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Modelling Living Processes with Nonlinear Dynamics

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† he author confirms sole responsibility for the following: study conception and design, data collection, analysis and interpretation of results, and manuscript preparation.

Abstract: Living processes are integral to open systems for multiple reasons. From a thermodynamic perspective, they reduce local entropy at the cost of increasing environmental entropy. Furthermore, these processes are inherently nonlinear owing to their feedback interactions with the environment. Nonlinear systems exhibit diverse behaviours: some stabilize at a constant value, whereas others display bifurcations and chaotic dynamics. This study introduces a model that describes living processes as the dynamics of nonlinear single-value diverging maps, whereas chaotic evolution is associated with the increasing entropy environment. This model enables us to describe the lifetime of different species simply by varying the model parameters. This model also simplifies explaining the transition from life to death.

Keywords: nonlinear map; living process; life curve

Introduction

Life processes have been acknowledged as being spontaneous and adhering to the second law of thermodynamics [**?**]. From this perspective, living processes cause a decrease in local entropy. However, owing to the interaction between the system and the environment, disorder within the environment increases. Consequently, an overall increase in total entropy is observed [1–3]. The dynamic relationship between a living system and its environment is characterized by feedback loops, which are essential for the sustainability of life processes [4]. Such systems demonstrate nonlinear dynamics, which implies that life processes inherently display nonlinear behavior [5–7]. Nonlinear systems typically exhibit universal behaviors. In certain regions, a system may converge toward a single stable value. In contrast, bifurcations may emerge in other regions, whereas under different conditions, the system may exhibit chaotic behaviors [8,9]. Several properties that characterize nonlinear systems render them suitable for describing living processes. First, the tendency of regular systems to strive for a constant value is ideal for describing living processes that maintain a stable parameter, including the temperature of homeotherms (birds and mammals) [10–12]. However, nonlinear systems for certain parameters become mathematically indeterminate. For instance, in the logistic map $x_{n+1} = Rx_n(1 - x_n)$, for $R > 4$, the system becomes indeterminate. This study associates the lack of definition of the mapping with death.

The proposed model correlates living processes with single-valued maps and associates chaotic dynamics with environmental interactions. If the living process is considered as a map with a constant final value, then this value can be linked to a parameter that must remain stable to sustain life, such as that occurring in homeotherms. As chaotic dynamics are intrinsically tied to an increase in entropy [13–17]; they can be considered as the environment.

The proposed mathematical model focuses on two main issues:

1. Similar processes in different animal species exhibit a typical lifetime. The last five years have seen publications on various factors, including *Genetic Factors* [18], *Metabolic Rate and Energy Expenditure* [19], *Evolutionary Pressures* [20,21], *Reproductive Strategies* [22] and *Comparative Biology and Genetic Models* [23].
   In our model, the same type of map characterized by parameters describes various processes. Lifetime is determined by setting the parameter values.

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2. The transition between life and death. In biology, death is a process through which biological systems transform from one state to another [24?]. This transition is characterized by critical points at which the systems move from order to disorder, akin to phase changes in physical systems. However, there are several problems with this approach [25]: First, biological systems are inherently complex, which renders the application of the simplified models used in physical systems challenging. Second, the interactions within biological systems are often nonlinear and involve multiple feedback loops. Consequently, the identification of phase transitions becomes a complicated task. The last difficulty offers an advantage to our approach, which already implements non-linearity. In our model, instead of implementing the complicated phase transition scenario, the transition from life to death occurs when the mapping ceases to be mathematically defined, which is prevalent in nonlinear dynamics, such as in a logistic map, inverse sine map, square root map, reciprocal map, and exponential map with logarithm [26–30].

Beyond the phenomenological aspect of describing life processes with a mathematical model, another research platform may be facilitated. A researcher can implement all the justifications for the different lifetimes of life processes to study their effect on the parameters of the mathematical model.

