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# Six Dimension for Proof of Riemann Hypothesis

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Posted Date: 22 August 2024

doi: 10.20944/preprints202408.1612.v1

Keywords: Riemann hypothesis; prime numbers; six-dimensional space time

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# Six Dimension for Proof of Riemann Hypothesis

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**Abstract:** In this paper, Riemann's hypothesis is proved based on the golden ratio and eccentricity of the ellipse in six dimensions. The proof of Riemann's hypothesis affects all fields of science. In this article, logical and mathematical rules are proposed for the distribution, identification and classification of prime numbers. The prime numbers are the product of the complex conjugate of five prime numbers in dimensions. Also, even numbers and... are also classified based on prime numbers. Based on the findings of this research, identifying the properties and laws related to prime numbers establishes a deep connection between quantum mechanics and general relativity.

**Keywords:** Riemann hypothesis; prime numbers; six-dimensional space time

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## 1. Introdoction

Riemann's hypothesis along with twin primes is one of Hilbert's unsolved problems. also, this problem is one of the most important problems of the millennium [1,2]. There are many proposed methods to prove this hypothesis [2–5]. Examining the zeta function in different dimensions is a more logical approach [6–8]. Proving the Riemann hypothesis with six-dimensional space-time is a new approach. This research is based on this approach [9]. Examining the eccentricity of the ellipse of the zeta function determines the relationship between the variables of the Riemann function [10]. Hypergeometry for the distribution of prime numbers is a useful step to prove the Riemann hypothesis [11].

In this article, the real Riemann function is introduced based on the eccentricity of the ellipse. The relationship between the eccentricity of the ellipse in two imaginary dimensions and the zeta function is investigated. The existence of obvious zeros for negative even numbers indicates the periodic fluctuation of the function. It has also been investigated using the golden ratio based on the doubling ratio and the 1/2 point for the real part of the zeta function. The logical connection between the eccentricity of the ellipse and the golden ratio paves the way for understanding the distribution of prime numbers. The proof of Riemann's hypothesis based on the structure of the Möbius space in six dimensions is the proof of the nature of prime numbers based on sequences such as the Fibonacci sequence.

## 2. Method

Ellipse eccentricity can be expressed trigonometrically. (2.1)

$$\sqrt{1-\frac{x^2}{y^2}} = \sin(\theta) = \sin\left(cos^{-1}\frac{x}{y}\right) \quad 2.1$$

A radian on the circumference of a circle is less than 60 degrees. (2.2) Therefore, the sum of one radian and ((π-2)/4π) is equal to 1/4 of the circumference of the circle.

$$60^{\circ} - \left(\left(\frac{1}{2\pi}\right)360\right) = 2.70422048692^{\circ} \quad 2.2$$

$$\left(\frac{1}{2\pi}\right) + \left(\frac{\pi-2}{4\pi}\right) = \frac{1}{4}$$

The eccentricity of the ellipse can indicate changes in the circumference of the circle (density). (2.3)

$$\mathrm{x} = 1 \Rightarrow \sin^2(\theta) = 1 - \frac{x}{y} = 1 - y^{-1} \quad 2.3$$

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The golden ratio uses the doubling ratio. (2.4)

$\frac{1+\sqrt{5}}{2} = \varphi$ 2.4

Möbius space transfers properties of higher dimensions to lower dimensions. (2.5)

$w = \frac{az+b}{cz+} \; b \neq d$ 2.5

Riemann's zeta function is convergent for values greater than one. (2.6)

$\zeta(s) = \frac{1}{\Gamma(s)} \int_0^\infty \frac{x^{s-1}}{e^x - 1} dx \to \Gamma(s) = \int_0^\infty x^{s-1} e^{-x} dx$ 2.6

Based on the study of Leonhard Euler, the Riemann zeta function is coded based on prime numbers. (2.7)

$\zeta(s) = \sum_{n=1}^{\infty} n^{-s} = \frac{1}{1^s} + \frac{1}{2^s} + \frac{1}{3^s} + \frac{1}{4^s} + \cdots$ 2.7

$$\zeta(s) = \prod_{p \in \mathbb{P}} \frac{1}{1-p^{-s}} = \frac{1}{1-2^{-s}} \cdot \frac{1}{1-3^{-s}} \cdot \frac{1}{1-5^{-s}} \cdot \frac{1}{1-7^{-s}} \cdots$$

