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Article

Interpretation of Gravity by Entropy

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Abstract: In this paper, we attempt to interpret gravity by entropy. We first introduce generalized entropy, acceleration of its entropy and its partial entropy, and assume that generalized entropy can represent as a second-order polynomial by applying the idea of the logistic function to its entropy. Besides, we define the inverse of partial entropy as the gravitational potential. By applying these concepts, we attempt to explain that 1) gravity becomes constant values within small distance under certain conditions. It is possible that gravity has 5-states within small enough distance. There may exist anti-gravity which is the opposite of Newton's gravity among 5-states. Furthermore, within small distance, we show the possibility that the gravitational potential and the Coulomb potential can treat in the same way, that 2) the rotation speed of the galaxy does not depend on its radius if the radius is within the size level of the universe (The galaxy rotation curve problem), and that 3) the gravitational acceleration toward the center may change at long distance compared to Newton's gravity. We show that it becomes an expansion of Newton's gravity, and that the possibility of the existence of certain constants which control gravity and the speed of galaxies, and that gravity may relate to entropy. It also describes the relationship between the Yukawa type potential and generalized partial entropy with negative. Using equations proposed in this paper, it attempts to propose 11-types of forces (accelerations) including the gravitational acceleration g and compare the ratios of the fundamental 4-forces in nature (strong-force, electromagnetic force, weak-force, and gravity). Furthermore, it suggests that there may exist new forces, and that the gravitational constant G can fluctuate if entropy changes. Thermodynamics, quantum, gravity, electromagnetic and ecology may unify through entropy.

Keywords: entropy; gravity; galaxy rotation curve; MOND; planck's law; coulomb's law; Inverse square law; bose-einstein distribution; fermi-dirac distribution; dynamical system; logistic function; yukawa potential; theory of everything

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1. Introduction

1. First, we define generalized entropy(+) $S_{D_+}(x,k)$ and generalized partial entropy(+) $S_{D_+}(x)$ partitioned by the partition function $D_+(x)$, and introduce acceleration of partial entropy $S''_{D_+}(x)$ and the positive function $Q_{D_+}(x)$ as satisfied $Q_{D_+}(x) = \xi x / D_+(x)$, where x is a positive variable and ξ is a positive constant.
2. Second, by applying the idea of the logistic function to generalized entropy, we derive a function $Q_{D_+}(x)$ that defines the partition function $D_+(x)$. Moreover, we assume that generalized entropy(+) $S_{D_+}(x,k)$ approximate by a second-degree polynomial, that is, the formula $\lambda_2 x^2 + \lambda_1 x$. In other words, we assume that the second derivative of $S_{D_+}(x,k)$ is a constant $\lambda_2/2$.
3. Third, the inverse of partial entropy(+) $S_{D_+}(x)$ is defined as potential $V_{D_+}(x,k)$, and the first derivative of potential $V_{D_+}(x,k)$ is defined as acceleration $V'_{D_+}(x,k)$. Namely, it assumes that potential and acceleration are derived from entropy.
4. Forth, for application to gravity theory, the inverse $1/\lambda_2$ is interpreted as mass m, the constant k is interpreted as the gravitational constant G, and a variable x is interpreted as distance R, etc. Thereby, potential $V_{D_+}(x,k)$ and acceleration $V'_{D_+}(x,k)$ are interpreted as the gravitational potential $V_{D_+}(R,G)$ and the gravitational acceleration $\bar{g}_\pm = -V'_{D_+}(R,G)$. Therefore, we show and propose some conclusions:

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(a) If distance R is small enough, gravity becomes a constant value regardless of R, and may not go to infinity under certain conditions. It is possible that gravity has 5-states within distance R is small enough. Among 5-states, there may exist anti-gravity, which is the opposite of Newton's gravity. Furthermore, within small distance, we show that the possibility that the gravitational potential and the Coulomb potential can treat in the same way.

(b) At distance large enough to be within the size of the universe, gravity follows the adjusted inverse square law. Within this distance, the rotation speed of the galaxy v follows the gravitational constant G, mass m and certain constants, not depend on the galaxy radius R. (the galaxy rotation curve problem)

(c) At large distance, gravity follows an adjusted inverse square law. Comparing to conventional gravity g, the adjusted gravitational acceleration $\tilde{g}_{\pm}$ towards the center of rotation becomes slightly weaker or stronger. This means that the gravitational acceleration towards the center of a rotating object can change slightly with distance (the pioneer Anomaly).

The adjusted gravitational acceleration $V'_{D_+}(R, G)$ can be shown as an expansion of Newton's gravity theory. Therefore, it is possible that there exist certain constants which control gravity and the speed of galaxies.