1. Terminology

For clarity, the following terminologies are presented:

1. ***Thermodynamics*** [31,32]:
   (a) *System:* A specific part (generally small) of the universe under consideration.
   (b) *Surroundings (or Environment):* All external elements that can interact with the system.
   (c) *Universe:* The total combination of the system and its surroundings.
2. ***Biology*** [33]
   (a) *Animate processes:* Processes that emphasize mobility, behavior, and sensory responses.
   (b) *Living processes:* Processes that emphasize fundamental life functions and biochemical activities.

As our theory is more closely related to systems that reduce entropy, the term “living process” is more appropriate [34,35]

2. Logistic Map

The logistic map is $x_{n+1} = Rx_n(1 - x_n)$ where R is a parameter determining the nature of the map, regular or chaotic [36,37]. In our approach, living processes are characterized by a system that converges for a predetermined fixed value, denoted as C. For that reason, we express the logistic map as follows:

$$x_{n+1} = x_n \frac{1 - x_n}{1 - C} \tag{1}$$

We are interested in a model with at least two adjustable parameters to define diverse life and animate processes. A single parameter characterizes the logistic model and is, therefore, inappropriate. In the next section, we present a new, more adjustable model.

3. Nonlinear Recursive Model

In the system-environment interaction cycle, two processes co-occur through a regular interaction, thereby producing the map striving for a single value (type 1) and a process that drives the system into chaos (type 2). We associated inanimate systems with convergence toward a chaotic state, whereas living processes were characterized by stabilization at a constant value. The nonlinear recursive relation was designed such that the constant value could be predicted as far in advance as possible.

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3.1. Regular process-living process

We formulate the following recursive relation describing a type 1-interaction:

$$x_{n+1} = \left[C^p - (Rx_n(C - x_n))^p\right]^{\frac{1}{p}} \tag{2}$$

where x is the term life parameter, with $x = C$ indicating that the life parameter is in the best possible life status. Further, n is the time (e.g., to years), and p is another parameter that enables better control in adjusting the parameters to different biological phenomena, such as adjusting the lifetime characteristics of varying living species. In the simulations conducted in this study, we used the simplest model; that is, $p = 1$.

If, during the iteration, $x = C$, the system stabilizes at that predefined value. Thus, the system is designed to reach the best life parameter value. The parameter R defines the map type, characterizing the process type. Numerical calculations, illustrated in Figs. 1 and Figure 2, demonstrate that the final value, once achieved, remained as expected regardless of the initial condition.

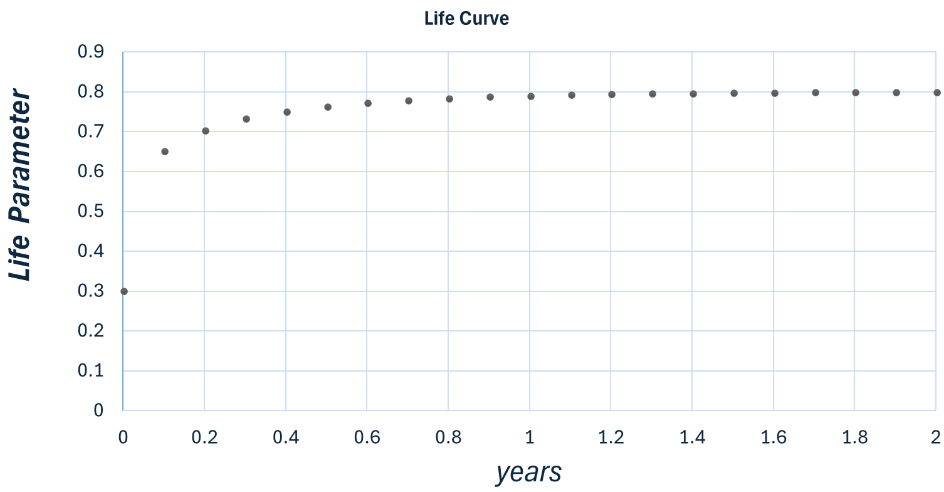

Figure 1. Plot for $C = 0.8$, $R = 1$, and $x_0 = 0.2$. The map stabilizes at $C = 0.8$, as expected.

