordinates when given in pure form using either metric.

Substituting the potential forms (68) and (72) of the forward and reverse shift tensors into (79) and (77), the potential forms

$$F^{\alpha}{}_{\bar{\mu}} = \exp(2w^\alpha{}_\mu), \qquad F^{\bar{\mu}}{}_\alpha = \exp(-2w^\mu{}_\alpha), \tag{81}$$

are yielded for the forward and reverse squared shift tensors. Use was made of (66) for commuting matrices to obtain (81). Using (81) in the squared form metric relations (76) and (78) yields

$$g_{\alpha\beta} = a_{\alpha\mu} \exp(-2w^\mu{}_\beta), \qquad g^{\alpha\beta} = \exp(2w^\alpha{}_\nu) a^{\nu\beta}. \tag{82}$$

These "(squared) potential forms" for the natural metric and its inverse provide a convenient and powerful means of expressing them in terms of gravity shifts, since they are compactly provided in forms such that the overlap restriction and therefore the 1-to-1 gravity shifting is automatically imposed. Similar to above for (75), the natural metric given by (82) is *devoid of event horizons*, since in an IC eigensystem at a location, application of the Minkowski absolute metric $\eta_{\mu\nu}$ in (82) yields a diagonal natural metric with non-zero and finitely large diagonal terms.

3.12. The "Star Case"

The natural field equation is developed such that it yields the observed post-Newtonian approximation for the natural metric (as discussed in the summary). The PN metric is provided in the harmonic gauge in Poisson and Will [20] (Chap. 8) (hence referred to as "PW"), along with the definitions of the utilized potentials. Consider the "star case" giving the field for a static spherically symmetric mass approximating stars as well as planets. The harmonic gauge PN metric in the star case may be readily shown to reduce to

$$\mathrm{d}s_N^2 = \left[-1 + 2M/r - 2(M/r)^2\right]\mathrm{d}t^2 + \left[1 + 2M/r\right](\mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2), \tag{83}$$

where M is the gravitational mass that is yielded when combining the Newtonian potential with the potentials resulting from pressure, internal energy, and the gravitational potential energy. This is the post-Newtonian expansion of the Schwarzschild metric from general relativity when given in isotropic coordinates, and it also serves as the star-case PN metric when given in the *standard gauge* (see PW [20], Chap. 13).

Consider the potential form natural metric (82) as given by $g_{\alpha\beta} = \eta_{\alpha\mu}\exp(-2w^\mu{}_\beta)$ in global ICs. With the star case being static, the generated potential field has no time-space components $w^0{}_j$ or $w^j{}_0$. Then $g_{00} = -\exp(-2w^0{}_0)$, $g_{0j} = 0$, and $g_{jk} = \exp(-2w^j{}_k)$. Applying the exponential expansion (64) to second order with $w^0{}_j = w^j{}_0 = 0$, the temporal metric term g_{00} is readily shown to be $-1 + 2w^0{}_0 - 2(w^0{}_0)^2$. If then $w^0{}_0$ is set to M/r, the second-order temporal term in (83) is generated. The temporal term in (83) is assumed then for the star-case PN metric given in global ICs, as generated using $w^0{}_0 = M/r$ in $g_{00} = -\exp(-2w^0{}_0)$. To obtain the spatial term in (83) utilizing $g_{jk} = \exp(-2w^j{}_k)$, the spatial subspace $w^j{}_k$ of the potential may be set to be diagonal with all diagonal terms equal to $-M/r$. This yields $1 - 2w^n{}_n = 1 + 2M/r$ for the first-order expansion of the spatial diagonal metric terms $g_{nn} = \exp(-2w^n{}_n)$, and zero for the off-diagonal terms. The spatial term in (83) is assumed then for the star-case PN metric given in global ICs, as generated using the diagonal $w^j{}_k = -\delta^j{}_k M/r$ in $g_{jk} = \exp(-2w^j{}_k)$. The global ICs are therefore assumed to be *isotropic coordinates*.

Summarizing the potential components from above, the global IC given potential tensor for the star case is

$$w^\alpha{}_\mu = \mathrm{diag}\left[M/r, -M/r, -M/r, -M/r\right]. \tag{84}$$

Utilizing the potential metric form (82), and applying the exponential expansion (64) to the star-case potential, yields the *exact* star-case metric given by

$$\mathrm{d}s_N^2 = -\exp(-2M/r)\,\mathrm{d}t^2 + \exp(2M/r)(\mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2), \tag{85}$$

which is *the assumed exact natural metric for the star case when given in global ICs*. The natural field equation is purposely constructed such that it will yield this metric for the star case. This results then in the natural field equation yielding the observed post-Newtonian approximation (83) of the star-case metric as its solution. The validity of the star-case natural metric solution is justified by successfully predicting the "classical tests" in our Solar System consisting of the deflection of light by the Sun, the Shapiro time delay for radar signals, and the perihelion advance for the orbit of Mercury (where, as established below, satisfaction of the SEP holds for the natural field equation, so the combined galactic and cosmological background system may be ignored).

Applying the potential form (68) to (84), the global IC star-case shift tensor is

$$S^\alpha{}_{\bar\mu} = \mathrm{diag}\left[e^{M/r}, e^{-M/r}, e^{-M/r}, e^{-M/r}\right]. \tag{86}$$

As can be seen, the global ICs utilized for the star case is indeed an *eigensystem* (though no tilde is used here). The star-case shift tensor applies for the Earth-generated shift tensor field utilized in the above Schild argument based evaluation given in global ICs, so the discussed temporal gravity shifting is *intrinsic* temporal shifting given by $\mathrm{d}\tilde{x}_S^0 = \tilde{S}^0{}_{\bar 0}\,\mathrm{d}\tilde{x}_{US}^0$ where $\tilde{S}^0{}_{\bar 0} = e^{M/r}$. Use of this value for $\tilde{S}^0{}_{\bar 0}$ yields

the naturally observed frequency shifting for light travelling up the tower with height h, as well as the corresponding ratio of the bottom clock rate over the top, being given by $\tilde{S}^0_{\bar{0}}(\text{top})/\tilde{S}^0_{\bar{0}}(\text{bottom})$ approximated as $1 - gh/c^2$ in laboratory units. This is the same as in the equivalent inertial case where the entire apparatus is accelerated at 1g relative to an absolute inertial frame, explicitly demonstrating equivalence principle satisfaction for the predicted gravitational redshifting. Recall that the Schild argument for gravitational redshifting was used to establish the existence of temporal gravity shifts, but the existence of corresponding spatial gravity shifts was only inferred. The spatial gravity shifts in the star-case shift tensor (86) are required though to obtain the post-Newtonian approximation (83) of the star-case metric. The successful predictions of the classical tests (and others), made using the star-case PN metric, are invoked *to establish the existence of spatial gravity shifts*.

Using $\mathrm{d}\tau_N = 0$ for natural interval measurement of gravity shifted light (from above), then the star-case metric (85) yields $c_S = \mathrm{e}^{-2M/r}$ for IC shifted light speed *in all directions*. Gravity shifted light outside a static spherical star or planet behaves then as if travelling through *amorphous glass* with an index of refraction $\mathrm{n} = \mathrm{e}^{2M/r}$, which may be used to predict the observed bending of light paths by the Sun, and to obtain the variable light speed in the Schild argument case.

A notable property for the star-case metric is the lack of an event horizon, as expected due to event horizons being forbidden in general. With the lack of event horizons "closing off" causality, observed "black holes" are renamed "black stars" when using GS theory to model them. Modelling of "black stars" is provided in the Supplement, where it is demonstrated that such stars will still be effectively "black," justifying their namesake. Similar to when utilizing general relativity to model them, it is shown that when using GS theory modelling, nongravitational forces are again not sufficient to prevent complete gravitational collapse of a black star into a singularity. But as discussed in the summary, the surface of a black star cannot move faster than the shifted light speed $c_S = \mathrm{e}^{-2M/R}$ where R is the surface radius (having applied Birkhoff's theorem for the natural metric outside the star so it is given by (85)), and it can readily be shown that collapse at this *exponentially small* light speed would take an *infinite* amount of time. Therefore, *black stars have finitely large sizes* given the finite age of our universe.

Similar to black stars, for *any* collapsing object, it may be reasonably inferred that the exponential relation (82) (or (75)) between the potential $w^\alpha{}_\mu$ and the natural metric $g_{\alpha\beta}$ results in a shifted light speed c_S at its surface, obtainable using the light equation (56) given in global ICs, that becomes infinitesimally small exponentially as the object collapses towards a singularity. Under this exponential "light speed governor" acting to limit collapse speeds, all collapsing objects remain finitely large at all finite ages, *barring singularity formation from collapsing objects in general*. This property (discussed in the summary) is examined in further detail in the Supplement. As can be seen, the exponential potential form (68) for the shift tensor results in *singularities of any kind being barred*, whether they be collapse-based singularities or the gravity shift overlap singularities discussed above.

3.13. Physical Plausibility

As demonstrated below, GS theory is compatible with quantum theory due to explicit formulation in absolute flat spacetime. Then with the above-established elimination of event horizons, singularities, and causality violations, *all physical law and modelling using gravity shift theory is physically plausible*. The validity of this statement depends on satisfaction of all of the above gravity shifting constraints, which have been shown to result in gravity shifting such that the "implausibilities"—consisting of event horizons, singularities, and causality violations—have been barred. The shifting constraints have been shown to result from application of the absolute flat spacetime and SEP postulates, so adherence to these postulates bars the implausibilities.

A requirement for the natural and absolute field equations is that, for all cases, they yield solutions that adhere to the gravity shifting constraints, thereby yielding solutions without the implausibilities. The physical plausibility of GS theory rests then on the field equation solutions satisfying the shifting constraints. It will be shown that the provided natural field equation (8) yields real-valued symmetric potential solutions $w^\alpha{}_\mu$ (when given in pure form) such that the shift tensor $S^\alpha{}_{\bar{\mu}} = \exp(w^\alpha{}_\mu)$ generates

gravity shifting that satisfies all of the constraints. This result is achieved so long as the natural matter (and nongravitational field) sources are limited such that the solutions $w^\alpha{}_\mu$ do not yield a shifted light speed c_S that exceeds the null speed v_{Null}, referred to as the "natural energy condition." (This is the "energy condition" discussed in the summary prior to identifying the "field equation" as the natural field equation.) As discussed later, the natural energy condition is evidently satisfied for the commonly accepted energy condition in general relativity applicable for ordinary natural matter. For any case, the equivalent absolute field equation utilizes the same potential $w^\alpha{}_\mu$ solution as the natural field equation, so again all gravity shifting constraints are satisfied.

3.14. Gravity Shifted Quantities

The quantities $Z^{\mu\nu}_{US}$ and $Z^{\alpha\beta}_S$ are utilized to depict partner properties for unshifted and shifted local partner objects (which includes the unshifted and shifted local partner regions of extended objects using the isolated unshifting condition), where "$Z^{\alpha\beta}$" represents quantities in general. From above, gravity shifting on the uniform scale may be depicted by using the local partner event fields x_{US} and x_S tied to the shared material content of local partner objects, with the partner event fields given by the event partner relation (16). As is well understood, application of a coordinate transform results in tensor quantities, $Z^{\mu\nu}$, depicting objects (meaning their properties) in the original coordinates, being given by $Z'^{\alpha\beta} = L^{\alpha'}{}_\mu L^{\beta'}{}_\nu Z^{\mu\nu}$ in the new coordinates, where $L^{\alpha'}{}_\mu$ is the coordinate transform Jacobian matrix. As can be seen, the event partner relation (16) *formally* takes the mathematical form of a coordinate transform. But as opposed to "passive" coordinate transforms where the coordinates are changed but the locations of events are not, the event partner relation is an "active transform" (as often referred to) where the locations of events are moved as expressed in common coordinates, so the coordinates are not changed. As understood in gravitational physics, similar to passive coordinate transforms, application of an active transform results in tensor quantities, $Z^{\mu\nu}_{UT}$, depicting "untransformed" objects, being given by $Z^{\alpha\beta}_T = T^\alpha{}_{\bar\mu} T^\beta{}_{\bar\nu} Z^{\mu\nu}_{UT}$ depicting the "transformed" objects, where $T^\alpha{}_{\bar\mu}$ is the active transform Jacobian tensor. Therefore, application of the event partner relation (16) as an active transform for a tensor quantity, $Z^{\mu\nu}_{US}$, depicting an unshifted object, results in $Z^{\alpha\beta}_S$ depicting the shifted partner object as given by the representative tensor "shift quantity (partner) relation"

$$Z^{\alpha\beta}_S = S^\alpha{}_{\bar\mu} S^\beta{}_{\bar\nu} Z^{\mu\nu}_{US} \quad \text{(zero order)}, \tag{87}$$

where the shift tensor $S^\alpha{}_{\bar\mu}$ is the active transform Jacobian tensor (the "zero order" indicator is explained below). Note that since (16) holds only for the *infinitesimally sized* region about any location x utilized as the shift origin $X = x$ for (16), then for the quantities $Z^{\mu\nu}_{US}$, $Z^{\alpha\beta}_S$, and $S^\alpha{}_{\bar\mu}$ used in (87), their location x is the shift origin $X = x$ for the active transform (16) applied in order to obtain (87). For instance, $S^\alpha{}_{\bar\mu}(x) = S^\alpha{}_{\bar\mu}(X = x)$. Similar to the Jacobian matrix $L^\mu{}_{\alpha'}$ for the inverse passive coordinate transform being applied to the lowered indices of tensor quantities to transform them, the Jacobian tensor $S^{\bar\mu}{}_\alpha(X = x)$ for the reverse event partner relation, (25), is applied to the lowered indices of unshifted tensor quantities to obtain their shifted partners. For instance, $Z^S_\alpha = S^{\bar\mu}{}_\alpha Z^{US}_\mu$.

Technically, the shift quantity relation (87) is *only applicable for "zero-order" tensor quantities $Z^{\alpha\beta}$ that do not contain derivatives*, as indicated. This is because the event partner relation (16) used to obtain (87) is a *nondifferentiable* homeomorphism between partner event fields on the uniform scale, so it may only be applied to tensor quantities $Z^{\alpha\beta}$ that do not contain derivatives. The quantity relation (87) is applicable then for all zero-order tensor quantities $Z^{\alpha\beta}$ utilized to depict object properties that are subject to gravity shifting. This includes all zero-order native tensor quantities depicting matter and the nongravitational fields, where a "native" quantity is one that does not contain a metric. An example is the de Broglie wave 1-form k_α for a particle (discussed later), with $k^S_\alpha = S^{\bar\mu}{}_\alpha k^{US}_\mu$ relating its partner values. The displacements tied to objects qualify as zero-order tensor quantities subject to gravity shifts, so (87) is applicable yielding $dx^\alpha_S = S^\alpha{}_{\bar\mu} dx^\mu_{US}$, recovering the partner relation (14) itself.

As it is the shifted objects that are actually present in a gravitational field, the shifted quantities $Z_S^{\alpha\beta}$ are the *actual* such quantities $Z^{\alpha\beta}$. This identification is formally expressed by the "shifted/actual quantity equality"

$$Z_S^{\alpha\beta} = Z^{\alpha\beta}, \tag{88}$$

which will be shown to hold for *all shifted quantities* $Z_S^{\alpha\beta}$, *including differentiated ones, except those explicitly containing the shift or potential tensors and their derivatives*. The equality (88) is consistent with the identification of the shifted events with the actual events as stated by (13), so as expected (88) holds for the shifted displacements as explicitly stated by $dx_S^{\alpha} = dx^{\alpha}$ (15). On the other hand, the unshifted partner quantities, $Z_{US}^{\mu\nu}$, depicting the unshifted partner objects, are the *hypothetical* quantities obtained if gravitation were removed in theory.

The gravitational field itself may be treated as an object subject to gravity shifting, as follows. Removal of the shifting present implies removal of the actual gravitational field to obtain the hypothetical partner "unshifted field" with *vanishing* field strength. The "unshifted (field) shift tensor" $S^{\alpha}{}_{\bar{\mu}}{}^{US} = \delta^{\alpha}_{\bar{\mu}}$ is therefore equal to the delta tensor as stated. So the equivalent "unshifted potential tensor" $w^{\alpha}{}_{\mu}{}^{US} = 0$ is zero as stated. Use of the unshifted shift tensor in the metric relation (35) yields the "unshifted (field) natural metric" $g_{\mu\nu}^{US}$ equal to the absolute metric $a_{\mu\nu}$. Application of gravity shifting to the unshifted field yields the partner "shifted field." This results in application of the shift quantity relation (87) to the unshifted metric $g_{\mu\nu}^{US}$ to yield the partner "shifted natural metric"

$$g_{\alpha\beta}^{S} = (g_{\mu\nu}^{US} = a_{\mu\nu})S^{\bar{\mu}}{}_{\alpha}S^{\bar{\nu}}{}_{\beta} = g_{\alpha\beta}. \tag{89}$$

The shifted natural metric $g_{\alpha\beta}^{S}$ is seen to equal the *actual* natural metric $g_{\alpha\beta}$ obtained via the metric relation (35) (as indicated), satisfying (88). The shifted/actual quantity equality (88) therefore holds for all zero-order matter and nongravitational field quantities that either are native quantities or contain the natural metric.

Application of gravity shifting to obtain the partner shifted field also results in application of the quantity relation (87) to the unshifted shift and potential tensors, yielding $S^{\alpha}{}_{\bar{\mu}}{}^{S} = \delta^{\alpha}_{\bar{\mu}}$ for the "shifted shift tensor," and $w^{\alpha}{}_{\mu}{}^{S} = 0$ for the "shifted potential tensor." These have the same *vanishing* field strength values as their unshifted partners, so they do not satisfy (88) since the actual values of the shift and potential tensors are $S^{\alpha}{}_{\bar{\mu}}$ and $w^{\alpha}{}_{\mu}$ for the actual gravitational system with nonvanishing field strength. This inequality can be understood in the context of $S^{\alpha}{}_{\bar{\mu}}{}^{S}$ and $w^{\alpha}{}_{\mu}{}^{S}$ being the shift and potential tensors for the shifted partner of an unshifted *local* gravitational system where the actual system acts as a "background system" applying the shifting. So $S^{\alpha}{}_{\bar{\mu}}{}^{S}$ and $w^{\alpha}{}_{\mu}{}^{S}$ are the shift and potential tensors for the *partner shifted local gravitational system only*, which are not then the shift and potential tensors of the actual system inducing the shifting. But as shown the natural metric $g_{\alpha\beta}^{S}$ for the partner shifted local gravitational system, obtained by applying (87) to the metric $g_{\mu\nu}^{US} = a_{\mu\nu}$ for the unshifted local system, does indeed match the metric $g_{\alpha\beta}$ of the actual system. Therefore, the only zero-order shifted quantities $Z_S^{\alpha\beta}$ for which the shifted/actual quantity equality, $Z_S^{\alpha\beta} = Z^{\alpha\beta}$ (88), does not hold, are ones explicitly containing the shifted shift tensor $S^{\alpha}{}_{\bar{\mu}}{}^{S} = \delta^{\alpha}_{\bar{\mu}}$ and/or the shifted potential tensor $w^{\alpha}{}_{\mu}{}^{S} = 0$.

The absolute metric $a_{\mu\nu}$ *is not subject to gravity shifting*, since it is an *absolute* quantity that does not depict a property of matter or fields that are tied to the local partner event fields x_{US} and x_S. Since the absolute metric $a_{\mu\nu}$ does not shift, *any quantities $Z^{\alpha\beta}$ that contain the absolute metric are not considered shifted quantities $Z_S^{\alpha\beta}$*, even if they consist of shifted native matter or nongravitational field quantities for which the absolute metric has then been applied. However, consider the following. *Prior to shift application*, the unshifted natural metric $g_{\mu\nu}^{US}$ may be substituted for the equal valued absolute metric $a_{\mu\nu}$ for any zero-order unshifted partner quantity $Z_{US}^{\mu\nu}$ containing the absolute metric. Therefore, when (87) is applied to $Z_{US}^{\mu\nu}$, (89) is applied to the contained unshifted natural metric $g_{\mu\nu}^{US} = a_{\mu\nu}$, yielding the shifted natural metric $g_{\alpha\beta}^{S}$ utilized in place of the absolute metric $a_{\alpha\beta}$. The resultant quantities formed using this "absolute (metric) replacement method" are therefore *shifted quantities $Z_S^{\alpha\beta}$*. As can

be seen, use of the absolute replacement method effectively enables *all* zero-order quantities to be subject to shifting, yielding *universal applicability* of the shift quantity partner relation (87). In applying the absolute replacement method, the absolute metric $a_{\mu\nu}$ contained in any $Z^{\mu\nu}_{US}$ is *interpreted* as the unshifted natural metric $g^{US}_{\mu\nu} = a_{\mu\nu}$. If $Z^{\mu\nu}_{US}$ is $a_{\mu\nu}$ itself, use of the absolute replacement method yields (89) giving the shifted natural metric $g^S_{\alpha\beta}$, which can be seen to also reproduce the metric relation $g_{\alpha\beta} = a_{\mu\nu}S^{\bar\mu}{}_\alpha S^{\bar\nu}{}_\beta$ (35) used to provide the *actual* natural metric $g_{\alpha\beta} = g^S_{\alpha\beta}$. The absolute metric in the metric relation $g_{\alpha\beta} = a_{\mu\nu}S^{\bar\mu}{}_\alpha S^{\bar\nu}{}_\beta$ may be interpreted then as the unshifted natural metric $g^{US}_{\mu\nu} = a_{\mu\nu}$, so the actual natural metric $g_{\alpha\beta} = g^S_{\alpha\beta}$ may be considered to be a shifted quantity $Z^{\alpha\beta}_S$, with again (88) applicable. Now if the absolute metric is applied to an *already formed* shifted quantity $Z^{\alpha\beta}_S$, then the resultant quantity $Z^{\alpha\beta}$ is not a shifted quantity due to the presence of $a_{\mu\nu}$.

The global IC given unshifted quantities $\check{Z}^{\mu\nu}_{US}$ are *inertially valued*, since they depict unshifted objects posed in the absolute inertial frames as given by their global ICs (using a "check" to indicate global IC use for clarity). The global IC values of the unshifted quantities $\check{Z}^{\mu\nu}_{US}$ are therefore their *known* inertial values, such as the known inertial values for quantities depicting matter and the nongravitational fields. The global IC values for the unshifted gravitational field quantities are $\check{g}^{US}_{\mu\nu} = \check{a}_{\mu\nu} = \eta_{\mu\nu}$ for the unshifted natural metric, and $\check{S}^\alpha{}_{\bar\mu}{}^{US} = \delta^\alpha_{\bar\mu}$ and $\check{w}^\alpha{}_\mu{}^{US} = 0$ for the unshifted shift and potential tensors. Use of the known inertial global IC values, $\check{Z}^{\mu\nu}_{US}$, for the unshifted partner quantities, enables a determination of the global IC values $\check{Z}^{\alpha\beta}_S$ for zero-order tensor shifted quantities via use of (87). This yields $\check{g}^S_{\alpha\beta} = \eta_{\mu\nu}\check{S}^{\bar\mu}{}_\alpha \check{S}^{\bar\nu}{}_\beta$, $\check{S}^{\alpha S}_{\bar\mu} = \delta^\alpha_{\bar\mu}$, and $\check{w}^{\alpha S}_\mu = 0$. The values of the zero-order shifted quantities, $Z^{\alpha\beta}_S$, in any coordinates, may be obtained then via coordinate transformation from their known global IC values $\check{Z}^{\alpha\beta}_S$. Finally, application of the equality $Z^{\alpha\beta}_S = Z^{\alpha\beta}$ (88) yields the actual values for all zero-order tensor quantities subject to gravity shifting, except those explicitly containing the shift or potential tensors, in which case the actual values $S^\alpha{}_{\bar\mu}$ and $w^\alpha{}_\mu$ may be utilized in the expressions for the shifted quantities.

All "higher-order" shifted quantities, $Z^{\alpha\beta}_S$, containing derivatives of arbitrarily high order, may be provided by differentiating zero-order tensor quantities $Z^{\alpha\beta}_S$ obtained via use of the shift quantity partner relation (87). As a result, the shifted/actual quantity equality $Z^{\alpha\beta}_S = Z^{\alpha\beta}$ (88) holds for all shifted quantities $Z^{\alpha\beta}_S$, including differentiated ones, except those explicitly containing the shift or potential tensors and their derivatives, as stated when (88) was given. Note that due to the actual and shifted event equality (13), differentiation with respect to the shifted event locations x^μ_S is the same as with respect to the actual event locations x^μ, supporting the equality $Z^{\alpha\beta}_S = Z^{\alpha\beta}$ for differentiated quantities. The differentiated shifted quantities $Z^{\alpha\beta}_S$ are not limited to being *tensor* quantities, though they must be formed from zero-order tensor quantities $Z^{\alpha\beta}_S$ obtained via (87). A key example is the natural metric compatible connection given by its Christoffel symbol

$$\Gamma^\alpha_{\mu\nu} = \frac{1}{2}g^{\alpha\sigma}(g_{\sigma\nu,\mu} + g_{\mu\sigma,\nu} - g_{\mu\nu,\sigma}), \tag{90}$$

where $\Gamma^\alpha_{\mu\nu} = \Gamma^{\alpha\,S}_{\mu\nu}$ as constructed from the zero-order $g_{\alpha\beta} = g^S_{\alpha\beta}$. From this the natural metric curvature tensor $R^\alpha{}_{\beta\mu\nu} = R^{\alpha\,S}_{\beta\mu\nu}$ may be formed as provided by the usual

$$R^\alpha{}_{\beta\mu\nu} = \frac{\partial\,\Gamma^\alpha_{\beta\nu}}{\partial x^\mu} - \frac{\partial\,\Gamma^\alpha_{\beta\mu}}{\partial x^\nu} + \Gamma^\alpha_{\sigma\mu}\Gamma^\sigma_{\beta\nu} - \Gamma^\alpha_{\sigma\nu}\Gamma^\sigma_{\beta\mu} = 0. \tag{91}$$

Differentiation of the zero-order tensor quantities $Z^{\alpha\beta}_S$ may be performed using ordinary derivatives, natural covariant derivatives utilizing the natural metric connection $\Gamma^\alpha_{\mu\nu}$, or absolute covariant derivatives using the absolute metric connection $A^\alpha_{\mu\nu}$. Since it is not subject to shifting though, absolute covariant differentiation yields a quantity that is no longer considered a shifted quantity $Z^{\alpha\beta}_S$. To obtain

then shifted quantities $Z_S^{\alpha\beta}$, differentiation of the zero-order tensor quantities $Z_S^{\alpha\beta}$ is limited to use of ordinary derivatives and natural covariant derivatives.

Application of the shift quantity relation to zero-order scalar quantities yields

$$Z_S = Z_{US} \quad \text{(zero order)}, \tag{92}$$

stating that zero-order scalar shifted quantities have values equal to their unshifted partners. This is applicable for "intrinsically" scalar quantities such as electric charge, and for zero-order scalar quantities formed by tensor contractions such as $Z_\alpha^{\alpha S} = S^\alpha_{\bar\mu} S^{\bar\nu}_\alpha Z^{\mu US}_\nu = Z^{\mu US}_\mu$.

4. The Local Diffeomorphism and Satisfaction of the Equivalence Principle

4.1. The Local Diffeomorphism

As will be shown, if a finitely large local system is surrounded by a gravitational "background system," the background system induces a *local diffeomorphism* applied to the local system so long as the effects of background system curvature may be neglected. The local diffeomorphism ("morph") between partner event fields takes the form

$$x_M^\alpha = M^\alpha(x_{UM}^\mu), \tag{93}$$

where $M^\alpha(x_{UM}^\mu)$ is a continuously differentiable function such that the locations x_M^α of the "morphed" partner events, x_M, are in 1-to-1 relation to the locations x_{UM}^μ of the "unmorphed" partner events, x_{UM}, over the extended spacetime region of the finitely large local system. This implies the existence of a 1-to-1 reverse morph given by

$$x_{UM}^\mu = M^{\bar\mu}(x_M^\alpha), \tag{94}$$

with the "bar" over the indice used for M indicating the reverse morph. The resultant differential forms of the forward and reverse morphs are

$$\mathrm{d}x_M^\alpha = M^\alpha_{\bar\mu}\,\mathrm{d}x_{UM}^\mu, \qquad \mathrm{d}x_{UM}^\mu = M^{\bar\mu}_\alpha\,\mathrm{d}x_M^\alpha, \tag{95}$$

where $M^\alpha_{\bar\mu}$ and $M^{\bar\mu}_\alpha$ are the forward and reverse morph Jacobian tensors defined by

$$M^\alpha_{\bar\mu} \equiv \partial M^\alpha(x_{UM}^\rho)/\partial x_{UM}^\mu = \partial x_M^\alpha/\partial x_{UM}^\mu,$$
$$M^{\bar\mu}_\alpha \equiv \partial M^{\bar\mu}(x_M^\sigma)/\partial x_M^\alpha = \partial x_{UM}^\mu/\partial x_M^\alpha. \tag{96}$$

Similar to the shift tensors, the quantities $M^\alpha_{\bar\mu}$ and $M^{\bar\mu}_\alpha$ are indeed *tensors*, since they transform as tensors when coordinate transforms are applied to the differential morphs (95). Similar to the local event partner relation (16), the morph (93) *formally* takes the mathematical form of a passive coordinate transform, but is instead an *active transform* where the locations of events are moved as expressed in common coordinates. However, the event partner relation is a *nondifferentiable homeomorphism* between partner event fields over an *infinitesimally sized* spacetime region, whereas the morph is a *continuously differentiable diffeomorphism* between partner event fields over an *extended* region.

4.2. Lack of Integrability for the Partner Relation and the Induction of Natural Metric Curvature

For background, see Schutz [18] (Chap. 5) for a discussion on integrability conditions for coordinate versus noncoordinate bases treated as the Jacobian matrices for passive coordinate transforms. Since the math is formally the same, this material is applicable for $M^{\bar\mu}_\alpha$ versus $S^{\bar\mu}_\alpha$ treated as Jacobian tensors for the active transforms consisting of the reverse morph (94) and the reverse partner relation

(24). Adapting this discussion to the morph versus partner relation case, similar to coordinate bases, $M^{\bar{\mu}}{}_\alpha$ for the differential form (95) of the reverse morph satisfies the "integrability condition"

$$\frac{\partial}{\partial x_M^\beta} M^{\bar{\mu}}{}_\alpha = \frac{\partial^2 x_{UM}^\mu}{\partial x_M^\beta \partial x_M^\alpha} = \frac{\partial^2 x_{UM}^\mu}{\partial x_M^\alpha \partial x_M^\beta} = \frac{\partial}{\partial x_M^\alpha} M^{\bar{\mu}}{}_\beta, \tag{97}$$

having used (96) and the commutivity of partial derivatives. This implies that the differential form may be *integrated* to yield the reverse morph (94) itself (see [21], Chap. 4, for mathematical proof), as expected since the differential form was obtained from the reverse morph. The reverse morph may then be inverted to yield the forward morph (93).

If the reverse partner relation (24) were integrable, the integrability condition for $S^{\bar{\mu}}{}_\alpha$ would hold, obtainable by substituting "S" for "M" throughout (97). However, similar to noncoordinate bases, $S^{\bar{\mu}}{}_\alpha$ does not generally satisfy the integrability condition, formally stated by the generally applicable "nonintegrability condition"

$$\partial S^{\bar{\mu}}{}_\alpha / \partial x_S^\beta \neq \partial S^{\bar{\mu}}{}_\beta / \partial x_S^\alpha. \tag{98}$$

An example of nonintegrability is the star case, where from (86) the reverse shift tensor in global ICs is

$$S^{\bar{\mu}}{}_\alpha = \text{diag}\left[e^{-M/r}, e^{M/r}, e^{M/r}, e^{M/r} \right]. \tag{99}$$

Then $\partial S^{\bar{0}}{}_0 / \partial x_S^n$ for the reverse shift tensor equals $\partial S^{\bar{0}}{}_0 / \partial x^n \neq 0$ for any n, where $dx_S^n = dx^n$ was utilized to convert from $\partial S^{\bar{0}}{}_0 / \partial x_S^n$ to $\partial S^{\bar{0}}{}_0 / \partial x^n$. But using $dx_S^0 = dx^0$, $\partial S^{\bar{0}}{}_n / \partial x_S^0$ equals $\partial S^{\bar{0}}{}_n / \partial x^0 = 0$. The nonintegrability condition applies then for the star case, so the reverse partner relation cannot be integrated over an extended spacetime region to yield a reverse morph in the form of (94), barring then formation of a forward morph (93) by inverting a reverse morph. The single star case may be used as a counterexample to establish that *the reverse partner relation* (24) *is not generally integrable, barring its integration to form a morph in either reverse or forward form* (as discussed in the summary).

On a technical note, when examining integrability above, use of the *reverse* partner relation (24) was selected in order to utilize the equality, $dx_S^\mu = dx^\mu$, of the shifted partner and actual displacements, so that $\partial S^{\bar{\mu}}{}_\alpha / \partial x_S^\beta$ may be conveniently converted to the equal valued $\partial S^{\bar{\mu}}{}_\alpha / \partial x^\beta$ to take the derivatives of the reverse shift tensor $S^{\bar{\mu}}{}_\alpha$, which is given as a function of the *actual* event locations x^σ (such as in the reverse star-case shift tensor (99)). On the other hand, attempted use of the forward partner relation (14) would require taking the derivatives $\partial S^\alpha{}_{\bar{\mu}} / \partial x_{US}^\beta$ in order to form the nonintegrability condition, which is inconvenient since $\partial S^\alpha{}_{\bar{\mu}} / \partial x_{US}^\beta$ does not equal $\partial S^\alpha{}_{\bar{\mu}} / \partial x^\beta$ due to $dx_{US}^\mu \neq dx^\mu$. For this reason, the reverse partner relation, and the corresponding differential form of the reverse morph, are utilized when evaluating integration.

Consider substitution of $M^{\bar{\mu}}{}_\alpha$ for $S^{\bar{\mu}}{}_\alpha$ in the metric relation (35) to form $g_{\alpha\beta} = a_{\mu\nu} M^{\bar{\mu}}{}_\alpha M^{\bar{\nu}}{}_\beta$. As this is formally the same as the passive coordinate transform of the absolute metric $a_{\mu\nu}$ where $M^{\bar{\mu}}{}_\alpha$ is the reverse Jacobian matrix, then the natural metric $g_{\alpha\beta}$ in this case *has no curvature* since the absolute metric has no curvature. This may be verified by forming the curvature tensor $R^\alpha{}_{\beta\mu\nu}$ from the metric $a_{\mu\nu} M^{\bar{\mu}}{}_\alpha M^{\bar{\nu}}{}_\beta$ and showing that it vanishes due to $M^{\bar{\mu}}{}_\alpha$ satisfying the integrability condition (97) applicable for coordinate transform Jacobians. On the other hand, returning to the actual metric relation $g_{\alpha\beta} = a_{\mu\nu} S^{\bar{\mu}}{}_\alpha S^{\bar{\nu}}{}_\beta$, adherence to the nonintegrability condition (98) for $S^{\bar{\mu}}{}_\alpha$ results in a *nonzero* curvature tensor $R^\alpha{}_{\beta\mu\nu}$. The tedious formal proof of this result is not shown here, but is verified for the star-case metric (85) obtained from use of the metric relation with a $S^{\bar{\mu}}{}_\alpha$ that has been shown to adhere to the nonintegrability condition. As the lack of integrability for the reverse partner relation (24) is equivalent to the nonintegrability condition, then the lack of integrability for the reverse partner relation *induces natural metric curvature*. The lack of partner relation integrability results in the metric relation $g_{\alpha\beta} = a_{\mu\nu} S^{\bar{\mu}}{}_\alpha S^{\bar{\nu}}{}_\beta$ *not* taking the form of a passive coordinate transform for the absolute metric, as consistent with the induction of natural metric curvature under the nonintegrability condition. Since natural observers characterize absolute spacetime using the natural metric $g_{\alpha\beta}$ due to their use

of gravity shifted instruments, *the lack of partner relation integrability for gravity shifts is the mechanism underlying natural observers perceiving the absolute flat spacetime manifold as being curved* (as stated in the summary).

4.3. Partner Paths

Even though the partner relation (in either forward or reverse form) cannot in general be integrated to obtain a diffeomorphism between partner event *fields*, partner unshifted and shifted event *paths* can be constructed. This extends the existence of partner events from the uniform scales for partner event fields, generally limited to infinitesimal spacetime regions, to *extended spacetime regions* containing partner event paths tied to partner shifted and unshifted objects *running along the partner paths*.

In preparation for the construction of partner paths, the formulation of unshifted and shifted geodesics is helpful. Consider an "unshifted path" constructed from unshifted displacements dx^μ_{US}, which is then parameterized by using the unshifted path parameter λ_{US} along its length, yielding an unshifted curve $x^\mu_{US}(\lambda_{US})$ (i.e., a parameterized path) with tangent vectors $U^\mu_{US} \equiv dx^\mu_{US}/d\lambda_{US}$. The length s_{US} of the unshifted path is obtained by integrating the unshifted proper intervals ds_{US} for the unshifted displacements dx^μ_{US}, as given by the unshifted line element (31), yielding

$$s_{US} = \int |a_{\mu\nu} \, dx^\mu_{US} \, dx^\nu_{US}|^{1/2} = \int |a_{\mu\nu} U^\mu_{US} U^\nu_{US}|^{1/2} \, d\lambda_{US}. \tag{100}$$

The "unshifted geodesic" running between two events is the unshifted curve that yields an extremum for its length s_{US}, resulting in the unshifted geodesic equation

$$\frac{dU^\mu_{US}}{d\lambda_{US}} + A^\mu_{\rho\sigma} U^\rho_{US} U^\sigma_{US} = 0, \tag{101}$$

where $A^\mu_{\rho\sigma}$ is the absolute metric Christoffel symbol given by (12). As is understood in standard treatments of this material (such as in MTW [16]), geodesics are curves that parallel transport their own tangent vectors under the given connection, with the geodesic equation depicting the parallel transport. This is the case then for the parallel transport of U^μ_{US} under the absolute metric connection $A^\mu_{\rho\sigma}$, with its parallel transport depicted by (101). The parameter λ_{US} becomes an affine parameter for unshifted geodesics, which may be replaced by s_{US} for absolutely spacelike geodesics and τ_{US} for absolutely timelike geodesics, since they are also affine parameters proportional to λ_{US} plus a constant. The absolutely timelike unshifted geodesics give the geodesic motions of hypothetical unshifted particles when gravitation is removed, which are dictated then by the absolute metric connection $A^\mu_{\rho\sigma}$. For the geodesic motion of unshifted light, the more general λ_{US} is utilized since $\tau_{US} = 0$. With $\check{A}^\mu_{\rho\sigma} = 0$ in global ICs, the geodesic motions of unshifted particles and light are *inertial* in the global ICs of absolute inertial frames, and the absolutely spacelike unshifted geodesics are *straight lines*.

Consider a "shifted path" constructed from shifted displacements dx^α_S, which is then parameterized by using the shifted path parameter λ_N along its length, yielding a shifted curve $x^\alpha_S(\lambda_N)$ with tangent vectors $U^\alpha_S \equiv dx^\alpha_S/d\lambda_N$. As will be shown, the length of interest for the shifted path is the *naturally measured* length s_N obtained by integrating the naturally measured shifted proper intervals $ds_N = ds_{S(N)}$ for the shifted displacements dx^α_S, as given by the natural metric line element (34), yielding

$$s_N = \int |g_{\alpha\beta} \, dx^\alpha_S \, dx^\alpha_S|^{1/2} = \int |g_{\alpha\beta} U^\alpha_S U^\beta_S|^{1/2} \, d\lambda_N. \tag{102}$$

The "shifted geodesic" running between two events is the shifted curve that yields an extremum for its natural length s_N, resulting in the shifted geodesic equation

$$\frac{dU^\alpha_S}{d\lambda_N} + \Gamma^\alpha_{\mu\nu} U^\mu_S U^\nu_S = 0, \tag{103}$$

where $\Gamma^\alpha_{\mu\nu}$ is the natural metric Christoffel symbol given by (90). Similar to the unshifted geodesics, the shifted geodesics are curves that parallel transport the tangent vector U^α_S under the natural metric connection $\Gamma^\alpha_{\mu\nu}$, with its parallel transport depicted by (103). The parameter λ_N becomes an affine parameter for shifted geodesics, which may be replaced by s_N for naturally spacelike geodesics and τ_N for naturally timelike geodesics, since they are also affine parameters proportional to λ_N plus a constant. This is the reason for using the natural designation "N" for λ_N. For the geodesic motion of shifted light, the more general λ_N is utilized since $\tau_N = 0$. The naturally timelike shifted geodesics depict the geodesic motions of shifted/actual "test particles" (ones that do not modify the field created by the surrounding gravitational sources) under the action of the gravitational field as given by the natural metric connection $\Gamma^\alpha_{\mu\nu}$. This result agrees with natural observations of free-particle gravitational motions, which is the reason why the naturally measured path length s_N was utilized when the principle of least action was applied to obtain the shifted geodesics. When the least action principle is used to obtain universal geodesic motions for all free-moving test particles, the same motions are yielded independent of their mass and composition, in agreement with the weak form of the postulated equivalence principle as required.

To construct *arbitrary* partner paths running from a common shift origin X, a "segmented construction" technique is employed where partner paths are segmented into partner pairs of infinitesimal incremental segments δx^μ_{US} and δx^α_S. Partner segments are related to each other via parallel transport back to the common shift origin X, where at X the partner segments adhere to the shift origin partner relation

$$\delta x^\alpha_S|_X = S^\alpha_{\bar\mu}|_X \delta x^\mu_{US}|_X, \tag{104}$$

as is required for partner segments at a single location. The partner paths are constructed then by adding their respective partner segments. The arbitrary path parameter λ_{US} may be utilized for unshifted paths, yielding curves where the tangent vectors for the segments δx^μ_{US} are $V^\mu_{US} \equiv \delta x^\mu_{US}/\delta\lambda_{US}$ with $\delta\lambda_{US}$ the infinitesimal incremental intervals for the segments. The unshifted segment tangent vectors parallel transport under the absolute directional covariant derivative as given by

$$\frac{D_A V^\mu_{US}}{d\lambda_{US}} \equiv \frac{dV^\mu_{US}}{d\lambda_{US}} + A^\mu_{\rho\sigma} U^\rho_{US} V^\sigma_{US} = 0, \tag{105}$$

where $U^\mu_{US} = dx^\mu_{US}/d\lambda_{US}$ is the usual tangent vector, for unshifted path displacements dx^μ_{US}, that is utilized to form the directional covariant derivative. The form of the unshifted segment parallel transport equation is consistent with the unshifted geodesic equation (101), as required to yield unshifted geodesics in the infinitesimal limit where the segment tangent vector $V^\mu_{US} = \delta x^\mu_{US}/\delta\lambda_{US}$ becomes $U^\mu_{US} = dx^\mu_{US}/d\lambda_{US}$. The arbitrary path parameter λ_N may be utilized for shifted paths, yielding curves where the tangent vectors for the segments δx^α_S are $V^\alpha_S \equiv \delta x^\alpha_S/\delta\lambda_N$ with $\delta\lambda_N$ the infinitesimal incremental intervals for the segments. The shifted segment tangent vectors parallel transport under the natural directional covariant derivative as given by

$$\frac{D_N V^\alpha_S}{d\lambda_N} \equiv \frac{dV^\alpha_S}{d\lambda_N} + \Gamma^\alpha_{\mu\nu} U^\mu_S V^\nu_S = 0, \tag{106}$$

where $U^\alpha_S = dx^\alpha_S/d\lambda_N$ is the usual tangent vector, for shifted path displacements dx^μ_S, that is utilized to form the directional covariant derivative. The form of the shifted segment parallel transport equation is consistent with the shifted geodesic equation (103), as required to yield shifted geodesics in the infinitesimal limit where the segment tangent vector $V^\alpha_S = \delta x^\alpha_S/\delta\lambda_N$ becomes $U^\alpha_S = dx^\alpha_S/d\lambda_N$.

The detailed segmented construction of partner paths is developed below, which utilizes the key "interval constancy property," established here. Since an unshifted or shifted path segment "δx^α" parallel transports via $D\,\delta x^\alpha/d\lambda = 0$ the same as any vector under parallel transport, then $\delta\lambda$ in the tangent vector $V^\alpha = \delta x^\alpha/\delta\lambda$ must be constant under parallel transport. Therefore, the interval $\delta\lambda$ for an unshifted or shifted path segment δx^α *does not change* when its tangent vector $\delta x^\alpha/\delta\lambda$ is parallel

transported. This is similar to the proper intervals δs for segments δx^α remaining constant under parallel transport using the applicable metric connection, noting that proper path distances s may be utilized to parameterize any portions of paths that are timelike or spacelike. Summarizing, $\delta\lambda_{US}$ for unshifted segments δx^μ_{US} along an arbitrary unshifted path, and $\delta\lambda_N$ for shifted segments δx^α_S along an arbitrary shifted path, do not change when the tangent vectors $V^\mu_{US} = \delta x^\mu_{US}/\delta\lambda_{US}$ and $V^\alpha_S = \delta x^\alpha_S/\delta\lambda_N$ for these segments are parallel transported under their applicable metric connections via (105) or (106), regardless of the path parameters λ_{US} and λ_N utilized.

Up to this stage in the partner path development, the path parameters λ_{US} and λ_N have been treated as *independent* arbitrary parameters. Going forward, the "general interval equality"

$$\delta\lambda_N|_X = \delta\lambda_{US}|_X \tag{107}$$

is applied for partner segments δx^μ_{US} and δx^α_S when transported back to the shift origin. This equality is obtained using the partner equivalence property for natural measurement of local partner objects, where $\delta\lambda_{US}|_X$ is interpreted as the naturally measured arbitrary interval $\delta\lambda_{US(N)}|_X$ for $\delta x^\mu_{US}|_X$ using unshifted instruments, and $\delta\lambda_N|_X$ is interpreted as the *partner* naturally measured interval $\delta\lambda_{S(N)}|_X$ for $\delta x^\alpha_S|_X$ using the raw shifted partner instruments. So (107) holds since $\delta\lambda_{S(N)}|_X = \delta\lambda_N|_X$ equals $\delta\lambda_{US(N)}|_X = \delta\lambda_{US}|_X$ under partner equivalence. As a verification, when $\delta\lambda_{US}|_X$ is a proper interval, (107) yields the expected partner equivalence based proper interval equality $\delta s_N|_X = \delta s_{US}|_X$ (as per (32)) applicable for partner segments at X.

Utilizing the interval constancy property and the general interval equality (107), the segmented construction of arbitrary partner paths is as follows. An arbitrary unshifted path is run from the shift origin, X, and parameterized with an arbitrary parameter λ_{US} to obtain the curve $x^\mu_{US}(\lambda_{US})$. It is then divided into infinitesimal incremental segments δx^μ_{US} along its length, each with its interval $\delta\lambda_{US}$ so that the tangent vector $V^\mu_{US} = \delta x^\mu_{US}/\delta\lambda_{US}$ may be formed. The first unshifted path segment at X is shifted, via (104), to form the first shifted path segment δx^α_S at X. Application of (107) to the shift origin partner relation, (104), yields partner tangent vectors $V^\mu_{US}|_X = [\delta x^\mu_{US}/\delta\lambda_{US}]_X$ and $V^\alpha_S|_X = [\delta x^\alpha_S/\delta\lambda_N]_X$ that satisfy the "tangent vector" partner relation

$$V^\alpha_S|_X = S^\alpha_{\bar\mu}|_X V^\mu_{US}|_X. \tag{108}$$

This provides the tangent vector V^α_S for the first shifted path segment δx^α_S, located at X. The second unshifted path segment, δx^μ_{US}, starting from the head of the first segment, is then parallel transported using (105) back to the shift origin X, with $\delta\lambda_{US}$ for this segment not changing under its parallel transport via its tangent vector. The shift origin partner relation (104) is applied to the second segment δx^μ_{US} at X to form the shifted partner segment δx^α_S, and then $V^\alpha_S = \delta x^\alpha_S/\delta\lambda_N$ is formed where $\delta\lambda_N = \delta\lambda_{US}$ as per (107), yielding satisfaction of (108). The shifted partner segment at X is then parallel transported, using (106), so that its tail is joined to the head of the first shifted path segment. Since $\delta\lambda_N$ for a shifted segment δx^α_S does not change when parallel transported via its tangent vector, then it has the same value $\delta\lambda_{US}$ as the second unshifted segment at X and therefore in its original location. With $\delta\lambda_N = \delta\lambda_{US}$, this second segment for the shifted path is considered to be the shifted partner of the second segment of the unshifted partner path, since it was obtained by applying the rules of unshifted and shifted parallel transport combined with uniform-scale gravity shifting at the shift origin X. This process is repeated for the third unshifted segment and so on, yielding the shifted partners of the unshifted segments, which are successively added to each other to form the partner shifted path $x^\alpha_S(\lambda_N)$ to the arbitrary unshifted path $x^\mu_{US}(\lambda_{US})$.

As $\delta\lambda_N = \delta\lambda_{US}$ for the partner segments along the entire lengths of partner paths, then for a given unshifted segment δx^μ_{US} located at λ_{US} from the shift origin X, the location of its partner shifted segment δx^α_S has the *same* value

$$\lambda_N = \lambda_{US}. \tag{109}$$

When the partner paths are exclusively timelike or spacelike, substitution of s_N for λ_N, and s_{US} for λ_{US}, yields $s_N = s_{US}$ for the proper path distances to partner segments. This is as expected since both δs_N and δs_{US} remain constant for partner segments under their respective parallel transports, and $\delta s_N|_X = \delta s_{US}|_X$ when each unshifted partner segment transported back to X is then shifted to obtain its shifted partner segment, yielding $\delta s_N = \delta s_{US}$ for all partner segments, and therefore $s_N = s_{US}$ when combined.

Shifted paths, tied to shifted objects running along them, are the paths that *actually* exist in a gravitational field, with path locations $x_S^\alpha(\lambda_N)$ that are the locations x^α of actual events x in the absolute spacetime manifold as per (13). Whereas the unshifted partner paths, tied to the unshifted partner objects running along them, are the *hypothetical* paths obtained when gravitation is removed, yielding path locations $x_{US}^\mu(\lambda_{US})$ that are the locations of the hypothetical unshifted path events $x_{US}(\lambda_{US})$ that are partners to the shifted/actual path events $x_S(\lambda_N = \lambda_{US}) = x$ with locations $x_S^\alpha(\lambda_N = \lambda_{US}) = x^\alpha$.