The Riemann conjecture of convergence for real points is 1/2. (2.8)

$\zeta(s) = \zeta\left(\frac{1}{2} + it\right) = 0$ 2.8

A one-dimensional line in the Möbius space equal to the created eccentric radius rotating a quarter in the circumference of the circle, which also applies to higher dimensions. (2.9)

$$\left(\frac{1}{2}\right)^2 2\pi r \; \left(\frac{1}{2}\right)^3 4\pi r^2 \; \left(\frac{1}{2}\right)^4 2\pi^2 r^3 \; \left(\frac{1}{2}\right)^5 \frac{8}{3}\pi^2 r^4 \; \left(\frac{1}{2}\right)^6 \pi^3 r^5$$

$\left(\frac{1}{2}\right)^6 \pi^3 r^5 \cong \ln(\varphi)$ 2.9

**3. Results**

Based on the Möbius function and Möbius space in 5 dimensions, we check the Riemann zeta function. (3.1)

$M\{1, -1, -1, 0, -1, 1\}$ 3.1

$$\frac{1}{\zeta(s)} = \sum_{n=1}^{\infty} \frac{\mu(n)}{n^s}$$

Ellipse eccentricity for a circle embedded on a sphere is equal to the complex conjugate product of Riemann's zeta function at the point 1/2. (3.2)

$\zeta\zeta^* = sin^2(\theta) \Rightarrow sin\left(cos\left(\frac{1}{x^n}\right)\right) \Rightarrow s \longrightarrow \left(\frac{1}{4} \pm 0\right)$ 3.2

Based on Taylor's expansion around point 0, the sine x function is expressed based on odd numbers. (3.3)

$\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots$ 3.3

Considering the eccentricity of the ellipse from the perspective of the surface of a four-dimensional sphere, we rewrite sin^2x. (3.4)

$sin^2(\theta) = 1 - x^{-1}$

$$a_{\mu\nu} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & sin^2\theta & 0 \\ 0 & 0 & sin^2\theta sin^2\phi \end{bmatrix} \Rightarrow 3.4$$

Considering the zeros of Riemann's zeta function for negative even numbers, we express this metric in cosine form. (3.5)

$$a_{\mu\nu} = \begin{bmatrix} cos^2\theta \, cos^2\phi & 0 & 0 \\ 0 & cos^2\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \Rightarrow |a_{\mu\nu}| = cos^2(60)cos^2(120)cos^2(120) = \left(\frac{1}{2}\right)^6 \; 3.5$$

According to the Riemann hypothesis, in a real period for the Riemann function, a six-dimensional sphere is defined with five degrees of freedom and two imaginary vectors. (3.6)

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$$g_{\mu\nu} = \begin{bmatrix} a^2cos^2\theta\, cos^2\phi & 0 & 0 & 0 & 0 & 0 \\ 0 & a^2cos^2\theta & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{a^2}{r^2} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{r^2}{a^2} & 0 & 0 \\ 0 & 0 & 0 & 0 & r^2sin^2\theta & 0 \\ 0 & 0 & 0 & 0 & 0 & r^2sin^2\theta\, sin^2\phi \end{bmatrix} \quad 3.6$$

According to the Möbius space, the eccentricity of the ellipse, Riemann's hypothesis, and the golden ratio, we state the equality condition. (3.7)

$$(3+6i)(3-6i) = 9+36 = 45 \Rightarrow \sin(45) = \cos(45) = \frac{1}{\sqrt{2}} \quad 3.7$$

$$\Rightarrow sin^{-1}(45) = 90 - 257.81302785878i$$

$$\left(\frac{2\,sin(60)+1}{2\,cos(60)+2i}\right)\left(\frac{2\,sin(60)-1}{2\,cos(60)-2i}\right) = 0$$

Based on this, the convergence of the function is natural for real numbers greater than 1. (3.8) Figure 1

$$\sum_{n=0}^{\infty} \frac{\left(\frac{1+\sqrt{5}}{2}\right)^n \left(\frac{1}{2}\right)}{n!e^{\frac{1+\sqrt{5}}{2}}}(x-0.5) \quad 3.8$$