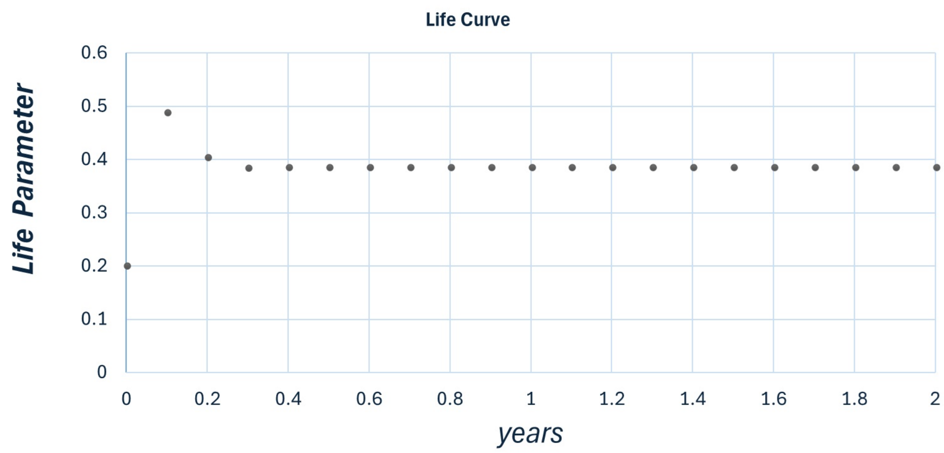

Figure 2. Plot for r $C = 0.8$, $x_0 = 0.2$, and $R = 0.6$. The map stabilizes at $c = 0.369$, contrary to the expected value of 0.8.

3.2. System with both interactions

The second type of interaction involves the following modifications:

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1. We hypothesize a gradual increase in R. This adjustment facilitates two primary effects:
   (a) The system progresses toward chaotic behavior. Associating this increase in chaos with increasing entropy suggests that the increased R aligns with the second law of thermodynamics.
   (b) Beyond a certain threshold of R, the map becomes mathematically undefined (as observed in other maps, such as logistic maps, where for $R > 4$, the map becomes mathematically undefined [36,37]). We interpret this critical condition as indicative of the process of death.
2. The influence of the environment is modeled by introducing slight randomness in x and C.

A plot of x during the years is referred as to *life plot*, as shown in Figure 3 with the parameters $R_0 = 2.5$ and $R_{n+1} - R_n = 0.002$. $C = 1.5 \pm 0.0075$ uniformly distributed, $p = 1.2$ and $x_0 = 0.55$. We identify four distinct phases:

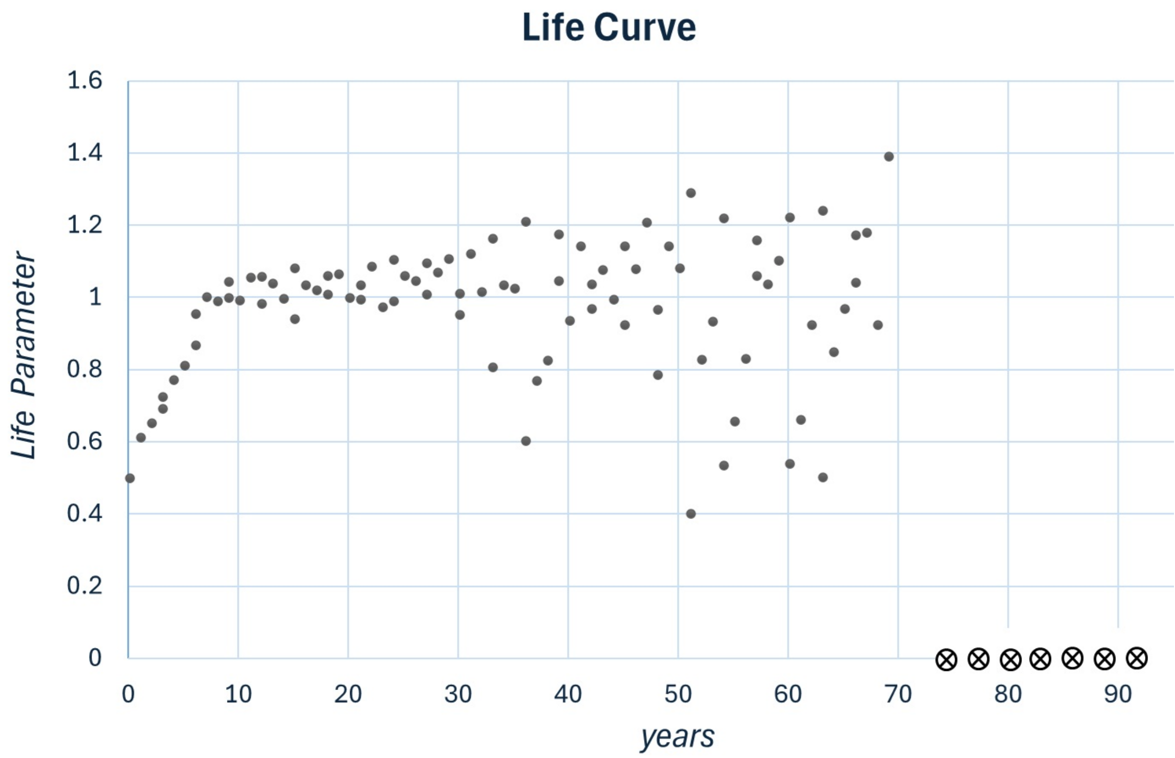

Figure 3. Plot for $c = 1$, $R = 2.5 \pm 0.02.$, x and C are uniformly distributed in the range $\pm 5 \times 10^{-5}$. The simulation illustrates the four stages in the evolution of a living process (The symbol $\otimes$ indicates an undefined map).

1. **Evolving section:**
   For $n = 0 - 8$, x approaches the constant value $c = 1 \pm 0.0075$.
2. **Sustainable zone:**
   For $n = 8 - 25$ x is oscillating randomly around $c = 1 \pm 0.0075$.
3. **Aging:**
   For $n = 30 - 60$, x represents a process wherein any disorder increases.
4. **Death:**
   For $n > 70$ the system becomes mathematically undefined. If the progression of x toward a constant value signifies a living process, the absence of a definable evolution of x, whether decreasing, increasing, or stable, indicates that this living process has ceased to exist.

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4. Example for Adjusting Parameters to Describe Life Process Type

In this study, we proposed a nonlinear map characterized by the parameters R and C. To each of them, we added a random component describing the influence of the environment. We considered it more appropriate to use the current formalism to describe living processes rather than combining all processes describing the animal itself. However, to simplify the example, we describe the life curve of a dog. In Figure 3, we demonstrated a life curve that fits the description of a human who has lived for approximately 70 years. The parameters to describe different animal species could be adjusted. For example, considering a specific dog with a life expectancy of 14.5 years, we adjusted the parameters as $R = 2.5$ and $C = 1$, as presented in Figure 4.