If an unshifted path is a geodesic satisfying (101), then application of the above segmented construction yields a shifted partner path that is a geodesic satisfying (103). To see that this is the case, when each segment δx_{US}^μ for the unshifted geodesic is parallel transported via (105) back to the shift origin, it runs parallel to the first unshifted segment located at X. When the transported segment at X is shifted via (104) to yield its shifted partner δx_S^α, the shifted partner runs parallel to the first shifted segment. Therefore, using (106), parallel transporting the second shifted partner segment, initially at X, to its place beyond the first shifted segment, yields a segment parallel to the first segment as required for a geodesic. Repeating this for the third shifted partner segment, and so on, yields segments parallel to their preceding segments as the shifted path is constructed, resulting in a partner shifted path that is a shifted geodesic satisfying (103). As is generally the case for geodesics, each partner geodesic may be generated by continuously parallel transporting the first segment at the shift origin X, yielding partner geodesics in their entirety (they may be run to arbitrarily high lengths) *uniquely* determined by parallel transporting the first partner segments at X as "generators." If the first unshifted partner segment $\delta x_{US}^\mu|_X$ is absolutely timelike with proper interval $\delta \tau_{US}|_X$, then via (32) the first shifted partner segment $\delta x_S^\alpha|_X$ is naturally timelike with interval $\delta \tau_N|_X = \delta \tau_{US}|_X$, resulting in timelike partner geodesics in their entirety since their generating segments at the shift origin are timelike. Similarly, an absolutely spacelike $\delta x_{US}^\mu|_X$ yields a naturally spacelike $\delta x_S^\alpha|_X$ with $\delta s_N|_X = \delta s_{US}|_X$ (here specifically spacelike intervals), resulting in spacelike partner geodesics in their entirety since their generating segments at X are spacelike. Finally, an absolutely null $\delta x_{US}^\mu|_X$, with proper interval $\delta \tau_{US}|_X = 0$, yields a naturally null $\delta x_S^\alpha|_X$ with $\delta \tau_N|_X = \delta \tau_{US}|_X = 0$, resulting in null geodesics depicting the motions of partner unshifted and shifted light. The timelike and null partner geodesics are *as expected* for partner particles and light moving from a common shift origin with path segments $\delta x_{US}^\mu|_X$ and $\delta x_S^\alpha|_X$ related by the uniform-scale partner relation (104) applicable at a single location. The generation of these expected partner geodesics by the given segmented construction technique verifies its validity.

4.4. Establishing the Local Diffeomorphism

A "nongravitational system" is a system consisting of matter and nongravitational fields with weak enough source strength that their contribution to the gravitational field may be considered negligible (the usual definition). So if a local nongravitational system is posed in the gravitational field of a surrounding "background system," the only field present is *the background system field*. The background system field is therefore the field for the *total system* consisting of the background system and the local nongravitational system combined. This equality is formally stated by

$$g_{\alpha\beta}^B = g_{\alpha\beta}, \qquad S_{\bar\mu}^{\alpha B} = S_{\bar\mu}^\alpha, \qquad w_\mu^{\alpha B} = w_\mu^\alpha, \quad \text{(local nongravitational system)} \tag{110}$$

where "B" is used to indicate the background system field quantities, and the total system field quantities are unlabeled. When establishing the local diffeomorphism below based on use of local nongravitational systems, equation (110) is "automatically" utilized to substitute background field quantities for the total field quantities found in previously given formulations (unless otherwise stated).

Therefore, for a local nongravitational system, the nongravitational material content is shifted by the background system shift tensor field $S^{\alpha B}_{\check{\mu}}$, yielding *background shifted paths* tied to the background shifted nongravitational material content. Consider the actual "gravitational case" consisting of a background shifted path tied to the background shifted nongravitational material content of a finitely large local nongravitational system, as posed in the gravitational field of a surrounding background system. Removal in theory of the background field yields the hypothetical partner "inertial case" consisting of the *unshifted partner path* tied to the same nongravitational material content as the shifted path, with the partner material content now unshifted. The "inertial case" is referred to as such since the lack of a background field results in *inertial conditions* for the unshifted partner path, and for the unshifted nongravitational material content it is tied to.

Starting with the inertial-case unshifted partner paths utilizing global ICs, then via (105) with a zero-valued absolute metric connection $\check{A}^{\mu}_{\rho\sigma}$, the IC components $\check{V}^{\mu}_{US}$ of an unshifted path segment tangent vector $\vec{V}_{US} = \delta\vec{x}_{US}/\delta\lambda_{US}$ remain *fixed* when parallel transported. With the general interval $\delta\lambda_{US}$ remaining fixed under the interval constancy property when $\vec{V}_{US}$ is parallel transported, then the global IC components $\delta\check{x}^{\mu}_{US}$ of the unshifted segment vector $\delta\vec{x}_{US}$ itself remain fixed. So when parallel transported back to the shift origin,

$$\delta\check{x}^{\mu}_{US}|_X = \delta\check{x}^{\mu}_{US}(\lambda_{US}), \tag{111}$$

where the segment on the right is at its original unshifted path location λ_{US}.

Any gravitational-case shifted path may be obtained by applying the background system field to the partner inertial-case unshifted partner path. Using the tangent vector partner relation (108), it may be readily shown that when parallel transported back to the shift origin, the dot products of different partner path segment tangent vectors are related via the equality

$$\left[\vec{V}_S \overset{N}{\cdot} \vec{W}_S\right]_X = \left[\vec{V}_{US} \overset{A}{\cdot} \vec{W}_{US}\right]_X, \tag{112}$$

where N over a dot is the inner product using the natural metric, in this case the background system natural metric $g^B_{\alpha\beta}$, and A over a dot is the inner product using the absolute metric. Consider arbitrary unshifted paths in a given global IC system, and their shifted partner paths. At the shift origin, the tangent vectors $\vec{e}_{(\check{\mu})US}|_X \equiv [\delta\vec{x}_{(\check{\mu})US}/\delta s_{US}]_X$ for any unshifted path segments $\delta\vec{x}_{(\check{\mu})US}|_X$ running along the four IC axis directions, as parameterized by their proper displacements $\delta s_{US}|_X$ (one temporal and three spatial), are equal to the fixed global IC basis vectors $\vec{e}_{(\check{\mu})}$ at all locations. Then with $\vec{e}_{(\check{\mu})US}|_X = \vec{e}_{(\check{\mu})}$ for a given $\vec{e}_{(\check{\mu})US}|_X$, its global IC components are $e^{\check{\alpha}}_{(\check{\mu})US}|_X = \delta^{\alpha}{}_\mu$. The shifted partner segments for each of the four $\delta\vec{x}_{(\check{\mu})US}|_X$ are designated as $\delta\vec{x}_{(\hat{\mu})S}|_X$. The tangent vectors for the four $\delta\vec{x}_{(\hat{\mu})S}|_X$ are given by $\vec{e}_{(\hat{\mu})S}|_X \equiv [\delta\vec{x}_{(\hat{\mu})S}|_X/\delta s_N]_X$ with $\delta s_N|_X = \delta s_{US}|_X$ via (32), where $\vec{e}_{(\hat{\mu})S}|_X$ will be utilized as coordinate basis vectors below with a "hat" designating their coordinates. This is why the tetrad designator $(\hat{\mu})$ is used for the shifted partner segments $\delta\vec{x}_{(\hat{\mu})S}|_X$. Application of (108), utilizing the background shift tensor, yields the global IC given "basis vector" partner relation

$$e^{\check{\alpha}}_{(\hat{\mu})S}|_X = \check{S}^{\alpha B}_{\check{\sigma}}|_X\, e^{\check{\sigma}}_{(\check{\mu})US}|_X \tag{113}$$

in component form. Substituting $\delta^{\sigma}{}_\mu$ for $e^{\check{\sigma}}_{(\check{\mu})US}|_X$ yields the IC values

$$e^{\check{\alpha}}_{(\hat{\mu})S}|_X = \check{S}^{\alpha B}_{\check{\mu}}|_X \tag{114}$$

for the shifted basis vectors at X. Summing the products with $\vec{e}_{(\check{\alpha})}$ on both sides of (114), and using the equality $\vec{e}_{(\check{\alpha})} = \vec{e}_{(\check{\alpha})US}|_X$ on the right, results in the basis vector partner relation

$$\vec{e}_{(\hat{\mu})S}|_X = \check{S}^{\alpha B}_{\check{\mu}}|_X\, \vec{e}_{(\check{\alpha})US}|_X \tag{115}$$

in vector form. Finally, substitution of either (113) or (115) into the partner tangent vector dot product equality, (112), yields the partner basis dot products

$$\left[\vec{e}_{(\hat{\mu})S} \overset{N}{\cdot} \vec{e}_{(\hat{\nu})S} \right]_X = \left[\vec{e}_{(\check{\mu})US} \overset{A}{\cdot} \vec{e}_{(\check{\nu})US} \right]_X = \eta_{\mu\nu} \tag{116}$$

at the shift origin, where use was made of $\vec{e}_{(\check{\mu})US}|_X = \vec{e}_{(\check{\mu})}$, and of $\vec{e}_{(\check{\mu})} \overset{A}{\cdot} \vec{e}_{(\check{\nu})} = \breve{a}_{\mu\nu} = \eta_{\mu\nu}$ for the global IC basis vectors.

Utilizing the gravitational-case shifted geodesics run from the shift origin, as given by the shifted geodesic equation (103) using the background natural metric $g^B_{\alpha\beta}$ to form the Christoffel symbol, the set of naturally orthonormal $\vec{e}_{(\hat{\mu})S}|_X$ (as per (116)) may be used to establish *Riemann normal coordinates* such as shown in PW [20] (Chap. 5). The shift origin X is therefore the Riemann coordinate origin. The quantities $\vec{e}_{(\hat{\mu})S}|_X$ are the Riemann coordinate basis vectors at the coordinate origin X, with Riemann coordinate values

$$e^{\hat{\alpha}}_{(\hat{\mu})S}|_X = \delta^{\alpha}_{\mu}, \tag{117}$$

justifying the above identification of the shifted partners $\vec{e}_{(\hat{\mu})S}|_X$ to $\vec{e}_{(\check{\mu})US}|_X$ as coordinate basis vectors. The utilized "hat" designates then the Riemann coordinates. With their basis vectors $\vec{e}_{(\hat{\mu})S}|_X$ and $\vec{e}_{(\check{\mu})US}|_X = \vec{e}_{(\check{\mu})}$ being partners, the constructed Riemann coordinates are "partner coordinates" to the utilized global ICs. As shown in PW, the natural metric near X is given by

$$\hat{g}^B_{\mu\nu} = \eta_{\mu\nu} - \frac{1}{3} \hat{R}^B_{\mu\alpha\nu\beta}|_X \hat{x}^{\alpha} \hat{x}^{\beta} + O(\hat{x}^3) \tag{118}$$

in Riemann normal coordinates, where its Christoffel symbol obeys

$$\hat{\Gamma}^{\mu B}_{\alpha\beta}|_X = 0, \qquad (\partial_{\hat{\beta}} \hat{\Gamma}^{\mu B}_{\nu\alpha})_X = -\frac{1}{3}(\hat{R}^{\mu B}_{\nu\alpha\beta} + \hat{R}^{\mu B}_{\alpha\nu\beta})_X, \tag{119}$$

noting use of the background natural metric for the formulation being made here. The Riemann coordinates represent *gravitational free-fall frames of reference*, specifically the free-fall frames of the surrounding background systems. The Riemann coordinates provide *locally inertial coordinates* under the background natural metric since

$$\hat{g}^B_{\mu\nu}|_X = \eta_{\mu\nu}, \qquad (\partial_{\hat{\beta}} \hat{g}^B_{\mu\nu})_X = 0, \tag{120}$$

and since the shifted geodesics are *straight lines* when close enough to X that the curvature-induced change in $\hat{\Gamma}^{\mu B}_{\nu\alpha}$ from zero, as per (119), may be considered to induce negligible "deflections" from straight lines. The term "Riemann inertial coordinates (ICs)" refers to Riemann coordinates when specifically utilized to provide locally inertial coordinates (as opposed to "Riemann coordinates" not limited to being inertial coordinates). The Riemann IC systems provide then *locally inertial frames of reference* as concerns the inertial geodesic motions of shifted/actual particles and light.

Consider the actual "gravitational case" consisting of a finitely large local nongravitational system for which the curvature of the surrounding background system's field yields negligible "segment transport effects," meaning effects on the parallel transport of path segments for shifted paths running throughout the local system. Then background system locally inertial free-fall frames exist, as provided by their Riemann ICs, that *subtend the finitely large local nongravitational system*, with therefore a zero-valued background natural metric connection $\hat{\Gamma}^{\mu B}_{\nu\alpha}$ over each inertial frame as per (119) with background curvature neglected. So in any given inertial free-fall frame, via (106) utilizing $\hat{\Gamma}^{\mu B}_{\alpha\beta} = 0$, the components $\hat{V}^{\mu}_S$ of a shifted path segment tangent vector $\vec{V}_S = \delta\vec{x}_S / \delta\lambda_N$ remain *fixed* when parallel transported. With the general interval $\delta\lambda_N$ remaining fixed under the interval constancy property when $\vec{V}_S$ is

parallel transported, then the Riemann IC components $\delta\hat{x}_S^\mu$ of the shifted segment vector $\delta\vec{x}_S$ itself remain fixed. So when parallel transported back to the shift origin,

$$\delta\hat{x}_S^\mu|_X = \delta\hat{x}_S^\mu(\lambda_N), \tag{121}$$

where the segment on the right is at its original shifted path location λ_N. In summary, for a finitely large local nongravitational system where background curvature yields negligible segment transport effects, equation (121) holds in the Riemann ICs of the background system locally inertial free-fall frames subtending the local system.

Utilizing the methodology in Weinberg [22] (Chap. 3), the "partner coordinate transform" between the partner global ICs and Riemann coordinates is established as follows. Applying the shifted geodesic equation (103) in the partner Riemann coordinates, with $\hat{\Gamma}_{\alpha\beta}^{\mu B}|_X = 0$ from (119) and $\hat{U}_S^\mu = \mathrm{d}\hat{x}_S^\mu/\mathrm{d}\tau_N$, yields

$$\left.\frac{\mathrm{d}^2\hat{x}_S^\mu}{\mathrm{d}\tau_N^2}\right|_X = 0 \tag{122}$$

for the geodesic motions of shifted particles at the shift origin. Transforming this into the partner global ICs yields

$$\left[\frac{\mathrm{d}^2\check{x}_S^\alpha}{\mathrm{d}\tau_N^2} + \check{\Gamma}_{\rho\sigma}^{\alpha B}\frac{\mathrm{d}\check{x}_S^\rho}{\mathrm{d}\tau_N}\frac{\mathrm{d}\check{x}_S^\sigma}{\mathrm{d}\tau_N}\right]_X = 0 \tag{123}$$

for the geodesic motions as per (103) with $\check{U}_S^\alpha = \mathrm{d}\check{x}_S^\alpha/\mathrm{d}\tau_N$, where via the coordinate transform the Christoffel symbol may be given by

$$\check{\Gamma}_{\rho\sigma}^{\alpha B}|_X = \left[\frac{\partial\check{x}_S^\alpha}{\partial\hat{x}_S^\nu}\frac{\partial^2\hat{x}_S^\nu}{\partial\check{x}_S^\rho\partial\check{x}_S^\sigma}\right]_X, \tag{124}$$

as opposed to the equal valued $\check{\Gamma}_{\rho\sigma}^{\alpha B}|_X$ in metric form (90) with $\check{x}^\alpha = \check{x}_S^\alpha$ and $\check{g}_{\alpha\beta} = \check{g}_{\alpha\beta}^B$. Multiplying both sides of (124) by the coordinate transform $[\partial\hat{x}_S^\mu/\partial\check{x}_S^\alpha]_X$ yields

$$\left[\frac{\partial^2\hat{x}_S^\mu}{\partial\check{x}_S^\rho\partial\check{x}_S^\sigma}\right]_X = \check{\Gamma}_{\rho\sigma}^{\alpha B}|_X\left[\frac{\partial\hat{x}_S^\mu}{\partial\check{x}_S^\alpha}\right]_X. \tag{125}$$

Equation (125) may be utilized as a differential equation to obtain the solution $\hat{x}_S^\mu$ as a function of $\check{x}_S^\alpha$ in the neighborhood of X, yielding the form

$$\hat{x}_S^\mu = P^{\hat{\mu}}(\check{x}_S^\alpha) = P^{\hat{\mu}}_{\check{\alpha}}|_X(\check{x}_S^\alpha - \check{X}^\alpha) + \frac{1}{2}P^{\hat{\mu}}_{\check{\rho}}|_X\check{\Gamma}_{\alpha\beta}^{\rho\,B}|_X(\check{x}_S^\alpha - \check{X}^\alpha)(\check{x}_S^\beta - \check{X}^\beta) + \cdots \tag{126}$$

for the partner coordinate transform, where $P^{\hat{\mu}}_{\check{\alpha}}|_X = [\partial\hat{x}_S^\mu/\partial\check{x}_S^\alpha]_X$ is the partner transform Jacobian at the shift origin.

In order that the Riemann coordinate shifted basis values $e_{(\hat{\alpha})S}^{\hat{\mu}}|_X = \delta^\mu_\alpha$ (117) are obtained from the partner global IC given shifted basis values $e_{(\hat{\alpha})S}^{\check{\mu}}|_X = \check{S}^{\mu B}_{\check{\alpha}}|_X$ (114), then it must be the case that the partner transform Jacobian at X is

$$P^{\hat{\mu}}_{\check{\alpha}}|_X = \check{S}^{\bar{\mu} B}_\alpha|_X, \tag{127}$$

since then $e_{(\hat{\alpha})S}^{\hat{\mu}}|_X$ is given by $P^{\hat{\mu}}_{\check{\sigma}}|_X\,e_{(\hat{\alpha})S}^{\check{\sigma}}|_X = \check{S}^{\bar{\mu} B}_\sigma|_X\check{S}^{\sigma B}_{\check{\alpha}}|_X = \delta^\mu_\alpha$ as required. Substituting this into (126) yields the explicitly given partner coordinate transform

$$\hat{x}_S^\mu = P^{\hat{\mu}}(\check{x}_S^\alpha) = \check{S}^{\bar{\mu} B}_\alpha|_X(\check{x}_S^\alpha - \check{X}^\alpha) + \frac{1}{2}\check{S}^{\bar{\mu} B}_\rho|_X\check{\Gamma}_{\alpha\beta}^{\rho\,B}|_X(\check{x}_S^\alpha - \check{X}^\alpha)(\check{x}_S^\beta - \check{X}^\beta) + \cdots, \tag{128}$$

where if desired the substitutions $\hat{x}^\mu = \hat{x}_S^\mu$ and $\check{x}^\alpha = \check{x}_S^\alpha$ (as per (13)) may be made since the actual and shifted events are the same. The third-order and higher terms in the expansion (not shown)

provide the partner transform beyond the solution of (125), which holds at the shift origin only, with these higher-order terms generally dependent on the background system curvature. For instance, the third-order term contains $\partial_{\check\beta}\check\Gamma^{\mu B}_{\nu\alpha}(X)$, which is similar to the Riemann coordinate value $\partial_{\hat\beta}\hat\Gamma^{\mu B}_{\nu\alpha}(X)$ given by (119). The explicitly given terms in (128) therefore provide the partner transform between the global ICs and the partner Riemann ICs, with the Riemann ICs applicable when the background system's curvature may be neglected. The absolute inertial frame given by the partner global ICs, and the background system locally inertial free-fall frame given by the partner Riemann ICs, are referred to as being "partner inertial frames."

With the shift origin partner relation (104) given in the partner global ICs by $\delta\check x^\alpha_S|_X = \check S^{\alpha B}_{\check\sigma}|_X \delta\check x^\sigma_{US}|_X$, application of the partner transform $P^{\hat\mu}_{\check\alpha}|_X = \check S^{\bar\mu B}_\alpha|_X$ on both sides yields the component equality

$$\delta\hat x^\mu_S|_X = \delta\check x^\mu_{US}|_X \tag{129}$$

of partner path segments at the shift origin, holding when the shifted partner segment components $\delta\hat x^\mu_S|_X$ are given in the partner Riemann coordinates, and the unshifted partner segment components $\delta\check x^\mu_{US}|_X$ are given in the partner global ICs. Combining (129) with the global IC equality (111) for the parallel transport of unshifted partner path segments to the shift origin, then $\delta\hat x^\mu_S|_X = \delta\check x^\mu_{US}(\lambda_{US})$. Applying to this the Riemann IC local inertial frame equality (121) for the parallel transport of shifted partner path segments to the shift origin, the component equality

$$\delta\hat x^\mu_S(\lambda_N = \lambda_{US}) = \delta\check x^\mu_{US}(\lambda_{US}) \tag{130}$$

is obtained for partner path segments at their original locations, holding for arbitrary partner paths when the background system's curvature yields negligible segment transport effects. Note the use of $\lambda_N = \lambda_{US}$ (109) for the locations of partner path segments in (130). Equation (130) may be integrated along the partner paths to obtain the key partner path locational value equality

$$\hat x^\mu_S(\lambda_N = \lambda_{US}) = \check x^\mu_{US}(\lambda_{US}) - \check X^\mu. \tag{131}$$

Therefore, over the region subtended by a finitely large local nongravitational system where the background system's curvature yields negligible segment transport effects, *any unshifted partner path given in global ICs, as specified relative to the shift origin $\check X^\mu$, yields an identically valued shifted partner path given in the Riemann ICs of a background system locally inertial free-fall frame subtending the region, where the global and Riemann ICs are related by the partner coordinate transform.* Of course the partner paths themselves are generally not the same event paths, but the global and Riemann IC locations of their partner events $x_{US}(\lambda_{US})$ and $x_S(\lambda_N = \lambda_{US})$ are the same.

Under conditions such that (131) holds, *all possible* global IC given unshifted partner paths running from X out to a selected arbitrary unshifted event location, $\check x^\mu_{US}$, yield identically valued partner Riemann IC given shifted partner paths running from X out to a *single* shifted event location $\hat x^\mu_S$. As a result, *over the region subtended by a finitely large local nongravitational system where the surrounding background system's curvature yields negligible effects on the parallel transport of shifted path segments, a local diffeomorphism is yielded between partner event fields*, which may be formally given by

$$\hat x^\mu_M = \check x^\mu_{UM} - \check X^\mu, \tag{132}$$

obtained by utilizing (131) for arbitrary partner paths run to their respective partner endpoints

$$x_{UM} \equiv x_{US}(\lambda_{US}), \qquad x_M \equiv x_S(\lambda_N = \lambda_{US}). \tag{133}$$

The relation between $\check x^\mu_{UM}$ and $\hat x^\mu_M$ given by (132) is indeed a diffeomorphism, since (by inspection) it is 1-to-1 and continuously differentiable, and it is also onto. Below, (132) will be converted into the usual form of a diffeomorphism given in common coordinates for both x^μ_{UM} and x^α_M, with then the

material in Section 4.1 being applicable. Note that the diffeomorphism (132) itself is specified without the need of path parameters, as the morph holds as a partner event *field* relation independent of any *particular* paths utilized to establish it. However, as per (133), unmorphed and morphed partner events x_{UM} and x_M have as their *origins* unshifted and shifted partner events $x_{US}(\lambda_{US})$ and $x_S(\lambda_N = \lambda_{US})$ obtained by utilizing partner paths. As partner path construction involves the use of metric connection based parallel transport, the partner path based partner events $x_{US}(\lambda_{US})$ and $x_S(\lambda_N = \lambda_{US})$ are not obtained by simply integrating the partner relation (14), which has been shown to not be integrable in general. For this reason, the notation x_{UM} and x_M is utilized to specify the unmorphed and morphed partner events $x_{UM} \equiv x_{US}(\lambda_{US})$ and $x_M \equiv x_S(\lambda_N = \lambda_{US})$ obtained via the use of arbitrary partner paths.

The morph was constructed utilizing partner unshifted and shifted objects running along partner event paths tied to the partner objects. Therefore, under the morph, *unmorphed and morphed partner objects are yielded* where their shared material content is tied to the unmorphed and morphed partner event fields x_{UM} and x_M. The local diffeomorphism therefore *extends gravity shifting to finitely large spacetime regions*, but in a form different from the partner relation based shifting. The term "gravity shifting" applies then to both the morph and partner relation based shifting, with discernment between the two made by context.

Since the morphed partner events x_M are obtained by running shifted paths from X to x_M that consist of infinitesimal shifted increments $\delta x_S^\alpha = \mathrm{d} x_S^\alpha$ summed together, then the morphed events x_M are the shifted events x_S as well, as stated by $x_M = x_S(\lambda_N)$ from (133). In addition, the shifted events x_S are the actual events x obtained when the gravitational field is present as per (13), with the gravitational field here the background system field. Therefore,

$$x_M = x = x_S, \tag{134}$$

stating that *the morphed, actual, and shifted events are the same*. The unmorphed events x_{UM} are the *hypothetical* events obtained if the gravitational field of the background system were removed in theory. At first sight, it may be thought that the parallel transported shifted path segments, $\delta x_S^\alpha = \mathrm{d} x_S^\alpha$, utilized to obtain the morph, could not be the same as shifted displacements $\mathrm{d} x_S^\alpha$ from the partner relation (14) using $S^{\alpha B}_{\bar\mu}$, so that $x_M = x_S(\lambda_N)$ obtained by summing the parallel transported shifted path segments $\delta x_S^\alpha = \mathrm{d} x_S^\alpha$ could not be the same as x_S obtained by summing shifted displacements $\mathrm{d} x_S^\alpha$ from the partner relation. However, due to the lack of integrability of the partner relation, for successive shifted displacements $\mathrm{d} x_S^\alpha$ added end-to-end along a shifted path, their hypothetical unshifted partners $\mathrm{d} x_{US}^\mu = S^{\bar\mu B}_{\alpha}\, \mathrm{d} x_S^\alpha$ are "stand-alone" displacements that are not connected end-to-end. This provides the *degree of freedom* required to allow the parallel transported shifted path segments $\delta x_S^\alpha = \mathrm{d} x_S^\alpha$ to also be shifted displacements $\mathrm{d} x_S^\alpha$ from the partner relation. Now the *unmorphed* partner displacement $\mathrm{d} x_{UM}^\mu$ to $\mathrm{d} x_S^\alpha = \mathrm{d} x_M^\alpha$ is located at the hypothetical unmorphed event x_{UM}. On the other hand, as per the practice followed for partner relation based gravity shifting, the infinitesimal *unshifted* partner displacement to $\mathrm{d} x_S^\alpha$, namely $\mathrm{d} x_{US}^\mu = S^{\bar\mu B}_{\alpha}\, \mathrm{d} x_S^\alpha$, is considered to be located at the shifted/actual event $x_S = x$ where uniform-scale gravity shifting is being evaluated, which via (134) is the morphed event x_M. If desired, $x_S = x_M$ may be used as the common shift origin X for the partner displacements $\mathrm{d} x_{US}^\mu$ and $\mathrm{d} x_S^\alpha$, yielding the infinitesimal partner event fields x_{US} and x_S depicting stand-alone uniform-scale shifting as given by (16) where $X = x_M$ and $S^\alpha_{\bar\mu} = S^{\alpha B}_{\bar\mu}$.

Assuming negligible background curvature segment transport effects, then (134) may be substituted into the partner coordinate transform (128) to obtain the "morph partner (coordinate) transform"

$$\hat{x}_M^\mu = P^{\hat\mu}(\check{x}_M^\alpha) = \check{S}^{\bar\mu B}_{\alpha}|_X(\check{x}_M^\alpha - \check{X}^\alpha) + \frac{1}{2}\check{S}^{\bar\mu B}_{\rho}|_X \check{\Gamma}^{\rho B}_{\alpha\beta}|_X(\check{x}_M^\alpha - \check{X}^\alpha)(\check{x}_M^\beta - \check{X}^\beta) \tag{135}$$

between the global ICs and the partner Riemann ICs, which is simply the partner coordinate transform applied to the locations of the morphed events x_M. Note that the higher-order curvature-dependent terms have been *dropped* from the partner coordinate transform (128), as occurs for morphs since

the background curvature segment transport effects are considered negligible. The utilized partner Riemann ICs for $\hat{x}_M^\mu$ therefore provide the locally inertial frame used to form the morph as given by (132). So the right-hand side of (135) may be substituted for $\hat{x}_M^\mu$ in (132), yielding the *reverse* morph

$$\check{x}_{UM}^\mu = \check{M}^{\bar{\mu}}(\check{x}_M^\alpha) = \check{X}^\mu + \check{S}_\alpha^{\bar{\mu}B}|_X \Delta\check{x}_M^\alpha + \frac{1}{2}\check{S}_\rho^{\bar{\mu}B}|_X \check{\Gamma}_{\alpha\beta}^{\rho\,B}|_X \Delta\check{x}_M^\alpha \Delta\check{x}_M^\beta \tag{136}$$

given *exclusively* in the partner global ICs, where $\Delta\check{x}_M^\alpha \equiv \check{x}_M^\alpha - \check{X}^\alpha$. Finally, inverting (136) yields the *exclusively* global IC given *forward* morph

$$\check{x}_M^\alpha = \check{M}^\alpha(\check{x}_{UM}^\mu) = \check{X}^\alpha + \check{S}_{\bar{\mu}}^{\alpha B}|_X \Delta\check{x}_{UM}^\mu - \frac{1}{2}\check{\Gamma}_{\rho\sigma}^{\alpha B}|_X \check{S}_{\bar{\mu}}^{\rho B}|_X \check{S}_\nu^{\sigma B}|_X \Delta\check{x}_{UM}^\mu \Delta\check{x}_{UM}^\nu, \tag{137}$$

where $\Delta\check{x}_{UM}^\mu \equiv \check{x}_{UM}^\mu - \check{X}^\mu$. Similar to the reverse morph (136), higher-order terms in the IC-given forward morph (137) would have curvature dependence, so they do not appear since background curvature segment transport effects are considered negligible for obtaining the morph.

Equations (137) and (136) are the explicit global IC forms for the generally given forward and reverse morphs (93) and (94). The forward and reverse morphs, in any coordinates, may be obtained via application of coordinate transforms to $\check{x}_{UM}^\mu$ and $\check{x}_M^\alpha$ in the global IC forms (137) and (136). Use of the global IC forward morph form (137) is helpful though for the following reason. Application of the morph partner transform (135) to the global IC location $\check{x}_M^\alpha$ of a morphed partner event x_M, as given by (137) (and dropping higher-order terms mixing with the curvature-dependent terms being neglected), yields the partner Riemann IC given location $\hat{x}_M^\mu$ equal to the global IC location $\check{x}_{UM}^\mu - \check{X}^\mu$ of the unmorphed partner event x_{UM}, recovering the key morph relation (132) equating the morph partner event locations $\hat{x}_M^\mu$ and $\check{x}_{UM}^\mu - \check{X}^\mu$. The morph partner transform (135) is therefore a *passive* coordinate transform that *formally* acts to "reverse" the global IC given morph, (137), which is an *active* transform between the partner events as given by $\check{x}_{UM}^\mu$ and $\check{x}_M^\alpha$. Since though the morph partner transform is not the actual reverse morph (136), it is referred to as the "pseudo-reverse" of the global IC given morph.

In the infinitesimally sized region surrounding the shift origin X, the morph (137) reduces to $\Delta\check{x}_M^\alpha = \check{S}_{\bar{\mu}}^{\alpha B}|_X \Delta\check{x}_{UM}^\mu$. Now the infinitesimal morph partner displacements $\Delta\check{x}_{UM}^\mu$ and $\Delta\check{x}_M^\alpha$ at the shift origin may be equated with the *first* infinitesimal unshifted and shifted partner path segments $\delta\check{x}_{US}^\mu|_X$ and $\delta\check{x}_S^\alpha|_X$ of the partner paths utilized to construct the morph, which satisfy the shift origin partner relation (104) using $S_{\bar{\mu}}^{\alpha B}|_X$. Then $\Delta\check{x}_{UM}^\mu = \delta\check{x}_{US}^\mu|_X = \check{x}_{US}^\mu - \check{X}^\mu$ and $\Delta\check{x}_M^\alpha = \delta\check{x}_S^\alpha|_X = \check{x}_S^\alpha - \check{X}^\alpha$. Substitution into $\Delta\check{x}_M^\alpha = \check{S}_{\bar{\mu}}^{\alpha B}|_X \Delta\check{x}_{UM}^\mu$ yields the local event partner relation (16) given in global ICs, and therefore in any coordinates under arbitrary coordinate transform (where $S_{\bar{\mu}}^\alpha|_X = S_{\bar{\mu}}^{\alpha B}|_X$). Similarly, the reverse morph (136) reduces to the reverse event partner relation (25) (where $S_\alpha^{\bar{\mu}}|_X = S_\alpha^{\bar{\mu}B}|_X$) in the infinitesimal region surrounding the shift origin X. Concluding, *in the infinitesimal region surrounding the shift origin X, the morph-given gravity shifting reduces to the event form of partner relation based gravity shifting*, as expected for gravity shifting at a single location.

Applying (96) to (137) and (136), the global IC given morph Jacobian tensors are

$$\check{M}_{\bar{\mu}}^\alpha = \check{S}_{\bar{\mu}}^{\alpha B}|_X - \check{\Gamma}_{\rho\sigma}^{\alpha B}|_X \check{S}_{\bar{\mu}}^{\rho B}|_X \check{S}_\nu^{\sigma B}|_X \Delta\check{x}_{UM}^\nu,$$
$$\check{M}_\alpha^{\bar{\mu}} = \check{S}_\alpha^{\bar{\mu}B}|_X + \check{S}_\rho^{\bar{\mu}B}|_X \check{\Gamma}_{\alpha\beta}^{\rho\,B}|_X \Delta\check{x}_M^\beta. \tag{138}$$

The differential forms (95) of the forward and reverse morphs may be explicitly given in global ICs utilizing the Jacobians (138). Note that *at the shift origin X, the morph Jacobians become the shift tensors*, as expected for gravity shifting at a single location. With infinitesimal morph partner displacements at the shift origin shown above to be shift partner displacements, then at the shift origin the differential forms of the forward and reverse morphs reduce to the forward and reverse partner relations (14) and (24) (where $S_{\bar{\mu}}^\alpha|_X = S_{\bar{\mu}}^{\alpha B}|_X$), as consistent with the reduction of the event-given morph to the event partner relation in the infinitesimal region surrounding the shift origin. Unlike the symmetric shift

tensors, the morph Jacobian tensors are generally *not symmetric* (away from the shift origin) when put in pure indice form utilizing the absolute metric or background natural metric. Since the morph partner (coordinate) transform (135) is the pseudo-reverse of the global IC given forward morph (137) as an active transform, their Jacobians are in *inverse relation* as stated by

$$P^{\hat{\mu}}{}_{\check{\alpha}} = \check{M}^{\bar{\mu}}{}_{\alpha},$$
$$P^{\check{\alpha}}{}_{\hat{\mu}} = \check{M}^{\alpha}{}_{\bar{\mu}}. \tag{139}$$

Converting $\check{\Gamma}^{\rho B}_{\alpha\beta}|_X$ for the reverse morph Jacobian in (138) into its explicit shift tensor and absolute metric form, the resultant differential reverse morph (from (95)) is

$$d\check{x}^{\mu}_{UM} = \check{S}^{\bar{\mu}B}{}_{\alpha}|_X d\check{x}^{\alpha}_{M} + \left[\check{S}^{\sigma\bar{\mu}}_{B} \check{S}^{B}_{\bar{\nu}\alpha} (\check{S}^{\bar{\nu}B}_{\sigma,\beta} - \check{S}^{\bar{\nu}B}_{\beta,\sigma}) + \check{S}^{\bar{\mu}B}_{\alpha,\beta} \right]_X \Delta\check{x}^{\beta}_{M} d\check{x}^{\alpha}_{M}. \tag{140}$$

The reverse morph Jacobian $\check{M}^{\bar{\mu}}{}_{\alpha} = \partial\check{x}^{\mu}_{UM}/\partial\check{x}^{\alpha}_{M}$, as given by (138), satisfies the integrability condition (97) where $x_M = x$, as expected since it is derived from the reverse morph (136) itself. So of course the differential reverse morph (140) may be integrated to yield the reverse morph. If the reverse shift tensor $\check{S}^{\bar{\mu}B}_{\alpha}$ satisfied the morph integrability condition (97), the quantity $\check{S}^{\bar{\nu}B}_{\sigma,\beta} - \check{S}^{\bar{\nu}B}_{\beta,\sigma}$ in (140) would vanish, yielding a differential reverse morph equal to the first-order expansion of the reverse partner relation $d\check{x}^{\mu}_{US} = \check{S}^{\bar{\mu}B}_{\alpha} d\check{x}^{\alpha}_{S}$ about X. The lack though of shift tensor integrability, as per the nonintegrability condition (98), yields a nonzero $\check{S}^{\bar{\nu}B}_{\sigma,\beta} - \check{S}^{\bar{\nu}B}_{\beta,\sigma}$, resulting in a differential reverse morph (140) that is not the reverse partner relation expansion, explicitly confirming that the reverse morph (136) itself is not the integrated reverse partner relation. In general then, *the local diffeomorphism does not take the form of the integrated partner relation* (which does not generally exist), as stated in the summary.

Due to the presence of background system curvature, the equality $\delta\hat{x}^{\alpha}_{S}|_X = \delta\hat{x}^{\alpha}_{S}(\lambda_N)$ (121) for the transport of shifted path segments is actually an approximation, resulting in a morph $\hat{x}^{\mu}_{M} = \check{x}^{\mu}_{UM} - \check{X}^{\mu}$ (132) that is an *approximation*. The event equality $x_M = x$ on the left of (134) is therefore an approximation. But in the limit where background curvature segment transport effects vanish, such as for *infinitesimally sized* local nongravitational systems, the equality (121) becomes exact, yielding the morph (132) as an *exact* equality, and therefore $x_M = x$ as an exact equality. The morph $\hat{x}^{\mu}_{M} = \check{x}^{\mu}_{UM} - \check{X}^{\mu}$, and also the event equality $x_M = x$, are *treated* then as equalities under the assumption that the segment transport effects of background system curvature may be considered negligible, which is the methodology utilized going forward (unless stated otherwise). The equality $x = x_S$ on the right of (134) is an *exact* equality applicable at any location, since it repeats the exact equality (13).

4.5. Application of the Morph to Local Systems and the Resultant Equivalence Principle Satisfaction

Consider the actual "gravitational case" where a local system is posed in the gravitational field of a surrounding background system, where it is assumed that the effects of the background system curvature are negligible for the local system. The ability to neglect background system curvature effects is case dependent, depending on the strength of the background curvature, the configuration of the local system (including its size, which may be finitely large), the phenomena being evaluated, and the measurement precisions for the observations made on the evaluated phenomena, similar to when determining equivalence principle applicability under general relativity. The effects of background system curvature are varied, with the above-discussed background curvature segment transport effects included in the inventory. Therefore, for any given gravitational case where background system curvature effects are negligible for a local system posed in the gravitational field of a surrounding background system, *the above-established background system induced diffeomorphism is applicable over the spacetime region subtended by the local system*. If the background system gravitational field acting on the local system is removed in theory, the hypothetical partner "inertial case" is yielded consisting of the *unmorphed partner local system only*, referred to as such since the lack of a background field yields *inertial conditions* that the unmorphed partner system is posed in. This implies that when background

curvature effects may be considered negligible, *the actual gravitational case consists of the morphed partner local system*, obtained by applying the background morph to the unmorphed partner local system.

The morph was established above utilizing local *nongravitational* systems only. As will be shown, the background system provided morph is additionally applicable for local *gravitational* systems. Assuming here morph applicability for local gravitational systems for the sake of discussion, the general material provided in this section is applicable for *all* local systems. An unmorphed partner system may consist then of a local gravitational system, so application of a background system morph implies that the morph is applied to the *gravitational field* of the unmorphed partner system when obtaining the morphed partner system. The discussion and formulation in this section takes this into account. Since morph application for only local nongravitational systems has been proven at present, only morph-based gravity shifting for nongravitational partner objects tied to the partner event fields x_{UM} and x_M has been proven, with then their shared material content consisting of only matter and the nongravitational fields. But since the morph also holds for local gravitational systems as shown below, the discussion here is applicable for morph-based gravity shifting of *all* local objects, including the local gravitational field itself *treated as material content* tied to the partner event fields x_{UM} and x_M.

The quantities $Z^{\mu\nu}_{UM}$ and $Z^{\alpha\beta}_{M}$ are utilized to depict partner properties for unmorphed and morphed partner objects in morph partner systems (where "$Z^{\alpha\beta}$" represents quantities in general). Since the morph is an active transform, its application on an unmorphed tensor quantity, $Z^{\mu\nu}_{UM}$, is similar to a passive coordinate transform with the morph Jacobian tensor $M^{\alpha}{}_{\bar{\mu}}$ used in place of the coordinate Jacobian matrix $L^{\alpha'}{}_{\mu}$, yielding the morphed partner tensor quantity $Z^{\alpha\beta}_{M}$ as given by the representative "morph quantity (partner) relation"

$$Z^{\alpha\beta}_{M} = M^{\alpha}{}_{\bar{\mu}} M^{\beta}{}_{\bar{\nu}} Z^{\mu\nu}_{UM}. \tag{141}$$

This is similar to the shift quantity partner relation (87) developed in Section 3.14 utilizing the local event partner relation (16) as the active transform, with the concepts similar for the morph and shift cases. For example, similar to the shift case utilizing the reverse shift tensor $S^{\bar{\mu}}{}_{\alpha}$, in the morph case the reverse morph Jacobian $M^{\bar{\mu}}{}_{\alpha}$ is applied to the lowered indices of unmorphed tensor quantities to obtain their morphed partners, such as $Z^{M}_{\alpha} = M^{\bar{\mu}}{}_{\alpha} Z^{UM}_{\mu}$. However, recall that the shift quantity relation (87) is only applicable for *zero-order* quantities due to the event partner relation (16) being a *nondifferentiable* homeomorphism. In contrast, since the morph (93) is a *continuously differentiable* diffeomorphism, the morph quantity relation (141) is applicable for *all* tensor quantities $Z^{\alpha\beta}$ utilized to depict the properties of objects subject to gravity shifting, including then tensor quantities *containing derivatives of arbitrarily high order*. This includes all zero-order and differentiated tensor quantities depicting matter, the nongravitational fields, and the gravitational field. The gravitational field quantities consist of the natural metric $g_{\alpha\beta}$, shift tensor $S^{\alpha}{}_{\bar{\mu}}$, and potential tensor $w^{\alpha}{}_{\mu}$, as well as their derivatives. When using the morph quantity relation (141), unmorphed quantities $Z^{\mu\nu}_{UM}$ are given at the unmorphed event location x_{UM}, and morphed quantities $Z^{\alpha\beta}_{M}$ are given at the morphed event location x_M. This requires that the forward and reverse morph Jacobians applied to $Z^{\mu\nu}_{UM}(x_{UM})$ are both given as functions of x_{UM}. This is already the case for the global IC given forward Jacobian in (138), but the reverse Jacobian must be converted to have x_{UM} dependence. Utilizing (137) to provide $\Delta\check{x}^{\alpha}_M$ in terms of $\Delta\check{x}^{\mu}_{UM}$ for the reverse Jacobian, then (138) becomes

$$\check{M}^{\alpha}{}_{\bar{\mu}} = \check{S}^{\alpha B}{}_{\bar{\mu}}|_X - \check{\Gamma}^{\alpha B}{}_{\rho\sigma}|_X \check{S}^{\rho B}{}_{\bar{\mu}}|_X \check{S}^{\sigma B}{}_{\bar{\nu}}|_X \Delta\check{x}^{\nu}_{UM},$$

$$\check{M}^{\bar{\mu}}{}_{\alpha} = \check{S}^{\bar{\mu}B}{}_{\alpha}|_X + \check{S}^{\bar{\mu}B}{}_{\rho}|_X \check{\Gamma}^{\rho B}{}_{\alpha\sigma}|_X \check{S}^{\sigma B}{}_{\bar{\nu}}|_X \Delta\check{x}^{\nu}_{UM}, \tag{142}$$

having dropped the higher-order terms mixing with the curvature-dependent terms. Application of arbitrary coordinate transforms to (142) yields $M^{\alpha}{}_{\bar{\mu}}$ and $M^{\bar{\mu}}{}_{\alpha}$ in any coordinates, allowing use of (141) in any coordinates. Note that the forward and reverse Jacobians $\check{M}^{\alpha}{}_{\bar{\mu}}$ and $\check{M}^{\bar{\mu}}{}_{\alpha}$ are *inverses* of each other (dropping higher-order terms mixing with curvature dependence), as expected.

The gravitational field for a morphed *nongravitational* system is obtained by applying the morph to the field for the unmorphed partner nongravitational system with vanishing field strength. But as established above, the background system field is the gravitational field for a local nongravitational system in the gravitational case. Therefore, the gravitational field for a morphed nongravitational system is identified as *the morphed background system field*, obtained by applying the background morph to the unmorphed partner nongravitational system field identified as *the unmorphed background system field* with vanishing field strength. The "unmorphed background (system field) shift tensor" $S^\alpha{}_{\bar\mu}{}^{UMB} = \delta^\alpha{}_{\bar\mu}$ is therefore equal to the delta tensor as stated. Use of the unmorphed background shift tensor in the metric relation (35) yields the "unmorphed background (system field natural) metric" $g_{\mu\nu}^{UMB} = a_{\mu\nu}$ equal to the absolute metric as stated. Application of the background morph, to obtain the morphed background system field from the unmorphed background field, results in application of the morph quantity relation (141) to the unmorphed background metric $g_{\mu\nu}^{UMB}$ to yield the partner "morphed background (system field natural) metric"

$$g_{\alpha\beta}^{MB} = (g_{\mu\nu}^{UMB} = a_{\mu\nu})M^{\bar\mu}{}_\alpha M^{\bar\nu}{}_\beta, \tag{143}$$

similar to (89) for obtaining the shifted natural metric. Note that similar to the absolute metric, *both the unmorphed and morphed background metrics have no curvature* (from Section 4.2, $a_{\mu\nu}M^{\bar\mu}{}_\alpha M^{\bar\nu}{}_\beta$ on the right of (143) has no curvature).

The absolute metric $a_{\mu\nu}$ *is not subject to the morph*, since it is an *absolute* quantity that does not depict a property of matter or fields that are tied to the morph partner event fields x_{UM} and x_M, similar to the absolute metric not being subject to partner relation based gravity shifts. Since the absolute metric $a_{\mu\nu}$ does not morph, *any quantities $Z^{\alpha\beta}$ that contain the absolute metric are not considered morphed quantities $Z_M^{\alpha\beta}$*. However, similar to the shifted case, consider the following. For *both* nongravitational and gravitational local systems, the unmorphed background metric $g_{\mu\nu}^{UMB}$ may be substituted for the equal valued absolute metric $a_{\mu\nu}$ for all unmorphed partner system formulation *prior* to morph application. Therefore, when the background morph is applied to any unmorphed system to yield the morphed partner system, the morph quantity relation (141), and therefore (143), is applied to the unmorphed background metric $g_{\mu\nu}^{UMB} = a_{\mu\nu}$ used in the unmorphed system formulation, yielding the morphed background metric $g_{\alpha\beta}^{MB}$ utilized in place of the absolute metric $a_{\alpha\beta}$ for the morphed partner system formulation. The resultant quantities formed using this "absolute (metric) replacement method" (as similarly coined for the shifted case) are therefore *morphed quantities $Z_M^{\alpha\beta}$*. As can be seen, use of the absolute replacement method effectively enables *all* quantities to be subject to the morph, yielding *universal applicability* of the morph quantity partner relation (141). In applying the absolute replacement method, the absolute metric $a_{\mu\nu}$ contained in any $Z_{UM}^{\mu\nu}$ is *interpreted* as the unmorphed background metric $g_{\mu\nu}^{UMB} = a_{\mu\nu}$. Now if the absolute metric is applied to an *already formed* morphed quantity $Z_M^{\alpha\beta}$, then due to the presence of $a_{\mu\nu}$ the resultant quantity $Z^{\alpha\beta}$ is not a morphed quantity.

With the morph partner transform (135) from global ICs to the partner Riemann ICs the pseudo-reverse of the global IC given morph (137), then application of the partner transform, $\hat{Z}_M^{\mu\nu} = P^{\hat\mu}{}_{\check\alpha}P^{\hat\nu}{}_{\check\beta}\check{Z}_M^{\alpha\beta}$, yields the key representative "partner quantity equality"

$$\hat{Z}_M^{\mu\nu} = \check{Z}_{UM}^{\mu\nu} \tag{144}$$

for the partner Riemann and global IC values of morphed and unmorphed partner quantities, having used the inverse relation (139) for the partner transform and morph Jacobians. The partner quantity equality is commensurate with the event locational equality (132) for the morph itself. Applying (144) to relate the morphed and unmorphed background metrics, then

$$\hat{g}_{\mu\nu}^{MB} = \check{g}_{\mu\nu}^{UMB} = \check{a}_{\mu\nu} = \eta_{\mu\nu}, \tag{145}$$

providing their values for morph partner system formulation in the partner Riemann and global ICs.

When a morph is applicable (i.e., when background system curvature effects are negligible), the morphing of objects making up a local system is *universal*, since the shared material content of all of the morph partner objects is tied to the unmorphed and morphed partner event fields x_{UM} and x_M. The universal morphing occurs then for any *instruments* present utilized to make measurements, morphing along with the objects being measured. So any instrument has both a morphed and unmorphed partner, which may be utilized to measure morphed and unmorphed partner objects respectively. Similar to natural measurement under partner relation gravity shifting, natural observers utilize "raw" morphed instruments "as is" to make measurements of morphed objects. Therefore, natural observers use raw morphed instruments to measure morphed objects for the actual "morphed partner case" when background system gravitation is present, whereas when background gravitation is removed in theory, the hypothetical "unmorphed partner case" is yielded where the unmorphed partners of the instruments make the same measurements on the unmorphed partners of the objects. As instruments measuring objects morph the same as the objects, then there is no difference between the morphed and unmorphed partner cases except the universal morph applied to their shared material content consisting of both the instruments and the objects being measured. With the morphed partner case just a morphed version of the unmorphed partner case, then *for natural observers, any raw morphed instrument reading for the morphed partner case, which is the actual case, is the same as the reading of the unmorphed partner instrument for the hypothetical unmorphed partner case*. Therefore, the partner equivalence property, shown to be applicable for the natural measurement of shifted objects under partner relation gravity shifting, is similarly applicable for the natural measurement of morphed objects under morph gravity shifting. Note that in the unmorphed partner case for a local gravitational system, the unmorphed instruments are still shifted instruments measuring shifted objects, yielding both morphed and shifted instruments measuring both morphed and shifted objects in the morphed partner case. But with the unmorphed and morphed partner cases differing by *only the morph application*, the instrument readings will be the same for the unmorphed and morphed partner instruments even though shifting is additionally present for both the instruments and the objects being measured.