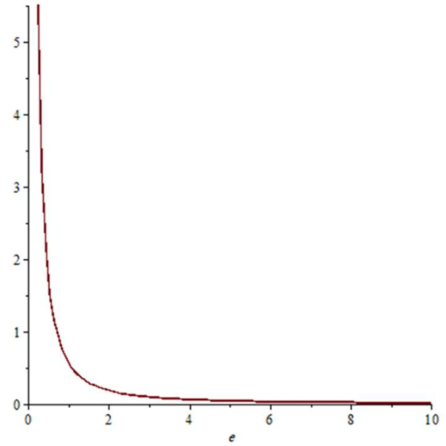


**Figure 1.** Zeta function based on golden ratio and Fibonacci sequence.

But it diverges at point 1/2 in an infinite period. (3.9)

$$\Psi(x) \sim \Upsilon(k) = \sum_{n=0}^{\infty} \frac{\gamma^n(k)a}{n!}(x-a)^n$$

$$f = e^{-2x}\ \Psi(x) \sim \Upsilon(k) =$$

$$\left(\sum_{n=0}^{\infty} \frac{\left(\frac{1+\sqrt{5}}{2}\right)^n \left(\frac{1}{2}\right)}{n!\,e^{\frac{1+\sqrt{5}}{2}}}\left(\left(\frac{1}{2\pi}\right)-0.5\right)\right)\left(\sum_{n=0}^{\infty} \frac{\left(\frac{1+\sqrt{5}}{2}\right)^n \left(\frac{1}{2}\right)}{n!\,e^{\frac{1+\sqrt{5}}{2}}}\left(\frac{1}{2\pi}+0.5\right)\right)$$

Based on the golden ratio and two imaginary vectors, the hypergeometric distribution is the probability distribution rate of the prime numbers in this space. (3.10)

$$1 + \sum_{x=1}^{n}\left(\left(\frac{x!+2x!}{2}\right)\right) = 1 + \frac{3}{2}HG([1,2],[\ \ ],1) - \frac{3}{2}\Gamma(n+2)HG([1,n+2],[\ \ ],1) \quad 3.10$$

Therefore, by using Khayyam-Pascal's expansion, it is possible to calculate the repeatable properties of prime numbers. For example, The fifty-second number of the prime numbers. 3.11

$$P_{52} = 233 \Rightarrow 2+3+3 = 8 \Rightarrow \left(\frac{1}{2^2}+\frac{1}{2^3}+\frac{1}{2^3}\right) = 0.5 \Rightarrow cos^{-1}(0.5) = 60\ \sin(60) = \sqrt{\frac{3}{2}}$$

According to the equality condition (3.7), the properties of prime numbers repeat periodically. Logical relationships are established for different groups of prime numbers. (3.8) Table 1

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**Table 1.** Every prime number can be expressed based on equality conditions.

| | | | | |
|---|---|---|---|---|
| 1 | 2 | 3 | 5 | 7 |
| $\frac{1}{\sqrt{2^1}}$ | $\frac{1}{2}$ | $\frac{1}{\sqrt{2^2 2^1}}$ | $\frac{1}{4}\frac{1}{\sqrt{2^1}}$ | $\frac{1}{8}\frac{1}{\sqrt{2^1}}$ |
| 11 | 17 | 19 | 23 | 29 |
| $\frac{1}{32}\frac{1}{\sqrt{2^1}}$ | $\frac{1}{256}\frac{1}{\sqrt{2^1}}$ | $\frac{1}{512}\frac{1}{\sqrt{2^1}}$ | $\frac{1}{2048}\frac{1}{\sqrt{2^1}}$ | $\frac{1}{16384}\frac{1}{\sqrt{2^1}}$ |
| 31 | 37 | 41 | 43 | 47 |
| $\frac{1}{32768}\frac{1}{\sqrt{2^1}}$ | $\frac{1}{262144}\frac{1}{\sqrt{2^1}}$ | $\frac{1}{1048576}\frac{1}{\sqrt{2^1}}$ | $\frac{1}{2097152}\frac{1}{\sqrt{2^1}}$ | $\frac{1}{8388608}\frac{1}{\sqrt{2^1}}$ |
| A | B | C | D | E |
| $3+2+7+6+8=26$<br>$2+6=8$ | $2+6+1+4+4=17$<br>$1+7=8$ | $1+0+4+8+5+7+6=31$<br>$1+3=4$ | $2+0+9+7+1+5+2=26$<br>$2+6=8$ | $8+3+8+8+8+6+0+8=41$<br>$4+1=5$ |
| $\log_8(32768)=5$ | $\log_8(262144)=6$ | $\log_4(1048576)=10$ | $\log_8(2097152)=7$ | $\log_5(8388608)=9.9\ldots$ |
| $P_5=7$ | $P_6=11$ | $P_{10}=29$ | $P_7=17$ | $P_9=23$ |
| | | | | |
| | | | | |
| 1 | 2 | 3 | 5 | 7 |
| $\frac{1}{\sqrt{2^1}}$ | $\frac{1}{\sqrt{2^2}}$ | $\frac{1}{\sqrt{2^3}}$ | $\frac{1}{\sqrt{2^4 2^5}}$ | $\frac{1}{\sqrt{2^5 2^2}}$ |
| | | | | |
| 11 | 17 | 19 | 23 | 29 |
| $\frac{1}{\sqrt{2^6 2^5}}$ | $\frac{1}{\sqrt{2^7 2^9 2^1}}$ | $\frac{1}{\sqrt{2^8 2^{10} 2^1}}$ | $\frac{1}{\sqrt{2^9 2^{13} 2^1}}$ | $\frac{1}{\sqrt{2^{10} 2^{16} 2^2 2^1}}$ |
| $\frac{1}{\sqrt{2^6 2^5}}$ | $\frac{1}{\sqrt{2^7 2^6 2^4}}$ | $\frac{1}{\sqrt{2^8 2^7 2^4}}$ | $(23)\times 2=46$ | |
| | | | $4+6=10$<br>$=X$ | |
| | | | $P_{X-1}=9$ | |