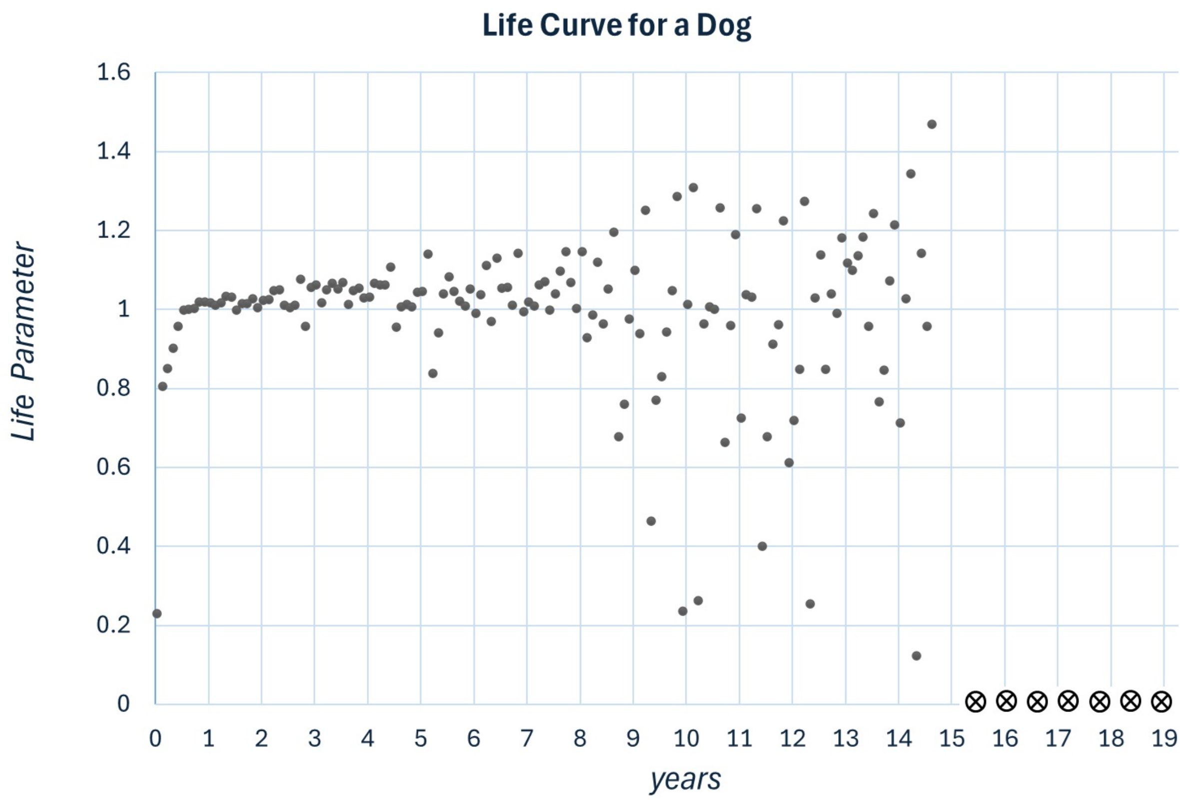

Figure 4. Plot for $c = 1$ and $R = 2.5 \pm 0.02$.. x and C are uniformly distributed in the range $\pm 5 \times 10^{-5}$. Simulation illustrating the four stages in the evolution of a living process.

5. Summary

This study formulated a nonlinear recursion equation based on established characteristics of living processes; specifically, their capacity to reduce entropy locally while interacting with the environment, thus upholding the second law of thermodynamics. This equation facilitates the depiction of a living system that operates in compliance with physical laws as a spontaneous process. The proposed mathematical model offers several advantages for the scientific description of phenomena [38–40]. The benefits pertinent to our research are as follows:

1. Conceptual Understanding: The application of nonlinear dynamics to the study of living processes yields insights that are often not discernible through qualitative analysis alone.
2. Simulation: The proposed model facilitates the simulation of unobserved life forms. Although our current study neglects the bifurcation phenomenon common to nonlinear systems, the identification of living processes that exhibit this behavior could significantly contribute to the field and refine our model.

3. Theoretical framework. The model provides a robust theoretical framework that assists in hypothesis generation, experimental design, and data interpretation. In our approach, various life processes were classified according to distinct nonlinear mappings. The evolution of a system within a specific category is determined by the numerical values of the mapping parameters. In the case of our mode, this may indicate characteristics such as the lifespan of a process.

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