The morph partner equivalence property may be applied to (144) to obtain the "naturally measured partner quantity equality"

$$\hat{Z}^{\mu\nu}_{M(N)} = \check{Z}^{\mu\nu}_{UM(N)},\tag{146}$$

which states that the natural morphed instrument measurement of a morphed quantity in the locally inertial free-fall frame of a background system, as specified utilizing the free-fall frame's Riemann ICs, yields a value equal to the hypothetical natural unmorphed partner instrument measurement of the unmorphed partner quantity in the partner absolute inertial frame, as specified utilizing the inertial frame's partner global ICs. Note that instrument readings are part of the set of quantities $Z^{\alpha\beta}$ depicting objects. So the readings, $\hat{Z}^{\mu\nu}_{M(N)}$ and $\check{Z}^{\mu\nu}_{UM(N)}$, from the raw morphed and unmorphed partner instruments used by natural observers, are subject to the equality (144) just as any morphed and unmorphed partner quantities are, justifying the validity of (146).

Consider the actual gravitational case where a natural observer is using raw morphed instruments to measure a morphed local system in a locally inertial free-fall frame for the background system, with the measurements specified using the free-fall frame's Riemann ICs. Removal of the background system's gravitational field in theory yields the hypothetical partner inertial case where the natural observer uses the unmorphed partner instruments to make the same measurements on the unmorphed partner system in the partner absolute inertial frame, with then the measurements specified using the inertial frame's partner global ICs. Under applicability of the partner equivalence property, the morphed instrument readings for the gravitational case will be identical to the unmorphed partner instrument readings for the partner inertial case, as per (146). Therefore, so long as the background system curvature effects are negligible so that a morph is applicable, natural observers perceive (via measurements) any morphed local system for the actual gravitational case, as posed in a Riemann IC specified background system locally inertial free-fall frame subtending the local system spacetime region, as being *the same* as its hypothetical partner inertial case consisting of the unmorphed partner

system as posed in the partner global IC specified partner absolute inertial frame, yielding *satisfaction of the equivalence principle for natural observers*. The *EEP is satisfied* then for naturally observed local nongravitational systems, and the *SEP is satisfied* for naturally observed local gravitational systems.

Consider an arbitrary coordinate transform applied to a Riemann IC given morphed local system, so the transform Jacobian $L^{\alpha'}{}_{\hat{\mu}}$ is applied to the morphed tensor quantities $\hat{Z}^{\mu\nu}_M$ to obtain their values $Z'^{\alpha\beta}_M$ in the new "primed" coordinates. Under the equality (144), this operation is equivalent to applying a *formally identical* "mirror" coordinate transform to the partner global IC given unmorphed partner system, with then an identically valued transform Jacobian

$$L^{\alpha''}{}_{\check{\mu}} = L^{\alpha'}{}_{\hat{\mu}} \quad \text{(mirror transform)} \tag{147}$$

applied to the unmorphed partner tensor quantities $\check{Z}^{\mu\nu}_{UM}$ to obtain their values $Z''^{\alpha\beta}_{UM}$ in the new "double-primed" coordinates. With the original Riemann and global ICs being partner coordinates, then under the formally identical mirror coordinate transforms, the new primed and double-primed coordinates are also *partner coordinate systems*. Application of the mirror coordinate transforms to their respective sides of the partner quantity equality, (144), applicable in the partner Riemann and global ICs, yields the partner quantity equality

$$Z'^{\alpha\beta}_M = Z''^{\alpha\beta}_{UM} \tag{148}$$

applicable in the partner primed and double-primed general coordinates.

To obtain the absolutely or naturally measured value $\mathbf{Z}_{(M)}$ of a quantity given in geometric form, $\mathbf{Z}$, generally requires selection of a frame of reference as the state of motion for the instrumentation employed to make the measurement, as well as selection of available coordinates for the frame to give component values $Z^{\alpha\beta}_{(M)}$ for the measured quantity (unless it is a scalar, which requires no coordinates), yielding $\mathbf{Z}_{(M)} = Z^{\alpha\beta}_{(M)}$. If a quantity is expressed in coordinate form $Z^{\alpha\beta}$ though, the practice followed in this paper is to *use the same coordinates and their frame of reference for the instrumentation employed to perform the measurement, as the coordinates and frame utilized for expressing the quantity*, the same practice as in previous discussion. Following this methodology, when a coordinate transform is applied to a coordinate-given tensor quantity $Z^{\mu\nu}$ to obtain $Z'^{\alpha\beta}$ in the new coordinates, its measured value is changed to $Z'^{\alpha\beta}_{(M)}$ as yielded using the new coordinates and their frame for the instrumentation employed, as opposed to the measured valued $Z^{\mu\nu}_{(M)}$ using the old coordinates and frame. As shown below, the measured value $Z^{\mu\nu}_{(M)}$ of a tensor quantity does not in all cases "directly" transform as a tensor the way the tensor quantity $Z^{\mu\nu}$ itself does, so

$$Z'^{\alpha\beta}_{(M)} \neq L^{\alpha'}{}_{\mu} L^{\beta'}{}_{\nu} Z^{\mu\nu}_{(M)} \quad \text{(generally)} \tag{149}$$

is applicable. To reliably obtain in all cases the measured value $Z'^{\alpha\beta}_{(M)}$ of a tensor quantity $Z^{\mu\nu}$ under coordinate transform, *first* the quantity is transformed and *then* measurement is applied in the new coordinates and frame for the instrumentation, as stated by the representative

$$Z'^{\alpha\beta}_{(M)} = [Z'^{\alpha\beta}]_{(M)} = [L^{\alpha'}{}_{\mu} L^{\beta'}{}_{\nu} Z^{\mu\nu}]_{(M)}. \tag{150}$$

Based on the above discussion, the partner Riemann and global IC naturally measured partner quantity equality, $\hat{Z}^{\mu\nu}_{M(N)} = \check{Z}^{\mu\nu}_{UM(N)}$ (146), is generally not "directly" transformed under respective application of mirror coordinate transforms. But with respective partner morphed and unmorphed instrument measurements on both sides of (144) yielding (146), and with respective mirror coordinate transforms of (144) yielding (148), the respective partner morphed and unmorphed instrument measurements on both sides of (148) yields the naturally measured partner quantity equality

$$Z'^{\alpha\beta}_{M(N)} = Z''^{\alpha\beta}_{UM(N)}, \tag{151}$$

applicable in the partner primed and double-primed general coordinates and their associated frames of reference. This is as expected under the partner equivalence property applicable for the natural measurement of morphed objects under morph gravity shifting, so again any raw morphed instrument reading for the morphed partner case is the same as the reading of the unmorphed partner instrument for the unmorphed partner case, here given in the partner primed and double-primed general coordinates. Therefore, similar to (146) yielding satisfaction of the "baseline form" of the equivalence principle as given in the Riemann ICs for the locally inertial free-fall frames of background systems, the equality (151) yields satisfaction of the equivalence principle in *any coordinates*, and therefore in *any frames of reference* as specified using any coordinates. This establishes the "corollary" to the baseline free-fall frame form of the equivalence principle: As long as the effects of background system curvature are negligible so that a morph is applicable, natural observers perceive any morphed local system for the actual gravitational case, as given in a local frame accelerating and/or rotating with respect to a background system inertial free-fall frame, as being *the same* as its hypothetical partner inertial case consisting of the unmorphed partner system as given in the mirror local frame accelerating and/or rotating with respect to the partner absolute inertial frame in the same manner. Concluding, *equivalence principle satisfaction for natural observers in gravity shift theory is the same as in general relativity*, in that for both theories, the equivalence principle is satisfied for all local systems given in all coordinates and frames so long as background system curvature effects are considered negligible.

The general discussion and formulation in this section applies for *all* local systems, as previously stated. EEP satisfaction will be examined in further detail below for local nongravitational systems, and separately SEP satisfaction will be further examined for local gravitational systems. The established morph, shown above to hold for nongravitational systems, will be shown to be applicable for local gravitational systems when detailing their SEP satisfaction. As can be seen, for GS theory, there is a significant amount of gravity shift based "mechanism" underlying the satisfaction of the equivalence principle as a property of the gravitational field, including the use of raw gravity shifted instrument based natural observation, as opposed to the much simpler curved spacetime approach for general relativity. The complexity for GS theory is the "price to pay" to obtain equivalence principle satisfaction in absolute flat spacetime.

4.6. Satisfaction of the Einstein Equivalence Principle for Local Nongravitational Systems

For a morphed local *nongravitational* system in particular, the unmorphed partner system has *no partner relation based gravity shifting*, since the local gravitational field present has vanishing field strength. As a result, for a morphed nongravitational system posed in a Riemann IC given locally inertial free-fall frame for the background system, *inertial behavior* is yielded for the inertial-case unmorphed partner nongravitational system as posed in the absolute inertial frame for the partner global ICs, thereby *obeying the laws of special relativity*. The global IC given quantities $\check{Z}^{\mu\nu}_{UM}$ in the unmorphed nongravitational system are *inertially valued*, since they depict unmorphed and unshifted objects. In addition, the instruments utilized by natural observers in the unmorphed partner case are both unmorphed and unshifted, so they *accurately* measure the inertially valued quantities as formally stated by $\check{Z}^{\mu\nu}_{UM(N)} = \check{Z}^{\mu\nu}_{UM}$. Combining this with (144) and (146) yields

$$\hat{Z}^{\mu\nu}_{M(N)} = \hat{Z}^{\mu\nu}_{M} = \check{Z}^{\mu\nu}_{UM} = \check{Z}^{\mu\nu}_{UM(N)}. \tag{152}$$

In its entirety, (152) states that the natural measurement, $\hat{Z}^{\mu\nu}_{M(N)}$, of the morphed quantities $\hat{Z}^{\mu\nu}_{M}$ in a morphed local nongravitational system, *yields values equal to the morphed quantities $\hat{Z}^{\mu\nu}_{M}$ themselves*, which are *inertially valued* since they are equal to the inertially valued partner unmorphed quantities $\check{Z}^{\mu\nu}_{UM}$ in the inertially behaved unmorphed partner nongravitational system, where similarly their natural measurement $\check{Z}^{\mu\nu}_{UM(N)}$ *yields values equal to the unmorphed quantities $\check{Z}^{\mu\nu}_{UM}$ themselves*. Therefore, so long as the background system curvature effects are negligible so that a morph is applicable, natural observers perceive (via measurements) any morphed local nongravitational system for the actual

gravitational case, as posed in a Riemann IC specified background system locally inertial free-fall frame subtending the local system spacetime region, as being *the same* as its hypothetical partner inertial case consisting of the inertially behaved unmorphed partner nongravitational system as posed in the partner global IC specified partner absolute inertial frame, yielding *satisfaction of the Einstein equivalence principle for natural observers*.

Utilizing (152), some key examples of naturally measured quantities for morphed nongravitational systems are the natural metric and Christoffel symbol

$$\hat{g}_{\mu\nu}^{M(N)} = \hat{g}_{\mu\nu}^{M} = \check{g}_{\mu\nu}^{UM} = \eta_{\mu\nu}, \qquad \hat{\Gamma}_{\alpha\beta}^{\mu\,M(N)} = \hat{\Gamma}_{\alpha\beta}^{\mu\,M} = \check{\Gamma}_{\alpha\beta}^{\mu\,UM} = 0, \tag{153}$$

and the displacements and locations

$$\mathrm{d}\hat{x}_{M(N)}^{\mu} = \mathrm{d}\hat{x}_{M}^{\mu} = \mathrm{d}\check{x}_{UM}^{\mu}, \qquad \hat{x}_{M(N)}^{\mu} = \hat{x}_{M}^{\mu} = \check{x}_{UM}^{\mu} - \check{X}^{\mu}, \tag{154}$$

where the event location equality is obtained by integrating the displacement equality, recovering (132) as expected. As the gravitational field over a local nongravitational system in the gravitational case is the background system field as per (110), then the natural metric $g_{\alpha\beta}^{M}$ for a morphed nongravitational system is identified as the morphed background metric $g_{\alpha\beta}^{MB}$, so the unmorphed natural metric $g_{\mu\nu}^{UM}$ is the unmorphed background metric $g_{\mu\nu}^{UMB} = a_{\mu\nu}$. Their values given by (153) and (145) indeed agree as expected. Comparison of (153) with (118), which is applicable for actual local nongravitational systems surrounded by background systems, shows that to first differential order, the Riemann coordinate Minkowski metric valued morphed natural metric, $\hat{g}_{\mu\nu}^{M}$, is also the *actual* natural metric $\hat{g}_{\mu\nu}$ given by the actual background metric $\hat{g}_{\mu\nu}^{B}$. But at higher order, the morphed natural metric $\hat{g}_{\mu\nu}^{M}$ for a local nongravitational system *has no curvature*, since it equals the morphed background metric $\hat{g}_{\mu\nu}^{MB}$, which has no curvature as previously shown. In contrast, the actual natural metric $\hat{g}_{\mu\nu} = \hat{g}_{\mu\nu}^{B}$ has curvature as per (118).

Physical law for morphed local systems may be expressed by the representative statement $Z_{M}^{\alpha\beta} = Y_{M}^{\alpha\beta}$ in any coordinates. Utilizing (152), the natural measurement of physical law for morphed nongravitational systems, as posed in the Riemann IC specified locally inertial free-fall frames of surrounding background systems, may be expressed by

$$\hat{Z}_{M(N)}^{\mu\nu} = \check{Z}_{UM}^{\mu\nu} = \check{Y}_{UM}^{\mu\nu} = \hat{Y}_{M(N)}^{\mu\nu}. \tag{155}$$

The "middle" equality $\check{Z}_{UM}^{\mu\nu} = \check{Y}_{UM}^{\mu\nu}$ represents physical law for the inertial-case unmorphed partner nongravitational systems exhibiting inertial behavior in the absolute inertial frames for the partner global ICs. So physical law is *inertial* for morphed nongravitational systems as naturally observed in the locally inertial frames, thereby *obeying the laws of special relativity*. Equation (155) is then a formal statement of EEP satisfaction for natural observers measuring morphed local nongravitational systems. Included in this is the use of the naturally measured "inertial" Minkowski metric valued natural metric, and the resulting naturally measured vanishing Christoffel symbol, as per (153). This along with the naturally measured displacements and locations (154), results in naturally measured geodesics being *straight lines* in the locally inertial free-fall frames, which is inertial behavior as expected under EEP satisfaction. Due to the inertial form of physical law being yielded for naturally observed local nongravitational systems so long as background system curvature effects are negligible, the background system locally inertial free-fall frames are the "preferred" frames of reference for natural observers, with the utilized Riemann ICs the preferred local "inertial coordinates" due to explicit inertial expression of nongravitational physical law.

Utilizing the above argument establishing equivalence principle satisfaction for all morphed local systems in all coordinates and frames, then applied for morphed local *nongravitational* systems, the EEP is satisfied for natural observers in all coordinates and frames so long as the effects of background curvature are negligible. As a result, naturally measured physical law $Z_{M(N)}^{\alpha\beta} = Y_{M(N)}^{\alpha\beta}$ for morphed

nongravitational systems satisfies the EEP in all coordinates and frames, obtainable via application of (150) to Riemann IC given physical law $\hat{Z}_M^{\mu\nu} = \hat{Y}_M^{\mu\nu}$, which when naturally measured yields $\hat{Z}_{M(N)}^{\mu\nu} = \hat{Y}_{M(N)}^{\mu\nu}$ (155) satisfying the EEP in the baseline locally inertial frames (from above). Therefore, when background curvature effects may be neglected so that a morphed local nongravitational system is yielded, its behavior, as naturally observed in a frame accelerating and/or rotating with respect to a locally inertial free-fall frame subtending the system, appears the same as its inertial behavior in deep space without gravitation present, as observed in a frame accelerating and/or rotating relative to the partner absolute inertial frame in the same manner. This is the above-established corollary to the baseline free-fall frame form of the equivalence principle, specialized here for local nongravitational systems. The EEP corollary holds then for the naturally observed behavior in the Schild argument, where shifted light, travelling up a tower on Earth, is naturally measured to be redshifted the same as if the entire apparatus were in deep space and accelerated at 1g. Note that the development has gone "full circle," where the Schild argument was combined with the EEP as a *postulate* (as contained in the SEP postulate) to establish the *existence* of universal gravity shifts in the first place, which were then utilized to establish the existence of a local diffeomorphism yielding the EEP as a *deduced* property *given* the existence of universal gravity shifts.

As is understood in general relativity, for any gravitational system (treated utilizing continuous distributions of matter as is common practice), the local system contained within any selected *infinitesimally sized* spacetime region, referred to here as a "micro system," may be treated as a local *nongravitational* system due to its gravitational field strength vanishing in the infinitesimal size limit, with the surrounding gravitational system treated as the *background system* for the micro system. In addition, in the infinitesimal size limit, all background system curvature effects on the micro system *vanish entirely*. These same properties can be seen to hold in GS theory, so *the morph is always applicable over the infinitesimal spacetime region subtended by a micro system, with the micro system treated as a morphed local nongravitational system*. The term "micro morph" is used to designate the morph for a micro system, as opposed to a "local morph," which may be applicable over an extended spacetime region. The above local morph formulation, applicable for finitely sized local nongravitational systems when background curvature effects are negligible, is therefore universally applicable for morph formulation of any micro system selected from an arbitrary gravitational system. For example, since a micro system has no gravitational field strength similar to a finitely sized nongravitational system, the gravitational field over the region of a micro system is the field of the surrounding gravitational system acting as the background system, which is the *actual* field for the *total system* consisting of the background system plus the micro system. This equality is formally stated by

$$g_{\alpha\beta}^B = g_{\alpha\beta}, \qquad S_{\bar{\mu}}^{\alpha B} = S_{\bar{\mu}}^{\alpha}, \qquad \text{(micro system)}. \tag{156}$$

Any event may be used as the shift origin X for a micro morph, with the micro morph applied to a micro system considered to occupy an infinitesimally sized region surrounding the shift origin. In the applicable infinitesimal size limit, all objects for a micro system are located at the shift origin location X. So the *single* event x is used to specify the location of all quantities $Z^{\alpha\beta}(x)$ depicting a micro system's objects, with x located at the shift origin X, as stated by

$$Z^{\alpha\beta}(x) = Z^{\alpha\beta}(x = X) \quad \text{(micro system)}. \tag{157}$$

This location specification is assumed for all micro system quantities subsequently given. The micro morph Jacobian tensor, $M_{\bar{\mu}}^{\alpha}$, becomes the background system shift tensor $S_{\bar{\mu}}^{\alpha B}$ at X as per (138), and therefore the actual shift tensor for the total system from (156). This equality is formally given at $x = X$ by

$$\left[M_{\bar{\mu}}^{\alpha} = S_{\bar{\mu}}^{\alpha} \right]_{x = X} \quad \text{(micro morph)}. \tag{158}$$

Since a micro system is nongravitational, its partner unmorphed quantities are also unshifted as stated by

$$\left[Z_{UM}^{\mu\nu} = Z_{US}^{\mu\nu} \right]_{x=X} \quad \text{(micro system)}, \tag{159}$$

where this equality of the unmorphed and unshifted values is applicable for quantities of all differential orders. Recall that for finitely large morphed systems the location of unshifted quantities $Z_{US}^{\mu\nu}$ is at the morphed event location x_M as opposed to the unmorphed event location x_{UM}, but for the infinitesimally sized micro systems, $x_M = x_{UM} = X$, so $Z_{US}^{\mu\nu}$ is at the same location $x = X$ as $Z_{UM}^{\mu\nu}$. For brevity, the term "micro free-fall frame" may be used when referring to an "infinitesimal free-fall frame." Combining (159) with (152) applied to micro systems, then

$$\left[\hat{Z}_{M(N)}^{\mu\nu} = \hat{Z}_{M}^{\mu\nu} = \check{Z}_{UM}^{\mu\nu} = \check{Z}_{US}^{\mu\nu} \right]_{x=X} \quad \text{(micro system)}. \tag{160}$$

So micro free-fall frame natural measurement of quantities, of all differential orders, yields their inertial unmorphed/unshifted values as given in the partner absolute inertial frame. A key example is light, which via use of (160) has a naturally measured velocity $d\hat{x}_{M(N)}^{n}/d\hat{t}_{M(N)}$ equal to $\hat{c}_{M(N)}^{n} = \check{c}_{US}^{n}$. This results in

$$\hat{c}_{M(N)} = \check{c}_{US} = 1 \quad \text{(micro system)}, \tag{161}$$

which establishes that in a micro fee-fall frame, the naturally measured speed of gravity shifted light is equal to the inertial fixed unshifted light speed, in accordance with the EEP.

Utilizing (158) and (159), application of the morph quantity partner relation, (141), for *zero-order* micro system quantities at $x = X$, yields the shift quantity partner relation (87) with the actual shift tensor $S^{\alpha}{}_{\bar{\mu}}|_X$ utilized, where the zero-order unmorphed and morphed partner quantities $Z_{UM}^{\mu\nu}$ and $Z_{M}^{\alpha\beta}$ in (141) are identified as the *unshifted and shifted partner quantities $Z_{US}^{\mu\nu}$ and $Z_{S}^{\alpha\beta}$*. From Section 3.14, "first-order" shifted tensor quantities $Z_{S}^{\alpha\beta}$ may be obtained by applying the natural covariant derivative to the zero-order shifted tensor quantities. At the shift origin X for a micro system given in Riemann ICs, the applicable zero-valued morph connection, (153), for nongravitational systems, is equal to the shifted/actual connection $\hat{\Gamma}_{\mu\nu}^{\alpha\,S} = \hat{\Gamma}_{\mu\nu}^{\alpha} = 0$ as constructed from $\hat{g}_{\mu\nu}^{S} = \hat{g}_{\mu\nu} = \eta_{\mu\nu}$, having used (118) applicable for local nongravitational systems and $g_{\alpha\beta}^{B} = g_{\alpha\beta}$ (156) for micro systems. Therefore, $\hat{Z}_{M}^{\mu\nu} = \hat{Z}_{S}^{\mu\nu}$ for up to *first-order* tensor quantities at any location x utilized as a micro morph shift origin $X = x$, yielding $Z_{M}^{\alpha\beta} = Z_{S}^{\alpha\beta}$ in any coordinates. Combining this with $Z_{S}^{\alpha\beta} = Z^{\alpha\beta}$ (88) yields the tensor equality

$$\left[Z_{M}^{\alpha\beta} = Z_{S}^{\alpha\beta} = Z^{\alpha\beta} \right]_{x=X} \quad \text{(micro system, to 1st order)}, \tag{162}$$

where the only metric utilized for the shifted/actual quantities is the natural metric and its connection (again the right-hand equality does not hold for quantities explicitly containing the shift and potential tensors). A key example is the equality $g_{\alpha\beta}^{M} = g_{\alpha\beta}^{S} = g_{\alpha\beta}$ of the micro-morphed, shifted, and actual values of the natural metric, as well as their covariant derivatives (using their respective equal metric connections) which all vanish. The equality (162) generally does not hold for the second-order or higher micro-morphed quantities, due to background curvature dependence in $Z_{S}^{\alpha\beta} = Z^{\alpha\beta}$ that is not present in $Z_{M}^{\alpha\beta}$.

With a micro morph always applicable for a micro system, *the EEP always holds for natural observers* over any infinitesimally sized locally inertial free-fall frame subtending a micro system selected from any gravitational system, with the micro system *naturally perceived as an inertially behaved nongravitational system* consisting of matter and nongravitational fields exclusively. So in the micro free-fall frames of gravitational systems, due to EEP satisfaction, the only naturally perceived field presence is the inertial Minkowski metric valued natural metric *universally coupled* to matter and the nongravitational fields. The micro free-fall frames are therefore the *universally applicable* "preferred" frames of reference for natural observers due to these properties holding. Now the use of the universally applicable preferred micro inertial frames, under EEP satisfaction, forms the basis of what is considered "naturally

observable" in GS theory, similar to establishing what is considered "observable" in general relativity, with again the "observers" in general relativity identified as natural observers in GS theory. Therefore, as in general relativity, *natural observers in gravity shift theory only perceive matter and the nongravitational fields as universally coupled to the natural metric* $g_{\alpha\beta}$. The natural observers perceive then *the natural metric itself* via its coupling to the perceived matter and nongravitational fields, as well as to the perceived displacements tied to the nongravitational material content. As a result, natural observers do not perceive any gravitational field action other than the action induced by the natural metric universally coupled to matter and the nongravitational fields, as discussed further below. Therefore, *natural observers do not perceive the gravity shifting taking place*, either in the form of partner relation based shifting or morph-based shifting, as can be seen under naturally observed EEP satisfaction in the universally applicable micro free-fall frames. Natural observers do not perceive then the shift tensor $S^{\alpha}_{\bar{\mu}}$ or potential tensor w^{α}_{μ}, even though they are coupled to matter and the nongravitational fields via the gravity shifting they dictate. In addition, natural observers do not perceive the absolute metric $a_{\mu\nu}$, even though it couples to matter and the nongravitational fields as absolutely observed.

As a key example, natural observers do not perceive the absolute metric or shift tensor contained in the metric relation $g_{\alpha\beta} = a_{\mu\nu}S^{\bar{\mu}}_{\alpha}S^{\bar{\nu}}_{\beta}$ (35) used to provide the natural metric, so the coupling of $a_{\mu\nu}$ and $S^{\alpha}_{\bar{\mu}}$ to matter and the nongravitational fields, via the natural metric given by the metric relation, is perceived by natural observers as only the natural metric coupling. The natural measurement based equations given above (everything containing an (N)) *only apply for the naturally observable quantities*, consisting then of quantities depicting the naturally observed matter and nongravitational fields universally coupled to the natural metric. In addition to what is considered observable, for the identified natural observers in general relativity, the *entire worldview* is based on the perceived behavior of gravitational systems as observed from their universally applicable preferred micro free-fall frames, which is therefore the "natural worldview" for the class of natural observers in GS theory. For example, since geodesic motion in any micro free-fall frame is dictated by the natural metric connection given by (119), naturally perceived inertial motion is yielded over the EEP-applicable "inertial region" about the frame origin. So there does not exist a naturally perceptible "gravitational force," with perceived deflection from inertial motion beyond the inertial region considered due to natural metric curvature. Therefore, as in general relativity, *natural observers in GS theory consider natural metric curvature to be the basis for gravitational action as opposed to a force*. The above general properties for natural observers, established via use of the universally applicable micro systems, were stated (for the most part) in the summary.

With gravity shifting consisting of dimensional shifts along the global IC axis directions for eigensystems, the natural measurement of a shifted displacement, $d\tilde{x}^{\mu}_{S}$, running along an IC axis using a shifted clock or ruler, yields a value $d\tilde{x}^{\mu}_{S(N)}$ equal to the unshifted partner $d\tilde{x}^{\mu}_{US}$ running along the same IC axis, as consistent with the partner equivalence property. For instance, $d\tilde{x}^{0}_{S(N)} = d\tilde{x}^{0}_{US}$ holds for the temporal displacements used in the Schild argument, where the global ICs employed is an eigensystem. Utilizing *separate* natural measurements along the four orthogonal IC axis directions, $d\tilde{x}^{\mu}_{S(N)} = d\tilde{x}^{\mu}_{US}$ is yielded for shifted displacements running in any directions. Following suit, natural measurement of a zero-order shifted quantity, $\tilde{Z}^{\mu\nu}_{S}$, utilizing shifted instruments, can be seen to yield its unshifted value $\tilde{Z}^{\mu\nu}_{US}$, as stated by the representative

$$\tilde{Z}^{\mu\nu}_{S(N)} = \tilde{Z}^{\mu\nu}_{US} \quad \text{(zero order, IC eigensystem).} \tag{163}$$

Combining (163) with (162) and (159) yields $\tilde{Z}^{\mu\nu}_{M(N)} = \tilde{Z}^{\mu\nu}_{UM}$ for zero-order micro-morphed quantities when the partner global ICs *are an eigensystem*, so $\tilde{Z}^{\mu\nu}_{M(N)} \neq \tilde{Z}^{\mu\nu}_{M}$ in general. From this micro morph based example,

$$Z^{\alpha\beta}_{M(N)} \neq Z^{\alpha\beta}_{M} \quad \text{(generally),} \tag{164}$$

stating that for morphed local systems (including finitely large and gravitational ones), the natural measurement of a morphed quantity, when given in arbitrary coordinates, may not yield a value equal

to the morphed quantity itself. However, it is always the case that $\hat{Z}^{\mu\nu}_{M(N)} = \hat{Z}^{\mu\nu}_M$ for micro systems given in the Riemann ICs, as per (160). If then partner Riemann and global ICs are utilized where the global ICs are an eigensystem, the inequality, $\check{Z}^{\mu\nu}_{M(N)} \neq \check{Z}^{\mu\nu}_M$, for the zero-order micro system quantities given in the partner global ICs, results in the inequality (149) (applied here for natural measurement) if $\hat{Z}^{\mu\nu}_{M(N)} = \hat{Z}^{\mu\nu}_M$ was "directly" transformed from the Riemann ICs to the eigensystem partner global ICs. This example demonstrates that (150) should be used in general to obtain measured tensor quantities under coordinate transform as opposed to directly transforming them, applicable for both natural and absolute measurement.

The use of "(N)" in $Z^{\alpha\beta}_{(N)}$ designates the *naturally measured value* for a quantity $Z^{\alpha\beta}$ in particular, whereas the use of "N" without the parentheses designates a "natural quantity" $Z^{\alpha\beta}_N$ *utilized by natural observers to perform modelling* where its origin is obtained via natural measurement. A previous example of this methodology is the use of the natural proper interval ds_N by natural observers to perform modelling, where the origin of ds_N is its equality to the naturally measured absolute proper interval as designated by $ds_{A(N)}$. Natural measurement of a quantity *in practice* may be made using any convenient frame of reference. However, the natural measurement of micro system quantities is *best understood and depicted* utilizing the preferred micro inertial free-fall frames, so that their naturally measured morphed values, $\hat{Z}^{\mu\nu}_{M(N)}$, are both their Riemann IC values $\hat{Z}^{\mu\nu}_M$ and their global IC inertial unmorphed/unshifted partner values $\check{Z}^{\mu\nu}_{UM} = \check{Z}^{\mu\nu}_{US}$ as per (160). Once naturally measured, coordinate transformation of $\hat{Z}^{\mu\nu}_M = \hat{Z}^{\mu\nu}_{M(N)}$ may be applied to obtain its value $Z^{\alpha\beta}_N = Z^{\alpha\beta}_M$ as utilized by natural observers to perform modelling in any coordinates, so $Z^{\alpha\beta}_N$ for micro system quantities is given by the representative

$$Z^{\alpha\beta}_N = L^\alpha{}_{\hat\mu} L^\beta{}_{\hat\nu} (\hat{Z}^{\mu\nu}_M = \hat{Z}^{\mu\nu}_{M(N)}) = Z^{\alpha\beta}_M \quad \text{(micro system)}, \tag{165}$$

noting that $\hat{Z}^{\mu\nu}_N = \hat{Z}^{\mu\nu}_{M(N)}$ in the micro inertial frames. With $Z^{\alpha\beta}_{M(N)} \neq Z^{\alpha\beta}_M$ (164) occurring for micro morphs, it may be the case that $Z^{\alpha\beta}_{N(N)} = Z^{\alpha\beta}_{M(N)} \neq Z^{\alpha\beta}_M = Z^{\alpha\beta}_N$ for naturally measured micro systems given in arbitrary coordinates. So though the natural quantities $Z^{\alpha\beta}_N$ are utilized by natural observers to perform modelling, *their values in arbitrary coordinates may not be their naturally measured values $Z^{\alpha\beta}_{N(N)}$*.

Now the micro inertial frame expression, $\hat{Z}^{\mu\nu}_{M(N)} = \hat{Y}^{\mu\nu}_{M(N)}$, contained in (155), is an expression of *nongravitational physics law* for matter and the nongravitational fields, since it is law applicable for the nongravitational micro systems. Using (165), arbitrary coordinate transform from $\hat{Z}^{\mu\nu}_{M(N)} = \hat{Y}^{\mu\nu}_{M(N)}$ results in covariantly given "natural" nongravitational physics law

$$Z^{\alpha\beta}_N = Z^{\alpha\beta}_M = Y^{\alpha\beta}_M = Y^{\alpha\beta}_N \tag{166}$$

utilized by natural observers to model systems in any coordinates. For matter and the nongravitational fields under the influence of gravitation, all physical law may be given via use of the natural observer based covariant form, (166), as obtained from transformation of the inertial form (155) of nongravitational physics law applicable for natural observation of local nongravitational systems in micro inertial free-fall frames. *Gravity shift theory is therefore a complete theory of gravitation* (as stated in the summary). As can be seen, the EEP-based "standard formulations," available in general relativity textbooks, may also be utilized in GS theory to provide the natural physical laws (166) for matter and the nongravitational fields under gravitation. As is understood in general relativity, differential-form physical laws for matter and the nongravitational fields may be given using no more than first-order differentials, which is applicable as well then in GS theory. Therefore, as per (162), *the quantities $Z^{\alpha\beta}_N = Z^{\alpha\beta}_M$ utilized in the natural physical laws $Z^{\alpha\beta}_N = Y^{\alpha\beta}_N$ for matter and the nongravitational fields under gravitation, as obtained via use of micro systems, are the shifted/actual quantities $Z^{\alpha\beta}_S = Z^{\alpha\beta}$ formed exclusively with the natural metric and its connection as field quantities*. With naturally measured physical law $Z^{\alpha\beta}_{M(N)} = Y^{\alpha\beta}_{M(N)}$ for morphed nongravitational systems satisfying the EEP in all coordinates and frames (from above), then via $Z^{\alpha\beta}_N = Z^{\alpha\beta}_M$ (165), natural measurement of the natural physical laws, $Z^{\alpha\beta}_N = Y^{\alpha\beta}_N$, for matter and the

nongravitational fields under gravitation, yields satisfaction of the EEP for their naturally measured form $Z_{N(N)}^{\alpha\beta} = Y_{N(N)}^{\alpha\beta}$ given in any coordinates and frames.

The natural metric is the *inertial* Minkowski metric $\hat{\eta}_{\mu\nu}^{(N)}$ in the micro inertial frame expressions $\hat{Z}_{M(N)}^{\mu\nu} = \hat{Y}_{M(N)}^{\mu\nu}$ of naturally observed nongravitational physics law, as established above for nongravitational systems. Coordinate transformation from the micro inertial frames generally yields "non-inertial" (i.e., non-Minkowski) values for the natural metric $g_{\alpha\beta} = g_{\alpha\beta}^{N}$ in the natural nongravitational physics laws (166) given in arbitrary coordinates, resulting in emergent gravitational field *action* induced by the natural metric due to its non-inertial values. With then the natural metric universally coupled to matter and the nongravitational fields obeying the natural nongravitational physics laws, the generally non-inertial natural metric is identified as the *universally coupled "gravitational metric" providing all naturally perceived action of the gravitational field on matter and the nongravitational fields*, similar to general relativity (as stated in the summary). Since all physical law for matter and the nongravitational fields may be given via the natural observer based form (166) with universal coupling to the natural/gravitational metric exclusively, GS theory falls into the category of a "metric theory of gravity" as commonly understood in gravitational theory, but with the proviso that this is part of the *natural worldview* as opposed to the more encompassing *absolute worldview*, which includes coupling to the shift tensor and morph fields as well as the absolute metric.

4.7. Satisfaction of the Strong Equivalence Principle for Local Gravitational Systems

As discussed above, the local morph was established where no gravitational source was within the spacetime region subtended by the morph. The background system provided morph may not be applicable then for local *gravitational* systems surrounded by background systems. Now in the nongravitational limit of local gravitational systems, the morph is indeed applicable as is always the case for local nongravitational systems assuming negligible background system curvature effects. The morph is applicable then in the asymptotic limit of a local gravitational system far from the system's gravitational sources, since nongravitational conditions are yielded. In addition, the morph is always applicable in the micro free-fall frames of a local gravitational system combined with a surrounding background system, since as discussed above, the gravitational source strength vanishes for the micro system contained within a micro free-fall frame, and background curvature effects vanish. In this case, the background system consists of the "original" background system combined with the rest of the local gravitational system surrounding the micro free-fall frame evaluated. With the morph applicable in all of the different ways nongravitational limits can be reached for local gravitational systems surrounded by background systems (assuming negligible background curvature effects), it is not unreasonable to expect that the morph is applicable as well when the gravitational strength of local gravitational systems is finitely large. Now as shown above, the SEP would indeed hold for a naturally observed local gravitational system so long as the background system morph were applicable. Here, the SEP is invoked as a *postulate* for GS theory to *infer* that so long as background system curvature effects are negligible, *the established background morph is indeed applicable for any local gravitational system surrounded by a background system, yielding satisfaction of the strong equivalence principle for natural observers* in agreement with the SEP postulate.

It is then the *success* of the established morph *as a means to achieve the postulated SEP satisfaction*, that is used as a "powerful" basis for inferring the morph's applicability for local gravitational systems, with the proven applicability in the nongravitational limit also supporting the inference of morph applicability. Without the morph applicability, it does not appear possible that the SEP could be satisfied, since the metric relation (35) based coupling of the absolute metric and shift tensor to the natural/gravitational metric, which in turn is universally coupled to matter and the nongravitational fields acting as gravitational source matter, would otherwise be expected to yield a *gravitational mass* for a source, as posed in a free-fall frame set up by a surrounding background system, to be dependent on the absolute metric and shift tensor values in the free-fall frame, and therefore on the background system field in violation of the SEP (see Will [1], Chap. 3, for background). Assuming that this

would be the case, then it is *required* that the background morph be applicable in order to circumvent gravitational mass sensitivity to the background system field, providing another compelling reason for morph applicability. The discussion and formulation in Section 4.5 provides the "basics" for morph application on local gravitational systems and the resultant SEP satisfaction.

If the gravitational field generated by a local gravitational system perturbed the sources of the surrounding background system, the background system field acting back on the local system would be perturbed, preventing the formation of a universal diffeomorphism acting on the local system. It is therefore required that the gravitational field of the local system not perturb the sources of the background system. This "background (source) nonperturbation requirement" is the *same* as in general relativity for the SEP to be in effect, where similarly if the local system field perturbed the background sources, then a perturbed background field would result that would act on the local system to prevent SEP satisfaction. In the development that follows, it is assumed that the background nonperturbation requirement is met as an operating condition unless stated otherwise. The requirement in GS theory that background system curvature effects on the local gravitational system be negligible for morph applicability, and therefore SEP satisfaction, is the same as the requirement in general relativity that background system curvature effects be negligible for SEP satisfaction. Therefore, the required "SEP conditions" in GS theory for SEP satisfaction, consisting of the background nonperturbation requirement and negligible background curvature effects, *are the same as the SEP conditions in general relativity*. Concluding, satisfaction of the SEP for both theories is "equally applicable" since the required SEP conditions are the same.

As discussed above, the gravitational field of a morphed local gravitational system is subject to the morph the same as any matter or nongravitational field. This implies that the field metric, shift, and potential tensors for a morphed gravitational system adhere to the tensor morph quantity partner relation (141), yielding

$$g^M_{\alpha\beta} = M^{\bar\mu}{}_\alpha M^{\bar\nu}{}_\beta g^{UM}_{\mu\nu}, \qquad S^{\beta M}_{\bar\nu} = M^\beta{}_{\bar\alpha} M^{\bar\mu}{}_\nu S^{\alpha\,UM}_{\bar\mu}, \qquad w^{\beta M}_\nu = M^\beta{}_{\bar\alpha} M^{\bar\mu}{}_\nu w^{\alpha\,UM}_\mu. \qquad (167)$$

Application of the morph partner (coordinate) transform, (135), to the morphed field tensors given in the partner global ICs, yields

$$\hat g^M_{\mu\nu} = \hat g^{UM}_{\mu\nu}, \qquad \hat S^{\alpha M}_{\bar\mu} = \check S^{\alpha\,UM}_{\bar\mu}, \qquad \hat w^{\alpha M}_\mu = \check w^{\alpha\,UM}_\mu, \qquad (168)$$

as expected under the morph partner quantity equality (144), stating the equality of the Riemann IC given field for a morphed gravitational system in the gravitational case, and the partner global IC given field for the unmorphed partner gravitational system in the partner inertial case. Substituting the unmorphed background metric $g^{UMB}_{\mu\nu} = a_{\mu\nu}$ for the absolute metric in all unmorphed partner system formulation (as discussed above), use of the metric relation (35) yields the "unmorphed metric relation,"

$$g^{UM}_{\alpha\beta} = g^{UMB}_{\mu\nu} S^{\bar\mu\,UM}_\alpha S^{\bar\nu\,UM}_\beta, \qquad (169)$$

giving the natural metric for the unmorphed partner system. Application of the morph to the tensors in the unmorphed metric relation, as per (141), yields the "morphed metric relation"

$$g^M_{\alpha\beta} = g^{MB}_{\mu\nu} S^{\bar\mu M}_\alpha S^{\bar\nu M}_\beta, \qquad (170)$$

having used (167) and (143). As can be seen, the unmorphed and morphed background metrics are used in place of the absolute metric in the unmorphed and morphed metric relations, as expected since this follows the absolute replacement method established above.

Similar to the practice followed using general relativity, when evaluating or utilizing SEP satisfaction, the "boundary" of a local gravitational system, when posed in the field of a surrounding background system, is taken as the "asymptotic (limit) region" far enough away from the local system's sources that their gravitational field contribution may be considered negligible, but still close enough

that the entire local system out to its boundary may be posed in a locally inertial free-fall frame of the background system. So only the background system contributes to the field at the boundary of the local system. For the partner inertial case with no background field present, the field strength is therefore negligible at the boundary of the unmorphed partner local gravitational system, so the unmorphed system may be treated as having *vanishing* field strength at its asymptotic region boundary. The unmorphed shift tensor $S^{\alpha}{}_{\bar{\mu}}{}^{UM} = \delta^{\alpha}{}_{\bar{\mu}}$ is equal to the delta tensor at the boundary as stated. The equivalent unmorphed potential tensor $w^{\alpha}{}_{\mu}{}^{UM} = 0$ is therefore zero as stated. For the gravitational case with the background system present, the shift and potential tensors for the morphed partner local gravitational system again have the vanishing field strength values $S^{\alpha}{}_{\bar{\mu}}{}^{M} = \delta^{\alpha}{}_{\bar{\mu}}$ and $w^{\alpha}{}_{\mu}{}^{M} = 0$ at its boundary, as obtained by applying (167) to the boundary values for the unmorphed partner system. Therefore, they do not approximate the *actual* values $S^{\alpha}{}_{\bar{\mu}} = S^{\alpha B}{}_{\bar{\mu}}$ and $w^{\alpha}{}_{\mu} = w^{\alpha B}{}_{\mu}$ of the shift and potential tensors at the boundary, which are equal to the background system shift and potential tensors (as indicated) due to the vanishing local system field contribution. With their boundary values being so different, it can be seen that *throughout* a morphed gravitational system, the morphed values of its shift and potential tensors will in general be quite different from their actual values, as stated by

$$S^{\alpha M}{}_{\bar{\mu}} \not\approx S^{\alpha}{}_{\bar{\mu}}, \qquad w^{\alpha M}{}_{\mu} \not\approx w^{\alpha}{}_{\mu}. \tag{171}$$

These "non-approximations" are due to $S^{\alpha M}{}_{\bar{\mu}}$ and $w^{\alpha M}{}_{\mu}$ being the shift and potential tensors for the *local morphed partner of the unmorphed local gravitational system only*, which are not then the shift and potential tensors of the *actual total system* consisting of the local and background system combined. Note the similarity with the shifted case in Section 3.14, where indeed in the infinitesimal micro-system limit with then the local system becoming nongravitational, the shifted case is yielded.

Consider an *actual* local gravitational system as posed in the field of a surrounding background system. The ability to neglect background curvature effects in an actual system is generally an *approximation*, so even in this case there are still "small" background curvature effects for the actual quantities $Z^{\alpha\beta}$ depicting the objects present. Therefore, when background curvature effects may be considered negligible so that a morphed gravitational system is yielded from the actual system, the resultant morphed values of quantities are generally *approximations* of their actual values, as stated by the representative

$$Z^{\alpha\beta}_M \approx Z^{\alpha\beta}. \tag{172}$$

Similar to the equality $Z^{\alpha\beta}_S = Z^{\alpha\beta}$ (88) in the shifted case, the morph case approximation (172) applies for all quantities subject to shifting except those explicitly containing the shift or potential tensors, which will generally have significantly different morphed and actual values due to (171) holding. For the natural metric, $g^M_{\alpha\beta} \approx g_{\alpha\beta}$ holds as per (172). As a check, at the local system boundary with then vanishing local system field contribution, the field is again the background system field, so $\hat{g}_{\mu\nu} = \hat{g}^B_{\mu\nu} = \eta_{\mu\nu}$ for the actual metric in Riemann ICs, and $\hat{g}^M_{\mu\nu} = \hat{g}^{MB}_{\mu\nu} = \eta_{\mu\nu}$ for the morphed metric, yielding $\hat{g}^M_{\mu\nu} = \hat{g}_{\mu\nu} = \eta_{\mu\nu}$, and therefore $g^M_{\alpha\beta} = g_{\alpha\beta}$ at the boundary in any coordinates. With their boundary values being equal, it can be seen that *throughout* an actual local gravitational system when background curvature effects may be considered negligible, the resultant morphed value $g^M_{\alpha\beta}$ of the natural metric will indeed approximate its actual value $g_{\alpha\beta}$. Combining the approximation $g^M_{\alpha\beta} \approx g_{\alpha\beta}$ with the morphed and actual metric relations (170) and (35) (and their equivalent potential forms), then

$$\begin{aligned} g^{MB}_{\mu\nu} S^{\bar{\mu}M}{}_{\alpha} S^{\bar{\nu}M}{}_{\beta} &= g^{MB}_{\alpha\mu}\exp(-2w^{\mu M}{}_{\beta}) = g^M_{\alpha\beta} \\ &\approx \quad g_{\alpha\beta} = a_{\alpha\mu}\exp(-2w^{\mu}{}_{\beta}) = a_{\mu\nu}S^{\bar{\mu}}{}_{\alpha}S^{\bar{\nu}}{}_{\beta}. \end{aligned} \tag{173}$$

Note that the shift and potential tensors on the bottom line of (173) are for the *actual total system* consisting of the local gravitational system combined with the background system, whereas the shift

and potential tensors on the top line are for the *morphed local gravitational system only*, with again these tensors for the actual and morphed cases generally having significantly different values as per (171).

When the natural field equation $H^{\alpha\beta}[w;a] = 8\pi T^{\alpha\beta}$ (8) is utilized by natural observers to model local gravitational systems surrounded by background systems, the contained field equation quantities are subject to morph application, via (141), so long as the SEP conditions are met (assumed throughout this discussion), where as is the case for morph formulation in general, the absolute replacement method is used so the unmorphed and morphed background natural metrics are used in place of the absolute metric. Therefore, if the natural field equation (NFE) is utilized in the partner inertial case consisting of the unmorphed partner local gravitational system as posed in the absolute inertial frame of the partner global ICs, application of the morph via (141) yields the NFE in the gravitational case where the morphed partner gravitational system is again given in the partner global ICs. Applying the morph partner transform, (135), yields the NFE in the gravitational case as given in the partner Riemann ICs providing the locally inertial free-fall frame of the surrounding background system. As per (144), the morphed NFE quantities in the gravitational case, when given in Riemann ICs, have the *same values* as the partner global IC given unmorphed partner NFE quantities in the inertial case, such as the key field equalities (168) (their quantities will be utilized in the provided NFE). This results in the morphed gravitational system modelled by the morphed NFE, and the unmorphed partner gravitational system modelled by the unmorphed partner NFE, having the same values for all of their partner quantities when given in their respective partner Riemann and global ICs, as per (144). Applying (146) yields satisfaction of the SEP for the morphed gravitational system when naturally observed. Summarizing, *assuming the SEP conditions are met, use of the natural field equation by a natural observer to model a local gravitational system surrounded by a background system, as posed in the Riemann ICs for a locally inertial free-fall frame of the background system, yields a predicted morphed gravitational system satisfying the SEP.* Based on the above discussion of equivalence principle satisfaction in any coordinates, under coordinate transformation from the baseline free-fall frame form of SEP satisfaction, the SEP is satisfied via natural field equation use in any coordinates and frames.

As can be seen from the above discussion, *any form* for the natural field equation will yield SEP satisfaction for a local gravitational system so long as the SEP conditions are met, since morph application to any form in the partner inertial case will yield a morphed field equation in the gravitational case, resulting in SEP satisfaction for the field equation modelled morphed gravitational system. However, consider use of the NFE $H^{\alpha\beta}[w;a] = 8\pi T^{\alpha\beta}$ for the *actual case* consisting of the actual local gravitational system surrounded by the background system. The *form* for the NFE must be such that its morphed form may be considered to be the approximation obtained from field equation use in the actual case when background curvature effects are *completely neglected* as an approximating assumption. To satisfy this "morph consistency requirement," then "in reverse," when background curvature effects may be considered negligible, the field equation for the actual case must be able to be put into a form that *approximates* its morphed form. As will be shown when developing the natural field equation below, the morph consistency requirement *significantly limits its form*. The morph consistency requirement also applies for the natural nongravitational physics laws under the influence of gravity, but will be shown to always hold for such laws without further constraining their forms.

5. Natural and Absolute Observation and Formulation

With morph-based equivalence principle satisfaction established for natural observers, the subject of natural and absolute observation and formulation may be systematically examined. This is found to be a deep subject, as is understood for the subject of "observation" in general physics. So only the basics are provided along with illustrative and useful examples.