$$\frac{(9^2\times 2^9)}{92}=\frac{C}{23}$$

$$\frac{(22^{2+22}\times 2^{22+2})}{2222}=\frac{C}{101}$$

$$\frac{(8^2\times 2^8)}{82}=\frac{C}{41}\Rightarrow 3.8$$

$$\frac{(2^{14+2}\times 2^{2+14})}{41+20}=\frac{C}{61}$$

Due to the existence of a natural difference between one radian and a quarter of the circumference of a circle, these relationships are obtained based on the golden constant growth rate in the five dimensions of six dimensions, of the ninth prime number. (3.9)

$$\left(\frac{1}{2\pi}\right)^3\left(\frac{1}{2}\right)^6\pi^3\left(\frac{1}{\sqrt{2}}\right)^5=\frac{1}{2048\sqrt{2}} \quad 3.9$$

Therefore, the ninth prime number represents the end of the first period in the cycle of prime numbers. The hypergeometric distribution of this period is expressed based on the point 1/2. (3.10)

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$$pf(X,u) = \begin{cases} 0 & u < 0 \\ 0 & 3 < u \\ (\frac{1}{6})((binomial(3,u))((binomial(3,5-u)) & other \end{cases}$$

$pf(X,2)$; $\frac{1}{2}$

$\bar{X} = \frac{5}{2}$ 3.10

The repetition of the properties of prime numbers determines how they are distributed. (3.11)

$$(\sum_{x=1}^{9}\left(\frac{\ln(2)}{\ln(10)}\right)^{9} \cong 60^{\circ} - \left(\left(\frac{1}{2\pi}\right)360\right) \quad 3.11$$

According to Riemann's assumption about the zeros of the zeta function for the real part of 1/2, prime numbers can be categorized into two distinct groups: symmetric primes and asymmetric primes. Symmetric primes, including pairs and mirror primes, exhibit properties such as 11, 17, and 23. On the other hand, asymmetric primes, such as 23, mark the beginning of a distinct period in prime number distribution. These asymmetric primes are characterized by being a combination of an even number and ending with the digit 3. (3.12)

$$\frac{P_n}{k} = \frac{x}{2^{\sigma}} + \frac{y}{2^{\sigma}} + \frac{z}{2^{\sigma}} + \cdots \quad 3.12$$

The nature of prime numbers:

Prime numbers are complex products of other prime numbers. (3.13)

$$(\sqrt{2}+7i)(\sqrt{2}-7i) = 51$$
$$(\sqrt{2}+3i)(\sqrt{2}-3i) = 11$$
$$(3\sqrt{2}+3i)(3\sqrt{2}-3i) = 43 \quad 3.13$$
$$(6\sqrt{2}+7i)(6\sqrt{2}-7i) = 121$$

…..