5. Fifth, it attempts to explain the relationship between the Yukawa type potential and generalized partial entropy(-) with negative. Similarly, we introduce generalized entropy(-) $S_{D_-}(x, k)$ and potential $V_{D_-}(R, G)$. Besides, we define that strong proximity acceleration(force) $g_{\pm}^{sp}$, weak proximity acceleration(force) $g_{\pm}^{wp}$, adjusted gravity $\tilde{g}_{\pm}$ and adjusted electromagnetic force $\bar{E}_{\pm}$. It attempts to propose 11-types of forces (accelerations) and compare the size of these forces. Moreover, we attempt to explain that the ratios of the fundamental 4-forces in nature (strong force, electromagnetic force, weak force, and gravity) are 1, $1E$-2, $1E$-5, and $1E$-39, respectively if the strong force is set to 1. By considering strong proximity acceleration $g_{\pm}^{sp}$ be regarded as strong force, weak proximity acceleration $g_{\pm}^{wp}$ as weak force, adjusted gravity $\tilde{g}_{\pm}$ as gravity and adjusted electromagnetic $\bar{E}_{\pm}$ as electromagnetic force.
6. Finally, it suggests that there may exist new forces, that mass m may represent by entropy and that the gravitational constant G can fluctuate if entropy changes. The gravitational acceleration G and Coulomb's constant k_e would simply be certain coefficients related to forces that humans can currently sense throughout the universe. Thermodynamics, quantum, gravity, electromagnetic and ecology may be unified through entropy.

2. Generalized Entropy and Application to Dynamical Systems

In this section, we introduce generalized entropy, generalized partial entropy and generalized acceleration entropy. We define generalized entropy(+) as follows. In this paper, the logarithm log represents the natural logarithm $\log_e$.

2.1. Generalized Partial Entropy $S_{D_+}(x)$ That Partitioned x by $D_+(x)$

We first define Generalized partial entropy(+) $S_{D_+}(x)$ and the number of states $W_{D_+}(x)$. Generalized partial entropy(+) $S_{D_+}(x)$ under $W_{D_+}(x)$ is defined by the number of states $W_{D_+}(x)$ (see [31] for the source of these ideas).

Definition 2.1. *We define that the number of states $W_{D_+}(x)$ that partitioned x by $D_+(x)$, and generalized partial entropy(+) $S_{D_+}(x)$ as follows:*

$$W_{D_+}(x) = \frac{(D_+(x) + x)^{D_+(x)+x}}{D_+(x)^{D_+(x)} x^x}, \tag{1}$$

$$S_{D_+}(x) = \log W_{D_+}(x), \tag{2}$$

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where x is a positive real variable, and $D_+(x)$ is a positive real valued function that partitioning x. If $x = 0$, then it defines as $D_+(0) = 1$, $W_{D_+}(0) = 1$ and $S_{D_+}(0) = 0$. □

(Note): Originally, the number of states should be defined as below equation(3). However, the equation(3) cannot define on real values well as follows:

$$W_{D_+}(x) = \frac{(D_+(x) + x - 1)!}{(D_+(x) - 1)!x!}. \tag{3}$$

Therefore, we adopt the definition of equation(1) by applying Stirling's approximation to the above equation(3). The partition $D_+(x)$ can be considered as the quantization $D_+(x)$. (End of Note)

2.2. Generalized Entropy(+) $S_{D_+}(x,k)$ and Generalized Partial Entropy(+) $S_{D_+}(x)$.

Definition 2.2. *Generalized entropy(+) $S_{D_+}(x,k)$ and generalized partial entropy(+) $S_{D_+}(x)$. Let $x > 0$ be a real variable, $k \gtrapprox 0$ and $\xi \gtrapprox 0$ be real constants. Let $D_+(x) > 0$ be a positive real valued function that partitioning x. $S_{D_+}(x,k)$, $S_{D_+}(x)$ and $Q_{D_+}(x)$ are defined as follows:*

$$S_{D_+}(x,k) = kD_+(x)S_{D_+}(x), \tag{4}$$

$$Q_{D_+}(x) = \frac{\xi x}{D_+(x)}, \tag{5}$$

$$\begin{aligned} S_{D_+}(x) &= (1 + \frac{x}{D_+(x)}) \log(1 + \frac{x}{D_+(x)}) - \frac{x}{D_+(x)} \log(\frac{x}{D_+(x)}) \\ &= (1 + \frac{Q_{D_+}(x)}{\xi}) \log(1 + \frac{Q_{D_+}(x)}{\xi}) - \frac{Q_{D_+}(x)}{\xi} \log(\frac{Q_{D_+}(x)}{\xi}), \end{aligned} \tag{6}$$

where for any positive variable $x > 0$, the function Q_{D_+} is satisfied $Q_{D_+} \gtrapprox 0$ and $Q'_{D_+} \gtrapprox 0$. □