5.1. Natural Observation, Quantities, and Formulation

A fair amount of general material has been provided above on the subject of natural observation and formulation. Summarizing for the development here, natural measurement of gravity shifted

objects, using then raw shifted/morphed instruments, yields the naturally measured values $Z^{\alpha\beta}_{(N)}$ of quantities $Z^{\alpha\beta}$ depicting the objects, with the naturally measurable quantities consisting of those depicting matter, the nongravitational fields, and the natural metric $g_{\alpha\beta}$ universally coupled to them. Once the naturally measured value $Z^{\alpha\beta}_{(N)}$ of a quantity is obtained, coordinate transformation may be applied to provide its value $Z^{\alpha\beta}_N$ as utilized by natural observers to model systems in any coordinates. In a given arbitrary coordinate system and its specified frame, it may not be the case that $Z^{\alpha\beta}_N$ equals its naturally measured value $Z^{\alpha\beta}_{(N)}$ (assuming the rule in this paper that natural measurement is made utilizing the same frame as specified by the coordinates). Natural measurement may be made utilizing any frames and coordinates. But natural measurement and depiction are best understood when using the micro free-fall frames due to morph-based EEP satisfaction, forming the basis of the "natural worldview," which is therefore the same as the EEP-based natural worldview in general relativity. The EEP is utilized in the micro free-fall frames in order to formulate the natural physical laws for matter and the nongravitational fields under the influence of gravitation.

Of interest for general formulation are *universally applicable* natural quantities $Z^{\alpha\beta}_N$ utilized to perform modelling for *any* systems. These may be obtained via natural measurement in micro free-fall frames so that nongravitational micro systems result where the EEP is satisfied due to micro morphs being universally applicable, yielding the inertially valued

$$\hat{Z}^{\mu\nu}_N = \hat{Z}^{\mu\nu}_{N(N)} = \hat{Z}^{\mu\nu}_{M(N)} = \hat{Z}^{\mu\nu}_M = \hat{Z}^{\mu\nu}_S$$
$$= \hat{Z}^{\mu\nu} = \hat{Z}^{\mu\nu}_{(N)} = \check{Z}^{\mu\nu}_{UM} = \check{Z}^{\mu\nu}_{US} \qquad \text{(micro system, to 1st order).} \qquad (174)$$

This list of up to first-order quantities is obtained by combining (160), (162), and (165), as well as the natural measurements of two quantities being the same if the two quantities are equal (using any coordinates). Coordinate transformation of (174) yields $Z^{\alpha\beta}_N = Z^{\alpha\beta}_M = Z^{\alpha\beta}_S = Z^{\alpha\beta}$ in any coordinates. These up to first-order universally applicable natural quantities are *the same* as the micro system based natural quantities developed above in order to formulate the natural nongravitational physics laws under gravitation. Higher differential order natural quantities $Z^{\alpha\beta}_N = Z^{\alpha\beta}_S = Z^{\alpha\beta}$ may then be obtained by applying natural covariant derivatives to the up to first-order quantities, but equality with the second-order and higher morphed quantities $Z^{\alpha\beta}_M$ no longer holds due to curvature effects in $Z^{\alpha\beta}_N = Z^{\alpha\beta}_S = Z^{\alpha\beta}$ that are not present in $Z^{\alpha\beta}_M$. Following this methodology, all universally applicable natural quantities

$$Z^{\alpha\beta}_N = Z^{\alpha\beta}_S = Z^{\alpha\beta} \quad \text{(natural metric use)} \qquad (175)$$

to arbitrarily high differential order may be obtained, which are shifted/actual quantities $Z^{\alpha\beta}_S = Z^{\alpha\beta}$ depicting matter and the nongravitational fields where the only metric utilized is the natural metric as specified. The natural metric use includes $g_{\alpha\beta}$ itself and its connection $\Gamma^\alpha_{\mu\nu}$ as utilized in the natural metric based covariant derivatives contained in $Z^{\alpha\beta}_S = Z^{\alpha\beta}$. This methodology is the same as the methodology employed in general relativity where first the EEP is utilized to formulate the inertial values of up to first-order natural quantities $\hat{Z}^{\mu\nu}_N$ in the micro free-fall frames, then coordinate transformation is used to obtain their values $Z^{\alpha\beta}_N$ in any coordinates, and finally natural covariant differentiation is applied to obtain higher-order natural quantities. Therefore, the available formulations of natural quantities in general relativity based on this method *are also applicable in gravity shift theory*, providing natural quantities $Z^{\alpha\beta}_N$ that are already "known." This is taken as a "given" in subsequent formulation. For example, the known natural matter SE tensor, $T^{\alpha\beta} = T^{\alpha\beta}_N$, developed and employed in general relativity, is the same in GS theory. The natural metric $g_{\alpha\beta}$ and its connection $\Gamma^\alpha_{\mu\nu}$ are natural quantities that may be specified by $g^N_{\alpha\beta}$ and $\Gamma^{\alpha N}_{\mu\nu}$, but for brevity, $g_{\alpha\beta}$ and $\Gamma^\alpha_{\mu\nu}$ continue to be used.

The above-established universally applicable natural quantities $Z^{\alpha\beta}_N$ *are utilized to construct all general formulations employed by natural observers to model systems*, with the only exception being the natural field equation containing the potential $w^\alpha{}_\mu$ as its operand, which is not a natural quantity.

What is meant by "general" formulations are ones that are *universally applicable* so that they may be utilized to perform modelling for *any* systems. This is the basis for requiring universal applicability of the natural quantities $Z_N^{\alpha\beta}$. Similar to construction of the universally applicable natural quantities, the universally applicable general formulations for natural observers may be obtained via natural measurement of same in micro free-fall frames, so that again nongravitational micro systems result where the EEP is satisfied due to micro morphs being universally applicable. As the goal is to construct general formulations utilizing natural quantities $Z_N^{\alpha\beta}$ that are shifted/actual quantities $Z_S^{\alpha\beta} = Z^{\alpha\beta}$, then as per (174), use of the EEP to initially construct general formulations limits the contained natural quantities to first-order quantities. After EEP-based formation, coordinate transformation may be applied to obtain up to first-order general formulations in any coordinates. Finally, natural covariant differentiation may be applied to obtain universally applicable natural general formulations containing natural/shifted/actual quantities $Z_N^{\alpha\beta} = Z_S^{\alpha\beta} = Z^{\alpha\beta}$ of *arbitrarily high* differential order, which is the desired goal. As can be seen, the methodology employed is the same as the EEP-based methodology employed in general relativity to obtain universally applicable general formulations. With all natural quantities being the same in both GS theory and general relativity, and with the EEP holding for constructing general formulations in both theories, then with the exception of their respective natural field equations, *all general formulations in gravity shift theory employed by natural observers to model systems are identical in form to the available such formulations in general relativity*. The available general relativity formulations may be utilized then to provide the natural general formulations in GS theory, as is subsequently done. Examples of general formulations are the natural physics laws for matter and the nongravitational fields under gravitation as formed via use of the EEP, which have been shown to be the available general relativity laws for same due to again EEP-based formation.

5.2. Absolute Observation, Quantities, and Formulation

A major difference between GS theory and general relativity is of course the additional existence of absolute observers in GS theory, along with then the existence of absolute quantities and formulations. As previously discussed, absolute observers utilize shift-corrected instruments to make observations with, which is the same as using hypothetical unshifted instruments, resulting in *accurate* measurements. Absolute measurement of gravity shifted objects yields the absolutely measured values $Z_{(A)}^{\alpha\beta}$ of quantities $Z^{\alpha\beta}$ depicting the objects. The absolutely measured values $Z_{(A)}^{\alpha\beta}$ of quantities are the *actual values* due to accurate measurement. *All quantities may be measured by absolute observers*, which includes then those depicting matter and the nongravitational fields, the absolute metric $a_{\mu\nu}$, and all quantities depicting the gravitational field, which includes the natural metric $g_{\alpha\beta}$, shift tensor $S^\alpha{}_{\bar\mu}$, and potential tensor $w^\alpha{}_\mu$. Note that the naturally measured quantities $Z_{(N)}^{\alpha\beta}$ are included in the "all encompassing" inventory of measured quantities for absolute observers, since as an option, absolute observers are free to utilize the same raw shifted/morphed instruments prior to shift-correction as natural observers use. But here the "explicit" absolutely measured values $Z_{(A)}^{\alpha\beta}$ of quantities using shift-corrected instruments are specifically evaluated.

Absolute measurement may be made utilizing any frames and coordinates. However, absolute measurement is *best understood and depicted* utilizing the absolute inertial frames for the following reasons. First and foremost, in the global ICs of absolute inertial frames, the absolutely measured value of a quantity is the same as its actual global IC value, as formally stated by the representative

$$\check{Z}_{(A)}^{\alpha\beta} = \check{Z}^{\alpha\beta}. \tag{176}$$

Equation (176) includes absolute measurement $\check{a}_{\mu\nu}^{(A)} = \eta_{\mu\nu}$ of the absolute metric yielding its Minkowski global IC value. This results in "absolute" geodesic motion

$$\frac{\mathrm{d}U_A^\alpha}{\mathrm{d}\tau_A} + A^\alpha{}_{\mu\nu} U_A^\mu U_A^\nu = 0, \tag{177}$$

as dictated by the absolute metric connection (where $U^\alpha_A \equiv \mathrm{d}x^\alpha/\mathrm{d}\tau_A$), to be perceived as being *inertial* by absolute observers in the global ICs of absolute inertial frames due to $\check{A}^\alpha_{\mu\nu} = 0$. The laws of special relativity, applicable for all formulation given in absolute inertial frames, are *explicitly perceived to hold* for absolute observers measuring all quantities utilizing the absolute inertial frames. The absolute inertial frames are therefore the "preferred" frames of reference for understanding and depicting measurement by absolute observers as well as absolutely measured behavior. Similar to the "natural worldview" being based on naturally measured quantities and behavior utilizing the preferred inertial free-fall frames for natural observers (the same as in general relativity), the "absolute worldview" is based on absolutely measured quantities and behavior utilizing the preferred absolute inertial frames for absolute observers. For example, absolute observers conceive of gravitation as an *ordinary force* due to absolutely perceived gravitational acceleration of objects relative to their preferred absolute inertial frames (as stated in the summary).

Absolute observers perceive the partner relation based gravity shifting (14) *taking place*, measured as $\mathrm{d}\check{x}^\alpha_{S(A)} = \check{S}^{\alpha\,(A)}_{\bar{\mu}}\,\mathrm{d}\check{x}^\mu_{US(A)}$ in their preferred absolute inertial frames. This provides a means by which absolute observers perceive the shift tensor $S^\alpha_{\bar{\mu}}$ and therefore the potential tensor w^α_{μ}. Similarly, when background curvature effects may be neglected for a local system, *absolute observers perceive the morphbased gravity shifting* (93) *as well*, measured as $\check{x}^\alpha_{M(A)} = \check{M}^\alpha_{(A)}(\check{x}^\mu_{UM(A)})$ in their preferred absolute inertial frames. Therefore, absolute observers perceive all gravity shifting of matter and fields tied to the partner event fields depicting gravity shifting, which includes gravity shifting of the local gravitational field by the background field as previously discussed. Any effects of gravity shifting on matter and fields is also absolutely perceived, such as gravity shifting induced dynamic shifts (discussed below). *The existence, properties, and effects of partner relation and morph-based gravity shifting are therefore part of the absolute worldview.* In contrast, natural observers do not perceive gravity shifting (as previously established), so its existence, properties, and effects are not part of the natural worldview. The shift and potential tensors $S^\alpha_{\bar{\mu}}$ and w^α_{μ} *couple* to matter and all fields via the gravity shift mechanism, so absolute observers perceive the coupling of the shift and potential tensors to matter and all fields. In addition, absolute observers perceive the coupling of the absolute metric $a_{\mu\nu}$ to matter and all fields, as well as the coupling of the natural metric $g_{\alpha\beta}$ to matter and the nongravitational fields. However, as established, natural observers only perceive the natural metric $g_{\alpha\beta}$ coupled to matter and the nongravitational fields.

Once the absolutely measured value $\check{Z}^{\alpha\beta}_{(A)} = \check{Z}^{\alpha\beta}$ of a quantity is obtained using a preferred absolute inertial frame, coordinate transformation may be applied to obtain its value $Z^{\alpha\beta}_A$ as utilized by absolute observers to model systems in any coordinates, referred to as an "absolute quantity." Similar to $Z^{\alpha\beta}_N$ not always equaling $Z^{\alpha\beta}_{(N)}$, it may not be the case that $Z^{\alpha\beta}_A$ equals its absolutely measured value $Z^{\alpha\beta}_{(A)}$ (assuming the rule in this paper that absolute measurement for a coordinate-given quantity $Z^{\alpha\beta}$ is made utilizing the same frame as specified by its coordinates). But with $Z^{\alpha\beta}_A$ *defined* via transform from the global IC value $\check{Z}^{\alpha\beta}_{(A)}$, then $\check{Z}^{\alpha\beta}_A = \check{Z}^{\alpha\beta}_{(A)}$ in the global ICs of absolute inertial frames. Combining this with (176) yields

$$\check{Z}^{\alpha\beta}_A = \check{Z}^{\alpha\beta}_{A(A)} = \check{Z}^{\alpha\beta}_{(A)} = \check{Z}^{\alpha\beta}, \tag{178}$$

which includes the recognition that the absolute measurement $\check{Z}^{\alpha\beta}_{A(A)}$ of $\check{Z}^{\alpha\beta}_A$ equals $\check{Z}^{\alpha\beta}_A$ itself in the global ICs. Now the absolute metric $\check{a}_{\mu\nu} = \eta_{\mu\nu}$ is the metric that is universally utilized in the laws of special relativity applicable in the absolute inertial frames. With the absolute observers explicitly perceiving adherence to the laws of special relativity in their preferred absolute inertial frames, then the absolute metric is utilized for *universally applicable* metric-incorporating formulation of absolute quantities $Z^{\alpha\beta}_A$. For absolute observers, the natural metric $g_{\alpha\beta}$ is used to depict the gravitational field only, so it is "just one more quantity" as opposed to the absolute metric universally used for all absolute quantities $Z^{\alpha\beta}_A$ when a metric is required (such as for raising and lowering indices). Beginning with

(178) in the absolute inertial frames, coordinate transformation yields $Z_A^{\alpha\beta} = Z^{\alpha\beta}$ in any coordinates. Following this methodology, all universally applicable absolute quantities

$$Z_A^{\alpha\beta} = Z^{\alpha\beta} \quad \text{(absolute metric use)} \tag{179}$$

(to arbitrarily high differential order) may be obtained, which are actual quantities $Z^{\alpha\beta}$ where the absolute metric is utilized for universal formulation as specified. Note that the shifted values $Z_S^{\alpha\beta}$ of quantities are not included in (179) as a general equality, since $Z_A^{\alpha\beta}$ may explicitly contain an absolute metric, in which case it would not be a shifted quantity (from above). The absolute metric $a_{\mu\nu}$ and its connection $A_{\mu\nu}^{\alpha}$ are absolute quantities that may be specified by $a_{\mu\nu}^A$ and $A_{\mu\nu}^{\alpha A}$, but for brevity, $a_{\mu\nu}$ and $A_{\mu\nu}^{\alpha}$ continue to be used.

The above-established universally applicable absolute quantities $Z_A^{\alpha\beta}$ *may be utilized to construct all general formulations employed by absolute observers to model systems.* The universal applicability of general formulations is the basis for requiring universal applicability of the absolute quantities $Z_A^{\alpha\beta}$. Similar to construction of the universally applicable absolute quantities, the universally applicable general formulations for absolute observers may be obtained via absolute measurement of same in absolute inertial frames, so that again the laws of special relativity hold. Coordinate transformation may then be applied to obtain general formulations in any coordinates. An option available for absolute observers is to utilize natural quantities $Z_N^{\alpha\beta}$ in absolute formulations, since as discussed above, these are part of the inventory of quantities available for absolute observers. Their use is more convenient in some cases, as demonstrated below. However, as will be shown, any natural quantity $Z_N^{\alpha\beta}$ may always be constructed using absolute quantities $Z_A^{\alpha\beta}$, so any absolute formulation utilizing convenient natural quantities may ultimately be considered to be based on absolute quantities exclusively.

5.3. Partner Quantities and Formulations

There exists what may be considered a "partner" absolute quantity $Z_A^{\alpha\beta}$ for every natural quantity $Z_N^{\alpha\beta}$, which are the respective absolute and natural values for a quantity of a particular type. This implies the existence of a "quantity partner relation" between any partner absolute and natural quantities $Z_A^{\alpha\beta}$ and $Z_N^{\alpha\beta}$. An example is the absolute metric $a_{\mu\nu} = a_{\mu\nu}^A$ considered the partner of the natural metric $g_{\alpha\beta} = g_{\alpha\beta}^N$, where the metric relation $g_{\alpha\beta} = a_{\mu\nu}S_{\alpha}^{\bar{\mu}}S_{\beta}^{\bar{\nu}}$ (35) is the quantity partner relation between them. Given the known formulation of a natural quantity $Z_N^{\alpha\beta}$ such as from general relativity, the formulation of the partner absolute quantity $Z_A^{\alpha\beta}$ may be obtained via their quantity partner relation. In addition, the *values* of natural quantities $Z_N^{\alpha\beta}$ may be considered "known" due to their (above-shown) formation via coordinate transformation and natural covariant differentiation applied to up to first-order *inertially valued* naturally measured quantities $\hat{Z}_N^{\mu\nu} = \hat{Z}_{(N)}^{\mu\nu} = \check{Z}_{US}^{\mu\nu}$ (as per (174)) in micro free-fall frames. So the value of an absolute quantity $Z_A^{\alpha\beta}$ may be obtained from the known value of the partner natural quantity $Z_N^{\alpha\beta}$ by applying their quantity partner relation. Note that absolute quantities exist that do not have natural partners, such as the shift tensor $S_{\bar{\mu}}^{\alpha} = S_{\bar{\mu}}^{\alpha A}$, but again every natural quantity has an absolute partner. A methodology for formulating partner quantities via use of quantity partner relations is developed below, along with various absolute quantities provided via their use given the known natural quantities.

Similar to the partner quantities, there exists a "partner" absolute formulation for every natural formulation, with the absolute quantities, $Z_A^{\alpha\beta}$, in the partner absolute formulation, the absolute partners of the natural quantities $Z_N^{\alpha\beta}$ in the natural formulation. The construction of a partner absolute formulation may be made then via application of partner quantity relations to the natural quantities $Z_N^{\alpha\beta}$ contained in the partner natural formulation. Since a physical law is a formulation, partner physical laws are included when generally discussing partner formulations here. Again, the natural general formulations in GS theory may be considered the known available natural general formulations in general relativity. Similar then to absolute quantities, absolute general formulations may be obtained

from the known partner natural general formulations. Examples are provided below. There exist though absolute formulations that do not have natural partners, but again every natural formulation, including then every natural physical law, has an absolute partner.

Consider "native" tensor quantities $Z^{\alpha\beta}$ that are defined *without the use of a metric*, such as displacements dx^α. Native tensor quantities are generally *zero-order* quantities, since a differentiated tensor quantity requires use of a metric connection based covariant derivative. Applying (174) yields $\hat{Z}^{\alpha\beta}_N = \hat{Z}^{\alpha\beta}_{(N)} = \hat{Z}^{\alpha\beta}_S = \hat{Z}^{\alpha\beta}$ for the zero-order native natural quantities in Riemann ICs, which when transformed into the partner global ICs results in $\check{Z}^{\alpha\beta}_N = \check{Z}^{\alpha\beta}_S = \check{Z}^{\alpha\beta}$ as per (175) given in global ICs, where no metric is utilized in $\check{Z}^{\alpha\beta}_S = \check{Z}^{\alpha\beta}$. But according to (178), a partner native absolute quantity satisfies $\check{Z}^{\alpha\beta}_A = \check{Z}^{\alpha\beta}_{(A)} = \check{Z}^{\alpha\beta}$, where again no metric is utilized in $\check{Z}^{\alpha\beta}$. Therefore, $\check{Z}^{\alpha\beta}_A = \check{Z}^{\alpha\beta}_N = \check{Z}^{\alpha\beta}_S = \check{Z}^{\alpha\beta}$ for native quantities, yielding in any coordinates or geometrically the "native equality"

$$Z^{\alpha\beta}_A = Z^{\alpha\beta}_N = Z^{\alpha\beta}_S = Z^{\alpha\beta}, \qquad \mathbf{Z}_A = \mathbf{Z}_N = \mathbf{Z}_S = \mathbf{Z}, \qquad \text{(native quantities)} \tag{180}$$

which is the quantity partner relation expressing the equality of partner native (and therefore zero-order) absolute and natural quantities. The value for a native absolute quantity $Z^{\alpha\beta}_A$ is equal then to the *known* value of the partner native natural quantity $Z^{\alpha\beta}_N$ (assuming it exists).

Applying the native equality (180) to displacements yields

$$dx^\alpha_A = dx^\alpha_N = dx^\alpha_S = dx^\alpha, \qquad d\vec{x}_A = d\vec{x}_N = d\vec{x}_S = d\vec{x}, \tag{181}$$

so both absolute and natural observers use the native shifted/actual displacements $dx^\alpha_S = dx^\alpha$ when modelling. Integrating (181) yields

$$x^\alpha_A = x^\alpha_N = x^\alpha_S = x^\alpha, \qquad x_A = x_N = x_S = x, \tag{182}$$

so both absolute and natural observers model with the native shifted/actual event locations $x^\alpha_S = x^\alpha$ as well as the events $x_S = x$ themselves. The coordinate systems utilized by absolute and natural observers are the same, so using (182), their basis vectors $\vec{e}_{(\alpha)} = \partial x / \partial x^\alpha$ are the same native quantities as stated by $\vec{e}^A_{(\alpha)} = \vec{e}^N_{(\alpha)} = \vec{e}^S_{(\alpha)} = \vec{e}_{(\alpha)}$. Similarly, the native coordinate basis 1-forms $\tilde{\omega}^{(\alpha)} = \tilde{d}x^\alpha$ are the same as stated by $\tilde{\omega}^{(\alpha)}_A = \tilde{\omega}^{(\alpha)}_N = \tilde{\omega}^{(\alpha)}_S = \tilde{\omega}^{(\alpha)}$. Using the basis vector and 1-form equalities in $\delta^\alpha_\mu = \langle \tilde{\omega}^{(\alpha)}, \vec{e}_{(\mu)} \rangle$ yields the native delta tensor equality $\delta^{\alpha N}_\mu = \delta^{\alpha A}_\mu = \delta^{\alpha S}_\mu = \delta^\alpha_\mu$, as expected from direct use of (180). For brevity when performing absolute or natural modelling, the actual value $Z^{\alpha\beta}$ for a native quantity may be used in place of its equal absolute or natural values, such as using dx^α for the absolute or natural displacement values for all modelling.

A key native quantity is the "de Broglie 1-form"

$$\tilde{k}^A = \tilde{k}^N = \tilde{k}^S = \tilde{k}, \qquad k^A_\alpha = k^N_\alpha = k^S_\alpha = k_\alpha, \tag{183}$$

which is utilized to express de Broglie waves for shifted/actual quanta and particles. A de Broglie wave may be depicted locally as a *native* geometric object consisting of a series of evenly spaced flat parallel surfaces in 4-spacetime—i.e., a geometric 1-form—as discussed in MTW [16] (Chap. 2). Therefore, the de Broglie wave geometric 1-form on the left of (183) has the *same* absolute and natural values as indicated, yielding the coordinate form on the right. The number of de Broglie wave surfaces "pierced" by a displacement vector is $\langle \tilde{k}^A, d\vec{x}_A \rangle = \langle \tilde{k}^N, d\vec{x}_N \rangle$, which is a native scalar quantity that must therefore have the same absolute and natural values, verifying that $\tilde{k}^A = \tilde{k}^N$ since $d\vec{x}_A = d\vec{x}_N$ from (181). In accordance with the EEP, for natural observers the micro free-fall frame Riemann IC components of a de Broglie 1-form are

$$\hat{k}^N_\mu = \hat{k}^{(N)}_\mu = 2\pi \left[-\hat{v}_{(N)}, \, \hat{r}^x_{(N)}/\hat{\lambda}_{(N)}, \, \hat{r}^y_{(N)}/\hat{\lambda}_{(N)}, \, \hat{r}^z_{(N)}/\hat{\lambda}_{(N)} \right] = \check{k}^{US}_\mu, \tag{184}$$

where $\hat{v}_{(N)} = \hat{v}^S = \check{v}^{US}$ is the inertial naturally measured frequency, $\hat{\tilde{r}}_{(N)} = \hat{\tilde{r}}^S = \check{\tilde{r}}^{US}$ is the inertial naturally measured unit 3-space travel direction for the de Broglie wave, and $\hat{\lambda}_{(N)} = \hat{\lambda}^S = \check{\lambda}^{US}$ is the inertial naturally measured wavelength along the travel direction $\hat{\tilde{r}}_{(N)}$ (having utilized (174) and thus the partner absolute inertial frame for the inertial unshifted values). The naturally measured de Broglie wavefronts run perpendicular to their travel direction $\hat{\tilde{r}}_{(N)}$ (in a vacuum). For absolute observers, the absolute inertial frame global IC components of the same de Broglie 1-form are

$$\check{k}^A_\alpha = \check{k}^{(A)}_\alpha = 2\pi\left[-\check{v}_{(A)}, \check{r}^x_{(A)}/\hat{\lambda}_{(A)}, \check{r}^y_{(A)}/\hat{\lambda}_{(A)}, \check{r}^z_{(A)}/\hat{\lambda}_{(A)}\right], \tag{185}$$

where $\check{v}_{(A)} = \check{v}^S$ is the absolutely measured frequency, $\check{\tilde{r}}_{(A)} = \check{\tilde{r}}_S$ is the absolutely measured unit 3-space travel direction for the de Broglie wave, and $\check{\lambda}_{(A)} = \check{\lambda}^S$ is the absolutely measured wavelength along the travel direction $\check{\tilde{r}}_{(A)}$. In general, the absolutely measured shifted de Broglie wavefronts do not run perpendicular to their travel direction $\hat{\tilde{r}}_{(A)}$, as is the case for gravity shifted light (from above).

The de Broglie 1-form is a *kinematic* quantity. Multiplication by $\hbar$ across (183) converts the de Broglie 1-form into the "momentum 1-form"

$$\tilde{p}^A = \tilde{p}^N = \tilde{p}^S = \tilde{p} = \hbar\tilde{k}, \qquad p^A_\alpha = p^N_\alpha = p^S_\alpha = p_\alpha = \hbar k_\alpha, \tag{186}$$

utilized to depict energy-momentum for shifted/actual quanta and particles, yielding then a native *dynamic* quantity. As stated in (183), the geometric de Broglie 1-form is the gravity shifted 1-form $\tilde{k}^S$. Utilizing a coordinate 1-form basis, $\tilde{k}^S$ may be given by $\tilde{k}_S = k^S_\alpha\tilde{\omega}^{(\alpha)}$, the sum of geometric 1-forms $k^S_\alpha\tilde{\omega}^{(\alpha)}$. Each shifted 1-form component value may be given by $k^S_\alpha = S^{\bar\mu}{}_\alpha k^{US}_\mu$, where k^{US}_μ are the component values for the geometric unshifted de Broglie 1-form $\tilde{k}_{US} = k^{US}_\mu\tilde{\omega}^{(\mu)}$. As can be seen, the application of gravity shifting as dimensional shifts applied to the parallel surfaces giving the unshifted geometric de Broglie 1-form, $\tilde{k}_{US}$, yields the parallel surfaces giving the shifted de Broglie 1-form $\tilde{k}^S$, representing then gravity shifting applied to unshifted de Broglie waves to yield the partner shifted waves. Under this dimensional shifting of de Broglie waves, the unshifted geometric momentum 1-form $\tilde{p}_{US} = \hbar\tilde{k}_{US}$ is shifted to become the shifted momentum 1-form $\tilde{p}_S = \hbar\tilde{k}_S$, noting that the dimensional shifting for the parallel surfaces of the geometric momentum 1-form is identical to the de Broglie wave dimensional shifting. Therefore, the dimensional shifting of the de Broglie waves for quanta and particles has yielded a "dynamic shift," meaning a shift in a dynamic property, in this case energy-momentum. Via (186), the various natural and absolute shifted dynamic quantities in GS theory may be formed via use of the shifted momentum 1-form $\tilde{p}^S = \tilde{p}^A = \tilde{p}^N$, as is done for the cases below. Using the dimensional shifting induced dynamic shifting to obtain $\tilde{p}^S$ as the basis for the dynamic shifting for all dynamic quantities, in general, *dynamic shifts accompany the dimensional shifts of unshifted objects when obtaining their shifted partners, with the dynamic shifts resulting from application of the dimensional shifts* (as stated in the summary).

The use of wave packets constructed from the de Broglie waveforms (184) and (185), as well as the use of (186), yields partner natural and absolute formulations of the Heisenberg uncertainty principle

$$\Delta\hat{p}^{(N)}_\alpha\Delta\hat{x}^\alpha_{(N)} \geq \hbar \text{ (no sum)}, \qquad \Delta\check{p}^{(A)}_\alpha\Delta\check{x}^\alpha_{(A)} \geq \hbar \text{ (no sum)}, \tag{187}$$

applicable along each IC axis direction (including the time axis direction to yield $\Delta E\Delta t \geq \hbar$) for *both* natural and absolute observers in their respective preferred Riemann and global ICs (see Beiser [23], Chap. 3, for background). The partner absolute uncertainty principle was obtained via quantity partner relation based conversion of the natural quantities contained in the known natural uncertainty principle available from EEP-based formulation in general relativity. In the usual manner, the wave packets depict shifted quanta and particles as they exist in actuality subject to the uncertainty principle. The naturally measured wave packets have group speeds $\hat{v}_{(N)}$ that do not exceed the fixed natural light speed $\hat{c}_{(N)} = \check{c}_{US} = 1$ (see (161)), whereas a hypothetical single wavelength de Broglie wave contributor has a natural speed that is not less than the natural light speed. This yields absolutely

measured wave packets with group speeds $\breve{v}_{(A)}$ that do not exceed the variable absolute light speed $\breve{c}_{(A)} = \breve{c}_S \leq 1$, where a single wavelength de Broglie wave contributor has an absolute speed that is not less than the absolute light speed. The momentum 1-form (186) holds for the average energy and momentum of the wave packets, so this 1-form depicts average energy-momentum for shifted/actual quanta and particles subject to the uncertainty principle.

The above discussion and formulation demonstrates that not only do the laws of quantum mechanics hold for natural observers using raw shifted/morphed instruments as expected due to EEP satisfaction, the quantum mechanics laws also hold *in actuality*, meaning for gravity shifted quantum phenomena posed in absolute flat spacetime as accurately measured by absolute observers using shift-corrected instruments. Employing then both absolute and natural observers in this manner, it can be seen that *the use of gravity shift theory yields adherence to quantum mechanical laws under gravitation in absolute flat spacetime, while satisfying the equivalence principle*.

Consider "metric quantities" formed by applying metrics to native quantities, either by raising/lowering their indices by metrics or by applying metric connection based covariant differentiation to the native quantities. What are considered "partner" natural and absolute metric quantities $Z_N^{\alpha\beta}$ and $Z_A^{\alpha\beta}$ are ones formed by applying "partner metric operations" to equal partner native quantities, which are the *same metric operations* with the natural metric utilized for the natural native quantity $Z_N^{\alpha\beta}$, and the partner absolute metric utilized for the equal partner native absolute quantity $Z_A^{\alpha\beta}$. In this manner, the natural metric is the expected metric utilized in the naturally measured partner natural metric quantity $\hat{Z}_{N(N)}^{\mu\nu} = \hat{Z}_N^{\mu\nu}$ in any micro free-fall frame, and the absolute metric is the expected metric utilized in the absolutely measured partner absolute metric quantity $\check{Z}_{A(A)}^{\alpha\beta} = \check{Z}_A^{\alpha\beta}$ in the partner absolute inertial frame, with the respective naturally and absolutely measured partner metric operations applied then to the partner native quantities in the same manner. The defined partner metric quantities $Z_N^{\alpha\beta}$ and $Z_A^{\alpha\beta}$ therefore result in *expected* partner metric quantities *as measured by "partner" natural and absolute observers respectively utilizing a micro free-fall frame and a partner absolute inertial frame*, justifying the identification of the defined partner metric quantities as indeed being "partners." The partner general formulations discussed above utilize partner quantities $Z_N^{\alpha\beta}$ and $Z_A^{\alpha\beta}$ consisting of both partner native and metric quantities.

An example of partner metric quantities is the partner natural and absolute lowered displacements, $dx_\nu^N = g_{\nu\alpha} \, dx^\alpha$ and $dx_\nu^A = a_{\nu\alpha} \, dx^\alpha$, obtained by lowering the equal partner native displacements $dx_A^\alpha = dx_N^\alpha = dx^\alpha$. Note that $dx_\nu^A = dx_\nu$, demonstrating that "A" labeling for absolute metric quantities may be utilized in place of absolute underscoring for all raised/lowered indices. Using *metric products*, the quantity partner relation between dx_ν^N and dx_ν^A may be readily shown to be $dx_\nu^A = a_{\nu\mu} g^{\mu\alpha} \, dx_\alpha^N$, yielding

$$dx_\nu^A = F^\alpha_{\bar{\nu}} \, dx_\alpha^N, \tag{188}$$

having used the squared shift tensor based inverse metric relation (78). The partner metrics $g_{\alpha\beta}^N = g_{\alpha\beta}$ and $a_{\alpha\beta}^A = a_{\alpha\beta}$ themselves may be obtained via a partner metric operation consisting of lowering the equal partner native delta tensors $\delta_\mu^{\alpha N} = \delta_\mu^{\alpha A} = \delta_\mu^\alpha$. A key example is natural and absolute proper intervals ds_N and ds_A, which are partner metric quantities since $ds_N^2 = g_{\alpha\beta} \, dx^\alpha \, dx^\beta$ (34) and $ds_A^2 = a_{\mu\nu} \, dx^\mu \, dx^\nu$ (28). Of interest is their quantity partner relation in the case of naturally timelike or null shifted particle motion, which is

$$\frac{d\tau_N}{d\tau_A} = \sqrt{\frac{g_{\alpha\beta} \, dx^\alpha \, dx^\beta}{a_{\mu\nu} \, dx^\mu \, dx^\nu}}. \tag{189}$$

This is a *coordinate invariant* quantity that is dependent on the *motion* of the particle, which may be given by its velocity $\breve{v}^n = d\breve{x}^n / d\breve{t}$ in global ICs. Recall that if a natural proper interval ds_N is timelike or null, the partner absolute proper interval ds_A is also timelike or null, so (189) is real valued for

naturally timelike or null particle motion. For shifted particle motion that is naturally timelike, and thus absolutely timelike, the partner 4-velocities are

$$U_N^\alpha = \mathrm{d}x^\alpha/\mathrm{d}\tau_N, \qquad U_A^\alpha = \mathrm{d}x^\alpha/\mathrm{d}\tau_A, \tag{190}$$

where (189) may be utilized in their quantity partner relation $U_A^\alpha = (\mathrm{d}\tau_N/\mathrm{d}\tau_A)U_N^\alpha$ to obtain U_A^α given U_N^α, and vice versa.

Representative partner covariant derivatives of partner quantities are given by

$$\nabla_\nu^N Z_{\ \mu}^{\alpha N} \equiv Z_{\ \mu;\nu}^{\alpha N}, \qquad \nabla_\nu^A Z_{\ \mu}^{\alpha A} \equiv Z_{\ \mu\,|\,\nu}^{\alpha A}, \tag{191}$$

where " ; " designates natural covariant derivatives using the natural metric connection $\Gamma_{\mu\nu}^\alpha$, and " | " designates absolute covariant derivatives using the absolute metric connection $A_{\mu\nu}^\alpha$. The quantity partner relation between the partner covariant derivatives of partner native quantities, $Z_{\ \mu}^{\alpha N} = Z_{\ \mu}^{\alpha A} = Z_{\ \mu}^\alpha$, may be obtained by adding $\nabla_\nu^N Z_{\ \mu}^\alpha - \nabla_\nu^A Z_{\ \mu}^\alpha$ to $\nabla_\nu^A Z_{\ \mu}^\alpha$ to form the representative

$$\nabla_\nu^N Z_{\ \mu}^\alpha = \nabla_\nu^A Z_{\ \mu}^\alpha + Z_{\ \mu}^\rho \Delta_{\ \rho\nu}^\alpha - Z_{\ \rho}^\alpha \Delta_{\ \mu\nu}^\rho, \tag{192}$$

where the "connection difference tensor"

$$\Delta_{\ \mu\nu}^\alpha \equiv \Gamma_{\ \mu\nu}^\alpha - A_{\ \mu\nu}^\alpha \tag{193}$$

is a *tensor quantity* (transforms as a tensor) from bimetric theory [13]. As a result, the quantity partner relations between partner covariant derivatives of partner native tensor quantities are indeed *covariant tensor relations*. If desired, any natural *metric* quantity $Z_{\ \mu}^{\alpha N}$ could be substituted throughout (192), followed by substituting on the right the quantity partner relation between $Z_{\ \mu}^{\alpha N}$ and its partner absolute metric quantity $Z_{\ \mu}^{\alpha A}$, yielding the covariant quantity partner relation between $\nabla_\nu^N Z_{\ \mu}^{\alpha N}$ and $\nabla_\nu^A Z_{\ \mu}^{\alpha A}$. Other indice forms of partner metric quantities follow suit. The above examples demonstrate how quantity partner relations are developed between partner metric quantities.

The partner formulations of the "weak constraint" on shifted particle motion are

$$\vec{U}_N \overset{N}{\cdot} \vec{U}_N = -1, \qquad \vec{U}_A \overset{A}{\cdot} \vec{U}_A = -1, \tag{194}$$

obtainable from the partner line elements $\mathrm{d}\tau_N^2 = -g_{\alpha\beta}\,\mathrm{d}x^\alpha\,\mathrm{d}x^\beta$ and $\mathrm{d}\tau_A^2 = -a_{\mu\nu}\,\mathrm{d}x^\mu\,\mathrm{d}x^\nu$, and use of (190). The absolute weak constraint may be readily derived via quantity partner relation application to the partner natural quantities in the partner natural weak constraint, and vice versa. Representative partner directional covariant derivatives of partner quantities are given by

$$\frac{\mathrm{D}_N}{\mathrm{d}\tau_N} Z_{\ \mu}^{\alpha N} \equiv U_N^\nu Z_{\ \mu;\nu}^{\alpha N}, \qquad \frac{\mathrm{D}_A}{\mathrm{d}\tau_A} Z_{\ \mu}^{\alpha A} \equiv U_A^\nu Z_{\ \mu\,|\,\nu}^{\alpha A}, \tag{195}$$

where naturally timelike paths are assumed here, which are then absolutely timelike. Applying the partner directional covariant derivatives to the partner weak constraints yields the partner "acceleration constraints"

$$\vec{a}_N \overset{N}{\cdot} \vec{U}_N = 0, \qquad \vec{a}_A \overset{A}{\cdot} \vec{U}_A = 0, \tag{196}$$

where the partner 4-accelerations are given by

$$a_N^\alpha \equiv \frac{\mathrm{D}_N U_N^\alpha}{\mathrm{d}\tau_N} = \frac{\mathrm{d}U_N^\alpha}{\mathrm{d}\tau_N} + \Gamma_{\mu\nu}^\alpha U_N^\mu U_N^\nu, \qquad a_A^\alpha \equiv \frac{\mathrm{D}_A U_A^\alpha}{\mathrm{d}\tau_A} = \frac{\mathrm{d}U_A^\alpha}{\mathrm{d}\tau_A} + A_{\mu\nu}^\alpha U_A^\mu U_A^\nu. \tag{197}$$

To obtain (196), use was made of $g_{\alpha\beta;\mu} = 0$ and $a_{\alpha\beta\,|\,\mu} = 0$. The right-hand side of the natural 4-acceleration equation gives the gravitational geodesic motions of shifted particles if set to zero, as per (103) where $U_S^\mu = U_N^\mu$ for the particles. So $a_N^\alpha = 0$ under gravitational geodesic motion, but

a_N^α is non-zero if a particle is under a nongravitational force. The right-hand side of the absolute 4-acceleration equation gives the absolute metric based geodesic motions of shifted particles if set to zero, as per (177). So $a_A^\alpha = 0$ under absolute geodesic motion, but a_A^α is non-zero if a particle is under a force, which includes the absolutely perceived *gravitational force* as well as any nongravitational force. Even when forces are present though, using their respective partner metrics, in both the natural and absolute partner cases the 4-accelerations are *orthogonal* to the 4-velocities as per their acceleration constraints (196). The above discussion and formulation establishes adherence to the weak and acceleration constraints for shifted particle motion in absolute flat spacetime, while adhering to the weak and acceleration constraints that result under satisfaction of the equivalence principle, with this self-consistency obtained due to respective use of the absolute and natural classes of observers.

Under EEP satisfaction, the naturally measured 4-momentum of a shifted massive particle, in a micro free-fall frame, is given by the inertial form $\hat{p}_{N(N)}^\mu = \hat{m}_{N(N)} \hat{U}_{N(N)}^\mu$, where as per (174), $\hat{m}_{N(N)} = \check{m}_{US}$ is the inertially valued naturally measured rest mass. The naturally measured rest mass may be obtained via $\hat{m}_{N(N)}^2 = -\hat{p}_{N(N)}^\mu \hat{p}_\mu^{N(N)} = \check{m}_{US}^2$ having used the natural weak constraint in (194), where $\hat{p}_\mu^{N(N)} = (\hat{g}_{\mu\nu}^{(N)} = \eta_{\mu\nu})\hat{p}_{N(N)}^\nu$ is the naturally measured momentum 1-form. The naturally measured rest mass $\hat{m}_{N(N)} = \check{m}_{US}$ of a shifted/actual particle is therefore a *fixed* scalar invariant equal to its fixed invariant unshifted rest mass, yielding

$$m_N = m_{US} \tag{198}$$

for the fixed natural rest mass in any coordinates. The natural momentum 1-form and 4-vector in any coordinates are then

$$p_\alpha^N = m_N U_\alpha^N, \qquad p_N^\alpha = m_N U_N^\alpha. \tag{199}$$

The above momentum and rest mass formulations are available general relativity formulations of same (if the unshifted rest mass m_{US} is interpreted as the fixed inertial value of rest mass).

As is understood in general relativity formulation of natural quantities, the "quantum" momentum 1-form $p_\alpha^N = \hbar k_\alpha$, established above via use of quantum de Broglie waves, equals the "mechanical" momentum 1-form $p_\alpha^N = m_N U_\alpha^N$. This equality holds then in GS theory, which may be established using the EEP in micro free-fall frames. Combining this equality with (186) stating the equality $p_\alpha^A = p_\alpha^N$ of partner quantum momentum 1-forms, implies that (186) *also holds for the equality of partner natural and absolute mechanical momentum 1-forms*. Therefore, $p_\alpha^A = p_\alpha^N = m_N U_\alpha^N$ for the absolute mechanical momentum 1-form for a shifted particle. Substituting $p_\alpha^A = a_{\alpha\mu} p_A^\mu$ and $p_\alpha^N = g_{\alpha\mu} p_N^\mu$ into $p_\alpha^A = p_\alpha^N$, and using (76), yields the partner relation

$$p_A^\mu = F_{\phantom{\bar{\mu}}\alpha}^{\bar{\mu}} p_N^\alpha \tag{200}$$

for the partner mechanical/quantum momentum 4-vectors. With $U_\alpha^N = dx_\alpha^N/d\tau_N$ and $U_\mu^A = dx_\mu^A/d\tau_A$ the partner velocity 1-forms (obtained by lowering the partner 4-velocities (190) by their respective metrics), then use of (188) yields $U_\alpha^N = (d\tau_A/d\tau_N)F_{\phantom{\bar{\mu}}\alpha}^{\bar{\mu}} U_\mu^A$. Substituting this partner relation in $p_\alpha^A = m_N U_\alpha^N$, and equivalently using $p_N^\alpha = m_N U_N^\alpha$ (199) in (200) where $U_N^\alpha = (d\tau_A/d\tau_N)U_A^\alpha$, respectively results in the absolute mechanical momentum 1-form and 4-vector

$$p_\alpha^A = m_{\alpha}^{\mu\,A} U_\mu^A, \qquad p_A^\mu = m_{\alpha}^{\mu\,A} U_A^\alpha, \tag{201}$$

where

$$m_{\alpha}^{\mu\,A} \equiv m_N (d\tau_A/d\tau_N) F_{\phantom{\bar{\mu}}\alpha}^{\bar{\mu}} \tag{202}$$

is the "absolute mass tensor" for a shifted/actual particle. Lowering $m_{\alpha}^{\mu\,A}$ by the absolute metric, and applying (76), yields $m_{\beta\alpha}^A = m_N(d\tau_A/d\tau_N)g_{\beta\alpha}$, so the absolute mass tensor is *symmetric* when given in pure indice form. The absolute mass tensor for a shifted particle is *field dependent* due to the presence of $F_{\phantom{\bar{\mu}}\alpha}^{\bar{\mu}}$ in (202), and is additionally dependent on the particle's *motion* due to the (above-discussed) motion dependence of $d\tau_A/d\tau_N$ given by (189).

The recognition of absolute mass as a *tensor*, as opposed to a scalar, is made clear by forming the scalar invariant $-p^{\alpha}_A \, p^A_{\alpha} = -m^2_N (\mathrm{d}\tau_A/\mathrm{d}\tau_N)^2 F^{\bar{\mu}}_{\alpha} F^{\bar{\alpha}}_{\nu} U^A_{\mu} U^{\nu}_A$, which reduces to the positive-valued $m^2_N = m^2_{US}$ in deep space as expected (using (198)). But within the gravitational field, $-p^{\alpha}_A \, p^A_{\alpha}$ can be *negative* depending on a particle's motion, as can be shown when using the above star-case field. A negative $-p^{\alpha}_A \, p^A_{\alpha}$ would yield an *imaginary* scalar absolute mass m_A that would not then be absolutely observable. A scalar absolute mass based absolute momentum form $p^{\mu}_A = m_A U^{\mu}_A$ is therefore not physically valid, requiring instead the tensor mass form $p^{\mu}_A = m^{\mu}{}^A_{\alpha} U^{\alpha}_A$ with a real-valued, and therefore absolutely observable, absolute mass tensor $m^{\mu}{}^A_{\alpha}$.

The natural partner to the absolute mass tensor $m^{\mu}{}^A_{\alpha}$ for a shifted/actual particle, as given by (202), is the "natural mass tensor"

$$m^{\mu}{}^N_{\alpha} \equiv m_N \delta^{\mu}{}_{\alpha}. \tag{203}$$

This is obtainable by taking the weak limit of (202) to yield the unshifted form $\breve{m}^{\mu}{}^{US}_{\alpha} = (m_{US} = m_N)\breve{\delta}^{\mu}{}_{\alpha}$ in an absolute inertial frame (using (198)), applying a micro morph to obtain $\hat{m}^{\mu}{}^N_{\alpha} \equiv m_N \hat{\delta}^{\mu}{}_{\alpha}$ in the partner micro free-fall frame, and then using a coordinate transform to obtain $m^{\mu}{}^N_{\alpha} = m_N \delta^{\mu}{}_{\alpha}$ in any coordinates. Utilizing GS theory then, natural mass is understood to be a mass *tensor* in actuality. However, in practice, such as making natural mass measurements or modelling naturally observed systems, the delta tensor for the natural mass $m^{\mu}{}^N_{\alpha}$ given by (203) is always *merged* with other quantities. A key example is use of the natural 4-momentum $p^{\mu}_N = m^{\mu}{}^N_{\alpha} U^{\alpha}_N$, so that $p^{\mu}_N = m_N \delta^{\mu}{}_{\alpha} U^{\alpha}_N = m_N U^{\mu}_N$, showing the delta tensor $\delta^{\mu}{}_{\alpha}$ used in the stand-alone natural mass tensor, $m^{\mu}{}^N_{\alpha}$, being merged with the 4-velocity in the momentum. As a result, natural mass is historically conceived of as a *scalar m_N* due to merger of the delta tensor in $m^{\mu}{}^N_{\alpha} = m_N \delta^{\mu}{}_{\alpha}$ with other quantities in practice. But as a *stand-alone* quantity, *the natural mass of a shifted/actual particle is actually a tensor* assuming GS theory is valid. The historical practice of convenient scalar conception and use is continued in this paper though, since in practice the delta tensor in $m^{\mu}{}^N_{\alpha} = m_N \delta^{\mu}{}_{\alpha}$ gets merged and may therefore be ignored.

The right of (202) for the absolute mass tensor, $m^{\mu}{}^A_{\alpha}$, may be put exclusively in terms of absolute quantities by the following: using m_N as given by $m^2_N = -p^{\alpha}_N \, p^N_{\alpha}$ where the natural momentum terms are provided by the quantum momentum 1-form $p^N_{\alpha} = \hbar k_{\alpha}$ and vector $p^{\alpha}_N = g^{\alpha\mu} \hbar k_{\mu}$, using (189) for $\mathrm{d}\tau_A/\mathrm{d}\tau_N$, and then using (76) and (78) to put all instances of the natural metric in terms of the absolute metric and squared shift tensor. Instead of using the resultant long expression, though, it is often convenient to use the right of (202) constructed with the natural quantities m_N and $\mathrm{d}\tau_N$, which is allowed for absolute observers since natural quantities are part of their inventory. A technique from available formulation for "handling" photons (see PW [20], Chap. 4), put into GS theory terms, is to utilize a *fixed* ratio $m_N/\mathrm{d}\tau_N$ while taking the natural light speed limit $\mathrm{d}\tau_N \to 0$ (yielding $\hat{v}_{(N)} \to \hat{c}_{(N)} = 1$ in micro free-fall frames), keeping then the natural energy $p^0_N = m_N \, \mathrm{d}t/\mathrm{d}\tau_N$ fixed as a naturally massive shifted particle "transitions" to a shifted photon of the same energy. This results in shifted photons having a zero-valued natural rest mass m_N, the same as unshifted photons having a zero-valued rest mass m_{US}, as expected under EEP satisfaction. On the other hand, application of this technique in (202) yields a *finitely large absolute mass tensor $m^{\mu}{}^A_{\alpha}$* for shifted/actual photons moving at less than the unshifted light speed (absolute manifold null speed) in a gravitational field, only becoming absolutely massless in the deep-space limit where then $\mathrm{d}\tau_A$ goes to zero as a shifted photon becomes a massless unshifted photon moving at the unshifted light speed.