Based on this, the nature of even, symmetrical, etc. numbers are also determined.3.14

$$(5\sqrt{2}+7i)(5\sqrt{2}-7i) = 99 \quad \ldots 3.14$$

**4. Disscoution**

Based on two imaginary dimensions in the six-dimensional space, for the real part of the zeta function 1/2 and also negative even numbers, the zeta function is zero. The cause of the zeroing of the function based on the doubling of the golden ratio is quite evident. The distribution of prime numbers is also expressed based on three vectors in the complex plane. Based on the condition of equal departure from the periodic centre for two imaginary vectors, the prime numbers are the areas of the unit and connected spheres that are expanding. Accordingly, at point 1/2 for a full period, the eccentricity is zero for two vectors. Figure 2

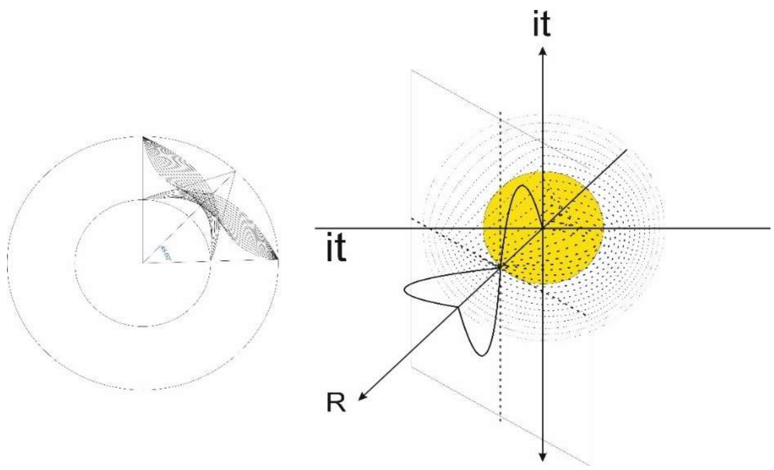


**Figure 2.** The 1.2 point of the phase limit tends to zero.

The ascending function determines the converging eccentricity of the period change points.

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$$\sum_{x=1}^{n}\left(\sin\left(\arccos\left(\frac{1}{x}\right)\right)\right)^2\ (\sin\ (\arccos\ (\frac{1}{n-1}))^2)\quad 4.1$$

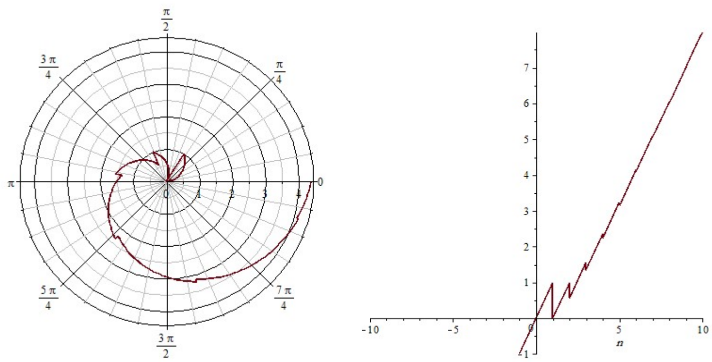


**Figure 3.** Sinusoidal phase shift based on ellipse eccentricity.

The distribution of prime numbers follows the geometric structure of nature. Periodic periods with dimensions greater than three dimensions lead to an increase in complexity. Möbius space transfers the properties of lower dimensions to higher dimensions. And also, it regularly transfers the properties of higher dimensions to lower dimensions. In the way of distribution of prime numbers, the volume of a five-dimensional sphere should be examined. Knowing the concentrated fields of information is possible by examining Einstein's equations in six dimensions. (4.2) [12]

$$\boldsymbol{\Psi_{\mu\nu}} + (\boldsymbol{R_{\mu\nu}} - \frac{1}{2}\boldsymbol{R g_{\mu\nu}} + \boldsymbol{\Lambda g_{\mu\nu}}) = \left(\frac{\pi-2}{2}\right)^6 \left(\frac{he}{c}\right)\boldsymbol{T_{\mu\nu}} + \boldsymbol{K_{\mu\nu}}\quad 4.2$$

**Appreciation:** I appreciate Lhm.Razzazi and all my dear professors for their education and guidance.

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