The above equation(6) can be obtain by applying the definition2.1 and Stirling's approximation. By the above definition, $S'_{D_+}(x)$ and $S''_{D_+}(x)$ are represented as follows:

$$S'_{D_+}(x) = \frac{Q'_{D_+}(x)}{\xi} \left(\log(1 + \frac{Q_{D_+}(x)}{\xi}) - \log(\frac{Q_{D_+}(x)}{\xi}) \right), \tag{7}$$

$$\begin{aligned} S''_{D_+}(x) &= \frac{Q'_{D_+}(x)}{\xi} \left(\frac{Q'_{D_+}(x)}{\xi + Q_{D_+}(x)} - \frac{Q'_{D_+}(x)}{Q_{D_+}(x)} \right) \\ &\quad + \frac{Q''_{D_+}(x)}{\xi} \left(\log(1 + \frac{Q_{D_+}(x)}{\xi}) - \log(\frac{Q_{D_+}(x)}{\xi}) \right). \end{aligned} \tag{8}$$

Let $S'_{D_+}(x)$ be named as entropy generation(+) (velocity) of $S_{D_+}(x)$ (see [13,14]), and $S''_{D_+}(x)$ be named as entropy acceleration(+) of $S_{D_+}(x)$. The function $Q_{D_+}(x)$ can be regard as the position partitioned a real value ξx by $Q_{D_+}(x)$. The first order derivative of $Q_{D_+}(x)$, that is, $Q'_{D_+}(x)$ can be regard as the change of the position by x and ξ (see [31] for details on how to derive generalized entropy, entropy acceleration and its partial entropy).

2.3. The Function $Q_{D_+}(x)$ and Approximation of Generalized Entropy(+) $S_{D_+}(x,k)$.

Next, we find the function $Q_{D_+}(x)$ using the ideas behind Planck's law and the logistic function for dynamical systems. Put the part of partial entropy $S''_{D_+}(x)$ as follows:

$$\frac{Q'_{D_+}(x)}{\xi} \left(\frac{1}{\xi + Q_{D_+}(x)} - \frac{1}{Q_{D_+}(x)} \right) = -\mu(x), \tag{9}$$

where $\mu(x) > 0$ is a positive real function. The left side of above equation(9) looks like spectra partitioned by $\xi x / Q_{D_+}(x)$ and the right side of (9) becomes an approximation by the function $\mu(x)$. We consider $Q'_{D_+}(x)$ as follows:

$$Q'_{D_+}(x) = \frac{dQ_{D_+}}{dx}. \tag{10}$$

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Transforming according to the equation(10), we can represent as follows:

$$dQ_{D_+}\left(\frac{1}{\xi + Q_{D_+}(x)} - \frac{1}{Q_{D_+}(x)}\right) = -\xi\mu(x)dx. \tag{11}$$

Integrating both sides gives as follows:

$$\log(\xi + Q_{D_+}(x)) - \log(Q_{D_+}(x)) = -\xi \int \mu(x)dx \pm \mu_1, \tag{12}$$

where $\mu_1 \geq 0$. Because the left side of the above equation is a positive number and $\xi > 0$, $\mu_1 > 0$. Therefore, we consider only the case where the sign of μ_1 is positive as follows:

$$-\xi \int \mu(x)dx \pm \mu_1 > 0. \tag{13}$$

Therefore, the following equation is satisfied:

$$\log(1 + \frac{\xi}{Q_{D_+}(x)}) = -\xi \int \mu(x)dx + \mu_1. \tag{14}$$

By transforming the above equation, it is satisfied as follows:

$$1 + \frac{\xi}{Q_{D_+}(x)} = \exp(-\xi \int \mu(x)dx + \mu_1). \tag{15}$$

Therefore, the function $Q_{D_+}(x)$ is represented as follows:

$$Q_{D_+}(x) = \frac{\xi}{\exp(-\xi \int \mu(x)dx + \mu_1) - 1}. \tag{16}$$

$Q_{D_+}(x)$ becomes the distribution function of the position which the real value ξx partitioned by $D_+(x)$. Let the distribution function $Q_{D_+}(x)$ named the Planck type distribution function. The equation(9) also looks like spectra partitioned by $\xi x/Q_{D_+}(x)$. If we actually take $Q_{D_+}(x)$ to $\log(x)$, we obtain the equation(16) similar to an expansion of the Planck distribution function (see [1,31]). Besides, the Planck type distribution function $Q_{D_+}(x)$ is thought of the expansion of the Bose-Einstein distribution function.