The partner 4-vector "force (based) motion laws" applicable for shifted/actual particle motion are

$$f^{\alpha}_N = \frac{\mathrm{D}_N \, p^{\alpha}_N}{\mathrm{d}\tau_N}, \qquad f^{\alpha}_A = \frac{\mathrm{D}_A \, p^{\alpha}_A}{\mathrm{d}\tau_A}, \tag{204}$$

obtainable from special relativity based force motion laws as measured by natural and absolute observers in their respective partner micro free-fall and absolute inertial frames. More convenient forms here for the partner force motion laws are the *lowered* forms

$$f_\alpha^N = \frac{D_N\, p_\alpha^N}{d\tau_N}, \qquad f_\alpha^A = \frac{D_A\, p_\alpha^A}{d\tau_A},$$

(205)

since then the 1-form equality $p_\alpha^A = p_\alpha^N$ (186) may be exploited. Applying the representative (192) and (195) for these 1-forms yields the "force partner relation"

$$f_\alpha^A = \frac{d\tau_N}{d\tau_A} f_\alpha^N + \Delta^\rho{}_{\alpha\mu} U_A^\mu p_\rho^A.$$

(206)

This relation holds for massive shifted particles as well as for shifted light moving in a gravitational field at less than the unshifted light speed. In a micro free-fall frame, the naturally measured gravitational force $\hat{f}_{\mu\,G}^{N(N)}$ is *zero* since $\hat{D}_N \hat{p}_\mu^{N(N)}/d\tau_{N(N)} = d\check{p}_\mu^{US}/d\tau_{US} = 0$ under EEP satisfaction when no nongravitational forces are present, yielding $f_{\alpha\,G}^N = 0$ for the natural gravitational force in any coordinates. Substitution of $f_{\alpha\,G}^N = 0$ into (206) yields

$$f_{\alpha\,G}^A = \Delta^\rho{}_{\alpha\mu} U_A^\mu p_\rho^A$$

(207)

when no natural nongravitational force is present, resulting (finally) in the formal expression for the *absolute gravitational force* applied to a shifted/actual particle (including shifted light). Absolute measurement of the absolute gravitational force is given by

$$\check{f}_{\alpha\,G}^{A(A)} = \check{\Gamma}_{\alpha\mu}^{\rho(A)} \check{U}_{A(A)}^\mu \check{p}_\rho^{A(A)}$$

(208)

in the preferred absolute inertial frames for absolute observers (using (193) with $\check{A}_{\alpha\mu}^\rho = 0$), confirming that *absolute observers perceive a gravitational force* (as stated in the summary). With the rightmost term in (206) identified as the absolute gravitational force $f_{\alpha\,G}^A$, then with no natural gravitational force, the absolute nongravitational force is given by the partner relation

$$f_{\alpha\,NG}^A = \frac{d\tau_N}{d\tau_A} f_{\alpha\,NG}^N.$$

(209)

Therefore, the total absolute force f_α^A, given by (206), is simply the *sum*

$$f_\alpha^A = f_{\alpha\,NG}^A + f_{\alpha\,G}^A$$

(210)

of the absolute nongravitational and gravitational forces without then cross coupling between them. Substitution of (201) into (207) yields

$$f_{\alpha\,G}^A = m^\nu{}_\rho^A \Delta^\rho{}_{\alpha\mu} U_A^\mu U_\nu^A$$

(211)

for the absolute gravitational force applied to a shifted particle (including shifted light) based on its absolute mass tensor. For the star case, the directional absolute force in the slow-moving limit is radially inward with magnitude $F_A = GMm_N/r^2$ in laboratory units, the expected Newtonian force in the weak limit, but remarkably also applicable when the star's field is strong.

As can be seen, particle motion in absolute flat spacetime is dictated by the usual force-based law of motion where both the gravitational and the nongravitational fields may apply the force, while the same motion is also dictated by motion law that adheres to the equivalence principle, which forbids the existence of a gravitational force, with this self-consistency obtained due to respective use of the absolute and natural classes of observers. Therefore, equivalent partner natural and absolute gravitational laws of motion are yielded, with the natural law obtained by using $f_N^\alpha = f_N^{\alpha\,G} = 0$ in (204)

(and dropping the fixed m_N) simply the usual law of geodesic motion utilizing the natural metric as the gravitational metric, and the equivalent partner absolute law obtained by using $f_A^\alpha = f_A^{\alpha\,G}$ in (204) a force-based law of motion where the gravitational field imposes the force. This equivalency resolves the conflict between gravitation seen as a force in flat spacetime as opposed to curvature without force under satisfaction of the equivalence principle.

The application of absolute gravitational force, $f_A^{\alpha\,G}$, to a particle moving in absolute flat spacetime, necessarily implies that the absolute gravitational force generally applies *absolute work* to the particle as it moves. This in turn implies that the gravitational field transfers absolute energy-momentum to the particle over the *local* spacetime region that the particle subtends, so *the gravitational field must possess a definable absolute energy-momentum density* as depicted by an "absolute field stress-energy tensor" $t_A^{\alpha\beta}$. Now natural observers do not detect the presence of $t_A^{\alpha\beta}$, for the following reasons. First, under satisfaction of the EEP in micro free-fall frames, natural observers do not detect the presence of $\hat{t}_A^{\mu\nu}$, only detecting the presence of the naturally observed matter SE tensor $\hat{T}_N^{\mu\nu} = \hat{T}_{N(N)}^{\mu\nu}$ as understood in general relativity. This includes no naturally perceived exchange of energy-momentum (EM) with the field and therefore $\hat{t}_A^{\mu\nu}$, since with only $\hat{T}_N^{\mu\nu}$ naturally perceived, EEP-based EM conservation is naturally perceived as $\partial_{\hat{\nu}}^{(N)} \hat{T}_{N(N)}^{\mu\nu} = 0$, so EM conservation holds "internal" to the naturally perceived matter without "outside" EM exchange. Second, the absolute field SE tensor $t_A^{\alpha\beta}$ is not present as a source in the natural field equation (8) utilized by natural observers to determine the gravitational field, so natural observers do not detect the presence of $t_A^{\alpha\beta}$ as a field source. These reasons combined implies that there is no means by which natural observers detect the presence of $t_A^{\alpha\beta}$, so *the presence of the absolute field SE tensor $t_A^{\alpha\beta}$ does not violate the equivalence principle* (either the EEP or SEP). Self-consistency is achieved by again using absolute and natural classes of observers.

The partner natural and absolute matter (plus nongravitational field) SE tensors are related by

$$T_{\alpha\,A}^{\ \ \mu} = |S^{-1}|\, T_{\alpha\,N}^{\ \ \mu}. \tag{212}$$

This relation may be obtained by starting with the partner dust SE tensors

$$T_{\alpha\,N}^{\ \ \mu} = p_\alpha^N N_N^\mu, \qquad T_{\alpha\,A}^{\ \ \mu} = p_\alpha^A N_A^\mu, \tag{213}$$

where $p_\alpha^A = p_\alpha^N$ for the momentum 1-form of a dust particle as per (186), and where the partner particle flux vectors are given in covariant form by

$$N_N^\mu = \frac{dN}{\sqrt{-g}\,d\Omega}\, dx^\mu, \qquad N_A^\mu = \frac{dN}{\sqrt{-a}\,d\Omega}\, dx^\mu. \tag{214}$$

The quantity $dN/d\Omega$ is the number of dust particles per infinitesimal 4-volume element $d\Omega = dx^0\,dx^1\,dx^2\,dx^3$. In the partner micro free-fall and absolute inertial frames for partner natural and absolute observers, the partner flux vectors both reduce to $N^\mu = (dN/dV)\,dx^\mu/dt$ where $dV = dx^1\,dx^2\,dx^3$, using $\sqrt{-\eta} = 1$ for both metrics. This yields the usual number flux definition $N^\mu = n\,dx^\mu/d\tau = n\,U^\mu$ where n is the number density in the inertial frame (in each partner case) where the dust particles are instantaneously stationary (see Schutz [18], Chap. 4), justifying (214), which is applicable in any coordinates. Then use of (214) in (213) yields (212) for partner dust SE tensors since $\sqrt{-g} = \sqrt{-a}\,|S^{-1}|$ (via (37)). For consistency with the dust SE tensors, (212) must hold relating the partner matter SE tensors for all configurations of matter and the nongravitational fields.

Applying metric products, the relation (212) becomes

$$T_A^{\alpha\beta} = |S^{-1}|\, F_{\ \mu}^{\bar\alpha}\, T_N^{\mu\beta} \tag{215}$$

in pure raised indice form. Now the natural matter SE tensor $T_N^{\alpha\beta}$ is *symmetric*. Its symmetry may be established using the EEP in micro free-fall frames, where then the *total* naturally measured SE tensor

is the natural matter SE tensor $\hat{T}^{\mu\nu}_{N(N)}$, allowing use of physical arguments for the total SE tensor to establish its symmetry, such as in Schutz [18] (Chap. 4). However, the right-hand side of (215) is not generally symmetric even though $T^{\alpha\beta}_N$ is, so *the absolute matter SE tensor* $T^{\alpha\beta}_A$ *is not generally symmetric.* But for absolute observers, the *total* SE tensor is

$$E^{\alpha\beta}_A \equiv T^{\alpha\beta}_A + t^{\alpha\beta}_A, \tag{216}$$

which is the sum of the absolute matter and gravitational field SE tensors, both of which are perceived by absolute observers in their preferred absolute inertial frames. The total SE tensor based physical arguments discussed above may similarly be applied to establish the symmetry of $\check{E}^{\mu\nu}_{A(A)}$ as absolutely measured in absolute inertial frames, yielding a *symmetric* "absolute total SE tensor" $E^{\alpha\beta}_A$ in any coordinates. Since $T^{\alpha\beta}_A$ is not symmetric, then via (216) with $E^{\alpha\beta}_A$ symmetric, *the absolute gravitational field SE tensor* $t^{\alpha\beta}_A$ *is also not symmetric.* But the lack of symmetry for $T^{\alpha\beta}_A$ and $t^{\alpha\beta}_A$ individually is not an issue, as only a *total* SE tensor, such as $T^{\alpha\beta}_N$ or $E^{\alpha\beta}_A$, must be symmetric to satisfy physical constraints as discussed in Schutz.

Since it is field dependent, the absolute total SE tensor $E^{\alpha\beta}_A$ is not known a priori for most gravitational systems. To obtain its value, the absolute field equation $H^{\alpha\beta}_A[w;a] = 8\pi E^{\alpha\beta}_A$ (9) may be applied given the potential solution $w^\alpha{}_\mu$ from the natural field equation $H^{\alpha\beta}_N[w;a] = 8\pi T^{\alpha\beta}_N$ (8). The value of the field SE tensor $t^{\alpha\beta}_A$ may then be obtained using (215) and (216) given the known natural matter SE tensor $T^{\alpha\beta}_N$. Alternately, using both the absolute and natural field equations combined with (212) and (216), $t_\alpha{}^\mu{}_A$ may be given by

$$t_\alpha{}^\mu{}_A = \frac{1}{8\pi}\left\{H_\alpha{}^\mu{}_A[w;a] - |S^{-1}|H_\alpha{}^\mu{}_N[w;a]\right\}, \tag{217}$$

providing an expression for the absolute field SE tensor in *pure field terms.*

Partner statements of energy-momentum conservation are

$$\nabla^N_\beta T^{\alpha\beta}_N = 0, \qquad \nabla^A_\beta E^{\alpha\beta}_A = 0. \tag{218}$$

The natural statement is the usual statement obtainable from required naturally measured total EM conservation $\hat{\nabla}^{N(N)}_\nu \hat{T}^{\mu\nu}_{N(N)} = \partial^{(N)}_{\hat{\nu}}\hat{T}^{\mu\nu}_{N(N)} = 0$ in micro free-fall frames under EEP satisfaction. The absolute statement is obtainable from required absolutely measured total EM conservation $\check{\nabla}^{A(A)}_\nu \check{E}^{\mu\nu}_{A(A)} = \partial^{(A)}_{\check{\nu}}\check{E}^{\mu\nu}_{A(A)} = 0$ in absolute inertial frames. The partner statements (218) establish that energy-momentum conservation holds locally in absolute flat spacetime for all matter and fields combined including the gravitational field, while also satisfying the equivalence principle, which forbids the inclusion of a gravitational field EM density in local EM conservation, with this self-consistency obtained due to respective use of the absolute and natural classes of observers. There is no energy-momentum conservation requirement for the individual absolute matter and field SE tensors $T^{\alpha\beta}_A$ and $t^{\alpha\beta}_A$, so energy-momentum may be *exchanged* between $T^{\alpha\beta}_A$ and $t^{\alpha\beta}_A$ as a system evolves, as is typical for components making up a total SE tensor ($E^{\alpha\beta}_A$). As understood from general relativity, $\nabla^N_\beta T^{\alpha\beta}_N = 0$ may not be integrated in general, so global natural matter energy-momentum is not generally conserved. However, utilizing any absolute inertial frame, local conservation $\partial_{\check{\nu}}\check{E}^{\mu\nu}_A = 0$ may be integrated to yield global absolute total EM conservation $d\check{P}^\alpha_A/d\check{t} = 0$ for any system, where $\check{P}^\alpha_A \equiv \int \check{E}^{\alpha 0}_A d\check{V}$ over all space is the total 4-momentum (with all emitted gravity waves included in the integral).

As has been demonstrated by multiple examples, self-consistency between formulation that results due to the absolute spacetime postulate, and formulation that results due to the equivalence principle postulate, is achieved through respective use of the absolute and natural classes of observers. The examples selected are ones that are typically examined in gravitational theory to establish in-consistencies between use of absolute flat spacetime and use of the equivalence principle, including

the Schild argument evaluated previously. It is through the emergence of universal gravity shifts that must exist when both postulates are assumed, combined with the recognition of absolute and natural observers utilizing shift-corrected and raw gravity shifted instruments respectively, that such potential inconsistencies are resolved (as discussed in the summary). Through the use of partner relations between partner quantities contained in partner formulations, *partner physical laws are equivalent*, establishing their self-consistency. The apparently "diametrically opposed" worldviews that result when assuming the absolute spacetime postulate, while also adhering to the equivalence principle, are resolved when recognized as the *equivalent worldviews* of the absolute and natural classes of observers. It is concluded that *gravity shift theory is a self-consistent theory of gravitation*, even though it rigidly adheres to both the absolute flat spacetime postulate and the strong equivalence principle.

6. The Natural Field Equation

6.1. General Form

As discussed previously, the natural field equation is utilized by natural observers to model gravitational systems. For predictive success, the NFE is assumed to yield in the linearized case the same observed natural metric $g_{\alpha\beta}$ as predicted by the linearized Einstein equation, as well as yield the observed post-Newtonian natural metric, which is also predicted by Einstein's equation. Einstein's equation $G^{\alpha\beta}[g] = 8\pi T_N^{\alpha\beta}$ is a relativistic tensor extension of Poisson's equation $\nabla^2 \phi = 4\pi \rho_N$ from Newtonian gravitation, where the symmetric natural matter SE tensor $T_N^{\alpha\beta}$ is the "most straightforward" relativistic tensor extension of the natural mass density ρ_N (see Schutz [18], Chap. 8, for a justification utilizing commonly accepted arguments). Similarly extending from Poisson's equation, and with the above-stated required agreements with Einstein's equation, the relativistic tensor NFE is therefore assumed to take the form $H^{\alpha\beta} = 8\pi T_N^{\alpha\beta}$, where $H^{\alpha\beta}$ is the symmetric natural field tensor functionally dependent on gravitational field quantities only. As established above, the field equation operand contained in $H^{\alpha\beta}$ must consist of the potential tensor $w^{\alpha}{}_{\mu}$, as required to satisfy the overlap restriction placed on gravity shifting, which bars forbidden matter and temporal singularities from occurring in absolute flat spacetime. All gravitational field quantities in GS theory ($S^{\alpha}{}_{\bar{\mu}}$, $g_{\alpha\beta}$, etc.) may be constructed using the potential $w^{\alpha}{}_{\mu}$ and the absolute metric $a_{\mu\nu}$. The most general form for $H^{\alpha\beta}$ is therefore $H^{\alpha\beta}[w; a]$, indicating a functional dependence on $w^{\alpha}{}_{\mu}$ and $a_{\mu\nu}$ with $w^{\alpha}{}_{\mu}$ as the operand. The resulting natural field equation takes the general form

$$H^{\alpha\beta}[w; a] = 8\pi\, T^{\alpha\beta}, \tag{219}$$

where the natural labeling for $T_N^{\alpha\beta}$ has been dropped for convenience (establishing (8)).

As discussed in Section 4.7, when background curvature effects may be considered negligible for a local system surrounded by a background system, the NFE in the actual case must be able to be put into a form that *approximates* its morphed form, so that the morphed form may be considered to be the approximation obtained from NFE use in the actual case when background curvature effects are completely neglected as an approximating assumption. This "morph consistency requirement" is applied here to significantly limit the form of the natural field equation.

To begin with, the natural matter SE tensor may be given in functional form by $T^{\alpha\beta} = T^{\alpha\beta}[q_\lambda, g]$, depicting native natural matter and nongravitational field quantities $q_\lambda = q_\lambda^N$ universally coupled to the natural metric $g_{\alpha\beta}$. The quantities q_λ are gravity shifted quantities in the actual and morphed cases. The general form (219) of the NFE may therefore be given by

$$H^{\alpha\beta}[w; a] = 8\pi\, T^{\alpha\beta}[q_\lambda, g] \tag{220}$$

to indicate this universal coupling. Now as stated by $Z_M^{\alpha\beta} \approx Z^{\alpha\beta}$ (172), when background curvature effects may be considered negligible, all actual case quantities $Z^{\alpha\beta}$ subject to gravity shifting approximate

their morph case values $Z^{\alpha\beta}_M$, with quantities explicitly containing the shift tensors $S^{\alpha}_{\bar{\mu}}$ or potentials w^{α}_{μ} being the exceptions due to (171) holding. Then

$$q_\lambda \approx q^M_\lambda \tag{221}$$

for the native natural matter and nongravitational field quantities, and $g_{\alpha\beta} \approx g^M_{\alpha\beta}$ for the natural metrics as per (173). Substituting these approximations into the matter SE tensor function $T^{\alpha\beta}[q_\lambda, g]$ yields

$$T^{\alpha\beta}[q_\lambda, g] \approx T^{\alpha\beta}_M[q^M_\lambda, g^M] \tag{222}$$

for the actual and morph case SE tensors. Note that in (222), the "M" label has been added to $T^{\alpha\beta}$ in the function $T^{\alpha\beta}[q^M_\lambda, g^M]$ to clarify that its morphed form $T^{\alpha\beta}_M = T^{\alpha\beta}[q^M_\lambda, g^M]$ has been obtained. This practice is continued below for all functions of morphed quantities. Based on the stated approximations, if the NFE had the form $H^{\alpha\beta}[g] = 8\pi T^{\alpha\beta}[q_\lambda, g]$, the actual case field equation $H^{\alpha\beta}[g] = 8\pi T^{\alpha\beta}[q_\lambda, g]$ would indeed approximate the morphed field equation $H^{\alpha\beta}_M[g^M] = 8\pi T^{\alpha\beta}_M[q^M_\lambda, g^M]$ (where $H^{\alpha\beta}_M[g^M]$ is $H^{\alpha\beta}[g^M]$ following the above practice), satisfying the morph consistency requirement. The Einstein equation $G^{\alpha\beta}[g] = 8\pi T^{\alpha\beta}[q_\lambda, g]$ is an example.

Consider a NFE form where the absolute metric $a_{\mu\nu}$ is explicitly expressed in $H^{\alpha\beta}[w; a]$ *outside* the natural metric $g_{\alpha\beta} = a_{\alpha\mu}\exp(-2w^\mu{}_\beta) = g_{\alpha\beta}(w, a)$. When given in the Riemann ICs of a background system free-fall frame, the actual case $\hat{H}^{\alpha\beta}[\hat{w}; \hat{a}]$ would contain explicit $\hat{a}_{\mu\nu}$ terms that are background system dependent, whereas use of the absolute metric replacement method yields $\hat{g}^{MB}_{\mu\nu} = \eta_{\mu\nu}$ in place of $\hat{a}_{\mu\nu}$ for the morphed case $\hat{H}^{\alpha\beta}_M[\hat{w}^M; \hat{g}^{MB}]$. Therefore, $\hat{H}^{\alpha\beta}[\hat{w}; \hat{a}]$ does not generally approximate $\hat{H}^{\alpha\beta}_M[\hat{w}^M; \hat{g}^{MB}]$ when $a_{\mu\nu}$ is explicitly expressed in $H^{\alpha\beta}[w; a]$, failing to satisfy the morph consistency requirement. So any $a_{\mu\nu}$ terms in $H^{\alpha\beta}[w; a]$ are assumed to be *implicitly* contained in $g_{\alpha\beta} = g_{\alpha\beta}(w, a)$ terms only, yielding $H^{\alpha\beta}[w; a] = H^{\alpha\beta}[w; g(w, a)]$. Utilizing (220), the general form for the natural field equation becomes

$$H^{\alpha\beta}[w; g(w, a)] = 8\pi\, T^{\alpha\beta}[q_\lambda, g(w, a)]. \tag{223}$$

This is *as expected* for the field equation utilized by natural observers, since the natural metric is used exclusively then to raise/lower indices and to form covariant derivatives with, where again the natural metric $g_{\alpha\beta} = g_{\alpha\beta}(w, a)$ is universally coupled to the native natural matter and nongravitational field quantities $q_\lambda = q^N_\lambda$ on the right. The morphed general form for the natural field equation is therefore

$$H^{\alpha\beta}_M[w^M; g^M(w^M, g^{MB})] = 8\pi\, T^{\alpha\beta}_M[q^M_\lambda, g^M(w^M, g^{MB})] \quad \text{with } w^{\alpha M}_{\mu} \to 0, \tag{224}$$

where $H^{\alpha\beta}_M[w^M; g^M]$ is $H^{\alpha\beta}[w^M; g^M]$ and $g^M_{\alpha\beta}(w^M, g^{MB})$ is $g_{\alpha\beta}(w^M, g^{MB})$ following the above "M" labeling practice. Note the zero-valued boundary condition for the morphed local system potential $w^{\alpha M}_{\mu}$. Both (223) and (224) provide *general forms* for the actual and morphed form natural field equation suitable for subsequent formulation.

The natural field tensor, $H^{\alpha\beta}[w; g(w, a)]$, in (223), consists of a sum of terms (or a single term) where each term is a product of a number of $w^\alpha{}_\mu$ and $g_{\alpha\beta}$ and/or their partial derivatives, generically represented by

$$H^{\alpha\beta}[w; g] = \sum_n f_n(w) h_n(g). \tag{225}$$

This representation includes terms that do not contain $w^\alpha{}_\mu$ or alternately $g_{\alpha\beta}$ entries, so a term could be given by $h_n(g)$ or $f_n(w)$ respectively. With $g_{\alpha\beta} \approx g^M_{\alpha\beta}$ (173) and $T^{\alpha\beta}[q_\lambda, g] \approx T^{\alpha\beta}_M[q^M_\lambda, g^M]$ (222), and $H^{\alpha\beta}[w; g(w, a)]$ given by (225), the only reason why the form (223) would not satisfy the morph consistency requirement is $w^\alpha{}_\mu \not\approx w^{\alpha M}_{\mu}$ as per (171). Recall that this non-approximation is due to the actual case $w^\alpha{}_\mu$ being the potential for the total system consisting of the local and background system combined, whereas the morphed-case $w^{\alpha M}_{\mu}$ is the potential for the morphed local system only, with

$w^{\alpha}{}_{\mu} \not\approx w^{\alpha M}{}_{\mu}$ formally established by utilizing $w^{\alpha}{}_{\mu} = w^{\alpha B}{}_{\mu}$ and $w^{\alpha M}{}_{\mu} = 0$ at the asymptotic boundary (region) of the local system.

Consider an $H^{\alpha\beta}[w; g(w, a)]$ form where any explicit instances of the potential $w^{\alpha}{}_{\mu}$ (i.e., outside of $g_{\alpha\beta}(w, a)$) are *linear* in the potential only, so any term $f_n(w)h_n(g)$ in (225) either has an $f_n(w)$ that is linear in $w^{\alpha}{}_{\mu}$, which may be partially differentiated, or does not contain a $w^{\alpha}{}_{\mu}$ entry so it consists of $h_n(g)$ only. Then $H^{\alpha\beta}$ may be given by

$$H^{\alpha\beta}[w; g] = H_g^{\alpha\beta}[g] + H_w^{\alpha\beta}[w; g], \tag{226}$$

where $H_g^{\alpha\beta}[g]$ is the sum of the $h_n(g)$ only terms, and $H_w^{\alpha\beta}[w; g]$ is the sum of the $f_n(w)h_n(g)$ terms linear in $w^{\alpha}{}_{\mu}$. The actual case NFE (223) may therefore be given by

$$H^{\alpha\beta}[w; g] = H_g^{\alpha\beta}[g] + H_w^{\alpha\beta}[w; g] = 8\pi\, T^{\alpha\beta}[q_{\lambda}, g] \quad \text{with } w^{\alpha}{}_{\mu} \to w^{\alpha B}{}_{\mu}, \tag{227}$$

where the condition on the right is the asymptotic boundary condition for the total system potential at the local system boundary. The "linear potential" form (226) for $H^{\alpha\beta}$ allows the actual case potential $w^{\alpha}{}_{\mu}$ to be "partitioned" into the sum

$$w^{\alpha}{}_{\mu} = w^{\alpha P}{}_{\mu} + w^{\alpha H}{}_{\mu}, \tag{228}$$

where $w^{\alpha P}{}_{\mu}$ is the solution to the "particular equation"

$$H^{\alpha\beta}[w^P; g] = H_g^{\alpha\beta}[g] + H_w^{\alpha\beta}[w^P; g] = 8\pi\, T^{\alpha\beta}[q_{\lambda}, g] \quad \text{with } w^{\alpha P}{}_{\mu} \to 0, \tag{229}$$

and $w^{\alpha H}{}_{\mu}$ is the solution to the "harmonic equation"

$$H_w^{\alpha\beta}[w^H; g] = 0 \quad \text{with } w^{\alpha H}{}_{\mu} \to w^{\alpha B}{}_{\mu}. \tag{230}$$

Utilizing (228), the sum of the particular and harmonic equations is the actual case (227), including the sum of their boundary conditions being the actual case boundary condition. As a result, starting with the actual case NFE (227), the harmonic equation (230) may be *subtracted*, yielding the particular equation (229).

Now the linear potential form (226) for $H^{\alpha\beta}$ also allows the morphed NFE (224) to be given in partitioned form as provided by

$$H_M^{\alpha\beta}[w^M; g^M] = H_g^{\alpha\beta}[g^M] + H_w^{\alpha\beta}[w^M; g^M] = 8\pi\, T_M^{\alpha\beta}[q_{\lambda}^M, g^M] \quad \text{with } w^{\alpha M}{}_{\mu} \to 0. \tag{231}$$

Therefore, the particular equation (229) for the actual case NFE has the *same form* as the morphed NFE (231), including the zero-valued boundary condition. Then with $g_{\alpha\beta} \approx g_{\alpha\beta}^M$ (173) and $T^{\alpha\beta}[q_{\lambda}, g] \approx T_M^{\alpha\beta}[q_{\lambda}^M, g^M]$ (222), the particular equation *approximates* the morphed equation with a solution $w^{\alpha P}{}_{\mu}$ that approximates $w^{\alpha M}{}_{\mu}$. Therefore, the morphed equation (231) may be considered to be the approximation obtained from the particular equation (229) when background curvature effects are completely neglected as an approximating assumption. Concluding, use of the linear potential form (226) for $H^{\alpha\beta}$ yields a natural field equation that *satisfies the morph consistency requirement*, since the morphed equation (231) may be considered to be the approximation obtained from the actual case NFE (227)—via the particular equation (229) obtained when the harmonic equation (230) is subtracted—when background curvature effects are completely neglected as an approximating assumption.

If the functions $f_n(w)$ for the $H_w^{\alpha\beta}$ terms $f_n(w)h_n(g)$ in (225) include various *non-linear* products of $w^{\alpha}{}_{\mu}$ or its derivatives, then the above "partition method" cannot be applied in general, so reduction of the NFE (223) in the actual case to a particular equation (229) approximating the morphed equation, (224), is generally not obtainable. A key property of the particular equation is its zero-valued boundary condition $w^{\alpha P}{}_{\mu} \to 0$, where with $T^{\alpha\beta} \to 0$, and $\hat{g}_{\alpha\beta} \to \eta_{\alpha\beta}$ in the Riemann ICs of background system

free-fall frames, the boundary conditions for the particular equation (229) have *no background system dependence* in agreement with the SEP. Without availability of the partition method to obtain the particular equation, it is not expected that the actual case NFE (223) would be reducible to a form where its potential solution would have a zero-valued boundary condition, with instead a non-zero boundary condition obtained that is background system dependent, violating the SEP, and not satisfying the morph consistency requirement since the non-zero boundary condition is inconsistent with the zero-valued morph boundary condition $w^{\alpha M}_{\mu} \to 0$. For this reason, it is concluded that *the natural field equation must be of a form that the partition method is applicable*, yielding the particular equation (229) in the actual case so that the morph consistency requirement is satisfied, as well as boundary conditions being obtained that are not background dependent in agreement with the SEP. With the partition method required, it is concluded that the natural field equation must be given by the general form (223) where any explicit instances of the potential w^{α}_{μ} (i.e., outside of $g_{\alpha\beta}(w, a)$) in the natural field tensor, $H^{\alpha\beta}[w; g(w, a)]$, are indeed *linear* in the potential only.

In order to satisfy the overlap restriction for gravity shifting, the form of the NFE must be such as to yield *real-valued* potential solutions w^{α}_{μ}. Due to the linearity of the explicitly given potential in $H^{\alpha\beta}[w; g(w, a)]$, use of a *given* real-valued natural metric $g_{\alpha\beta}$ in a NFE of the form (223) will yield real-valued potential solutions. If instead there were non-linear explicit potential instances, the potential solutions w^{α}_{μ} would not be expected to be real valued over the entire range of possible systems that could be modelled. Therefore, application of the morph consistency requirement prevents this possibility due to restricting explicit potential instances in $H^{\alpha\beta}[w; g(w, a)]$ to being linear. Even with this the case, at first sight, there appears to be the possibility that the non-linear use of w^{α}_{μ} as contained in the metric relation $g_{\alpha\beta} = a_{\alpha\mu} \exp(-2w^{\mu}_{\beta})$ utilized in the NFE, (223), may result in *complex-valued* w^{α}_{μ} solutions. However, use of an iterative technique shows that w^{α}_{μ} is real valued as follows. Starting with a given Minkowski metric valued natural metric $g_{\alpha\beta}$ in global ICs (equal then to the absolute metric $a_{\alpha\beta} = \eta_{\alpha\beta}$), the NFE (223) will yield a real-valued w^{α}_{μ} solution due to the linearity of the explicitly given potential in $H^{\alpha\beta}[w; g(w, a)]$. This solution is used in $g_{\alpha\beta} = a_{\alpha\mu} \exp(-2w^{\mu}_{\beta})$ to yield a real-valued natural metric, which is then reapplied in (223) to yield another real-valued w^{α}_{μ} solution, and so on. As it is expected that this iterative process will converge to the exact NFE solution, then the exact w^{α}_{μ} solution is real valued since it was so for every iteration. Coordinate transform again yields a real-valued w^{α}_{μ} solution for the natural field equation as given in any coordinates.

Summarizing, application of the morph consistency requirement results in the natural field tensor $H^{\alpha\beta}[w; a]$ being constrained to a form $H^{\alpha\beta}[w; g(w, a)]$ where any explicit instances of the potential w^{α}_{μ} are linear in the potential only, resulting in $H^{\alpha\beta}$ being given by $H^{\alpha\beta}_{g}[g] + H^{\alpha\beta}_{w}[w; g]$ (226) where $H^{\alpha\beta}_{w}[w; g]$ is a sum of terms $f_n(w)h_n(g)$ with $f_n(w)$ linear in w^{α}_{μ}. The linearity of the explicit potential instances in $H^{\alpha\beta}[w; g(w, a)]$ yields real-valued potential solutions w^{α}_{μ} for the NFE (223), as necessary in order to satisfy the gravity shifting overlap restriction. The initial $H^{\alpha\beta}[w; a]$ could in theory be any symmetric tensor function formed from w^{α}_{μ} and $a_{\mu\nu}$ such that real-valued solutions w^{α}_{μ} are obtained from the NFE as given by (220), resulting in an onerous number of possibilities for $H^{\alpha\beta}$. Application of the morph consistency requirement has severely limited the form for $H^{\alpha\beta}[w; a]$, resulting in a manageable number of possible terms contributing to $H^{\alpha\beta}$, as shown below.

The morph consistency requirement also applies for the natural nongravitational physics laws under the influence of gravitation. However, all natural nongravitational physics laws utilize only the natural metric $g_{\alpha\beta}$ universally coupled to the native natural matter and nongravitational field quantities $q_{\lambda} = q^{N}_{\lambda}$, so any actual case form approximates its morphed form since $q_{\lambda} \approx q^{M}_{\lambda}$ (221) and $g_{\alpha\beta} \approx g^{M}_{\alpha\beta}$ (173). Therefore, *all natural nongravitational physics laws satisfy the morph consistency requirement without further constraining their forms*.

A commonly accepted constraint on the field tensor $H^{\alpha\beta}$ is that it consists of "$N = 2$ terms," where N is the total number of derivatives taken on the field quantities in each term. This is established in Weinberg [22] (Chap. 7) using scaling arguments, with $N = 2$ coming about for Einstein's equation via extension from Poisson's equation $\nabla^2\phi = 4\pi\rho_N$ for Newtonian gravitation. Therefore, the $H^{\alpha\beta}[w; g]$

terms $f_n(w)h_n(g)$ (225) in the NFE (223) are assumed to be $N = 2$ terms due to its extension from Poisson's equation and the above-stated agreements with Einstein's equation. Using "w" and "g" to generically represent any raised/lowered index forms for $w^\alpha{}_\mu$ and $g_{\alpha\beta}$ as well as the scalar $w^\alpha{}_\alpha$, and using "g^m" to represent any product of $g_{\alpha\beta}$ with itself (including $g^0 \equiv 1$), the list of possible $N = 2$ $H^{\alpha\beta}[w; g]$ terms $f_n(w)h_n(g)$ where $f_n(w)$ is at highest order linear in $w^\alpha{}_\mu$ is as follows: $g_{,\alpha\beta}\, g^m$, $g_{,\alpha}\, g_{,\beta}\, g^m$, $w\, g_{,\alpha\beta}\, g^m$, $w\, g_{,\alpha}\, g_{,\beta}\, g^m$, $w_{,\alpha}\, g_{,\beta}\, g^m$, and $w_{,\alpha\beta}\, g^m$. Each one of these terms generically represents multiple possible terms in various index configurations of the contained $w^\alpha{}_\mu$ and $g_{\alpha\beta}$, with each configuration required to have free raised indices α and β to be an $H^{\alpha\beta}$ contributor.

6.2. Lagrangian Formulation

As is commonly assumed for field equations in gravitational physics, the natural field equation is assumed to result from use of a Lagrangian-based formulation. In particular, in order to satisfy the requirement that the NFE yields the observed post-Newtonian natural metric, a Lagrangian formulation is assumed so that as shown in Will [1] (Chap. 4), the predicted PN metric will have the correct observed "conservation" parameterized post-Newtonian (PPN) parameters $\alpha_3 \equiv \zeta_1 \equiv \zeta_2 \equiv \zeta_3 \equiv \zeta_4 \equiv 0$ in the standard gauge. A suitable background for the Lagrangian formulation made here is provided in Carroll [24] (Chaps. 1 & 4), with the NFE formulation patterned similarly. As such, the "natural action" is defined by

$$S_N \equiv \frac{1}{16\pi}S_H + S_M, \tag{232}$$

where $S_M = S_M[q_\lambda, g]$ is the matter action from general relativity for the naturally observed native matter and nongravitational fields q_λ universally coupled to the natural metric, and S_H the "(natural) field action" defined by

$$S_H \equiv \int \sqrt{-g}\, L_H\, \mathrm{d}^4x \tag{233}$$

with L_H the "natural field Lagrangian." With the potential $w^\alpha{}_\mu$ the field operand for the NFE $H^{\alpha\beta}[w; g(w, a)] = 8\pi T^{\alpha\beta}$ (223), the field variation applied to the natural action S_N is the potential variation $\delta w^\alpha{}_\mu$. Combining the field variation $\delta w^\alpha{}_\mu$ with the variation δq_λ for each native nongravitational quantity, q_λ, yields the complete natural action variation

$$\delta S_N = \int \mathrm{d}^4x \frac{\delta S_N}{\delta w^\alpha{}_\mu}\delta w^\alpha{}_\mu + \int \mathrm{d}^4x \frac{\delta S_N}{\delta q_\lambda}\delta q_\lambda. \tag{234}$$

Applying the principle of least action, the action variation δS_N is set to zero, yielding the natural field equation as the Euler-Lagrange (EL) equation

$$\frac{\delta S_N}{\delta w^\alpha{}_\mu} = 0 \tag{235}$$

giving the critical point for the natural action S_N under the field variation $\delta w^\alpha{}_\mu$. Similarly, the critical point for the natural action S_N under the variation δq_λ for each native natural nongravitational quantity, $q_\lambda = q_\lambda^N$, is the EL equation

$$\frac{\delta S_M[q_\lambda, g]}{\delta q_\lambda} = 0 \tag{236}$$

giving the natural equation of motion for q_λ, where $\delta S_N/\delta q_\lambda = \delta S_M[q_\lambda, g]/\delta q_\lambda$ was utilized since the field action S_H has no q_λ quantity dependence.

Using (233) to form S_H, the natural action functional derivative in the EL equation (235) is given by

$$\frac{\delta S_N}{\delta w^\alpha{}_\mu} = \frac{1}{16\pi}\frac{\delta S_H}{\delta w^\alpha{}_\mu} + \frac{\delta S_M}{\delta g^{\sigma\rho}}\frac{\partial g^{\sigma\rho}(w, a)}{\partial w^\alpha{}_\mu}, \tag{237}$$

having applied the chain rule to the matter action with functional dependence $S_M[q_\lambda, g(w, a)]$ due to universal coupling of the natural metric $g_{\alpha\beta}$ to the native natural matter and nongravitational field

quantities q_λ. The field tensor $H^{\alpha\beta}$ is obtained from the field action derivative $\delta S_H / \delta w^\alpha{}_\mu$, so the field action has the functional dependence $S_H[w; g(w, a)]$ as required to yield the natural field tensor with functional dependence $H^{\alpha\beta}[w; g(w, a)]$. Then from (233) the natural field Lagrangian similarly has the functional dependence $L_H[w; g(w, a)]$. Using the functional dependence $S_H[w; g(w, a)]$ in (237), the natural action functional derivative becomes

$$\frac{\delta S_N}{\delta w^\alpha{}_\mu} = \frac{1}{16\pi} \frac{\delta S_H[w; g]}{\delta w^\alpha{}_\mu} + \left[\frac{1}{16\pi} \frac{\delta S_H[w; g]}{\delta g^{\sigma\rho}} + \frac{\delta S_M}{\delta g^{\sigma\rho}} \right] \frac{\partial g^{\sigma\rho}(w, a)}{\partial w^\alpha{}_\mu}, \tag{238}$$

with the "partial derivative rule" used for $\delta S_H[w; g] / \delta w^\alpha{}_\mu$ where the functional derivative is for the explicit $w^\alpha{}_\mu$ terms in $S_H[w; g]$ outside of the natural metric, since the implicit functional dependence of $w^\alpha{}_\mu$ in $g_{\alpha\beta}$ is *not indicated* in the utilized functional form $S_H[w; g]$. The commonly utilized relation (see Carroll)

$$T_{\alpha\beta}[q_\lambda, g] = -2 \frac{1}{\sqrt{-g}} \frac{\delta S_M[q_\lambda, g]}{\delta g^{\alpha\beta}} \tag{239}$$

gives the natural matter SE tensor via the metric functional derivative of the matter action. Using (238) in (235), and applying (239), yields the EL natural field equation

$$\frac{1}{\sqrt{-g}} \frac{\delta S_H[w; g]}{\delta w^\alpha{}_\mu} + \left[\frac{1}{\sqrt{-g}} \frac{\delta S_H[w; g]}{\delta g^{\sigma\rho}} - 8\pi\, T_{\sigma\rho} \right] \frac{\partial g^{\sigma\rho}(w, a)}{\partial w^\alpha{}_\mu} = 0. \tag{240}$$

To evaluate $\partial g^{\sigma\rho}(w, a) / \partial w^\alpha{}_\mu$ where $g^{\sigma\rho}(w, a) = \exp(2w^\sigma{}_\nu)\, a^{\nu\rho}$ (82), a "direct" calculation applying the functional derivative $\partial w^\beta{}_\lambda / \partial w^\alpha{}_\mu = \delta^\beta{}_\alpha \delta^\mu{}_\lambda$ to the $w^\beta{}_\lambda$ terms in the expansion $\exp(2w^\sigma{}_\nu)$, given by (64), does not yield a compact closed form for $\partial g^{\sigma\rho}(w, a) / \partial w^\alpha{}_\mu$. However, use of the action functional derivative $\delta S_N / \delta w^\alpha{}_\mu$, as given by (238), in the action variation (234), results in $\partial g^{\sigma\rho}(w, a) / \partial w^\alpha{}_\mu$ being multiplied by the variation $\delta w^\alpha{}_\mu$ when forming the EL equation (235). Using the relation

$$\frac{\partial g^{\sigma\rho}(w, a)}{\partial w^\alpha{}_\mu}\, \delta w^\alpha{}_\mu = \delta_w\, g^{\sigma\rho}(w, a), \tag{241}$$

where $\delta_w\, g^{\sigma\rho}(w, a)$ is defined as the variation in $g^{\sigma\rho}(w, a)$ induced by a variation in $w^\alpha{}_\mu$, enables $\partial g^{\sigma\rho}(w, a) / \partial w^\alpha{}_\mu$ to be given in closed form, as follows. The matrix form of (82) for the inverse metric is $g^{-1}(w, a) = \exp(2w)\, a^{-1}$, yielding $\delta_w\, g^{-1}(w, a) = [\delta_w \exp(2w)]\, a^{-1}$ for $\delta_w\, g^{\sigma\rho}(w, a)$ in matrix form. As shown below, the potential tensor $w^\alpha{}_\mu$ and its variation $\delta w^\alpha{}_\mu$ *commute*, as expressed by $w\, \delta w = \delta w\, w$ in matrix from. Utilizing this commutivity when varying the expansion (63) for $\exp(2w)$, it can be readily shown that $\delta_w \exp(2w) = 2\, \delta w \exp(2w)$ (as if w were a scalar). Therefore, $\delta_w\, g^{-1}(w, a) = 2\, \delta w \exp(2w)\, a^{-1}$, or compactly $\delta_w\, g^{-1}(w, a) = 2\, \delta w\, g^{-1}$. In tensor form this becomes $\delta_w\, g^{\sigma\rho}(w, a) = 2\, \delta w^{(\sigma}{}_\mu\, g^{\mu\rho)}$, where the right side has been symmetrized since $g^{\sigma\rho}$ on the left-hand side is symmetric. Manipulating the tensor expression yields

$$\delta_w\, g^{\sigma\rho}(w, a) = 2\, \delta^{(\sigma}{}_\alpha\, g^{\rho)\mu}\, \delta w^\alpha{}_\mu. \tag{242}$$

Using (242) to substitute for $\delta_w\, g^{\sigma\rho}(w, a)$ in (241), and with $\delta w^\alpha{}_\mu$ arbitrary, then

$$\frac{\partial g^{\sigma\rho}(w, a)}{\partial w^\alpha{}_\mu} = 2\, \delta^{(\sigma}{}_\alpha\, g^{\rho)\mu} \tag{243}$$

is yielded for $\partial g^{\sigma\rho}(w, a) / \partial w^\alpha{}_\mu$ as contained in the natural action variation (234) with $\delta S_N / \delta w^\alpha{}_\mu$ given by (238). The closed form (243) is applicable then in the EL natural field equation $\delta S_N / \delta w^\alpha{}_\mu = 0$. A key property for this form is that it has *strictly a natural metric dependence*, so it does not introduce into the NFE either an explicit potential dependence or most importantly an explicit absolute metric dependence, with such dependence implicitly contained in $g^{\alpha\beta}(w, a)$ only.

To prove that the matrices w and δw commute, consider the metric $g_{\alpha\beta} = a_{\alpha\mu} \exp(-2w^\mu{}_\beta)$ formed from the potential $w^\alpha{}_\mu$, and the inverse "varied metric" $g_\delta^{\alpha\beta} \equiv \exp(2w^{\alpha\,\delta}_\mu) a^{\mu\beta}$ formed from the "varied potential" $w^{\alpha\,\delta}_\mu \equiv w^\alpha{}_\mu + \delta w^\alpha{}_\mu$. In the Riemann ICs of any micro free-fall frame the metric $\hat{g}_{\alpha\beta}$ is the diagonal Minkowski metric $\eta_{\alpha\beta}$. The inverse varied metric $\hat{g}_\delta^{\alpha\beta}$ may be diagonalized under a Lorentz transform while $\hat{g}_{\alpha\beta}$ remains the diagonal Minkowski metric. The diagonalization of $\hat{g}_\delta^{\alpha\beta}$ is possible since any physically valid variation $\delta w^\alpha{}_\mu$ is limited to yielding a potential $w^{\alpha\,\delta}_\mu$ such that the shifted light speed c_S^δ does not exceed the absolute manifold null speed v_{Null}, thereby allowing a Lorentz transformation to diagonalize $\hat{g}_\delta^{\alpha\beta}$ (which is generally not $\eta^{\alpha\beta}$). With $\hat{g}_{\alpha\beta}$ and $\hat{g}_\delta^{\alpha\beta}$ both diagonalized, then their matrices commute, as stated by $\hat{g}\,\hat{g}_\delta^{-1} = \hat{g}_\delta^{-1}\hat{g}$. The squared shift tensor metric relations (76) and (78) in matrix form are $g = a\,F^{-1}$ and $g^{-1} = F a^{-1}$. Since the natural metric is symmetric, then $g = g^T = F^{-1\,T} a$ and $g^{-1} = g^{-1\,T} = a^{-1} F^T$, where the symmetry of the absolute metric was utilized. Using these relations for the diagonalized metric and varied metric in Riemann ICs yields $\hat{F}^{-1\,T}\hat{F}_\delta^T = \hat{F}_\delta \hat{F}^{-1}$, noting that $\hat{F}^{-1}$ and $\hat{F}_\delta$ are generally not diagonal. Then $(\hat{F}_\delta \hat{F}^{-1})^T = \hat{F}_\delta \hat{F}^{-1}$, so $\hat{F}_\delta \hat{F}^{-1}$ is a symmetric matrix. Utilizing this symmetry, it may be shown that $\hat{F}^{-1}\hat{F}_\delta = \hat{F}_\delta \hat{F}^{-1}$. This commutation statement may be given in tensor form and coordinate transformed, yielding the matrix form $F^{-1}F_\delta = F_\delta F^{-1}$ in any coordinates. Similar to (66), $\ln\{B\}\ln\{C\} = \ln\{C\}\ln\{B\}$ if the square matrices $\{B\}$ and $\{C\}$ commute, and since $F^{-1} = \exp(-2w)$ and $F_\delta = \exp(2w_\delta)$ (with w_δ the varied potential $w^{\alpha\,\delta}_\mu$ in matrix form), then $w\,w_\delta = w_\delta\,w$. Substituting $w_\delta = w + \delta w$ yields $w\,\delta w = \delta w\,w$, establishing that w and δw commute.

Using (243) in (240) yields

$$H_{\alpha\beta}[w; g(w,a)] = \frac{1}{2}\frac{1}{\sqrt{-g}}\frac{\delta S_H[w;g]}{\delta w^\alpha{}_\mu}\,g_{\mu\beta} + \frac{1}{\sqrt{-g}}\frac{\delta S_H[w;g]}{\delta g^{\alpha\beta}} = 8\pi\,T_{\alpha\beta} \tag{244}$$

for the EL natural field equation, having identified the "middle" field action S_H based term with the natural field tensor $H_{\alpha\beta}[w; g(w,a)]$ via comparison with (223) (where all indices are raised/lowered by the natural metric). Utilizing (239), (243), and (244), it may be readily shown that

$$H_\alpha{}^\mu[w; g(w,a)] = \frac{1}{2}\frac{1}{\sqrt{-g}}\frac{\delta S_H[w; g(w,a)]}{\delta w^\alpha{}_\mu}, \tag{245}$$

$$T_\alpha{}^\mu[q_\lambda, g] = -\frac{1}{\sqrt{-g}}\frac{\delta S_M[q_\lambda, g(w,a)]}{\delta w^\alpha{}_\mu}, \tag{246}$$

which when used in (235) (with S_N given by (232)) quickly yields the natural field equation $H_{\alpha\beta} = 8\pi T_{\alpha\beta}$. The origin of the field action S_H functional derivatives in (244) is via their use in the natural action S_N variation (234) where the contained action, S_H, is given by (233) utilizing the provided field Lagrangian L_H. As such, to obtain the functional derivatives within (234), the usual variational techniques are employed (as discussed in Carroll [24], Chap. 4) where integration by parts is applied, and where Stokes's theorem is applied to convert divergences to boundary surface integrals that are then dropped. The generation of the S_H functional derivatives from the provided L_H therefore proceeds "automatically" employing the usual techniques, so what remains for the natural field equation development is to establish the natural field Lagrangian L_H. Since the functional derivatives in (244) are partial derivatives where the implicit $w^\alpha{}_\mu$ dependence in $g_{\alpha\beta}(w,a)$ is ignored, the Lagrangian for this exercise need only be given using the functional dependence $L_H[w; g]$. The $w^\alpha{}_\mu$ dependence in $g_{\alpha\beta}(w,a)$ has already been accounted for when $\partial g^{\sigma\rho}(w,a)/\partial w^\alpha{}_\mu$ was determined and applied to obtain (244).

Proceeding with the determination of the natural field Lagrangian $L_H[w; g]$, similar to $H^{\alpha\beta}[w; g]$ given by (225), based on their functional dependence $L_H[w; g]$ and $S_H[w; g]$ each consist of a sum of terms of the form $f_n(w)h_n(g)$, where again the contained $w^\alpha{}_\mu$ and $g_{\alpha\beta}$ may be partially differentiated. From above, all $H^{\alpha\beta}[w; g]$ terms $f_n(w)h_n(g)$ must be $N = 2$ where $f_n(w)$ is at highest order linear in $w^\alpha{}_\mu$. These same requirements hold for the natural metric lowered field tensor $H_{\alpha\beta}[w; g]$ in (244). Utilizing

(233) in (244) to obtain $H_{\alpha\beta}[w;g]$, then employing the usual variational techniques as discussed above, all $L_H[w;g]$ terms $f_n(w)h_n(g)$ must also be $N=2$, where again $f_n(w)$ is at highest order linear in $w^{\alpha}{}_{\mu}$. Note that $\delta S_H[w;g]/\delta w^{\alpha}{}_{\mu}$ in (244), with $S_H[w;g]$ given by (233), yields $H_{\alpha\beta}[w;g]$ terms $f_n(w)h_n(g)$ that are one factor of $w^{\alpha}{}_{\mu}$ less than in the "parent" $L_H[w;g]$ terms $f_n(w)h_n(g)$. As such, if $L_H[w;g]$ terms $f_n(w)h_n(g)$ were present where $f_n(w)$ consists of *quadratic* products of $w^{\alpha}{}_{\mu}$, $H_{\alpha\beta}[w;g]$ terms $f_n(w)h_n(g)$ would be generated that are *linear* in $w^{\alpha}{}_{\mu}$, as allowed. However, $\delta S_H[w;g]/\delta g^{\alpha\beta}$ in (244) does not reduce the number of $w^{\alpha}{}_{\mu}$ products used in the parent $L_H[w;g]$ terms $f_n(w)h_n(g)$, so quadratic $w^{\alpha}{}_{\mu}$ products are *not allowed* since they would generate quadratic $w^{\alpha}{}_{\mu}$ products for the $H_{\alpha\beta}[w;g]$ terms. An exception would be a quadratic $L_H[w;g]$ term that yields a vanishing $H_{\alpha\beta}[w;g]$ contribution when $\delta S_H[w;g]/\delta g^{\alpha\beta}$ is applied, so it would contribute to $H_{\alpha\beta}[w;g]$ via $\delta S_H[w;g]/\delta w^{\alpha}{}_{\mu}$ application with the allowed linear $w^{\alpha}{}_{\mu}$ dependence, without generating a quadratic contribution. However, a check of all possible quadratic $N=2$ $L_H[w;g]$ terms shows each would yield a non-vanishing quadratic $H_{\alpha\beta}[w;g]$ contribution when $\delta S_H[w;g]/\delta g^{\alpha\beta}$ is applied, so indeed no $L_H[w;g]$ term may contain quadratic $w^{\alpha}{}_{\mu}$ products.

Using "w" and "g" to again generically represent any raised/lowered index forms for $w^{\alpha}{}_{\mu}$ and $g_{\alpha\beta}$ as well as the scalar $w^{\alpha}{}_{\alpha}$, and "g^m" to represent any product of $g_{\alpha\beta}$ with itself (including $g^0 \equiv 1$), the list of possible $N=2$ $L_H[w;g]$ terms $f_n(w)h_n(g)$ where $f_n(w)$ is at highest order linear in $w^{\alpha}{}_{\mu}$ is as follows: $g_{,\alpha\beta}\,g^m$, $g_{,\alpha}\,g_{,\beta}\,g^m$, $w\,g_{,\alpha\beta}\,g^m$, $w\,g_{,\alpha}\,g_{,\beta}\,g^m$, $w_{,\alpha}\,g_{,\beta}\,g^m$, and $w_{,\alpha\beta}\,g^m$. This is the same as the above list for $H^{\alpha\beta}[w;g]$ terms, where similarly each one of these terms generically represents multiple possible terms in various index configurations of the contained $w^{\alpha}{}_{\mu}$ and $g_{\alpha\beta}$. However, whereas for $H^{\alpha\beta}[w;g]$ where each configuration is required to have free raised indices α and β to be an $H^{\alpha\beta}$ contributor, for the scalar $L_H[w;g]$ each configuration is required to have no free indices. This requirement significantly reduces the number of possible index configurations for each term in the provided list, which is a key advantage of Lagrangian-based formulation as opposed to constructing a field tensor $H^{\alpha\beta}$ without use of a Lagrangian (as is well known in gravitational physics). The list of possible terms may be shortened further by working in the Riemann ICs of a micro free-fall frame, since then the first metric derivatives $\hat{g}_{,\alpha}$ vanish, yielding $\hat{g}_{,\alpha\beta}\,\hat{g}^m$, $\hat{w}\,\hat{g}_{,\alpha\beta}\,\hat{g}^m$, and $\hat{w}_{,\alpha\beta}\,\hat{g}^m$. This list may be converted to covariant terms as follows. First, any generic $\hat{w}_{,\alpha\beta}$ is converted to equivalently $\hat{w}_{;\alpha\beta}$ plus terms consisting of vanishing $\hat{w}_{,\alpha}\,\hat{\Gamma}$ products and/or nonvanishing $\hat{w}\,\hat{\Gamma}_{,\alpha}$ products. The generic $\hat{w}\,\hat{\Gamma}_{,\alpha}$ products are terms of the form $\hat{w}\,\hat{g}_{,\alpha\beta}\,\hat{g}^m$. Then using (118) where $\hat{g}^B_{\alpha\beta} = \hat{g}_{\alpha\beta}$ (156) for micro free-fall frames, the generic $\hat{g}_{,\alpha\beta}$ may be given by $\hat{R}$, so in any coordinates the list of generically given terms becomes the covariant $R\,g^m$, $wR\,g^m$, and $w_{;\alpha\beta}\,g^m$. Generating the *specific* $L_H[w;g]$ terms from the generic terms, the lists of every possible independent scalar index configuration for each term in the generic list is as follows: R for $R\,g^m$, wR and $w^{\alpha}{}_{\sigma}R^{\sigma}{}_{\alpha}$ for $wR\,g^m$, and $w_{;}{}^{\sigma}{}_{\sigma}$ and $w^{\alpha}{}_{\sigma;}{}^{\sigma}{}_{\alpha}$ for $w_{;\alpha\beta}\,g^m$. Note that index configurations equal to the given ones are not listed as they would be redundant, such as $w_{\alpha\sigma}R^{\sigma\alpha} = w^{\alpha}{}_{\sigma}R^{\sigma}{}_{\alpha}$. As the terms $w_{;}{}^{\sigma}{}_{\sigma}$ and $w^{\alpha}{}_{\sigma;}{}^{\sigma}{}_{\alpha}$ are *divergences*, their use in (244) (via (233)) yields vanishing S_H functional derivatives, so they may be dropped. The list of possible specific $L_H[w;g]$ terms is therefore R, wR, and $w^{\alpha}{}_{\sigma}R^{\sigma}{}_{\alpha}$.

Using the list of possible $L_H[w;g]$ terms, the most general natural field Lagrangian that may be formed is

$$L_H[w;g] = a\,R + b\,wR + c\,w^{\alpha}{}_{\sigma}R^{\sigma}{}_{\alpha}, \tag{247}$$

where a, b, and c are arbitrary constants. This is the *most general possible Lagrangian* that may be formed under the assumed requirements consisting of the well-accepted assumptions for formulation of Einstein's equation (as applied above), as well as being linear for all explicit $w^{\alpha}{}_{\mu}$ use (to yield linear $w^{\alpha}{}_{\mu}$ in $H^{\alpha\beta}$) in order to self-consistently yield SEP satisfaction under morph application, i.e., satisfy the morph consistency requirement.

Substituting (247) into (233) and applying the usual variational techniques as discussed above, the potential functional derivative of S_H in the NFE (244) is given by

$$\frac{1}{\sqrt{-g}}\frac{\delta S_H[w;g]}{\delta w^{\alpha}{}_{\mu}} = b\,\delta^{\mu}{}_{\alpha}R + c\,R^{\mu}{}_{\alpha}, \tag{248}$$

and the metric functional derivative is given by

$$\frac{1}{\sqrt{-g}} \frac{\delta S_H[w;g]}{\delta g^{\alpha\beta}} = a\, G_{\alpha\beta} + b\left[\, g_{\alpha\beta}\, w \,;^{\sigma}{}_{\sigma} - w\,;_{(\alpha\beta)} + w\, G_{\alpha\beta}\,\right]$$
$$+ c\left[\frac{1}{2}\, w_{\alpha\beta}\,;^{\sigma}{}_{\sigma} + \frac{1}{2}\, g_{\alpha\beta}\, w^{\sigma}{}_{\lambda}\,;^{\lambda}{}_{\sigma} - w^{\sigma}{}_{(\alpha\,;\beta)\,\sigma} + w^{\sigma}{}_{(\alpha} R_{\beta)\,\sigma} - \frac{1}{2}\, g_{\alpha\beta}\, w^{\sigma}{}_{\lambda} R^{\lambda}{}_{\sigma}\right]. \tag{249}$$

Using these in (244) yields the natural field equation

$$H_{\alpha\beta}[w;g(w,a)] = a\, G_{\alpha\beta} + b\left[\frac{1}{2}\, g_{\alpha\beta} R + g_{\alpha\beta}\, w\,;^{\sigma}{}_{\sigma} - w\,;_{(\alpha\beta)} + w\, G_{\alpha\beta}\right]$$
$$+ c\left[\frac{1}{2} R_{\alpha\beta} + \frac{1}{2}\, w_{\alpha\beta}\,;^{\sigma}{}_{\sigma} + \frac{1}{2}\, g_{\alpha\beta}\, w^{\sigma}{}_{\lambda}\,;^{\lambda}{}_{\sigma} - w^{\sigma}{}_{(\alpha\,;\beta)\,\sigma} + w^{\sigma}{}_{(\alpha} R_{\beta)\,\sigma} - \frac{1}{2}\, g_{\alpha\beta}\, w^{\sigma}{}_{\lambda} R^{\lambda}{}_{\sigma}\right]$$
$$= 8\pi\, T_{\alpha\beta}[q_{\lambda},g], \tag{250}$$

providing the most general possible form for the natural field tensor $H_{\alpha\beta}$ under the assumed requirements (as applied thus far). Note that any explicit potential $w^{\alpha}{}_{\mu}$ use in (250) is indeed linear, satisfying the morph consistency requirement.

6.3. Application of the Observational Requirements

As stated above, for predictive success the natural field equation $H_{\alpha\beta} = 8\pi T_{\alpha\beta}$ is assumed to yield in the linearized case the same observed natural metric $g_{\alpha\beta}$ as predicted by the linearized Einstein equation $G_{\alpha\beta} = 8\pi T_{\alpha\beta}$, referred to as the "linearized case requirement." From available general relativity formulation (such as PW [20], Chap. 5), utilizing the quantity $p_{\alpha\beta}$ as defined by $g_{\alpha\beta} = \eta_{\alpha\beta} + p_{\alpha\beta}$, the linearized Ricci tensor is

$$R_{\alpha\beta}[p;\eta] = -\frac{1}{2}(p,_{\alpha\beta} + p_{\alpha\beta},^{\sigma}{}_{\sigma} - 2\, p^{\sigma}{}_{(\alpha\,,\beta)\,\sigma}), \tag{251}$$

yielding as its trace-reverse the linearized Einstein tensor

$$G_{\alpha\beta}[p;\eta] = -\frac{1}{2}(p,_{\alpha\beta} + p_{\alpha\beta},^{\sigma}{}_{\sigma} - 2\, p^{\sigma}{}_{(\alpha\,,\beta)\,\sigma}) + \frac{1}{2}\, \eta_{\alpha\beta}\left(p,^{\sigma}{}_{\sigma} - p^{\sigma}{}_{\lambda},^{\lambda}{}_{\sigma}\right), \tag{252}$$

where all indices are raised/lowered by the Minkowski metric, so for instance, $p^{\mu}{}_{\beta} = \eta^{\mu\alpha} p_{\alpha\beta}$. The linearized Einstein equation is posed *as if* absolute flat spacetime is being utilized, with the coordinates the global ICs of an absolute inertial frame, since the Minkowski metric functioning as the "background metric" is identified with the absolute metric $a_{\alpha\beta} = \eta_{\alpha\beta}$. The "metric perturbation" $p_{\alpha\beta}$ functions as the quantity depicting the gravitational field posed in flat spacetime, with $p_{\alpha\beta}$ very small. So all second-order or higher $p_{\alpha\beta}$ product terms are dropped for linearized formulation. For comparison of linearized formulation in GS theory with that in general relativity, again the global ICs of an absolute inertial frame are utilized, with the Minkowski metric valued absolute metric $a_{\alpha\beta}$ acting as the background metric as per usual in GS theory with its postulated absolute flat spacetime. The linearized form of $g_{\alpha\beta} = a_{\alpha\mu} \exp(-2w^{\mu}{}_{\beta})$ (82) in global ICs is $g_{\alpha\beta} = \eta_{\alpha\mu}(\delta^{\mu}{}_{\beta} - 2w^{\mu}{}_{\beta})$, so again using $g_{\alpha\beta} = \eta_{\alpha\beta} + p_{\alpha\beta}$ to define $p_{\alpha\beta}$, then $p_{\alpha\beta}$ and $w_{\alpha\beta} = \eta_{\alpha\mu} w^{\mu}{}_{\beta}$ for the linearized natural metric are related via

$$w_{\alpha\beta} = -\frac{1}{2}\, p_{\alpha\beta}. \tag{253}$$

Utilizing (253), linearized formulation in GS theory based on the potential, $w^{\alpha}{}_{\mu}$, as the field quantity, may be converted to using $p_{\alpha\beta}$ as the field quantity, allowing a direct comparison with corresponding general relativity formulation using solely the natural metric $g_{\alpha\beta} = \eta_{\alpha\beta} + p_{\alpha\beta}$. In addition, purely natural metric quantities (no explicit $w^{\alpha}{}_{\mu}$) may be directly given in $p_{\alpha\beta}$ based form, so for instance,

$R_{\alpha\beta}[p;\eta]$ and $G_{\alpha\beta}[p;\eta]$ in linearized GS theory are given by (251) and (252) from linearized general relativity.

Using the above background, it can be seen that in order for the linearized NFE $H_{\alpha\beta} = 8\pi T_{\alpha\beta}$ to yield the same observed natural metric $g_{\alpha\beta} = \eta_{\alpha\beta} + p_{\alpha\beta}$ as predicted by the linearized Einstein equation $G_{\alpha\beta} = 8\pi T_{\alpha\beta}$, then when (253) is applied to convert the $w^{\alpha}{}_{\mu}$ based linearized form $H_{\alpha\beta}[w;\eta]$ for the natural field tensor to its equal valued $p_{\alpha\beta}$ based form $H_{\alpha\beta}[p;\eta]$, the equivalence

$$H_{\alpha\beta}[p;\eta] \equiv G_{\alpha\beta}[p;\eta] \tag{254}$$

must hold, meaning that *their forms must be identical*. Only if (254) holds would it be expected that the linearized NFE would yield for all possible linearized cases the same observed natural metric $g_{\alpha\beta} = \eta_{\alpha\beta} + p_{\alpha\beta}$ as predicted by the linearized Einstein equation, as required. For this reason, there *must* exist a set of constants *a*, *b*, and *c* for the natural field tensor $H_{\alpha\beta}$ given by (250) that will satisfy (254), yielding the linearized Einstein equation from the NFE when linearized. Using (251), (252), and (253), it may be readily shown that when $c = -2b$ and *a* is arbitrary, the linearized $H_{\alpha\beta}$ for the NFE (250) is given by the *proportionality* $H_{\alpha\beta}[p;\eta] \propto G_{\alpha\beta}[p;\eta]$. The reason why *a* is *arbitrary* is due to its multiplier $G_{\alpha\beta}$ in (250), so of course any *a* value yields an $H_{\alpha\beta}$ contribution proportional to $G_{\alpha\beta}$. The condition $c = -2b$ is the *unique* relation between *b* and *c* that yields $H_{\alpha\beta}[p;\eta] \propto G_{\alpha\beta}[p;\eta]$, so this relation is *required*, but they may both be multiplied by the same arbitrary constant and maintain $H_{\alpha\beta}[p;\eta]$ proportionality with $G_{\alpha\beta}[p;\eta]$. Before proceeding to obtain the equivalence (254), the condition $c = -2b$ is applied to simplify the *exact* form for $H_{\alpha\beta}[w;g]$, which is then fitted to yield the star-case metric (85) for the NFE solution prior to fitting to (254) as the last step.

The symmetric natural metric based trace-reverse for a symmetric tensor $B_{\alpha\beta}$ is defined by

$$\overline{B}_{\alpha\beta} \equiv B_{\alpha\beta} - \frac{1}{2}g_{\alpha\beta}B, \tag{255}$$

where the scalar *B* is the trace $B = g^{\sigma\alpha}B_{\alpha\sigma} = B^{\sigma}{}_{\sigma}$. A convenient property of the natural trace-reverse is $\overline{\overline{B}}_{\alpha\beta} = B_{\alpha\beta}$, so the trace-reverse is its own inverse. Utilizing the trace-reverse, substitution of the condition $c = -2b$ in (250) yields

$$H_{\alpha\beta}[w;g(w,a)] = \overline{Q}_{\alpha\beta}[w;g(w,a)] = 8\pi\, T_{\alpha\beta}[q_{\lambda},g(w,a)] \tag{256}$$

for the natural field equation, where $Q_{\alpha\beta}[w;g]$ is the symmetric "natural potential Ricci tensor" defined by

$$Q_{\alpha\beta}[w;g] \equiv aR_{\alpha\beta} + b\left[-R_{\alpha\beta} + wR_{\alpha\beta} - K_{\alpha\beta} - P_{\alpha\beta}\right], \tag{257}$$

with $K_{\alpha\beta}$ and $P_{\alpha\beta}$ the symmetric tensors

$$K_{\alpha\beta}[w;g] \equiv 2\,w^{\sigma}{}_{(\alpha}R_{\beta)\,\sigma}, \tag{258}$$

$$P_{\alpha\beta}[w;g] \equiv w_{;(\alpha\beta)} + w_{\alpha\beta}{}^{;\sigma}{}_{\sigma} - 2\,w^{\sigma}{}_{(\alpha\,;\beta)\,\sigma}. \tag{259}$$

For convenient reference, the fully determined (natural) potential Ricci tensor, established below, is given by

$$Q_{\alpha\beta}[w;g] = 2\,R_{\alpha\beta} - P_{\alpha\beta} + wR_{\alpha\beta} - K_{\alpha\beta}, \tag{260}$$

which when used in (256) provides the *final form* of the natural field equation. Even though the "potential Ricci tensor" $Q_{\alpha\beta}$ is not an actual metric Ricci tensor $R_{\alpha\beta}$, the "Ricci" nomenclature is used due to its formal use being similar to use of $R_{\alpha\beta}$, with indeed $Q_{\alpha\beta}$ substituted for $R_{\alpha\beta}$ to change from Einstein's equation $G_{\alpha\beta} = \overline{R}_{\alpha\beta} = 8\pi T_{\alpha\beta}$ to the natural field equation. Note that if $P_{\alpha\beta}[w;g]$ is linearized and then (253) is used, the equivalence

$$P_{\alpha\beta}[p;\eta] \equiv R_{\alpha\beta}[p;\eta] \tag{261}$$

is yielded, where $R_{\alpha\beta}[p;\eta]$ is the linearized metric Ricci tensor given by (251).

As stated above, for predictive success the natural field equation is assumed to yield the observed post-Newtonian natural metric, referred to as the "PN metric requirement." As such, the NFE must yield the observed star-case PN metric given by (83). Recall that in Section 3.12, the potential-form metric relation (82) was utilized to infer from (83) that (85) is the *exact* global IC given metric for the star case, so its PN approximation is the star-case PN metric. Therefore, in order that it yield the observed PN approximation (83) of the star-case metric as its solution, the NFE must yield the exact metric (85) for the star case. For this reason, there *must* exist constants a and b for the potential Ricci tensor $Q_{\alpha\beta}$, given by (257), such that when utilized in the NFE (256) will yield the metric (85) for the star case. To aid in this task, the trace-reverse is applied across the natural field equation (256) to obtain its alternate trace-reversed form

$$\overline{H}_{\alpha\beta}[w;g(w,a)] = Q_{\alpha\beta}[w;g(w,a)] = 8\pi\,\overline{T}_{\alpha\beta}[q_\lambda, g(w,a)]. \tag{262}$$

Then in the vacuum outside any source, the natural field equation yields

$$Q_{\alpha\beta}[w;g(w,a)] = 0 \quad (\text{vacuum}) \tag{263}$$

for the potential Ricci tensor, whereas in general relativity $R_{\alpha\beta}[g] = 0$ for the metric Ricci tensor as obtained from Einstein's equation. Equation (263) holds then for the field outside a star, so $Q_{\alpha\beta}[w;g(w,a)]$ must vanish for the potential field (84) giving the star-case metric (85).

To conveniently solve for the constants a and b that yield $Q_{\alpha\beta} = 0$ for the star case, the global IC given star-case potential (84) is first converted to its value

$$w^\alpha{}_\mu = \text{diag}[\,M/r, -M/r, -M/r, -M/r\,] \tag{264}$$

in the inertial spherical coordinates centered on the star, where "inertial spherical coordinates (ISCs)" are obtained by converting the Cartesian spatial coordinates of absolute inertial frame global ICs to spherical coordinates. Note that (264) has the same value as when given in global ICs due to $w^\alpha{}_\mu$ being spatially isotropic. Utilizing (264) for the potential field in the tensors making up $Q_{\alpha\beta}[w;g(w,a)]$, as given by (257), yields the following list of ISC values: $R_{rr} = -2M^2/r^4$, $wR_{rr} = 4M^3/r^5$, $K_{rr} = 4M^3/r^5$, and $P_{rr} = -4M^2/r^4$. All other components are *zero*. Using these values in (257) *uniquely* requires the condition $a = 3b$ to hold in order to yield $Q_{\alpha\beta} = 0$ for the star case. Applying the condition $a = 3b$ in (257), the potential Ricci tensor form yielding $Q_{\alpha\beta} = 0$ becomes

$$Q_{\alpha\beta}[w;g] = b\,[\,2\,R_{\alpha\beta} - P_{\alpha\beta} + wR_{\alpha\beta} - K_{\alpha\beta}\,], \tag{265}$$

resulting in the NFE (262) and therefore (256) such that the ISC-given star-case potential (264) is its solution. Conversion of (264) back to global ICs yields (84) as the NFE global IC solution for the star case, resulting in the exact natural metric (85), as required to yield the observed star-case PN metric (83).

As the last step for setting the constants in (250), the constant b is set in (265) such that the required linearized field tensor equivalence (254) is satisfied. Applying the trace-reverse to both sides of (254) yields the required equivalence

$$Q_{\alpha\beta}[p;\eta] \equiv R_{\alpha\beta}[p;\eta], \tag{266}$$

having used (262) and $G_{\alpha\beta} = \overline{R}_{\alpha\beta}$. Linearizing $Q_{\alpha\beta}[w;g]$ given by (265), and substituting (253), yields $Q_{\alpha\beta}[p;\eta] = b\,R_{\alpha\beta}[p;\eta]$, since $wR_{\alpha\beta}$ and $K_{\alpha\beta}$ (given by (258)) become second-order or higher in $p_{\alpha\beta}$ (using (251)), and since $P_{\alpha\beta}[p;\eta] \equiv R_{\alpha\beta}[p;\eta]$ (261). Then *uniquely* setting $b = 1$ yields the required equivalence (266) and therefore (254), resulting in *the linearized natural field equation* $H_{\alpha\beta}[p;\eta] = 8\pi T_{\alpha\beta}$ *being the same as the linearized Einstein equation* $G_{\alpha\beta}[p;\eta] = 8\pi T_{\alpha\beta}$ (as stated in the summary).

Concluding, as required, *for all possible linearized cases, the linearized natural field equation yields the same observed natural metric $g_{\alpha\beta} = \eta_{\alpha\beta} + p_{\alpha\beta}$ as successfully predicted by the linearized Einstein equation.*

Applying $b = 1$ in (265), the fully determined potential Ricci tensor $Q_{\alpha\beta}$ is established, as already provided by (260) where $K_{\alpha\beta}$ and $P_{\alpha\beta}$ are given by (258) and (259). Use of the fully determined $Q_{\alpha\beta}$ completes the establishment of the natural field equation as given by (256) or its trace-reverse form (262). Using the uniquely determined $c = -2b$, $a = 3b$, and $b = 1$ from above, then the initially arbitrary constants in the most general possible form of the NFE, given by (250), have been constrained to be

$$a = 3, \quad b = 1, \quad c = -2. \tag{267}$$

Therefore, the resultant natural field equation is *unique*. Note that it is *parameterless*.

When applying the PN metric requirement, only the *particular* star case was utilized when setting the constants a, b, and c, resulting in values such that the observed star-case PN metric is yielded. However, as required, the established natural field equation indeed yields the observed PN natural metric for all possible post-Newtonian cases, as follows. The Lagrangian-based NFE yields a PN metric with the correct observed conservation PPN parameters $\alpha_3 \equiv \zeta_1 \equiv \zeta_2 \equiv \zeta_3 \equiv \zeta_4 \equiv 0$ in the standard gauge, since as discussed above this property holds for Lagrangian-based field equations as shown in Will [1] (Chap. 4). That proof relies on the existence of an "energy-momentum complex" $\Theta^{\mu\nu}$ that satisfies the conservation law $\Theta^{\mu\nu}{}_{,\nu} = 0$ where $\Theta^{\mu\nu}$ reduces to $T^{\mu\nu}$ in the absence of gravitation, which is shown to exist in Lee, Lightman, and Ni (LLN) [25] for Lagrangian-based metric theories of gravity such as GS theory. The one caveat to this rule is if the theory contains absolute variables, the symmetry group applicable for them must satisfy a few basic requirements. However, these requirements are satisfied for all known metric theories, which is the case as well for the Poincaré group applicable for the absolute metric $a_{\mu\nu}$ (as the sole absolute variable) in GS theory. (These requirements are discussed in the PPN formulation section of the Supplement, where it is shown that the $a_{\mu\nu}$ Poincaré group satisfies these requirements.) It is also shown in Will [1] (Chap. 4) that if a field equation satisfies the SEP, the correct observed "preferred-frame" PPN parameters $\alpha_1 \equiv \alpha_2 \equiv \alpha_3 \equiv 0$ will be yielded in the standard gauge (note that α_3 does "double duty" as both a conservation parameter and a preferred-frame parameter), as well as the correct observed "preferred-location" parameter $\xi \equiv 0$. Therefore, the NFE yields the correct observed preferred-frame and preferred-location PPN parameters due to SEP satisfaction under morph application. The observed star-case PN metric (83) holds in the standard gauge as discussed. As such, the observed "spatial curvature" PPN parameter $\gamma \equiv 1$ (see [1], Table 4.1) is "read off" the first-order spatial term in the star-case PN metric, and the observed "nonlinearity" PPN parameter $\beta \equiv 1$ is read off its second-order temporal term, with both parameters applicable for standard gauge expression of the observed PN natural metric for general cases. Since the observed star-case PN metric is predicted by the NFE, then it correctly predicts the observed PPN parameters $\gamma \equiv 1$ and $\beta \equiv 1$. Summarizing, *all ten* observed standard gauge PPN parameters are correctly predicted by the natural field equation, so as required, *the natural field equation yields the observed post-Newtonian natural metric for all possible post-Newtonian cases*. This result is verified in the Supplement by showing that the observed post-Newtonian natural metric is yielded from the established 1.5PN post-Minkowskian formulation.

Beginning with the most general possible form (250) of the natural field equation utilizing the arbitrary constants a, b, and c, application of the key observational requirements, consisting of the linearized case requirement and the PN metric requirement, resulted in uniquely setting their values to (267). With there being *only three* arbitrary constants in the most general possible form (250), the *ability* to set these constants to yield the observed linearized case metric, and to yield the observed PN metric, is a "powerful" verification of the validity of (250). Now care was taken to insure that any "intermediate form" of the NFE, at then any stage in the NFE's development, was the most general possible form subject to any assumptions that had been made up to that stage. Therefore, the NFE's final form (256) (or its trace-reverse (262)), with $Q_{\alpha\beta}$ given by (258)–(260), is *the most general possible form* subject to the assumptions made for its development. However, the final form of the NFE has

been shown to be *unique*. Therefore, *the natural field equation is uniquely obtained from the assumptions made for its development.*

The assumptions made for the NFE's development are the absolute flat spacetime and SEP postulates, the morph consistency requirement, the well-accepted assumptions for formulation of Einstein's equation, and satisfaction of the linearized case and PN metric requirements. Based on the assumed *physical validity* of the NFE's developmental assumptions, then with the NFE's final form uniquely obtained from them, *the natural field equation is assumed to be physically valid.* As stated in the summary, the ability of the natural field equation to satisfy the SEP under morph application, linearize to the observationally predictive linearized Einstein equation, and yield the observed post-Newtonian approximation for the natural metric, results in a wide variety of natural gravitational observations being successfully predicted (as discussed below), verifying the field equation's validity. Again, the Supplement extends the range of verification to cover all available natural observations of local systems utilized to test gravitational theories.

Even though the natural field equation $H^{\alpha\beta}[w; g(w,a)] = 8\pi T^{\alpha\beta}[q_\lambda, g(w,a)]$ (256) is utilized by natural observers to model naturally perceived gravitational systems, the contained potential tensor $w^\alpha{}_\mu$ and absolute metric $a_{\alpha\beta}$ are *absolutely observable quantities only*. The question arises as to whether or not use of the so-called "natural field equation" is indeed appropriate for natural observers. To answer this question, consider use of the equivalent morphed form, generally given by (224) representing the morphed form of (256) constructed using the morphed (258)–(260). For morph applicability, the ability to satisfy the SEP conditions is not just limited to local systems surrounded by background systems. In addition, the SEP conditions *always hold* for any "total system" modelled as a whole, since then the "background system" is *empty*, so there are no curvature effects imposed as there is no background curvature, and there are no background sources to be perturbed. The morphed natural field equation (224) may therefore be utilized to model *any possible system*, since either a system may be modelled as a local system surrounded by a background system when the SEP conditions hold, or a system may be modelled as a total system so that the SEP conditions automatically hold. The naturally observed background system free-fall frame may be utilized, in which case the Minkowski metric $\eta_{\alpha\beta}$ is used for the naturally observed Riemann IC morphed background natural metric $g^{MB}_{\alpha\beta}$ (equal to $a_{\alpha\beta}$ for a total system, with the Riemann ICs being the global ICs), or any other frame/coordinates may be used with the value of $g^{MB}_{\alpha\beta}$ known for natural observers via coordinate transform of $g^{MB}_{\alpha\beta} = \eta_{\alpha\beta}$ from the Riemann ICs. All of the quantities in the morphed NFE (224) are therefore naturally observable quantities with the exception of the morphed potential $w^{\alpha M}_\mu$, which is applicable for both local and total systems. For natural observer use, the morphed potential $w^{\alpha M}_\mu$ is treated as a *hypothetical* quantity utilized to obtain the naturally observable morphed natural metric solution $g^M_{\alpha\beta}(w^M, g^{MB}) = g^{MB}_{\alpha\mu}\exp(-2w^{\mu M}_\beta)$. With the morphed potential $w^{\alpha M}_\mu$ treated as a hypothetical quantity and all other quantities being naturally observable, use of the universally applicable morphed natural field equation (224) (or its trace-reverse) is considered appropriate for natural observers. Therefore, *use of the natural field equation is considered to be appropriate for natural observers modelling naturally perceived gravitational systems*, since for natural observer interpretation and use in practice, the "naturally appropriate" morphed form of the NFE, with $w^{\alpha M}_\mu$ treated hypothetically, may always be utilized.

6.4. Solution Properties, the Natural Energy Condition, and Conservation Application

In Section 3.7, the *heuristic argument* was utilized that at the fundamental level, the coupling of the shift tensor field to a symmetric SE tensor source charge, with both in the presence of the symmetric absolute and natural metrics (or at the very least the symmetric absolute metric), results in a symmetric shift tensor when given in pure (indice) form. As the natural field equation $H_{\alpha\beta}[w; g(w,a)] = 8\pi T_{\alpha\beta}$ is a *formal statement* depicting the shift tensor field (using its potential $w^\alpha{}_\mu$) being coupled to the symmetric natural matter SE tensor $T_{\alpha\beta}$ in the presence of the symmetric absolute and natural metrics, then the NFE *must* yield a symmetric shift tensor $S^\alpha{}_{\bar\mu}$ field when given in pure form, proven as follows. Utilizing the $T_{\alpha\beta}$ symmetry, the trace-reverse form (262) of the NFE yields a symmetric potential Ricci

tensor $Q_{\alpha\beta}$. Now regardless of whether or not the potential tensor $w^\alpha{}_\mu$ is symmetric in pure form, every term in $Q_{\alpha\beta}$, given by (258)–(260), is symmetric except for $w_{\alpha\beta}{}^{;\sigma}{}_\sigma$ in $P_{\alpha\beta}$. Therefore, $w_{\alpha\beta} = g_{\alpha\mu}w^\mu{}_\beta$ must indeed be symmetric to yield a symmetric $Q_{\alpha\beta}$. So any potential solution $w^\alpha{}_\mu$ for the natural field equation is symmetric when raised/lowered by the natural metric to yield a pure form. With $S^\alpha{}_{\bar\mu} = \exp(w^\alpha{}_\mu)$, the symmetric potential solution yields a symmetric shift tensor field when using the natural metric to yield a pure form ($g\,w = w^T g$ for the matrix form of the symmetry $w_{\alpha\beta} = w_{\beta\alpha}$ where $w_{\alpha\beta} = g_{\alpha\mu}w^\mu{}_\beta$, yielding $g\,w^n = w^{n\,T}g$ for each term in the matrix expansion $S = e^w$, so $g\,S = S^T g$, which in tensor form is the symmetry $S_{\alpha\bar\beta} = S_{\beta\bar\alpha}$). Utilizing (52), the shift tensor field is also symmetric when using the absolute metric to yield a pure form, completing the proof. It can be shown that the potential solution $w^\alpha{}_\mu$ is also symmetric when using the absolute metric to yield a pure form. Due to the symmetry of the potential tensor, the natural field equation $H_{\alpha\beta}[w; g(w,a)] = 8\pi T_{\alpha\beta}$ provides *ten algebraically independent conditions* to determine the ten algebraically independent components of the potential tensor $w^\alpha{}_\mu$.

Prior to discussion of energy conditions below, it is helpful to first examine gravity waves. Similar to the reduction of Einstein's equation to its linearized form under SEP application in a micro free-fall frame, the natural field equation is reduced to its linearized form under morph-based application of the SEP. The resulting linearized natural field equation, $\hat H_{\alpha\beta}[p; \eta] = 8\pi \hat T_{\alpha\beta}$, in the micro free-fall frame Riemann ICs, is the same as the linearized Einstein equation $\hat G_{\alpha\beta}[p; \eta] = 8\pi \hat T_{\alpha\beta}$, with the field tensor equivalence (254) holding. In a vacuum, natural field equation modelled gravity waves adhere, therefore, to $\hat G_{\alpha\beta}[p; \eta] = 0$ applicable for Einstein field equation modelled gravity waves (see Schutz [18], Chap. 9, for a background). The resultant gravity wave propagation speed in a micro free-fall frame is $\hat v_G = 1$ (in geometrized units) the same as the shifted light speed $\hat c_S = 1$. So in any coordinates,

$$v_G = c_S. \tag{268}$$

Therefore, the natural field equation in GS theory successfully predicts that *gravity waves propagate at the same speed as (shifted/actual) light*, as observed, the same as for Einstein equation prediction in general relativity. In addition, adherence to $\hat G_{\alpha\beta}[p; \eta] = 0$ in micro free-fall frames when using the natural field equation to model gravity waves, yields successful prediction of the observed *dual spin-2 polarization state*s, the same as for Einstein equation prediction.

Without use of an "energy condition" limiting the gravitational sources, the natural field equation admits to potential solutions $w^\alpha{}_\mu$ such that the shift tensor $S^\alpha{}_{\bar\mu} = \exp(w^\alpha{}_\mu)$ yields shifted light speeds c_S exceeding the null speed v_{Null}, so the gravity shifting would violate the speed constraint. An example would be use of a *negative* gravitational mass $M < 0$ for the above star case (Sec. 3.12), since with $c_S = e^{-2M/r}$ giving the global IC shifted light speed outside a star, a negative mass would yield a speed faster than the null speed $v_{Null} = 1$. Therefore, in GS theory the required "natural energy condition" for gravitational sources is as follows: *The natural energy-momentum of gravitational sources must be such that the natural field equation solutions produce shifted light speeds c_S that do not exceed the absolute manifold null speed v_{Null}.* Using (268), satisfaction of the natural energy condition also prevents gravitational field propagation speeds $v_G = c_S$ from exceeding the null speed v_{Null}. Therefore, *under the natural energy condition, causal connectedness of all types is limited by the null speed v_{Null}, preventing causality violation in absolute flat spacetime.* A listing of various popular natural energy conditions utilized in general relativity based modelling is provided in Carroll [24] (Chap. 4). Though a formal proof has not been performed, it is evident that the ones that prevent natural energy transport faster than light in general relativity, utilizing then Einstein's equation, yield a shifted light speed c_S that does not exceed the null speed v_{Null} when using the natural field equation in GS theory, satisfying the natural energy condition. These include, for instance, the commonly accepted "Dominant Energy Condition" applicable for ordinary forms of naturally observed matter, which is the energy condition discussed previously. For GS theory, *any form of matter is considered acceptable so long as the natural energy condition is satisfied*, so the natural energy condition provides a definitive means of deciding if a particular form of matter may exist.

Again, the natural field equation yields a symmetric potential $w^\alpha{}_\mu$ solution that results in a symmetric shift tensor field $S^\alpha{}_{\bar\mu}$ (when put in pure form), and as established above, the NFE yields a real-valued potential solution $w^\alpha{}_\mu$ due to any explicit potential use being linear. Satisfaction of the speed constraint, under the natural energy condition, enables the global IC given potential $w^\alpha{}_\mu$ to be *diagonalized* using Lorentz transforms (utilizing its pure form symmetry). This results in an IC eigensystem shift tensor $\tilde S^\alpha{}_{\bar\mu} = \exp(\tilde w^\alpha{}_\mu)$ that adheres to the shift factor range (61), yielding 1-to-1 gravity shifting satisfying the overlap restriction, as well as shifting satisfying the temporal constraint (see Section 3.6 for background). As previously established, combining satisfaction of the speed and temporal constraints yields satisfaction of the null constraint (forward null cone limited evolution). Therefore, with the assumed adherence to the natural energy condition for gravitational sources, *the natural field equation predicts gravity shifting that adheres to all of the established gravity shifting constraints* (as stated in Section 3.13).

As previously shown, satisfaction of the shifting constraints results in gravity shifting such that gravity shift overlap singularities are barred, and such that causality violations are prevented from occurring. In addition, the exponential potential form $S^\alpha{}_{\bar\mu} = \exp(w^\alpha{}_\mu)$ (68) for the shift tensor has been shown to prevent the formation of both event horizons and collapse-based singularities, with these properties applicable then for the shift tensor formed from the NFE potential solution $w^\alpha{}_\mu$. With the barring of event horizons, singularities, and causality violations as implausibilities, as well as explicit formulation in absolute flat spacetime resulting in compatibility with quantum theory (as demonstrated above), *all physical law and modelling is physically plausible when utilizing the natural field equation to predict the gravitational field*.

It can be shown that for an *arbitrary* potential field $w^\alpha{}_\mu$ (symmetric in pure indice form), the natural divergence of the established natural field tensor $H^{\alpha\beta}[w; g(w,a)]$ is non-zero in general, as stated by the "non-equivalency"

$$\nabla^N_\beta H^{\alpha\beta}[w; g(w,a)] \not\equiv 0 \quad \text{(arbitrary } w^\alpha{}_\mu\text{)}. \tag{269}$$

As shown in LLN [25], for Lagrangian-based metric theories of gravity such as GS theory, a non-zero divergence is generally expected for gravitational field tensors that contain absolute quantities, so (269) is expected for $H^{\alpha\beta}[w; g(w,a)]$. As discussed previously, EEP satisfaction for natural observers in micro free-fall frames yields energy-momentum satisfaction for the naturally observed matter and nongravitational fields, as expressed by $\partial^{(N)}_\nu \hat T^{\mu\nu}_{N(N)} = 0$ utilizing the natural matter SE tensor $T^{\alpha\beta}_N = T^{\alpha\beta}$. Therefore,

$$T^{\alpha\beta}{}_{;\beta} = 0, \tag{270}$$

providing the "natural matter conservation statement" in any coordinates. So in contrast to general relativity where $T^{\alpha\beta}{}_{;\beta} = 0$ may be obtained from the Bianchi identity $G^{\alpha\beta}{}_{;\beta} \equiv 0$ when applied in the Einstein equation $G^{\alpha\beta} = 8\pi T^{\alpha\beta}$, in GS theory $T^{\alpha\beta}{}_{;\beta} = 0$ is a condition *exclusively obtained independently from the natural field equation* $H^{\alpha\beta} = 8\pi T^{\alpha\beta}$. The available Lagrangian-based establishment of $T^{\alpha\beta}{}_{;\beta} = 0$ for metric-based theories applies for GS theory as well. The method employed is to apply coordinate transformations (or equivalently diffeomorphisms) to the matter action S_M utilized in the covariant matter action principle $\delta S_M = 0$, obtaining a Bianchi identity such that when the EL natural matter equations of motion (236) are substituted, $T^{\alpha\beta}{}_{;\beta} = 0$ is yielded. An example suitable for GS theory is provided in Will [1] (Chap. 3, using $\delta\mathscr{L}_M/\delta q_\lambda \equiv \delta S_M/\delta q_\lambda$ for the functional derivatives), where it is explained that $T^{\alpha\beta}{}_{;\beta} = 0$ is obtained as a consequence of universal coupling yielded under micro free-fall frame EEP satisfaction, independent of gravitational field equations.

Due to its independent origin from the natural field equation, the matter conservation statement $T^{\alpha\beta}{}_{;\beta} = 0$ (270) acts as an independent "ancillary condition" applied to the NFE $H^{\alpha\beta} = 8\pi T^{\alpha\beta}$, resulting in the "natural field constraint" (or "field constraint" for short)

$$\nabla^N_\beta H^{\alpha\beta}[w; g(w,a)] = 0 \tag{271}$$

acting to *constrain* the potential field $w^\alpha{}_\mu$ used to construct the natural field tensor $H^{\alpha\beta}[w; g(w, a)]$, since $H^{\alpha\beta}{}_{;\beta} \not\equiv 0$ (269) for arbitrary $w^\alpha{}_\mu$. In general relativity, application of $T^{\alpha\beta}{}_{;\beta} = 0$ to Einstein's equation $G^{\alpha\beta} = 8\pi T^{\alpha\beta}$ does *not constrain* the metric-given field $g_{\alpha\beta}$ due to the Bianchi identity $G^{\alpha\beta}{}_{;\beta} \equiv 0$ holding. This results in the Einstein equation providing only *six* functionally independent conditions when matter conservation $T^{\alpha\beta}{}_{;\beta} = 0$ is applied, so *four additional coordinate conditions* are applied to the natural metric $g_{\alpha\beta}$, setting the coordinates, in order to fully determine it. On the other hand, in GS theory, the *lack* of $H^{\alpha\beta}{}_{;\beta} \equiv 0$ holding as an identity, due to $H^{\alpha\beta}{}_{;\beta} \not\equiv 0$ (269) for arbitrary potential fields $w^\alpha{}_\mu$, implies that the natural field equation $H^{\alpha\beta} = 8\pi T^{\alpha\beta}$ provides *ten functionally independent conditions* for determining the $w^\alpha{}_\mu$ field when matter conservation $T^{\alpha\beta}{}_{;\beta} = 0$ is applied, the same as the number of algebraically independent conditions prior to $T^{\alpha\beta}{}_{;\beta} = 0$ application (from above). This is as necessary since *a priori* absolute manifold coordinates (such as global ICs) may be utilized when solving the NFE, so the coordinates have already been set, requiring the NFE to provide ten functionally independent conditions when $T^{\alpha\beta}{}_{;\beta} = 0$ is applied in order to *fully determine* the $w^\alpha{}_\mu$ field. For convenience when referring to the "natural field equation," the ancillary matter conservation statement $T^{\alpha\beta}{}_{;\beta} = 0$ may be included, with discernment from the natural field equation $H^{\alpha\beta} = 8\pi T^{\alpha\beta}$ alone made by context.

In general relativity, solution of the Einstein equation, $G^{\alpha\beta} = 8\pi T^{\alpha\beta}$, requires *simultaneous use* of the conservation statement $T^{\alpha\beta}{}_{;\beta} = 0$ due to the Bianchi identity $G^{\alpha\beta}{}_{;\beta} \equiv 0$ yielding $T^{\alpha\beta}{}_{;\beta} = 0$ while obtaining the solution. In contrast, due to $H^{\alpha\beta}{}_{;\beta} \not\equiv 0$ (269) holding for arbitrary potential fields $w^\alpha{}_\mu$, the natural field equation $H^{\alpha\beta} = 8\pi T^{\alpha\beta}$ in GS theory may be solved as a "relaxed form" where *first* it is solved given a $T^{\alpha\beta}[q_\lambda, g]$ in *functional form*, and *then* the conservation statement $T^{\alpha\beta}{}_{;\beta} = 0$ is applied to determine the field $w^\alpha{}_\mu$ as well as the matter and nongravitational field variables q_λ as constrained by $T^{\alpha\beta}{}_{;\beta} = 0$. This is similar to solution of the relaxed wave equation form of Einstein's equation in general relativity post-Minkowskian theory (see PW [20], Chap. 6), which is known as a convenient and powerful solution method.

As shown in Weinberg [22] (Chap. 7), due to satisfaction of the Bianchi identity $G^{\alpha\beta}{}_{;\beta} \equiv 0$ in Einstein's equation $G^{\alpha\beta} = 8\pi T^{\alpha\beta}$, the Cauchy problem in general relativity requires four additional coordinate conditions to be imposed in order to completely specify the initial conditions necessary for unique predicted evolution from an initial hypersurface $x^0 = t$. In contrast, due to the *lack of* $H^{\alpha\beta}{}_{;\beta} \equiv 0$ holding as an identity (as per (269)), in GS theory *only the natural field equation* $H^{\alpha\beta} = 8\pi T^{\alpha\beta}$ is required to completely specify the initial conditions for unique evolution from an initial hypersurface $x^0 = t$. The conservation condition $T^{\alpha\beta}{}_{;\beta} = 0$ is imposed both initially and during system evolution, but the natural field equation under this constraint again completely specifies the initial conditions, and the evolution under this constraint is uniquely determined from the initial conditions.

7. The Absolute Field Equation

7.1. General Form

As discussed previously, the absolute field equation is utilized by absolute observers to model gravitational systems, with it being the *partner formulation* to the natural field equation $H_N^{\alpha\beta} = 8\pi T_N^{\alpha\beta}$ utilized by natural observers (for clarity here, the "N" designation is used for natural quantities). As partner formulations are equivalent, the partner absolute field equation *must predict a gravitational field that is the same as predicted by the natural field equation*. So the AFE must have the same real-valued potential solution $w^\alpha{}_\mu$ as the NFE, referred to as the "matched solution requirement." In the development that follows, the partner absolute field equation is constructed such that this requirement is met.

With it being the partner formulation to the natural field equation $H_N^{\alpha\beta} = 8\pi T_N^{\alpha\beta}$, the absolute field equation is expected to take the same general form as the NFE, but with a partner "absolute field tensor" $H_A^{\alpha\beta}$ used in place of the natural field tensor $H_N^{\alpha\beta}$, and a partner absolute SE tensor used in place of the natural matter SE tensor $T_N^{\alpha\beta}$. Now in the NFE, $T_N^{\alpha\beta}$ depicts the total energy-momentum density for all naturally observed EM contributors combined. Therefore, in the partner AFE, the partner absolute SE tensor is expected to depict the total EM density for all absolutely observed EM contributors combined,

as appropriate for the field equation utilized by absolute observers to predict the gravitational field based on the absolutely observed gravitational sources. As established, the naturally observed EM contributors consist of only the naturally observed EM for matter and the nongravitational fields, with the natural matter SE tensor $T_N^{\alpha\beta}$ depicting their total EM density. But as shown in Section 5.3, the absolutely observed EM contributors consist of the absolutely observed EM for matter and the nongravitational fields as depicted by the absolute matter SE tensor $T_A^{\alpha\beta}$ (the absolute *matter* partner to $T_N^{\alpha\beta}$), as well as the absolutely observed EM for the gravitational field as depicted by the absolute field SE tensor $t_A^{\alpha\beta}$, with then the absolute total SE tensor $E_A^{\alpha\beta} \equiv T_A^{\alpha\beta} + t_A^{\alpha\beta}$ (216) depicting the total EM density for all absolute EM contributors. Therefore, when forming the partner AFE, it is the absolute *total* SE tensor $E_A^{\alpha\beta}$ that is identified as the partner to the natural *matter* SE tensor $T_N^{\alpha\beta}$. With $H_A^{\alpha\beta}$ and $E_A^{\alpha\beta}$ used in place of $H_N^{\alpha\beta}$ and $T_N^{\alpha\beta}$ respectively in the natural field equation $H_N^{\alpha\beta} = 8\pi T_N^{\alpha\beta}$, the partner absolute field equation is given by $H_A^{\alpha\beta} = 8\pi E_A^{\alpha\beta}$. Recall that the total SE tensor $E_A^{\alpha\beta}$ is symmetric, so the absolute field tensor $H_A^{\alpha\beta}$ must be symmetric as well.

In order to obtain a real-valued potential solution $w^\alpha{}_\mu$, then similar to the natural field equation, the potential must again be the *operand* in the absolute field equation. For this reason, the general functional form for the partner absolute field tensor must be given by $H_A^{\alpha\beta} = H_A^{\alpha\beta}[w;a]$, depicting all possible field tensors that may be formed where $w^\alpha{}_\mu$ is the operand, similar to $H_N^{\alpha\beta} = H_N^{\alpha\beta}[w;a]$ as the initially given general functional form for the natural field tensor as stated by (219). The absolute field equation therefore takes the general form

$$H_A^{\alpha\beta}[w;a] = 8\pi \, E_A^{\alpha\beta} \tag{272}$$

(establishing (9)). Due to the morph consistency requirement, the functional form for the natural field tensor was shown to be $H_N^{\alpha\beta}[w;a] = H_N^{\alpha\beta}[w;g(w,a)]$, as expected since in general, natural formulations utilize the natural metric $g_{\alpha\beta}$ to raise/lower indices and to form covariant derivatives. Since it applies for natural formulations only, the morph consistency requirement *does not apply* for the absolute field equation. So as is generally expected for absolute formulations, in the absolute field equation $H_A^{\alpha\beta}[w;a] = 8\pi E_A^{\alpha\beta}$ the absolute metric $a_{\alpha\beta}$ is assumed to raise/lower indices and to form covariant derivatives. This is consistent with the assumption that the classical absolute field equation being developed here *is the classical limit of an absolute quantum formulation for the field equation*. With the gravitational field treated by absolute observers as an ordinary force field similar to the nongravitational fields, then in the global ICs of absolute inertial frames, quantum gravitational field equation formulation is expected to utilize the Minkowski metric valued absolute metric $a_{\alpha\beta}$ for all metric use similar to field equation formulation for the nongravitational fields. With the classical potential $w^\alpha{}_\mu$ the operand in the classical AFE, then the quantized potential $\widehat{w}^\alpha{}_\mu$ is the gravitational field quantity coupled to the quantized source term $\widehat{E}_A^{\alpha\beta}$ in the quantized AFE, with all posed in absolute flat spacetime using then the classical absolute metric for all metric use. This implies that the absolute metric is *exclusively* utilized as the metric in the classical absolute field equation $H_A^{\alpha\beta}[w;a] = 8\pi E_A^{\alpha\beta}$, including raising/lowering the indices of $H_A^{\alpha\beta}[w;a]$ and $E_A^{\alpha\beta}$, such as to form $H_{\alpha\beta}^A[w;a] = 8\pi E_{\alpha\beta}^A$ from $H_A^{\alpha\beta}[w;a] = 8\pi E_A^{\alpha\beta}$, and *exclusive absolute metric use in the absolute field tensor* $H_A^{\alpha\beta}[w;a]$. The absolute field tensor $H_A^{\alpha\beta}[w;a]$ consists of a sum of terms (or a single term), where under exclusive absolute metric use, each term is a product of a number of $w^\alpha{}_\mu$ and $a_{\alpha\beta}$ and/or their partial derivatives, yielding the generic representation

$$H_A^{\alpha\beta}[w;a] = \sum_n f_n(w) h_n(a). \tag{273}$$

The absolute metric $a_{\alpha\beta}$ therefore appears *explicitly* in the absolute field equation, as opposed to being only *implicitly* contained within the natural metric $g_{\alpha\beta}(w,a)$ when utilized in the natural field equation.

The *internal* construction of $E_A^{\alpha\beta}$ was not considered when establishing exclusive absolute metric use in the absolute field equation $H_A^{\alpha\beta}[w;a] = 8\pi E_A^{\alpha\beta}$, so exclusive use of $a_{\alpha\beta}$ with $E_A^{\alpha\beta}$ pertained to use *external* to $E_A^{\alpha\beta}$. Lowering (216) by the absolute metric, and substituting (212) and (217), yields

$$E_{\alpha\,A}^{\ \ \mu} = T_{\alpha\,A}^{\ \ \mu} + t_{\alpha\,A}^{\ \ \mu} = |S^{-1}|\,T_{\alpha\,N}^{\ \ \mu} + \frac{1}{8\pi}\left\{H_{\alpha\,A}^{\ \ \mu} - |S^{-1}|H_{\alpha\,N}^{\ \ \mu}\right\}, \tag{274}$$

with the functional dependences on the right of (274) given by

$$T_{\alpha\,A}^{\ \ \mu}[q_\lambda^A, w, a] = |S^{-1}(w)|\,T_{\alpha\,N}^{\ \ \mu}[q_\lambda^N(q_\lambda^A, w), g(w, a)], \tag{275}$$

$$t_{\alpha\,A}^{\ \ \mu}[w, a] = \frac{1}{8\pi}\left\{H_{\alpha\,A}^{\ \ \mu}[w;a] - |S^{-1}(w)|H_{\alpha\,N}^{\ \ \mu}[w; g(w,a)]\right\}. \tag{276}$$

The quantities q_λ^A in (275) depict the native *absolute* matter and nongravitational field quantities, where each nongravitational quantity q_λ^A is the absolute partner of the native natural nongravitational quantity q_λ^N. As such, each pair of native nongravitational partner quantities are related by a purely gravity shift based partner relation $q_\lambda^N = q_\lambda^N(q_\lambda^A, w)$ (per the discussion in Section 5.3), which is utilized in the functional form $T_{\alpha\,N}^{\ \ \mu}[q_\lambda^N(q_\lambda^A, w), g(w, a)]$ for the natural matter SE tensor. Even though $q_\lambda^N = q_\lambda^N(q_\lambda^A, w)$, this functional dependence does not negate that each native natural nongravitational quantity q_λ^N is universally coupled to the natural metric $g_{\alpha\beta}$. Using the above functional dependences in (274), and raising by the absolute metric, the functional form for $E_A^{\alpha\beta}$ is given by

$$E_A^{\alpha\beta}[q_\lambda^A, w, a] = T_A^{\alpha\beta}[q_\lambda^A, w, a] + t_A^{\alpha\beta}[w, a], \tag{277}$$

which implicitly contains natural metric terms $g_{\alpha\beta}(w, a)$ since both $T_{\alpha\,A}^{\ \ \mu}[q_\lambda^A, w, a]$ and $t_{\alpha\,A}^{\ \ \mu}[w, a]$ do so as seen on the right of (275) and (276). Note that in addition to containing absolute metric terms implicitly inside the natural metric $g_{\alpha\beta}(w, a)$, $E_A^{\alpha\beta}[q_\lambda^A, w, a]$ contains absolute metric terms explicitly outside the natural metric, such as occurs for $H_{\alpha\,A}^{\ \ \mu}[w; a]$ in (276) providing $t_{\alpha\,A}^{\ \ \mu}[w, a]$. Using the functional form $E_A^{\alpha\beta}[q_\lambda^A, w, a]$ in (272) yields

$$H_A^{\alpha\beta}[w;a] = 8\pi\,E_A^{\alpha\beta}[q_\lambda^A, w, a] \tag{278}$$

for the general form of the absolute field equation, where again the absolute metric is used exclusively in $H_A^{\alpha\beta}[w;a]$ for all metric use as per (273), and is used exclusively for external manipulation of $H_A^{\alpha\beta}[w;a]$ and $E_A^{\alpha\beta}[q_\lambda^A, w, a]$.

The absolute field equation is directly derivable from its natural field equation partner, as follows. Starting with the NFE in the mixed indice form $H_{\alpha\,N}^{\ \ \mu}[w; g(w, a)] = 8\pi T_{\alpha\,N}^{\ \ \mu}$, multiplication by $|S^{-1}|$ yields $0 = 8\pi|S^{-1}|T_{\alpha\,N}^{\ \ \mu} - |S^{-1}|H_{\alpha\,N}^{\ \ \mu}$. This form of the NFE, combined with natural EM conservation $\nabla_\mu^N T_{\alpha\,N}^{\ \ \mu} = 0$, yields the complete potential solution $w^\alpha_{\ \mu}$ the same as $H_N^{\alpha\beta} = 8\pi T_N^{\alpha\beta}$ used in conjunction with $\nabla_\beta^N T_N^{\alpha\beta} = 0$. The absolute field tensor $H_{\alpha\,A}^{\ \ \mu}[w; a]$ is added to both sides to obtain $H_{\alpha\,A}^{\ \ \mu} = 8\pi|S^{-1}|T_{\alpha\,N}^{\ \ \mu} + H_{\alpha\,A}^{\ \ \mu} - |S^{-1}|H_{\alpha\,N}^{\ \ \mu}$. Note that the addition of $H_{\alpha\,A}^{\ \ \mu}[w; a]$ to both sides *does not interfere with the NFE solution $w^\alpha_{\ \mu}$*. Finally, application of (274) yields the absolute field equation $H_{\alpha\,A}^{\ \ \mu}[w; a] = 8\pi E_{\alpha\,A}^{\ \ \mu}$, or equivalently $H_A^{\alpha\beta}[w;a] = 8\pi E_A^{\alpha\beta}$, completing the proof. As can be seen, *the absolute field equation is simply the natural field equation in an alternate form*. Recall that the NFE and AFE general forms (219) and (272) were combined in order to establish (217) originally giving the absolute field SE tensor $t_{\alpha\,A}^{\ \ \mu}$ in terms of the field tensors $H_{\alpha\,A}^{\ \ \mu}$ and $H_{\alpha\,N}^{\ \ \mu}$, which is the reason *why* the AFE may be derived from the NFE as above.

In the above derivation, any arbitrary $H_A^{\alpha\beta}[w;a]$ form may be utilized where the contained potential is the NFE $w^\alpha_{\ \mu}$ solution, yielding then the same AFE $w^\alpha_{\ \mu}$ solution as the NFE, satisfying the matched solution requirement. However, use of an arbitrary $H_A^{\alpha\beta}[w;a]$ form results in an *arbitrary* $E_A^{\alpha\beta}$ via the AFE $H_A^{\alpha\beta}[w;a] = 8\pi E_A^{\alpha\beta}$, contrary to a given system possessing a *given* physical absolute total SE tensor $E_A^{\alpha\beta}$. Due to its field dependence, $E_A^{\alpha\beta}$ is not known a priori for most gravitational

systems, typically since the field dependent absolute field SE tensor $t_A^{\alpha\beta}$ is not known. However, there are specialized cases where $E_A^{\alpha\beta}$ is known a priori, such as static cases examined below. Whether or not $E_A^{\alpha\beta}$ for a system is known a priori, it is *required* that the form for $H_A^{\alpha\beta}[w;a]$ be such as to yield the system's $E_A^{\alpha\beta}$ value, via the AFE, when the NFE $w^\alpha{}_\mu$ solution is utilized in $H_A^{\alpha\beta}[w;a]$. This is referred to as the "total SE (tensor) requirement," which must be satisfied in order to satisfy the matched solution requirement while also yielding the given $E_A^{\alpha\beta}$ value for a system. Consider use of the AFE $H_A^{\alpha\beta}[w;a] = 8\pi E_A^{\alpha\beta}$ as a *stand-alone* field equation, which is solved for $w^\alpha{}_\mu$ based on the given $E_A^{\alpha\beta}$ value for a system (known a priori or not, so the AFE is being solved in theory). *Only if the $H_A^{\alpha\beta}[w;a]$ form satisfies the total SE requirement will the stand-alone AFE $w^\alpha{}_\mu$ solution equal the NFE solution*. In the development that follows, a *unique form* for $H_A^{\alpha\beta}[w;a]$ is established based on the requirements. It is then *assumed* that this form for $H_A^{\alpha\beta}[w;a]$ indeed *satisfies the total SE requirement*. So via the AFE, use of the NFE $w^\alpha{}_\mu$ solution for a system in the established $H_A^{\alpha\beta}[w;a]$ form is assumed to yield the system's $E_A^{\alpha\beta}$. The established $H_A^{\alpha\beta}[w;a]$ form is tested below for the static star case where $E_A^{\alpha\beta}$ is known a priori, where it is shown that use of the NFE $w^\alpha{}_\mu$ solution in $H_A^{\alpha\beta}[w;a]$ indeed yields the known $E_A^{\alpha\beta}$, verifying satisfaction of the total SE requirement. With the total SE requirement assumed to be met for the established $H_A^{\alpha\beta}[w;a]$ form, then the stand-alone AFE $w^\alpha{}_\mu$ solution for a system with a given $E_A^{\alpha\beta}$ (known or not) is indeed the same as the NFE $w^\alpha{}_\mu$ solution, satisfying the matched solution requirement.

As is well accepted in gravitational physics, in the weak limit the energy-momentum content of the gravitational field is negligible compared to the EM content of the source matter. This is assumed then in GS theory, so in the weak limit the field SE tensor $t_{\alpha A}^{\ \ \mu}$ is assumed to be negligible compared with the absolute matter SE tensor $T_{\alpha A}^{\ \ \mu}$ given by (275). Working in global ICs, use of (277) therefore yields $E_{\alpha A}^{\ \ \mu}[q^N, w, \eta] = |S^{-1}| T_{\alpha N}^{\ \ \mu}$ in the weak limit. The AFE (278) becomes $H_{\alpha A}^{\ \ \mu}[w;\eta] = 8\pi |S^{-1}| T_{\alpha N}^{\ \ \mu}$, where $H_{\alpha A}^{\ \ \mu}[w;\eta]$ is linearized in $w^\alpha{}_\mu$ for the weak limit. With $|S^{-1}| = e^{-w}$ (74), then when expanded in w and multiplied by $T_{\alpha N}^{\ \ \mu}$, all but the leading unity term in the e^{-w} expansion may be dropped in the weak limit, yielding the linearized form $H_{\alpha A}^{\ \ \mu}[w;\eta] = 8\pi T_{\alpha N}^{\ \ \mu}$ for the AFE. Using the Minkowski metric to raise/lower indices in the global IC given weak limit case, and applying the linearized substitution (253) to utilize $p_{\alpha\beta}$ as defined by $g_{\alpha\beta} = \eta_{\alpha\beta} + p_{\alpha\beta}$, then the absolute field equation in the weak limit is given by the linearized form

$$H_{\alpha\beta}^A[p;\eta] = 8\pi T_{\alpha\beta}^N. \tag{279}$$

In order to yield the same field solution $p_{\alpha\beta}$ as the natural field equation $H_{\alpha\beta}^N[p;\eta] = 8\pi T_{\alpha\beta}^N$ for all possible linearized cases in the weak limit, then the linearized absolute field tensor $H_{\alpha\beta}^A[p;\eta]$ *must* have the same form as the linearized natural field tensor, as stated by the required equivalence

$$H_{\alpha\beta}^A[p;\eta] \equiv H_{\alpha\beta}^N[p;\eta] \equiv G_{\alpha\beta}[p;\eta], \tag{280}$$

where the equivalence with the linearized Einstein tensor $G_{\alpha\beta}[p;\eta]$ has been added via (254). Satisfying this "linearized case requirement," the weak limit linearized absolute field equation $H_{\alpha\beta}^A[p;\eta] = 8\pi T_{\alpha\beta}^N$, natural field equation $H_{\alpha\beta}^N[p;\eta] = 8\pi T_{\alpha\beta}^N$, and Einstein equation $G_{\alpha\beta}[p;\eta] = 8\pi T_{\alpha\beta}^N$ are all identical, yielding the same natural metric solution $g_{\alpha\beta} = \eta_{\alpha\beta} + p_{\alpha\beta}$ for all possible linearized cases. Raising by the Minkowski metric, then $H_{\alpha A}^{\ \ \mu}[p;\eta] \equiv H_{\alpha N}^{\ \ \mu}[p;\eta]$, and with $|S^{-1}| = 1 - w = 1 + 2p$ when linearized (using (253)), then via (276) the absolute field SE tensor $t_{\alpha A}^{\ \ \mu}[w,a]$ *vanishes* when linearized, as stated by the equivalent identities

$$t_{\alpha A}^{\ \ \mu}[p,\eta] \equiv 0, \qquad t_{\alpha A}^{\ \ \mu}[w,\eta] \equiv 0. \tag{281}$$

So the *exact* $t_{\alpha A}^{\ \ \mu}[w,a]$, given by (276) in any coordinates, consists of *second-order or higher $w^\alpha{}_\mu$ products*, as consistent with the assumption that the absolute field SE tensor $t_{\alpha A}^{\ \ \mu}$ becomes negligible compared to absolute matter SE tensor $T_{\alpha A}^{\ \ \mu}$ (and therefore the natural matter SE tensor $T_{\alpha N}^{\ \ \mu}$) in the weak limit. With this property for the field SE tensor $t_{\alpha A}^{\ \ \mu}[w,a]$ established, then when the absolute field equation

$H^A_{\alpha\beta}[w;a] = 8\pi E^A_{\alpha\beta}$ is linearized, it *formally* has the same form $H^A_{\alpha\beta}[p;\eta] = 8\pi T^N_{\alpha\beta}$ as when the weak limit is taken, as expected.

The above-established natural field tensor $H^{\alpha\beta}_N[w;g]$ consists of $N = 2$ terms where again N is the total number of derivatives taken on the field quantities in each term. The linearized form $H^N_{\alpha\beta}[p;\eta] \equiv G_{\alpha\beta}[p;\eta]$ consists then of $N = 2$ terms as seen in (252), so via (280) $H^A_{\alpha\beta}[p;\eta]$ consists of $N = 2$ terms, and using (253) $H^A_{\alpha\beta}[w;\eta]$ does as well. From scaling arguments such as provided in Weinberg [22] (Chap. 7), then all of the terms in the *exact* $H^{\alpha\beta}_A[w;a]$ must have the same N, and since all of the terms in the linearized $H^A_{\alpha\beta}[w;\eta]$ are $N = 2$, then the exact absolute field tensor $H^{\alpha\beta}_A[w;a]$ *must* consist of $N = 2$ terms. Therefore, the $H^{\alpha\beta}_A[w;a]$ terms $f_n(w)h_n(a)$ (273), utilized in the absolute field equation, are required to contain $w^\alpha{}_\mu$ and/or $a_{\alpha\beta}$ partial derivatives in $N = 2$ combinations. Now if an $H^{\alpha\beta}_A[w;a]$ term $f_n(w)h_n(a)$ was given by $h_n(a)$ only, the $N = 2$ requirement would imply that contained $a_{\alpha\beta}$ would have to be partially differentiated. Such $h_n(a)$ may be converted to covariant form by substituting absolute metric covariant derivatives ∇^A_μ for the partial derivatives ∂_μ when given in global ICs, but with $\nabla^A_\mu a_{\alpha\beta} = 0$, the $h_n(a)$ vanish. The $N = 2$ requirement therefore *forbids terms that do not contain $w^\alpha{}_\mu$*, so all $H^{\alpha\beta}_A[w;a]$ terms $f_n(w)h_n(a)$ have an $f_n(w)$ that is linear in $w^\alpha{}_\mu$ or consists of higher-order $w^\alpha{}_\mu$ products.

Since the established NFE has been shown to yield only real-valued $w^\alpha{}_\mu$ solutions as required, then under the matched solution requirement the absolute field equation must yield only real-valued $w^\alpha{}_\mu$ solutions. Therefore, a key restriction on the form of $H^{\alpha\beta}_A[w;a]$ is that it must yield only *real-valued* $w^\alpha{}_\mu$ solutions for the absolute field equation $H^{\alpha\beta}_A[w;a] = 8\pi E^{\alpha\beta}_A$ over the *entire range* of possible systems that may be modelled, and therefore over the entire range of possible given real-valued source fields $E^{\alpha\beta}_A$ for the modelled systems. This restriction limits the form of $H^{\alpha\beta}_A[w;a]$, as follows.

Consider first the possible form for $H^{\alpha\beta}_A[w;a]$ prior to restricting the AFE $w^\alpha{}_\mu$ solutions to being real valued. Utilizing global ICs so that the absolute metric $a_{\alpha\beta}$ is the Minkowski metric $\eta_{\alpha\beta}$, then any $a_{\alpha\beta}$ partial derivatives vanish, yielding possible $H^{\alpha\beta}_A[w;a]$ terms $f_n(w)h_n(a)$ consisting of products of the following: $w^\alpha{}_\mu$ partial derivatives, undifferentiated $w^\alpha{}_\mu$, and fixed $\eta_{\alpha\beta}$. The $f_n(w)h_n(a)$ may consist of various index combinations of these contributors so long as free raised indices α and β are obtained in order to contribute to $H^{\alpha\beta}_A[w;a]$. The global IC given $f_n(w)h_n(a)$ may therefore be composed of $w^\alpha{}_\mu$ products of various orders (including being linear in $w^\alpha{}_\mu$) multiplied by fixed constants, where the $w^\alpha{}_\mu$ may be partially differentiated and/or undifferentiated. Applying the real-valued $w^\alpha{}_\mu$ solution restriction, consider an $H^{\alpha\beta}_A[w;a]$ form where any instances of the potential $w^\alpha{}_\mu$ are *linear* in the potential only, so with no terms allowed that do not contain $w^\alpha{}_\mu$ (from above), any term $f_n(w)h_n(a)$ is linear in $w^\alpha{}_\mu$ (which may be partially differentiated). In this case, the global IC given AFE $H^{\alpha\beta}_A[w;a] = 8\pi E^{\alpha\beta}_A$ yields a set of coupled linear partial differential equations in the four global IC coordinates, with all $w^\alpha{}_\mu$ multiplied by fixed constants on the left-hand sides, and with the right of each equation consisting of a generally variable $E^{\alpha\beta}_A$ component (times 8π). Regardless of the given real-valued (continuous) source field $E^{\alpha\beta}_A$ utilized, a real-valued global IC $w^\alpha{}_\mu$ solution is yielded due to the coupled differential equations being *linear*, so real-valued $w^\alpha{}_\mu$ solutions are yielded over the entire range of possible systems that may be modelled. Now if instead an $H^{\alpha\beta}_A[w;a]$ form has terms $f_n(w)h_n(a)$ where some $f_n(w)$ are *not* linear in $w^\alpha{}_\mu$, the coupled differential equations $H^{\alpha\beta}_A[w;a] = 8\pi E^{\alpha\beta}_A$ may yield a *complex-valued* global IC $w^\alpha{}_\mu$ solution due to the non-linearity of $w^\alpha{}_\mu$. Indeed, it is expected that non-linearity in the $H^{\alpha\beta}_A[w;a]$ terms would yield complex-valued solutions in some $E^{\alpha\beta}_A$ cases. Therefore, in order that only real-valued global IC $w^\alpha{}_\mu$ solutions are yielded over the entire range of possible systems that may be modelled, it is concluded that $H^{\alpha\beta}_A[w;a]$ *must consist of terms that are linear in $w^\alpha{}_\mu$ exclusively*. Note that achieved real-valued global IC $w^\alpha{}_\mu$ solutions, assuming an $H^{\alpha\beta}_A[w;a]$ linear in $w^\alpha{}_\mu$, coordinate transform to again yield real-valued $w^\alpha{}_\mu$ solutions for the absolute field equation given in any coordinates.

Summarizing from above, the $H^{\alpha\beta}_A[w;a]$ terms $f_n(w)h_n(a)$ are restricted to being linear in $w^\alpha{}_\mu$ and $N = 2$ only. Using "w" and "a" to generically represent any raised/lowered index forms for $w^\alpha{}_\mu$ and

$a_{\alpha\beta}$ as well as the scalar $w^\alpha{}_\alpha$, and "a^m" to represent any product of $a_{\alpha\beta}$ with itself (including $a^0 \equiv 1$), *all possible* global IC given $N = 2$ $H_A^{\alpha\beta}[w; a]$ terms $f_n(w)h_n(a)$, with $f_n(w)$ linear in $w^\alpha{}_\mu$, take the *single form* $\breve{w}_{,\alpha\beta}\,\eta^m$, where $\eta_{\alpha\beta} = \breve{a}_{\alpha\beta}$. Since absolute covariant derivatives $\nabla_{\breve\mu}^A$ in global ICs are simply partial derivatives $\partial_{\breve\mu}$, then the global IC $\breve{w}_{,\alpha\beta}\,\eta^m$ may be given in covariant form by $w_{|\alpha\beta}\,a^m$, applicable in *any coordinates*. The form $w_{|\alpha\beta}\,a^m$ generically represents multiple possible covariant terms in various index configurations of the contained $w^\alpha{}_\mu$ and $a_{\alpha\beta}$, with each configuration required to have free raised indices α and β to be an $H_A^{\alpha\beta}$ contributor. As can be seen, the combined requirements of $w^\alpha{}_\mu$ linearity and $N = 2$ result in a relatively simple form for $H_A^{\alpha\beta}[w; a]$.

As discussed in Section 5.3, required absolutely measured total EM conservation, given by $\partial_{\breve\nu}^{(A)}\breve{E}_{A(A)}^{\mu\nu} = 0$ in absolute inertial frames, yields $E_A^{\alpha\beta}{}_{|\beta} = 0$ (218) in any coordinates. When applied to the absolute field equation $H_A^{\alpha\beta}[w; a] = 8\pi E_A^{\alpha\beta}$, then $H_A^{\alpha\beta}{}_{|\beta} = 0$ must hold. Now it may be reasonably assumed that the vanishing absolute field tensor divergence, $\nabla_\beta^A H_A^{\alpha\beta}[w; a] = 0$, is a condition that is *independent of* the natural field equation $H_N^{\alpha\beta}[w; g] = 8\pi T_N^{\alpha\beta}$, natural EM conservation $T_N^{\alpha\beta}{}_{;\beta} = 0$, and the resultant natural field tensor based field constraint $\nabla_\beta^N H_N^{\alpha\beta}[w; g(w, a)] = 0$ (271) obtained by applying $T_N^{\alpha\beta}{}_{;\beta} = 0$ to the NFE. Therefore, unless $\nabla_\beta^A H_A^{\alpha\beta}[w; a] = 0$ is an identity, it would act to apply an *additional constraint* to the potential field $w^\alpha{}_\mu$ that is *completely determined* by the NFE $H_N^{\alpha\beta}[w; g] = 8\pi T_N^{\alpha\beta}$ used in conjunction with natural EM conservation $T_N^{\alpha\beta}{}_{;\beta} = 0$. The potential field $w^\alpha{}_\mu$ would therefore be *overdetermined* if $H_A^{\alpha\beta}{}_{|\beta} = 0$ were not an identity, contradicting use of the NFE fully determining $w^\alpha{}_\mu$. It is concluded that $H_A^{\alpha\beta}{}_{|\beta} = 0$ must indeed be an *identity*, as stated by the "absolute divergence identity"

$$\nabla_\beta^A H_A^{\alpha\beta}[w; a] \equiv 0, \tag{282}$$

in order that application of absolute total EM conservation $E_A^{\alpha\beta}{}_{|\beta} = 0$ to the absolute field equation $H_A^{\alpha\beta}[w; a] = 8\pi E_A^{\alpha\beta}$ not contradict use of the natural field equation. Turning this around, accepting that the absolute divergence identity (282) holds, its application to the AFE yields absolute total EM conservation in the form of $E_A^{\alpha\beta}{}_{|\beta} = 0$, *regardless* of the potential solution $w^\alpha{}_\mu$ applied in (282) such as if obtained from the NFE. As shown above, assumed satisfaction of the total SE requirement for the $H_A^{\alpha\beta}[w; a]$ form yields a stand-alone AFE $w^\alpha{}_\mu$ solution (for any given system with then a given $E_A^{\alpha\beta}$) that is the same as the NFE solution, so with the restriction (282) placed on the $H_A^{\alpha\beta}[w; a]$ form, absolute total EM conservation $E_A^{\alpha\beta}{}_{|\beta} = 0$ is achieved while *preserving* the same AFE $w^\alpha{}_\mu$ solution as for the NFE.

Since the potential $w^\alpha{}_\mu$ consists of ten algebraically independent components (under its pure form symmetry), then the absolute field equation $H_A^{\alpha\beta}[w; a] = 8\pi E_A^{\alpha\beta}$ provides *ten* algebraically independent conditions. The absolute divergence identity $\nabla_\beta^A H_A^{\alpha\beta}[w; a] \equiv 0$ (282) consists of four functionally independent conditions. As a result, if the AFE $H_A^{\alpha\beta}[w; a] = 8\pi E_A^{\alpha\beta}$ is utilized as a stand-alone field equation to obtain its solution $w^\alpha{}_\mu$ for a system with a given $E_A^{\alpha\beta}$ (known or not), adherence to the absolute divergence identity implies that the AFE consists of only *six* functionally independent conditions for determining $w^\alpha{}_\mu$. This is similar to Einstein's equation $G^{\alpha\beta} = 8\pi T^{\alpha\beta}$ providing only six functionally independent conditions for determining $g_{\alpha\beta}$ due to the Bianchi identity $G^{\alpha\beta}{}_{;\beta} \equiv 0$ holding. As discussed above, for Einstein equation use four additional coordinate conditions are applied to the natural metric $g_{\alpha\beta}$, setting the coordinates, in order to fully determine it. On the other hand, in GS theory, *a priori* absolute manifold coordinates (such as global ICs) may be utilized when solving the absolute field equation, so the coordinates have already been set. As a result, the six functionally independent conditions provided by the AFE under application of the absolute divergence identity, $\nabla_\beta^A H_A^{\alpha\beta}[w; a] \equiv 0$, *are not sufficient for fully determining the potential tensor field $w^\alpha{}_\mu$ consisting of ten algebraically independent components*. However, as required, the AFE $w^\alpha{}_\mu$ solution must be the same as the NFE solution. Since the NFE $w^\alpha{}_\mu$ solution is subject to the natural field constraint $\nabla_\beta^N H_N^{\alpha\beta}[w; g(w, a)] = 0$ (271), the natural field constraint *similarly applies* to the AFE $w^\alpha{}_\mu$ solution. *The*

natural field constraint (271) *provides the additional four functionally independent conditions required to fully determine the potential field $w^\alpha{}_\mu$ when using the absolute field equation to determine $w^\alpha{}_\mu$ based on a given $E_A^{\alpha\beta}$.* Since the $H_A^{\alpha\beta}[w;a]$ form is assumed to satisfy the total SE requirement, then when the NFE $w^\alpha{}_\mu$ solution is applied to the AFE $H_A^{\alpha\beta}[w;a] = 8\pi E_A^{\alpha\beta}$ to obtain $E_A^{\alpha\beta}$ for a given system, adherence to the natural field constraint $H_N^{\alpha\beta}{}_{;\beta} = 0$ *already applies.* This implies that "in reverse," when the natural field constraint is combined with the AFE $H_A^{\alpha\beta}[w;a] = 8\pi E_A^{\alpha\beta}$ to fully determine the AFE $w^\alpha{}_\mu$ solution for a system with a given $E_A^{\alpha\beta}$, the AFE $w^\alpha{}_\mu$ solution will indeed be the same as the NFE solution, satisfying the matched solution requirement. The *physical significance* of applying $H_N^{\alpha\beta}{}_{;\beta} = 0$ is that *adherence to both natural EM conservation $T_N^{\alpha\beta}{}_{;\beta} = 0$ and the natural field equation $H_N^{\alpha\beta} = 8\pi T_N^{\alpha\beta}$ is incorporated into solving the absolute field equation*, as both are required to obtain the natural field constraint $H_N^{\alpha\beta}{}_{;\beta} = 0$. In discussion above concerning stand-alone AFE $w^\alpha{}_\mu$ solution for a system with a given $E_A^{\alpha\beta}$, application of the natural field constraint $H_N^{\alpha\beta}{}_{;\beta} = 0$ was considered to be in effect since the AFE $w^\alpha{}_\mu$ solution is required to be the same as the NFE solution. For convenience when referring to the "absolute field equation," the natural field constraint $H_N^{\alpha\beta}{}_{;\beta} = 0$ may be included, with discernment from the absolute field equation $H_A^{\alpha\beta} = 8\pi E_A^{\alpha\beta}$ alone made by context.

7.2. Lagrangian Formulation

As is commonly assumed for field equations in gravitational physics, the absolute field equation is assumed to result from use of a Lagrangian-based formulation. This is expected since the AFE is both the partner of the NFE and derived from the NFE (as above), with the NFE constructed using a Lagrangian formulation. In addition, as discussed the classical AFE being developed here is assumed to be the classical limit of a quantum gravitational AFE where the gravitational field is treated as an ordinary force field similar to the nongravitational fields, and as such would be expected to be obtainable from a Lagrangian formulation the same as the field equations for the nongravitational fields. Since the AFE is the partner of the NFE, the AFE Lagrangian formulation here is constructed as the partner of the NFE formulation made above. Following this methodology, the "absolute action" is defined by

$$S_A \equiv \frac{1}{16\pi} S_{HA} + S_E, \tag{283}$$

where $S_E = S_E[q_\lambda^A, w, a]$ is the "(absolute total) energy action" for all absolutely observed energy-momentum sources consisting of all matter, all nongravitational fields, and the gravitational field, and where S_{HA} is the "(absolute) field action" defined by

$$S_{HA} \equiv \int \sqrt{-a}\, L_{HA}\, \mathrm{d}^4 x \tag{284}$$

with L_{HA} the "absolute field Lagrangian." With the potential $w^\alpha{}_\mu$ the field operand for the absolute field equation $H_A^{\alpha\beta}[w;a] = 8\pi E_A^{\alpha\beta}$ (278), the field variation applied to the absolute action S_A is the potential variation $\delta w^\alpha{}_\mu$. Combining the field variation $\delta w^\alpha{}_\mu$ with the variation δq_λ^A for each native absolute nongravitational quantity, q_λ^A, yields the complete absolute action variation

$$\delta S_A = \int \mathrm{d}^4 x\, \frac{\delta S_A}{\delta w^\alpha{}_\mu}\, \delta w^\alpha{}_\mu + \int \mathrm{d}^4 x\, \frac{\delta S_A}{\delta q_\lambda^A}\, \delta q_\lambda^A. \tag{285}$$

Applying the principle of least action, the action variation δS_A is set to zero, yielding the absolute field equation as the Euler-Lagrange (EL) equation

$$\frac{\delta S_A}{\delta w^\alpha{}_\mu} = 0 \tag{286}$$

giving the critical point for the absolute action S_A under the field variation $\delta w^\alpha{}_\mu$. Similarly, the critical point for the absolute action S_A under the variation δq^A_λ for each native absolute nongravitational quantity, q^A_λ, is the EL equation

$$\frac{\delta S_E[q^A_\lambda, w, a]}{\delta q^A_\lambda} = 0 \tag{287}$$

giving the absolute equation of motion for q^A_λ, where $\delta S_A / \delta q^A_\lambda = \delta S_E[q^A_\lambda, w, a] / \delta q^A_\lambda$ was utilized since the field action S_{HA} has no q^A_λ quantity dependence.

The absolute partner formulations for (245) and (246) are

$$H_{\alpha A}{}^\mu[w; a] = \frac{1}{2} \frac{1}{\sqrt{-a}} \frac{\delta S_{HA}[w; a]}{\delta w^\alpha{}_\mu}, \tag{288}$$

$$E_{\alpha A}{}^\mu[q^A_\lambda, w, a] = -\frac{1}{\sqrt{-a}} \frac{\delta S_E[q^A_\lambda, w, a]}{\delta w^\alpha{}_\mu}, \tag{289}$$

which are used to *define* the field action $S_{HA} = S_{HA}[w; a]$ and the energy action $S_E = S_E[q^A_\lambda, w, a]$, as well as via (284) to define the absolute field Lagrangian $L_{HA} = L_{HA}[w; a]$. Note the functional dependencies for S_{HA}, S_E, and L_{HA} are established via these definitions, which apply in the EL equations above. Substituting S_A given by (283) into the EL field equation (286), applying (288) and (289), and lowering by the absolute metric, yields

$$H^A_{\alpha\beta}[w; a] = \frac{1}{2} \frac{1}{\sqrt{-a}} \frac{\delta S_{HA}[w; a]}{\delta w^\alpha{}_\mu} a_{\mu\beta} = 8\pi E^A_{\alpha\beta} \tag{290}$$

for the EL absolute field equation, with the "middle" term giving $H^A_{\alpha\beta}[w; a]$. The derivation of the AFE via their use justifies the definitions (288) and (289).

The absolute total energy action may be given by

$$S_E[q^A_\lambda, w, a] = S_M[q^N_\lambda(q^A_\lambda, w), g(w, a)] - \frac{1}{16\pi}\{S_{HA}[w; a] - S_{HN}[w; g(w, a)]\}. \tag{291}$$

To show this is the case, applying the $w^\alpha{}_\mu$ functional derivative across (291), and using (246) (under universal natural metric coupling to $q_\lambda = q^N_\lambda$), (245), and (288), yields

$$\frac{\delta S_E}{\delta w^\alpha{}_\mu} = -\sqrt{-g}\, T_{\alpha N}{}^\mu - \frac{1}{16\pi}\left\{2\sqrt{-a}\, H_{\alpha A}{}^\mu - 2\sqrt{-g}\, H_{\alpha N}{}^\mu\right\}.$$

Substituting for $\delta S_E / \delta w^\alpha{}_\mu$ via (289), and using $\sqrt{-g} = \sqrt{-a}\,|S^{-1}|$ (from (37)), readily yields (274) giving $E_{\alpha A}{}^\mu$, completing the proof that application of (289) utilizing (291) for the energy action $S_E[q^A_\lambda, w, a]$ yields the absolute total SE tensor $E_{\alpha A}{}^\mu[q^A_\lambda, w, a]$. Substituting (291) into (283), and using (232), yields

$$S_A[q^A_\lambda, w, a] \equiv S_N[q^N_\lambda(q^A_\lambda, w), w, g(w, a)], \tag{292}$$

stating the *formal equivalence* of the absolute and natural actions. This interesting result comes about due to the AFE being an alternate form of the NFE as shown above, so the equivalence of their actions is not so unexpected. But similar to the AFE $H^{\alpha\beta}_A = 8\pi E^{\alpha\beta}_A$ providing new information about a system that the NFE does not provide, namely, its total SE tensor $E^{\alpha\beta}_A$ and its relation to the gravitational field, the "parent" absolute action $S_A = (1/16\pi)S_{HA} + S_E$ for the AFE again provides new information about a system that the natural action S_N does not provide, namely, its total energy action S_E and similarly its relation to the gravitational field.

Application of (287), utilizing (291) for $S_E[q^A_\lambda, w, a]$, yields

$$\frac{\delta S_E[q^A_\lambda, w, a]}{\delta q^A_\lambda} = \frac{\delta S_M[q^N_\lambda(q^A_\lambda, w), g(w, a)]}{\delta q^A_\lambda} = \frac{\delta S_M[q^N_\lambda, g]}{\delta q^N_\lambda} \frac{\partial q^N_\lambda(q^A_\lambda, w)}{\partial q^A_\lambda} = 0. \tag{293}$$

As can be seen, the absolute equations of motion (287) based on use of the absolute total energy action, S_E, reduce to absolute motion equations based on the familiar natural matter action S_M used in general relativity and metric-based theories in general (such as above for GS theory), where all that is required to obtain the absolute motion equations is to use the partner relations $q_\lambda^N = q_\lambda^N(q_\lambda^A, w)$ in $S_M[q_\lambda^N, g]$ to form $S_M[q_\lambda^N(q_\lambda^A, w), g]$ prior to taking the q_λ^A functional derivatives. Equation (293) shows that the absolute q_λ^A functional derivative of the matter action, $S_M[q_\lambda^N(q_\lambda^A, w), g]$, is simply the familiar natural q_λ^N functional derivative of $S_M[q_\lambda^N, g]$ multiplied by the partial functional derivative $\partial q_\lambda^N(q_\lambda^A, w)/\partial q_\lambda^A$, with $\delta S_M[q_\lambda^N, g]/\delta q_\lambda^N = 0$ the natural equation of motion (236) for the natural nongravitational quantity q_λ^N that is the partner of the absolute quantity q_λ^A. Therefore, (293) provides the *partner relation* between the partner absolute and natural equations of motion for the partner native nongravitational quantities q_λ^A and q_λ^N. Note that adherence to the familiar natural motion equation $\delta S_M[q_\lambda^N, g]/\delta q_\lambda^N = 0$ for a natural quantity, q_λ^N, yields via (293) adherence to the partner absolute motion equation $\delta S_M[q_\lambda^N(q_\lambda^A, w), g]/\delta q_\lambda^A = 0$ for the partner absolute quantity q_λ^A.

What remains for the absolute field equation development is to establish the absolute field Lagrangian $L_{HA}[w; a]$ used to construct the field action $S_{HA}[w; a]$ given by (284). The usual variational techniques (as discussed above for the NFE) are again employed to "automatically" generate the S_{HA} functional derivative utilized in the EL absolute field equation (290) employed to obtain $H_{\alpha\beta}^A[w; a]$.

Proceeding with the determination of $L_{HA}[w; a]$, similar to $H_A^{\alpha\beta}[w; a]$ given by (273), based on their functional dependence $L_{HA}[w; a]$ and $S_{HA}[w; a]$ each consist of a sum of terms of the form $f_n(w)h_n(a)$, where again the contained $w^\alpha{}_\mu$ and $a_{\alpha\beta}$ may be partially differentiated. From above, all $H_A^{\alpha\beta}[w; a]$ terms $f_n(w)h_n(a)$ must be $N = 2$ where $f_n(w)$ is linear in $w^\alpha{}_\mu$. These same requirements hold for the absolute metric lowered field tensor $H_{\alpha\beta}^A[w; a]$ in (290). Utilizing (284) in (290) to obtain $H_{\alpha\beta}^A[w; a]$, then employing the usual variational techniques, all $L_{HA}[w; a]$ terms $f_n(w)h_n(a)$ must also be $N = 2$. Now $\delta S_{HA}[w; a]/\delta w^\alpha{}_\mu$ in (290), with $S_{HA}[w; a]$ given by (284), yields $H_{\alpha\beta}^A[w; a]$ terms $f_n(w)h_n(a)$ that are one factor of $w^\alpha{}_\mu$ less than in the "parent" $L_{HA}[w; a]$ terms $f_n(w)h_n(a)$. As such, $L_{HA}[w; a]$ terms $f_n(w)h_n(a)$ where $f_n(w)$ consists of *quadratic* products of the potential, $w^\alpha{}_\mu$, will generate $H_{\alpha\beta}^A[w; a]$ terms $f_n(w)h_n(a)$ that are *linear* in $w^\alpha{}_\mu$ as required. Therefore, the absolute field Lagrangian $L_{HA}[w; a]$ must consist of $N = 2$ terms $f_n(w)h_n(a)$ where $f_n(w)$ consists of quadratic $w^\alpha{}_\mu$ products only. In contrast, recall that the natural field Lagrangian $L_{HN}[w; g]$ consists of $N = 2$ terms that are linear in $w^\alpha{}_\mu$ or do not contain it.

Using "w" and "a" to again generically represent any raised/lowered index forms for $w^\alpha{}_\mu$ and $a_{\alpha\beta}$ as well as the scalar $w^\alpha{}_\alpha$, and "a^m" to represent any product of $a_{\alpha\beta}$ with itself (including $a^0 \equiv 1$), *all possible* global IC given $N = 2$ $L_{HA}[w; a]$ terms $f_n(w)h_n(a)$, with $f_n(w)$ quadratic in $w^\alpha{}_\mu$ products, only take the *two forms* $\breve{w}_{,\alpha}\breve{w}_{,\beta}\eta^m$ and $\breve{w}\breve{w}_{,\alpha\beta}\eta^m$, where $\eta_{\alpha\beta} = \breve{a}_{\alpha\beta}$. Since $\partial_{\breve\mu} = \nabla_{\breve\mu}^A$ in global ICs, then these forms may be given covariantly by $w_{|\alpha} w_{|\beta} a^m$ and $w w_{|\alpha\beta} a^m$, applicable in *any coordinates*. Each of these terms generically represents multiple possible covariant terms in various index configurations of the contained $w^\alpha{}_\mu$ and $a_{\alpha\beta}$, with each configuration required to yield a scalar to be a contributor for the scalar $L_{HA}[w; a]$. Each possible $L_{HA}[w; a]$ contributor may be evaluated by using (284) to form its S_{HA} contribution, and then the $w^\alpha{}_\mu$ functional derivative is taken applying the usual variational techniques, yielding the contributor's $H_{\alpha\,A}{}^{\mu}$ contribution via (288). Using integration by parts and dropping boundary surface integrals, every possible $w w_{|\alpha\beta} a^m$ configuration for construction of the Lagrangian, $L_{HA}[w; a]$, yields an $H_{\alpha\,A}{}^{\mu}$ contribution that is a sign-reversed form of an $H_{\alpha\,A}{}^{\mu}$ contribution generated by a particular $w_{|\alpha} w_{|\beta} a^m$ configuration. Dropping the redundant $w w_{|\alpha\beta} a^m$ terms, only the "symmetric form" $w_{|\alpha} w_{|\beta} a^m$ based terms are utilized for $L_{HA}[w; a]$ construction. Generating the *specific* $L_{HA}[w; a]$ terms, every possible independent scalar index configuration for the generic $w_{|\alpha} w_{|\beta} a^m$ is as follows: $w^\alpha{}_{\mu|\sigma} w^\mu{}_\alpha{}^{|\sigma}$, $w^\alpha{}_{\mu|\sigma} w^\sigma{}_\alpha{}^{|\mu}$, $w^\mu{}_{\alpha|\mu} w^\sigma{}_\sigma{}^{|\alpha}$, and $w^\mu{}_{\mu|\alpha} w^\sigma{}_\sigma{}^{|\alpha}$. Note that index configurations equal to the given ones are not listed, as they would be redundant. Again due to redundancy, also not listed is any configuration that yields an $H_{\alpha\,A}{}^{\mu}$ contribution proportional to one generated by the provided list.

Using the list of possible $L_{HA}[w;g]$ terms, the most general absolute field Lagrangian that may be formed is

$$L_{HA}[w;a] = a\, w^{\alpha}{}_{\mu|\sigma} w^{\mu}{}_{\alpha}{}^{|\sigma} + b\, w^{\alpha}{}_{\mu|\sigma} w^{\sigma}{}_{\alpha}{}^{|\mu} + c\, w^{\mu}{}_{\alpha|\mu} w^{\sigma}{}_{\sigma}{}^{|\alpha} + d\, w^{\mu}{}_{\mu|\alpha} w^{\sigma}{}_{\sigma}{}^{|\alpha}, \tag{294}$$

where a, b, c, and d are arbitrary constants. This is the *most general possible Lagrangian* that may be formed under the assumed requirements for formulation of the absolute field equation.

Substituting (294) into (284), and applying the usual variational techniques as discussed above, the potential functional derivative of S_{HA} is given by

$$\frac{1}{\sqrt{-a}}\frac{\delta S_{HA}[w;a]}{\delta w^{\alpha}{}_{\mu}} = -2\,a\, w^{\mu}{}_{\alpha|}{}^{\sigma}{}_{\sigma} - b\,(w^{\sigma\mu}{}_{|\alpha\sigma} + w^{\sigma}{}_{\alpha|}{}^{\mu}{}_{\sigma}) - c\,(w_{|}{}^{\mu}{}_{\alpha} + \delta^{\mu}{}_{\alpha}\,w^{\sigma}{}_{\lambda|}{}^{\lambda}{}_{\sigma}) - 2\,d\,\delta^{\mu}{}_{\alpha}\,w_{|}{}^{\sigma}{}_{\sigma}. \tag{295}$$

Using this in (290) (and utilizing the $w_{\alpha\beta}$ symmetry) yields the absolute field equation

$$\begin{aligned}
H^{A}_{\alpha\beta}[w;a] &= -a\,w_{\alpha\beta|}{}^{\sigma}{}_{\sigma} - b\,w^{\sigma}{}_{(\alpha|\beta)\sigma} - \frac{1}{2}\,c\,(w_{|\alpha\beta} + a_{\alpha\beta}\,w^{\sigma}{}_{\lambda|}{}^{\lambda}{}_{\sigma}) - d\,a_{\alpha\beta}\,w_{|}{}^{\sigma}{}_{\sigma} \\
&= 8\pi\,E^{A}_{\alpha\beta}[q^{A}_{\lambda},w,a],
\end{aligned} \tag{296}$$

providing the most general possible form for the absolute field tensor $H^{A}_{\alpha\beta}$ under the assumed requirements (as applied thus far). Note that all of the $H^{A}_{\alpha\beta}[w;a]$ terms take the generic $N=2$ form $w_{|\alpha\beta}\,a^{m}$ linear in the potential $w^{\alpha}{}_{\mu}$, as expected from above.

7.3. Application of the Linearized Case and Absolute Divergence Identity Requirements, and Verification of the Total SE Requirement

As established above, the linearized absolute field tensor $H^{A}_{\alpha\beta}[p;\eta]$ must satisfy the field tensor equivalence (280) in order that in all possible linearized (and therefore weak limit) cases, the absolute field equation $H^{A}_{\alpha\beta}[p;\eta] = 8\pi T^{N}_{\alpha\beta}$ (279) yields the same field solution $p_{\alpha\beta}$ (as defined by $g_{\alpha\beta} = \eta_{\alpha\beta} + p_{\alpha\beta}$) as the natural field equation $H^{N}_{\alpha\beta}[p;\eta] = 8\pi T^{N}_{\alpha\beta}$ and therefore Einstein's equation $G_{\alpha\beta}[p;\eta] = 8\pi T^{N}_{\alpha\beta}$. Utilizing global ICs for linearized formulation (as per usual from above), then $a_{\alpha\beta} = \eta_{\alpha\beta}$, so the absolute metric covariant derivatives reduce to partial derivatives in (296) giving $H^{A}_{\alpha\beta}[w;a]$, yielding a global IC $H^{A}_{\alpha\beta}[w;\eta]$ that is *already in linearized form*. So the linearized relation (253) may be applied to yield

$$H^{A}_{\alpha\beta}[p;\eta] = \frac{1}{2}\,a\,p_{\alpha\beta,}{}^{\sigma}{}_{\sigma} + \frac{1}{2}\,b\,p^{\sigma}{}_{(\alpha,\beta)\sigma} + \frac{1}{4}\,c\,(p_{,\alpha\beta} + \eta_{\alpha\beta}\,p^{\sigma}{}_{\lambda,}{}^{\lambda}{}_{\sigma}) + \frac{1}{2}\,d\,\eta_{\alpha\beta}\,p_{,}{}^{\sigma}{}_{\sigma}. \tag{297}$$

A comparison of (297) with (252) giving the linearized Einstein tensor, $G_{\alpha\beta}[p;\eta]$, shows that in order for the equivalence $H^{A}_{\alpha\beta}[p;\eta] \equiv G_{\alpha\beta}[p;\eta]$ (280) to hold, the constants in (297) must have the *unique values*

$$a = -1, \quad b = 2, \quad c = -2, \quad d = 1. \tag{298}$$

These values are assumed then, satisfying the linearized case requirement (280).

Using the values (298) for the absolute field tensor in (296), the resultant absolute field equation is *unique*. Note that similar to the natural field equation, it is *parameterless*. The symmetric absolute metric based trace-reverse for a symmetric tensor $B_{\alpha\beta}$ is defined by

$$\overline{B}_{\alpha\beta} \equiv B_{\alpha\beta} - \frac{1}{2}\,a_{\alpha\beta}B, \tag{299}$$

where the scalar B is the trace $B = a^{\sigma\alpha}B_{\alpha\sigma} = B^{\sigma}{}_{\sigma}$. A convenient property of the absolute trace-reverse is $\overline{\overline{B}}_{\alpha\beta} = B_{\alpha\beta}$, so the trace-reverse is its own inverse. Utilizing the trace-reverse, substitution of the constant values (298) in (296) yields

$$H^{A}_{\alpha\beta}[w;a] = \overline{Q}^{A}_{\alpha\beta}[w;a] = 8\pi\,E^{A}_{\alpha\beta}[q^{A}_{\lambda},w,a] \tag{300}$$

for the absolute field equation, where $Q_{\alpha\beta}^A[w; a]$ is the symmetric "absolute potential Ricci tensor" defined by

$$Q_{\alpha\beta}^A[w; a] \equiv w_{|\alpha\beta} + w_{\alpha\beta}{}^\sigma{}_\sigma - 2\, w^\sigma{}_{(\alpha\,|\,\beta)\,\sigma}. \tag{301}$$

Note that $Q_{\alpha\beta}^A[w; a]$ has the same form as $P_{\alpha\beta}[w; g]$ (259) contained in the natural potential Ricci tensor $Q_{\alpha\beta}^N[w; g]$ (260), but with the *absolute metric*, as opposed to the natural metric, utilized to raise/lower indices and form covariant derivatives. Applying the absolute trace-reverse across (300) yields the alternate trace-reverse form

$$\overline{H}{}_{\alpha\beta}^A[w; a] = Q_{\alpha\beta}^A[w; a] = 8\pi\, \overline{E}{}_{\alpha\beta}^A[q_\Lambda^A, w, a] \tag{302}$$

of the absolute field equation. Unlike $Q_{\alpha\beta}^N[w; g] = 0$ (263) in a vacuum, $Q_{\alpha\beta}^A[w; a]$ is generally non-zero in a vacuum due to $\overline{E}{}_{\alpha\beta}^A = \bar{t}{}_{\alpha\beta}^A$ generally being non-zero.

A direct evaluation of $H_A^{\alpha\beta}{}_{|\beta} = \overline{Q}{}_A^{\alpha\beta}{}_{|\beta}$, with $Q_A^{\alpha\beta}$ given by (301) raised by the absolute metric, yields satisfaction of the absolute divergence identity $\nabla_\beta^A H_A^{\alpha\beta}[w; a] \equiv 0$ (282). So as required, *the absolute divergence identity is satisfied for the established absolute field tensor $H_A^{\alpha\beta}$*. Instead of going through a direct evaluation, satisfaction of $H_A^{\alpha\beta}{}_{|\beta} \equiv 0$ may be understood by using global ICs, in which case $H_A^{\alpha\beta}[w; n]$ takes a linearized form similar to $H_{\alpha\beta}^A[w; \eta]$ discussed above. This implies that via the use of (253), $H_A^{\alpha\beta}[w; n] \equiv -\frac{1}{2}H_A^{\alpha\beta}[p; n]$ as an equivalence of *forms*, and with (280) holding, then $H_A^{\alpha\beta}[w; n] \equiv -\frac{1}{2}G^{\alpha\beta}[p; n]$. As the linearized Bianchi identity $\partial_\beta G^{\alpha\beta}[p; n] \equiv 0$ holds in global ICs, then $\partial_\beta H_A^{\alpha\beta}[w; n] \equiv \nabla_\beta^A H_A^{\alpha\beta}[w; a] \equiv 0$ in global ICs, yielding $\nabla_\beta^A H_A^{\alpha\beta}[w; a] \equiv 0$ in any coordinates. Note that application of the absolute divergence identity as a requirement to constrain the form for $H_{\alpha\beta}^A[w; a]$ given by (296), did not in fact further constrain the form for $H_{\alpha\beta}^A[w; a]$ yielded after application of the linearized case requirement. However, the absolute divergence identity may still be considered to have been applied, since alternately it may be applied first to partially constrain the constants a through d in (296), and then the linearized case requirement is imposed to complete constraint application, yielding again $H_{\alpha\beta}^A[w; a]$ given by $\overline{Q}{}_{\alpha\beta}^A[w; a]$.

The remaining requirement for the $H_{\alpha\beta}^A[w; a]$ form is the total SE requirement, which is verified here for the static star case where the total SE tensor $E_{\alpha\beta}^A$ is known a priori for static systems (outside their sources). To establish $E_{\alpha\beta}^A$ for static systems, consider a mass particle moving under the gravitational field of a static system. From general relativity, the temporal component p_0^N of the natural momentum 1-form for a particle remains *constant* as it moves through a static field (see Schutz [18], Chap. 7). In GS theory, with the native absolute and natural momentum 1-forms $\tilde{p}^A$ and $\tilde{p}^N$ being equal (as per (186)), then p_0^A is constant as well. Utilizing global ICs so that the absolute metric is the fixed Minkowski metric, then the absolute energy $E_A = p_A^0 = m_\alpha^{0A} U_A^\alpha$ (using (201)) obtained by raising p_α^A by the absolute metric $a^{\alpha\beta} = \eta^{\alpha\beta}$, is given by $E_A = p_A^0 = -p_0^A$. So remarkably, *the global IC given absolute energy E_A of a mass particle remains constant as it moves through a static gravitational field*. The constancy of E_A for a mass particle is commensurate with the constancy of the global IC given $E_A = h\nu$ for a photon moving through a static field. Now the absolute kinetic energy K_A changes as a mass particle moves, but this is compensated by an equal but opposite change in the absolute mass-energy E_A^M (from all contained energy sources) under the locational change in the gravity shifting applied to the matter making up the particle (the influence of shifting is quantified by the $(d\tau_A/d\tau_N)F_\alpha^0$ factor in (202) giving m_α^{0A} contained in $E_A = m_\alpha^{0A} U_A^\alpha$), so the total energy $E_A = K_A + E_A^M$ remains constant. With E_A remaining constant, then when utilizing global ICs, *the gravitational field for a static system does not transfer absolute energy to a mass particle as it moves through the field, and similarly for a photon*.

Consider the construction of a static gravitational system by assembling it using the following process. Utilizing global ICs, an infinitesimally massed particle is set at a particular location. Then holding this particle at the fixed location, another infinitesimally massed particle is allowed to come in from an effectively infinite distance, free-falling until it merges with the fixed particle. This merged

"central mass" is held fixed while another infinitesimally massed particle is allowed to free-fall from an infinite distance and merge with the central mass. The process is repeated until the desired static system is yielded, consisting of a finitely sized and massed static central mass source with a surrounding static field (the central mass may be shaped by controlling the merge locations of the incoming particles as the central mass is built up). In constructing the system, the original locations of the incoming particles are all set at an effectively infinite distance from one another in addition to being set an infinite distance from the central mass location. With each particle having an infinitesimal mass, then the "original state" consisting of all of the particles combined, but each with an infinite separation from the rest, has a vanishing gravitational field strength. Therefore, $w^\alpha{}_\mu = 0$ for the original state, so via (276) the absolute field SE tensor $t_A^{\alpha\beta}$ is zero, yielding a zero-valued absolute gravitational field energy E_A^G by integrating $t_A^{\alpha\beta}$ over all space. Now since the gravitational field for a static system does not transfer absolute energy to a mass particle moving through it, then the field energy E_A^G *remains zero* as each incoming particle comes in through the static field of the central mass then present. This property holds in particular for the field *outside* the central mass, as any transfer of energy must come from the local region an incoming particle is passing through, with all local regions of travel located outside the central mass once the particle merges with it. Therefore, $E_A^G = 0$ outside the central mass of the assembled static system, so in general *the global IC given absolute gravitational field energy E_A^G is zero outside the source mass of a static system*. Note that the total energy $E_A^T = E_A^M(\text{orig}) + E_A^G$ of the original state consists of only the combined rest-mass energy $E_A^M(\text{orig})$ of all of the masses, since $E_A^G = 0$ for the entire original state. Then under absolute energy conservation, the total energy $E_A^T = E_A(\text{central}) + E_A^G$ of the assembled "final state" is the same as $E_A^M(\text{orig})$, where $E_A(\text{central})$ is the energy of the central mass from all contained energy sources, and E_A^G is the zero-valued field energy outside the central mass. Therefore, $E_A(\text{central}) = E_A^M(\text{orig})$, as expected since with the absolute energy E_A of a particle not changing as it travels though a static field, then when assembling the static system using the above process, each incoming particle contributes only its *original rest-mass energy $E_A^M(\text{particle})$* when it merges with the central mass.

Gravity wave generation due to mass particle acceleration was not considered in the above construction, which reduces the kinetic energy K_A of an incoming particle and therefore the assembled central mass energy $E_A(\text{central})$. But gravity wave generation may be considered to operate as an *independent* energy dissipation mechanism that does not impact the lack of absolute energy transfer between the static field and the incoming particle, preserving the above $E_A^G = 0$ result outside the assembled central mass. Note that an incoming particle will generate gravity waves on impact with the central mass due to sudden deceleration, but this reduces $E_A(\text{central})$ without affecting the lack of energy transfer between the static field and the incoming particle, again preserving the $E_A^G = 0$ result.

It may be reasonably assumed that the global IC given energy component t_A^{00} of the absolute field SE tensor, $t_A^{\alpha\beta}$, has the same sign at all locations outside the source mass of a static system. Then with $E_A^G = 0$ for the field energy outside the source mass, $t_A^{00} = 0$ at all outside locations. Note that inside the static source mass, the internal stresses and pressures present may result in a non-zero t_A^{00}, with $t_A^{00} \to 0$ at the surface since $t_A^{00} = 0$ in the vacuum outside the mass. Since at the quantum level the gravitational field may be considered to be constructed from virtual exchange (spin-2) gravitons, then with the graviton field having a zero-valued global IC classical energy density t_A^{00}, it may be inferred that the other "momentum components" of the classical field SE tensor $t_A^{\alpha\beta}$ are zero valued as well (such as would be expected if the graviton field were treated as a gas with a pressure proportional to its energy density). With all components of the global IC $t_A^{\alpha\beta}$ being zero valued, then $t_A^{\alpha\beta} = 0$ in any coordinates. Summarizing, for a static system, *the absolute field SE tensor $t_A^{\alpha\beta}$ is zero (in any coordinates) in the vacuum outside the static source mass*.

Since $T_A^{\alpha\beta} = 0$ in a vacuum, then the absolute total SE tensor $E_A^{\alpha\beta} = T_A^{\alpha\beta} + t_A^{\alpha\beta}$ *is also zero* in the vacuum outside the source mass of a static system, as stated by

$$E_A^{\alpha\beta} = t_A^{\alpha\beta} = 0 \quad \text{(vacuum, static system)}. \tag{303}$$

Therefore, $E_A^{\alpha\beta}$ is known a priori for static systems (outside their sources) as claimed above. Also, $E_A^{\alpha\beta} = 0$ reasonably approximates the vacuum $E_A^{\alpha\beta} = t_A^{\alpha\beta}$ for typical systems with slow-moving sources such as our Solar System, since their fields may be treated quasi-statically.

Lowering (303) by the absolute metric, then the vacuum $E_{\alpha\beta}^A$ is zero for static systems. This implies that in order to satisfy the total SE requirement, when the natural field equation potential solution $w^\alpha{}_\mu$ for a static system is substituted into the absolute field tensor $H_{\alpha\beta}^A[w;a]$, the vacuum $H_{\alpha\beta}^A[w;a]$ *must be zero*, yielding $E_{\alpha\beta}^A = 0$ via the absolute field equation $H_{\alpha\beta}^A[w;a] = 8\pi E_{\alpha\beta}^A$ as required. This is verified for the static star case as follows. Utilizing ISCs, the vacuum $w^\alpha{}_\mu$ field is given by (264), which is the natural field equation solution as shown above. Substitution of this $w^\alpha{}_\mu$ field into the established $H_{\alpha\beta}^A[w;a]$ form (300) indeed yields $H_{\alpha\beta}^A[w;a] = 0$ and therefore $E_{\alpha\beta}^A = 0$, *verifying total SE requirement satisfaction*. With the representative star case verifying satisfaction of the total SE requirement, then under the *uniqueness* of the established form for $H_{\alpha\beta}^A[w;a]$ as obtained from the previously applied assumptions for the AFE development, it is *assumed* that *the established form for the absolute field tensor $H_{\alpha\beta}^A[w;a]$ (given by (300) with (301) providing $Q_{\alpha\beta}^A[w;a]$) satisfies the total SE requirement for general cases*, even when $E_A^{\alpha\beta}$ is not known a priori.

Beginning with the most general possible form (296) of the absolute field equation utilizing the arbitrary constants *a* through *d*, application of the linearized case and absolute divergence identity requirements resulted in uniquely setting their values per (298). With there being *only four* arbitrary constants in the most general possible form (296), the *ability* to set these constants to yield both the observed linearized case metric and the absolute divergence identity, and additionally to satisfy the total SE requirement for the star case, is a "powerful" verification of the validity of (296). Now care was taken to insure that any "intermediate form" of the AFE, at then any stage in the AFE's development, was the most general possible form subject to any assumptions that had been made up to that stage. Therefore, the AFE's final form (300) (or its trace-reverse (302)), with $Q_{\alpha\beta}^A$ given by (301), is *the most general possible form* subject to the assumptions made for its development. However, the final form of the AFE has been shown to be *unique*. Therefore, *the absolute field equation is uniquely obtained from the assumptions made for its development*.

The assumptions made for the AFE's development are the absolute flat spacetime and SEP postulates, and the additional assumptions made (above) for establishing the AFE. Based on the assumed *physical validity* of the AFE's developmental assumptions, then with the AFE's final form uniquely obtained from them, *the absolute field equation is assumed to be physically valid*. The AFE's validity is verified by it predicting the same potential $w^\alpha{}_\mu$ (given use of $H_{N}^{\alpha\beta}{}_{;\beta} = 0$ (271)) as the successfully predictive natural field equation, assuming that the AFE $H_{\alpha\beta}^A[w;a] = 8\pi E_{\alpha\beta}^A$ successfully predicts the total SE tensor $E_{\alpha\beta}^A$ using the NFE's potential solution (one of the AFE's developmental assumptions).

7.4. Solution Properties, the Absolute Energy Condition, and Absolute Energy Conservation

As discussed above, due to its field dependence, the absolute total SE tensor $E_A^{\alpha\beta}$ is not known a priori for most gravitational systems, typically since the field dependent absolute field SE tensor $t_A^{\alpha\beta}$ is not known a priori. Therefore, for most gravitational systems, the absolute field equation $H_A^{\alpha\beta} = 8\pi E_A^{\alpha\beta}$ may not be utilized to determine the gravitational field. On the other hand, since the natural matter SE tensor $T_N^{\alpha\beta}$ *is known* by natural observers, the natural field equation $H_N^{\alpha\beta} = 8\pi T_N^{\alpha\beta}$ is *preferable* to the absolute field equation for determining the field for general systems (as stated in the summary). Also, the natural field equation is preferable for natural observer modelling of gravitational systems, since it directly predicts naturally observed gravitational phenomena, and is the field equation form for which morph application yields natural observer SEP compliance. However, given the NFE potential $w^\alpha{}_\mu$ solution for a general system, the absolute field equation $H_A^{\alpha\beta}[w;a] = 8\pi E_A^{\alpha\beta}$ is useful for determining $E_A^{\alpha\beta}$. In addition, the AFE's absolute field tensor $H_A^{\alpha\beta}[w;a]$ is utilized to construct (276) providing the value of the absolute field SE tensor $t_A^{\alpha\beta}$ given the NFE $w^\alpha{}_\mu$ solution.

In Section 3.7, the *heuristic argument* was utilized that, at the fundamental level, the coupling of the shift tensor field to a symmetric SE tensor source charge, with both in the presence of the symmetric absolute and natural metrics (or at the very least the symmetric absolute metric), results in a symmetric shift tensor when given in pure (indice) form. An evaluation was performed in Section 6.4 for the natural field equation $H_{\alpha\beta}^N[w; g] = 8\pi T_{\alpha\beta}^N$ as a formal statement depicting the shift tensor field being coupled to the symmetric natural matter SE tensor $T_{\alpha\beta}^N$, where it was shown that $w_{\alpha\beta} = g_{\alpha\mu}w^\mu{}_\beta$ is symmetric, from which it was shown that the shift tensor $S^\alpha{}_{\bar\mu}$ is symmetric when given in pure form using either metric. Similarly, since the absolute field equation $H_{\alpha\beta}^A[w; a] = 8\pi E_{\alpha\beta}^A$ is a *formal statement* depicting the shift tensor field (using its potential $w^\alpha{}_\mu$) being coupled to the symmetric absolute total SE tensor $E_{\alpha\beta}^A$ in the presence of the symmetric absolute metric, then it *must* yield a symmetric shift tensor $S^\alpha{}_{\bar\mu}$ when given in pure form. Utilizing the $E_{\alpha\beta}^A$ symmetry, the trace-reverse form (302) of the AFE yields a symmetric potential Ricci tensor $Q_{\alpha\beta}^A$. Now regardless of whether or not the potential tensor is symmetric in pure form, every term in $Q_{\alpha\beta}^A$, given by (301), is symmetric except for $w_{\alpha\beta}\vert^\sigma{}_\sigma$. Therefore, $w_{\alpha\beta} = a_{\alpha\mu}w^\mu{}_\beta$ must indeed be symmetric to yield a symmetric $Q_{\alpha\beta}^A$. So any potential solution $w^\alpha{}_\mu$ for the absolute field equation is symmetric when raised/lowered by the absolute metric to yield a pure form. To prove that $S^\alpha{}_{\bar\mu}$ is symmetric in pure form based on $w_{\alpha\beta} = a_{\alpha\mu}w^\mu{}_\beta$ symmetry, the absolute metric may be substituted for the natural one in the above partner NFE proof where $S^\alpha{}_{\bar\mu}$ symmetry was established based on $w_{\alpha\beta} = g_{\alpha\mu}w^\mu{}_\beta$ symmetry. The symmetry of the potential tensor was assumed in Section 7.1 in order to state that the AFE provides ten algebraically independent conditions, providing the basis for requiring application of the natural field constraint (271) to fully determine the $w^\alpha{}_\mu$ solution under satisfaction of the absolute divergence identity (282).

Without use of an "energy condition" limiting the gravitational sources, the absolute field equation (with use of the natural field constraint (271) assumed) admits to potential solutions $w^\alpha{}_\mu$ such that the shift tensor $S^\alpha{}_{\bar\mu} = \exp(w^\alpha{}_\mu)$ yields shifted light speeds c_S exceeding the null speed v_{Null}, so the gravity shifting would violate the speed constraint. Therefore, the required "absolute energy condition" for gravitational sources is as follows: *The absolute energy-momentum of gravitational sources must be such that the absolute field equation solutions produce shifted light speeds c_S that do not exceed the absolute manifold null speed v_{Null}.* Using (268), satisfaction of the absolute energy condition also prevents gravitational field propagation speeds $v_G = c_S$ from exceeding the null speed v_{Null}. Therefore, *energy transport of all types is limited by the null speed v_{Null} under the absolute energy condition.* The absolute energy condition limiting the absolute source, $E_A^{\alpha\beta}$, via use of the absolute field equation, can be seen to be the partner form of the natural energy condition limiting the partner source $T_N^{\alpha\beta}$ via use of the partner natural field equation. Since the partner AFE and NFE have the same solution $w^\alpha{}_\mu$, satisfaction of the natural energy condition yields absolute energy condition satisfaction, and vice versa. For (most) systems where $E_A^{\alpha\beta}$ is not known a priori, the natural energy condition is applied to limit the known $T_N^{\alpha\beta}$ when using the NFE to first obtain $w^\alpha{}_\mu$, and then the AFE is applied using this $w^\alpha{}_\mu$ to obtain the partner $E_A^{\alpha\beta}$, which automatically satisfies the absolute energy condition. But if a system is modelled using the AFE assuming a *given* $E_A^{\alpha\beta}$, then $E_A^{\alpha\beta}$ must satisfy the absolute energy condition, resulting in a partner $T_N^{\alpha\beta}$ that satisfies the natural energy condition.

Again, the absolute field equation yields a symmetric potential $w^\alpha{}_\mu$ solution that results in a symmetric shift tensor field $S^\alpha{}_{\bar\mu}$ (when put in pure form), where as established above, the AFE yields a real-valued potential solution $w^\alpha{}_\mu$ due to potential use being linear. Satisfaction of the speed constraint under the absolute energy condition enables the global IC given potential $w^\alpha{}_\mu$ to be *diagonalized* using Lorentz transforms. The same argumentation may be made then as above (Sec. 6.4) for the partner NFE yielding adherence to the gravity shifting constraints. Therefore, with the assumed adherence to the absolute energy condition for gravitational sources, *the absolute field equation predicts gravity shifting that adheres to all of the established gravity shifting constraints* (as stated in Section 3.13). This is expected since under assumed satisfaction of the total SE requirement, the AFE yields the same potential solution $w^\alpha{}_\mu$ as the partner NFE yielding adherence. But the evaluation made here also applies for *stand-alone* use of the AFE for given $E_A^{\alpha\beta}$.

As discussed in Section 6.4, satisfaction of the gravity shifting constraints, combined with the exponential potential form $S^\alpha_{\bar{\mu}} = \exp(w^\alpha{}_\mu)$ (68) for the shift tensor, bars event horizons, singularities, and causality violations as implausibilities. Then with explicit formulation in absolute flat spacetime resulting in compatibility with quantum theory (as demonstrated above), *all physical law and modelling is physically plausible when utilizing the absolute field equation to predict the gravitational field*. This result, combined with the similar physical plausibility when using the partner natural field equation, fully establishes that *all physical law and modelling using gravity shift theory is physically plausible* (as stated in Section 3.13 and the summary).

As discussed above, due to satisfaction of the Bianchi identity $G^{\alpha\beta}{}_{;\beta} \equiv 0$ in Einstein's equation $G^{\alpha\beta} = 8\pi T_N^{\alpha\beta}$, the Cauchy problem in general relativity requires four additional coordinate conditions to be imposed in order to completely specify initial conditions. Similarly, due to the identity $H_A^{\alpha\beta}{}_{|\beta} \equiv 0$ holding in the absolute field equation $H_A^{\alpha\beta} = 8\pi E_A^{\alpha\beta}$, four additional conditions are required to completely specify the initial conditions for deterministic evolution from an initial hypersurface $x^0 = t$. However, these may not be coordinate conditions, since *a priori* coordinates may be utilized in GS theory. Instead, the natural field constraint $\nabla^N_\beta H_N^{\alpha\beta} = 0$ (271) provides the four additional conditions, with evolution under the natural field constraint and AFE uniquely determined from the initial conditions. The AFE Cauchy problem is the partner to the NFE Cauchy problem discussed above, with the same deterministic evolution obtained utilizing the partner initial conditions for the partner AFE and NFE use.

As discussed in Section 6.4, in GS theory, natural matter EM conservation $T_N^{\alpha\beta}{}_{;\beta} = 0$ is a condition exclusively obtained independently from the natural field equation $H_N^{\alpha\beta} = 8\pi T_N^{\alpha\beta}$ by utilizing the available Lagrangian-based establishment of $T_N^{\alpha\beta}{}_{;\beta} = 0$ for metric-based theories, obtained by applying coordinate transformations (or equivalently diffeomorphisms) to the matter action S_M utilized in the covariant matter action principle statement, $\delta S_M = 0$, to yield a Bianchi identity from which $T_N^{\alpha\beta}{}_{;\beta} = 0$ is yielded. A similar formulation of the Bianchi identity obtained from the energy action S_E utilized in the covariant total energy action principle statement $\delta S_E = 0$, with S_E given by (291), results in a Bianchi identity where $E_A^{\alpha\beta}{}_{|\beta}$ *cannot be isolated* due to $E_A^{\alpha\beta}$ being coupled to the potential $w^\alpha{}_\mu$ via (289), preventing absolute total EM conservation $E_A^{\alpha\beta}{}_{|\beta} = 0$ from being obtained using the given S_E. It is suspected though that there does exist an action such that its Bianchi identity (or some other variant of Noether's theorem) yields $E_A^{\alpha\beta}{}_{|\beta} = 0$, but this has not been further pursued. Therefore, putting that possibility aside, *reliance is made on application of the absolute divergence identity* $\nabla^A_\beta H_A^{\alpha\beta}[w; a] \equiv 0$ (282) *to the absolute field equation* $H_A^{\alpha\beta} = 8\pi E_A^{\alpha\beta}$ *to obtain* $E_A^{\alpha\beta}{}_{|\beta} = 0$, yielding the required local absolutely measured total EM conservation $\partial^{(A)}_{\check{\nu}} \check{E}^{\mu\nu}_{A(A)} = 0$ in absolute inertial frames, thereby resulting in global total EM conservation $d\check{P}_A^\alpha / d\check{t} = 0$ (from Sec. 5.3).

8. Prediction Assessment

Due to the complete GS theory being *uniquely obtained* from the flat spacetime and SEP postulates as well as the additional assumptions made for development of the field equations, then for both natural and absolute observers, *all observational predictions made using the complete gravity shift theory are uniquely obtained from its postulates and the additional field equation assumptions* (as stated in the summary). Again, based on the assumed *physical validity* of the flat spacetime and SEP postulates as well as the natural field equation assumptions, then using the resulting unique natural field equation to predict natural gravitational observations, *the provided complete gravity shift theory is expected to successfully predict all natural observations of classical gravitational phenomena*, meaning in each case to obtain a prediction that "*formally agrees*" with the corresponding observation to within the uncertainty range obtained by combining the specified observation error with any astrophysical modelling uncertainties encountered. Will [1] categorizes the breadth of the available natural observations utilized to test gravitational theories, with the exception of the additional recent pulsar timing array (PTA) detections of low-frequency gravity waves [15]. As a verification, the corresponding natural predictions made

with GS theory are shown here to either formally agree with these available test cases or, at minimum, to approximately agree without formal agreement claimed. Note that the following evaluation excludes testing for cosmological prediction other than assumed use of the natural RW metric (as yet unspecified) to account for cosmological effects when observing distant local systems.

Below, the observational properties utilized to develop the natural field equation are first applied to establish predictions for a wide range of available test cases. Then predictions from the Supplement are added, extending the range of predictions to cover *all available* natural observations of local systems utilized to test gravitational theories. However, some of the predictions using the NFE observational properties involve black and neutron stars, with the detailed modelling for these compact objects provided in the Supplement. The Supplement modelling of black and neutron stars is assumed as background when predicting test cases involving them using the NFE observational properties. As discussed, a sizable fraction of most observed black stars have not formed as simply lone "native" black stars collapsing through their photon spheres undisturbed up to as presently observed, instead having a history of significant accretion accumulation and/or formation as remnants of earlier black stars merging below the photon spheres of the remnants. Such a star is referred to as an "accumulated and/or merged black star (AMBS)." The term "black star assembly (BSA)" refers to the assembly of matter that forms an AMBS, in particular the assembly of matter below the photon sphere of the subsequent AMBS. When a BSA is "mature"—meaning that the BSA is old enough that all of its matter is much smaller than its photon sphere—it is shown to have an appearance and a gravitational metric (near and above the photon sphere) *closely approximating* those of a native black star with the same gravitational mass M as the entire BSA. The same properties hold for a mature AMBS. As shown, a native black star continually collapses towards a singularity with its surface speed asymptotically approaching the exponential shifted light speed limit (which from above is $c_S = e^{-2M/R}$ for a non-spinning black star, approximating this light speed if it is spinning), resulting in the singularity never being reached over the star's finite age. The speed of accreted matter also asymptotically approaches the exponential shifted light speed as it approaches a collapsing native black star. Under this behavior, it is shown that even matter accreted early in the history of an observed native black star may not have impacted its collapsing surface, so the BSA may not have formed an AMBS yet. Similarly, merging native black stars may not have actually merged together, so the BSA may not have formed an AMBS yet. Definitive modelling has not been performed establishing when various BSA configurations form into AMBSs, so at present it may be the case that an observed AMBS is in actuality its BSA. Based on this uncertainty, an observed "black star" that is formed via a BSA eventually becoming an AMBS, but whose current BSA-versus-AMBS status is unknown, is referred to as a "BSA/AMBS." Going forward, the term "black star" may refer to either a native black star or a mature BSA/AMBS with an appearance and a gravitational metric (near and above its photon sphere) *closely approximating* those of a native black star, with the meaning discerned by context. Similar to native black stars, the collapse of BSA/AMBSs is limited by the exponential shifted light speed, so any BSA/AMBS takes an infinite amount of time to collapse to a singularity. See the Supplement for modelling of BSA/AMBSs.

As is commonly accepted, no prediction made with general relativity, utilizing then Einstein's equation, has been found that disagrees with observation. From the GS theory perspective, it is understood that the successful prediction using general relativity is *naturally observable* prediction, since the only "observers" in general relativity are natural observers. To determine if a naturally observable prediction made using GS theory agrees with its corresponding natural observation, the prediction may be compared against the successful prediction using GR theory, yielding agreement of the GS theory prediction with the observation if the predictions for both theories agree. This method is utilized as a convenient "tool" here and in the Supplement for establishing agreement of GS theory predictions with natural observations.

The natural field equation was developed so as to satisfy the following observational properties: satisfaction of the SEP, as obtained under morph application; linearization to the linearized Einstein equation, so as to yield in the linearized case the same natural metric $g_{\alpha\beta}$ as the observationally

predictive linearized Einstein equation; and prediction of the observed post-Newtonian approximation for the natural metric. A wide variety of natural gravitational phenomena are *successfully predicted* from these observational properties, as listed here via use of the corresponding test cases in Will [1].

To begin with, satisfaction of the SEP implies satisfaction of the EEP for the nongravitational limits of local systems. The test cases successfully predicted from EEP satisfaction are as follows (see [1], Chap. 2): tests verifying satisfaction of the Weak Equivalence Principle such as the Eötvös experiments; tests of Local Lorentz Invariance; and tests of Local Position Invariance, consisting of the gravitational redshift experiments (including the Pound-Rebka experiment and the clock "redshifting" discussed above) and the measurements of the constancy of the fundamental nongravitational constants.

As discussed above, NFE prediction of the observed post-Newtonian metric yields successful predictions of the "classical tests" in our Solar System (see [1], Chap. 7): the deflection of light by the Sun, the Shapiro time delay for radar signals, and the perihelion advance for the orbit of Mercury. Due to SEP satisfaction, these successful predictions hold even in the presence of the background system consisting of our galaxy (with our Solar System orbiting about the galactic center) combined with the cosmology of our universe.

The following cases are successfully predicted from SEP satisfaction (see [1], Chap. 8): tests verifying no Nordvedt effect occurring; tests showing no preferred frames or locations for the orbital motions of bodies; tests showing no preferred frames or locations for the structures of massive bodies, including a lack of variation of the locally measured gravitational constant G_L, and a lack of precession of the spin axes of massive bodies; and cosmological tests verifying the constancy of Newton's gravitational constant G as the universe evolves. These tests are satisfied not only for weak-field cases—such as in our Solar System—where the predicted observed PN metric is accurate, but also for the strong-field cases involving black and neutron stars, including involving "black stars" consisting of mature BSA/AMBSs. For the strong-field cases, the predictive Einstein-Infeld-Hoffman (EIH) formalism for GR theory (see [1], Chap. 10) is also applicable for GS theory, since the NFE satisfies the SEP and yields the observed PN metric. See the "Gravity shift post-Minkowskian and post-Newtonian theory" section in the Supplement, referred to as the "PM work," detailing establishment of the EIH formalism. Use of the EIH formalism yields satisfaction of the SEP motion tests involving black and neutron stars (including mature BSA/AMBSs).

Prediction with the NFE also satisfies the other tests of observed post-Newtonian gravity (see [1], Chap. 9). Tests on spin effects include geodetic and frame-dragging precessions, consisting of the Gravity Probe B experiment and binary pulsar precessions, and tests of spin effects on orbits using Earth-orbiting satellites. The Earth-Moon de Sitter precession is successfully predicted. The tests of conservation laws for observed PN gravity are satisfied, consisting of laboratory measurements on Earth, and both lunar and binary pulsar observations. The successfully predictive EIH formalism applies for the binary pulsar conservation cases.

Linearization of the NFE to the linearized Einstein equation, combined with satisfaction of the SEP, results in the predicted speed and polarization of gravity waves being the same as in GR theory. As shown in Will [1] (Chap. 11), the GR-predicted wave speed equals the speed of light, and the polarization is $E(2)$ class N_2 with then two polarization modes of helicity ± 2. These same derivations are applicable for GS theory using the linearized NFE, with it understood that gravity waves move at the shifted light speed, which in the free-fall frames is naturally measured as the fixed unshifted light speed. These derivations are discussed above and in the PM work (in the "Gravitational waves" section). The equality of the gravity wave and light speeds has been verified [26] (as discussed in [1], Chap. 12), as well as the $E(2)$ class N_2 polarization [27], so both the GS and GR theories successfully predict the observed wave speed and polarization. If the gravitational field were to be quantized, both theories would predict naturally massless spin-2 gravitons moving at light speed.

Beyond the above-discussed wide variety of natural gravitational phenomena successfully predicted from the observational properties utilized to develop the natural field equation, the Supplement extends predictive verification to cover the rest of the test cases for local systems discussed in Will [1]

(Chap. 12) plus the PTA gravity wave detections, extending then the range of verification to cover *all available* natural observations of local systems utilized to test gravitational theories. The Supplement's PM work provides a comprehensive post-Minkowskian formulation for GS theory, given to 1.5PN order both for near-zone systems and for gravitational radiation (using the "PN" designation system where quadrupole radiation is set to "1PN"). The section "Observational properties of black and neutron star systems," referred to as the "BNS work," provides strong-field predictions for black and neutron stars (including mature BSA/AMBSs), as well as nearby matter and photons when present. These works combined give the predictions extending coverage to all available test cases. The Supplement also provides (in the PM work) detailed developmental discussion for some of the above material: development and use of the post-Newtonian natural metric for near-zone systems, the EIH formalism for compact objects, and the gravity wave speed and polarization.

In both the near-zone and radiation cases, it is shown (in the PM work) that *the GS PM theory 1.5PN expansions yield the same naturally observable predictions as the corresponding 1.5PN expansions in GR PM theory*. These include the predictions utilizing the radiative EM balance equations for obtaining near-zone system behavior under 1.5PN radiation losses, such as the secular decay of compact binary orbits. Therefore, *for all successful naturally observable predictions made using the 1.5PN near-zone and radiation formulation in GR PM theory, the same predictions using the 1.5PN formulation in GS PM theory are also successful*.

As the near-zone 1PN post-Newtonian theory is embedded within the near-zone 1.5PN formulation given in the PM work, and the linearized NFE was evaluated as part of the PM work, some of the predictions in the PM work have already been listed above. The PM work extends near-zone predictions from 1PN to 1.5PN, and radiation predictions from only linear NFE predictions to additionally include 1.5PN predictions for both the radiation and the near-zone systems generating the radiation. Available test cases added by this extension only include cases involving black and neutron stars (including mature BSA/AMBSs), since the weak-field cases not involving compact objects are successfully predicted using only the 1PN post-Newtonian theory (to the author's knowledge), noting that the radiation test cases verifying the linearized NFE [26,27] utilize compact objects as the sources. As a result, the PM work extension only adds three available test cases, which are successfully predicted (as shown in detail): the orbital decays of observed binary pulsars due to radiation losses, the early "1.5PN parts" (as defined in the PM work) of the detected gravity wave signals generated by inspiralling black and neutron stars (including mature BSA/AMBSs), and the PTA detected low-frequency gravity waves generated by (the commonly assumed) supermassive black star binaries (which are actually BSA/AMBSs). (Note the PTA case is evaluated in the BNS work, but is shown to be successfully predicted using the 1.5PN PM theory.) These are key cases though, as most available gravitational theories fail to be predictive when these cases are encountered.

All of the test cases listed above in this section are successfully predicted by use of the linearized NFE and the GS post-Minkowskian theory given to 1.5PN, combined with use of the SEP/EEP. As can be seen, these cover an extensive range of naturally observed gravitational phenomena.

Only the strong-field cases predicted in the BNS work remain. The strong-field cases consist of naturally observable properties of black and neutron star systems, including systems containing mature BSA/AMBSs as "black stars." The predicted observable properties of the systems consist of the gross observational properties of the black and neutron stars themselves (including mature BSA/AMBSs), as well as the observable properties of nearby matter and photons when present. Included is the prediction of detected gravity waves generated by compact star mergers through merger and ringdown.

Prediction of the strong-field cases required structural modelling of black and neutron stars, including BSA/AMBSs. The GS theory neutron star structural modelling is shown to fairly well approximate the successfully predictive modelling using GR theory, so that given the present *significant modelling uncertainties* for the material properties of neutron stars, it was concluded that the GS theory modelling of neutron stars predicts appearances and observable structural properties that formally

agree with their corresponding observations to within the present observation and modelling combined uncertainty range for each case. The GS theory native black star and BSA/AMBS structural modelling is shown to significantly differ from the GR modelling of black holes. However, the GS theory predictions of the resultant observable properties of "mature" native black stars and BSA/AMBSs (meaning again ones that are old enough to have collapsed to be much smaller than their respective photon spheres), as well as the observable properties of nearby matter and photons, are shown to formally or approximately agree with observations.

As shown, GS theory modelling successfully predicts the observed "blackness" of mature native black stars and BSA/AMBSs, since they are predicted to be far fainter than presently available instruments can detect, including remaining effectively black under any possible impact heating due to accreted matter impacting them. As stated, the presently available modelling for both accretion disks and astrophysical jets has *significant modelling uncertainties*, similar to the material properties of neutron stars having significant modelling uncertainties. As established, due to the severe "darkening mechanisms" (see the BNS work for detailed modelling and discussion of them) that set in below the photon spheres of black stars (including mature BSA/AMBSs), predictions for *observed* phenomena are limited to near and above the photon spheres (excluding the black star effective blackness predictions). The gravitational metric of a mature BSA/AMBS closely approximates the metric of a native black star near and above its photon sphere. Near and above the photon sphere of a native black star or mature BSA/AMBS, the non-spinning star-case natural metric in GS theory closely approximates the GR theory star-case Schwarzschild metric, resulting in the closeness of various gross observational properties of mass particles and photons (as depicted by various figures in the BNS work). Again, the 1.5PN PM formulations for the GS and GR theories yield the same predictions, including the 1.5PN natural metrics being the same. With the mature BSA/AMBSs included when predicting the gravity waves generated by merging compact binaries, use of the linearized NFE is again shown to yield the same successful predictions for the measured gravity wave speed and polarization [26,27], through merger and ringdown, as use of the linearized Einstein equation. It is commonly assumed that GR modelling successfully predicts the observed properties of black and neutron star systems, including the observable properties of the black and neutron stars themselves, as well as the observable properties of nearby matter and photons.

Utilizing the above-listed properties, an argument is made claiming that, *with the exception of the "high-order parts" (beyond the early "1.5PN parts") of detected gravity wave signals generated by merging compact binaries through merger and ringdown, when made to the required accuracies, gravity shift theory predictions for all of the presently available observations of black and neutron star systems, including systems containing mature BSA/AMBSs as "black stars," formally agree with the corresponding observations to within the presently encountered observation/modelling uncertainty range for each case, establishing these predictions as being successful*. This conclusion holds then for the available strong-field test cases discussed in Will [1] plus the PTA gravity wave detections. The successfully predicted strong-field cases examined in the BNS work are listed here (repeating those discussed above): the appearances and observed structural properties of neutron stars; the observed effective blackness of mature native black stars and BSA/AMBSs, including remaining effectively black under any possible impact heating due to accreted matter impacting them; the observed behaviors of masses and photons near black and neutron stars (including mature BSA/AMBSs); the observed motions of stars about most observed supermassive black stars (MOSPMBS), such as about Sgr A* at our galactic center, noting that MOSPMBS are actually mature BSA/AMBSs (as discussed in the BNS work); the observed properties of accretion disks and astrophysical jets about black and neutron stars (including mature BSA/AMBSs); the accretion disk based images of Sgr A* and M87* (both mature BSA/AMBSs), including the sizes of the effectively black disk-shaped gravitational images of their photon spheres; the observed gravitational imaging yielded by black and neutron stars (including mature BSA/AMBSs), for sources beyond them; observations of near-zone phenomena sensitive to the structural properties of neutron stars; the observed orbital decays of binary pulsars; the early "1.5PN parts" of detected gravity waves generated by merging

compact binaries, which consist of native black stars, neutron stars, and mature BSA/AMBSs; the PTA detected low-frequency gravity waves generated by (the commonly assumed) supermassive black star binaries (which are actually BSA/AMBSs); and the measured speed and polarization, through merger and ringdown, for gravity waves generated by merging compact binaries.

The last examined BNS case is the "high-order parts" of detected gravity wave signals generated by merging compact binaries through merger and ringdown, meaning the detector *signals* (individual detector outputs, not providing then information on wave polarization obtained by examining the correlated signals from multiple detectors) later than the early 1.5PN parts of the signals successfully predicted. Without the as yet performed numerical and high-order PM modelling required to accurately predict the high-order parts of gravity wave signals, formal agreement of GS theory predictions with the full detected waves remains unverified. Using though the established analytical modelling, it is shown that, at minimum, the high-order parts of gravity waves are "grossly" predicted, meaning that their general features are predicted. Specifically, the general amplitude behavior is grossly predicted, from increasing amplitude during inspiral, to amplitude peak during merger, through smooth transition from merger to ringdown, and finally exponentially decreasing amplitude during ringdown. In addition, the evolution of the continually increasing frequencies of the quadrupole and higher modes is at minimum grossly predicted. So it can at least be said that no direct contradiction with the detected waves is apparent. Due to the expected agreement of all GS theory predictions with observations, it is expected that if the required numerical and high-order PM modelling is performed, GS theory will indeed successfully predict the entire detected signals, but only gross predictive agreement at minimum is claimed here. Again, the early 1.5PN parts of detected gravity waves are accurately predicted, so it is only the high-order parts of gravity wave signals where the present claim of only gross prediction applies. Again, the measured $E(2)$ class N_2 polarization state throughout the entire measured waves is predicted, and the speed-of-light wave speed is predicted.

The above complete listing in this section covers all of the available test cases for local systems, as categorized in Will [1] plus the PTA gravity wave detections. In each case, it has been shown that the prediction either formally agrees with the observation to within the presently encountered observation/modelling uncertainty range for the case or, at minimum, approximates the observation without formal agreement claimed. The latter case is only encountered for the high-order parts (beyond the early 1.5PN parts) of detected gravity wave signals generated by merging compact binaries through merger and ringdown, where using the established analytical modelling, only gross prediction at minimum is claimed.

Concluding, *all available local system test cases are successfully predicted using gravity shift theory with the exception of the high-order parts of detected gravity wave signals*, which at least are shown to be grossly predicted using the present analytical modelling. This result provides extensive verification supporting the above conclusion that the given complete GS theory is expected to successfully predict all natural observations of classical gravitational phenomena. Again, it is assumed that application of the natural RW metric, as yet unspecified, accounts for cosmological effects in the naturally observed properties of distant local systems.

As discussed in the PM and BNS works, assuming that the given GS theory will successfully predict all natural gravitational phenomena, it is predicted that once high PN order and numerical modelling are performed, the detected gravity wave signals generated by black star mergers will be better predicted by GS theory (characteristically higher S/N ratios when using template matching) than by GR theory, observationally proving the validity of gravity shift theory over general relativity.

9. Discussion

The provided gravity shift theory has been shown to be a physically plausible classical theory of gravity that successfully predicts all available local system test cases for naturally observed gravitational phenomena, with the exception of the high-order parts of detected gravity wave signals generated by merging compact binaries, which at least are shown to be grossly predicted using the

present analytical modelling. As such, *gravity shift theory is proffered here as a possible replacement for general relativity theory*.

Indeed, GR theory has serious plausibility issues. As discussed above, due to dual use of the metric $g_{\mu\nu}$ to determine gravitational effects and give the spacetime structure, GR theory is fundamentally incompatible with quantum theory. In addition, GR theory predicts the existence of event horizons, which concerning their plausibility are problematic (such as the classically predicted loss of entropy for objects passing through an event horizon), and more seriously predicts the existence of singularities, which are highly implausible. In contrast, due to explicit formulation in absolute flat spacetime, GS theory is compatible with quantum theory, as demonstrated above. Also, no event horizons or singularities are predicted in GS theory. In light of the GR theory plausibility issues versus the absence of them in GS theory, it is concluded that *gravity shift theory is preferred over general relativity theory as a classical theory of gravity*. This conclusion of course hinges on GS theory continuing to be predictive as further prediction and observational testing are performed, such as obtaining high S/N ratios for detected gravity waves generated by black and neutron star mergers/ringdowns once high PN order and numerical modelling are performed (hopefully the researchers geared to this work will take this on).

Given the acceptance of both the absolute flat spacetime and strong equivalence principle postulates, it is concluded that the general GS theory uniquely obtained from these postulates—i.e., everything other than the field equations and their predictions—is indeed valid, with no future changes anticipated. The additional natural and absolute field equations required for the complete GS theory are developed utilizing additional assumptions, and as such are subject to change if it is deemed that one or more of these assumptions is not physically valid. If this is found to be the case, such as if needed to maintain successful prediction, a future version of the complete GS theory would utilize the modified field equations, but the contained general GS theory would remain unchanged. Therefore, regardless of any future field equation changes, *general gravity shift theory depicts the workings of the classical gravitational field without future changes anticipated*.

Supplementary Materials: The following supporting information can be downloaded at the website of this paper posted on Preprints.org. S1: W. Northcutt, Gravity Shift Theory Observational Predictions, (2025).

Conflicts of Interest: The author declares no conflicts of interest.

References

1. C. M. Will, *Theory and Experiment in Gravitational Physics* (Cambridge University Press, Cambridge, 2018)
2. Y. Ben-Menahem, *Conventionalism* (Cambridge University Press, Cambridge, 1946)
3. H. Poincaré, *Science and Hypothesis* (Walter Scott Publishing, London, 1905)
4. H. von Helmholtz, *Hermann von Helmholtz: Epistemological Writings*, edited by R.S. Cohen and Y. Elkana (Reidel, Dordrecht, 1977) (Helmholtz produced the provided writings from 1868 to 1878.)
5. H. Reichenbach, *The Philosophy of Space & Time* (Dover, New York, 1957) (The Dover edition is an English translation, by M. Reichenbach and J. Freund, of Philosophie der Raum-Zeit-Lehre published in 1927.)
6. M. Jammer, *Concepts of Space: The History of Theories of Space in Physics* (Dover, New York, 1993)
7. J. Broekaert, A spatially-VSL gravity model with 1-PN limit of GRT. Found. Phys. **38**, 409 (2008)
8. E. A. Milne, *Relativity, Gravitation, and World-Structure* (Clarendon Press, Oxford, 1935)
9. E. A. Milne, *Kinematic Relativity* (Clarendon Press, Oxford, 1948)
10. C. Weisendanger, Poincaré gauge invariance and gravitation in Minkowski spacetime. Class. Quantum Grav. **13**(4), 681 (1996)
11. L. V. Verozub, Gravitation equations, and space-time relativity. arXiv:0805.2688
12. R. P. Feynman, F. B. Morinigo, and W. G. Wagner, *Feynman Lectures on Gravitation*, edited by B. Hatfield (Perseus Books, Reading, 1995)
13. N. Rosen, General relativity and flat space. I & II. Phys. Rev. **57**, 147 (1940)**;** N. Rosen, A bi-metric theory of gravitation. Gen. Relativ. Gravit. **4**(6), 435 (1973)**;** N. Rosen, A theory of gravitation. Ann. Phys. **84**, 455 (1974)**;** N. Rosen, A bi-metric theory of gravitation II. Gen. Relativ. Gravit. **6**(3), 259 (1975)
14. K. Hinterbichler and R. A. Rosen, Interacting spin-2 fields. J. High Energy Phys. **2012**, 47 (2012)

15. EPTA Collaboration and InPTA Collaboration, The second data release from the European Pulsar Timing Array - III. Search for gravitational wave signals. Astron. & Astrophys. **678**, A50 (2023)
16. C. W. Misner, K. S. Thorne, and J. A. Wheeler, *Gravitation* (Freeman, New York, 1973)
17. R. M. Wald, *General Relativity* (University of Chicago Press, Chicago, 1984)
18. B. F. Schutz, *A First Course in General Relativity* (Cambridge University Press, Cambridge, 2009)
19. R. A. Horn and C. R. Johnson, *Topics in Matrix Analysis* (Cambridge University Press, Cambridge, 1991)
20. E. Poisson and C. M. Will, *Gravity: Newtonian, Post-Newtonian, Relativistic* (Cambridge University Press, Cambridge, 2014)
21. B. F. Schutz, *Geometrical Methods of Mathematical Physics* (Cambridge University Press, Cambridge, 1980)
22. S. Weinberg, *Gravitation and Cosmology* (John Wiley & Sons, New York, 1972)
23. A. Beiser, *Concepts of Modern Physics* (McGraw-Hill, New York, 1967)
24. S. Carroll, *Spacetime and Geometry* (Addison Wesley, San Francisco, 2004)
25. D. L. Lee, A. P. Lightman, and W.-T. Ni, Conservation laws and variational principles in metric theories of gravity. Phys. Rev. D **10**(6), 1685 (1974)
26. The LIGO/Virgo/KAGRA Collaborations, Gravitational waves and gamma-rays from a binary neutron star merger: GW170817 and GRB 170817A. Astrophys. J. Lett. **848**, L13 (2017)
27. The LIGO/Virgo/KAGRA Collaborations, Tests of general relativity with GWTC-3. arXiv:2112.06861