Further, if we consider to partition $\xi + Q_{D_+}(x)$ and $Q_{D_+}(x)$ into squares of discrete integers, put $Q'_{D_+}(x) = 1$, $\mu(x)$ is represented the wave number, and $1/\xi$ is the Rydberg constant Ry, then the equation(11) resembles the Rydberg formula that represents a spectral series. Namely, entropy may be related to atomic spectra and its energy levels. We would like to make this a topic of research in the future. Next, we make assumption about approximation of generalized entropy(+) $S_{D_+}(x,k)$.

Assumption 2.3. *Assume generalized entropy(+) $S_{D_+}(x,k)$ can approximate by a second-degree polynomial. Hence set as follows:*

$$S_{D_+}(x,k) = \lambda_2 x^2 \pm \lambda_1 x, \tag{17}$$

where $\lambda_2 \gneqq 0$ $\lambda_1 \geq 0$ are real numbers, and $S_{D_+}(0,k) = 0$. □

Hence, the first derivative $S'_{D_+}(x,k)$ is represented a first-degree polynomial as follows:

$$S'_{D_+}(x,k) = 2\lambda_2 x \pm \lambda_1. \tag{18}$$

Besides, the second derivative $S''_{D_+}(x,k)$ is constant. Namely, it is satisfied as follows:

$$S''_{D_+}(x,k) = 2\lambda_2. \tag{19}$$

In other words, we assume that the second derivative of $S_{D_+}(x,k)$ is a constant.

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2.4. The Inverse of Generalized Partial Entropy(+) $S_{D_+}(x)$ and Potential $V_{D_+}(x,k)$.

Next, we focus on the inverse of generalized partial entropy(+) $S_{D_+}(x)$ as follows:

$$\frac{1}{S_{D_+}(x)} = k\frac{\xi x}{Q_{D_+}(x)}\frac{1}{\lambda_2 x^2 \pm \lambda_1 x}. \tag{20}$$

By the equation(16), we can represent as follows:

$$\frac{1}{S_{D_+}(x)} = k\frac{1}{\lambda_2 x \pm \lambda_1}(\exp(-\xi\int\mu(x)dx + \mu_1) - 1). \tag{21}$$

We define the inverse of $S_{D_+}(x)$ as potential $V_{D_+}(x,k)$:

$$V_{D_+}(x,k) = -k\frac{\frac{1}{\lambda_2}}{x \pm \frac{\lambda_1}{\lambda_2}}(1 - \exp(-\xi\int\mu(x)dx + \mu_1)). \tag{22}$$

In other words, the above potential $V_{D_+}(x,k)$ can be defined as the product of a constant k, the partition $D_+(x) = \xi/Q_{D_+}(x)$, and the inverse of generalized partial entropy $S_{D_+}(x)$. Here, let us reorganize the above, that is, we assume as follows:

$$S_{D_+}(x,k) = \lambda_2 x^2 \pm \lambda_1 x, \tag{23}$$

$$\frac{Q'_{D_+}(x)}{\xi}\left(\frac{1}{\xi + Q_{D_+}(x)} - \frac{1}{Q_{D_+}(x)}\right) = -\mu(x), \tag{24}$$

where $\lambda_2 \gneqq 0$ is a positive real number and $\mu(x) \gneqq 0$ is a positive real function. Therefore, we define the inverse of $S_{D_+}(x)$ as potential $V_{D_+}(x,k)$:

$$V_{D_+}(x,k) = -k\frac{\frac{1}{\lambda_2}}{x \pm \frac{\lambda_1}{\lambda_2}}(1 - \exp(-\xi\int\mu(x)dx + \mu_1)), \tag{25}$$

where $\xi \gneqq 0$, $\lambda_2 \gneqq 0$, $\lambda_1 \geq 0$, $\mu_1 \geq 0$ are real numbers and $\mu(x) \gneqq 0$ is a positive real function. The first derivative $V'_{D_+}(x,k)$ is satisfied as follows:

$$V'_{D_+}(x,k) = \frac{k\frac{1}{\lambda_2}}{(x \pm \frac{\lambda_1}{\lambda_2})^2}(1 - \exp(-\xi\int\mu(x)dx + \mu_1)) - \frac{k\frac{1}{\lambda_2}\xi\mu(x)}{x \pm \frac{\lambda_1}{\lambda_2}}\exp(-\xi\int\mu(x)dx + \mu_1). \tag{26}$$

Let $V_{D_+}(x,k)$ be named as potential of $S_{D_+}(x)$, and $V'_{D_+}(x,k)$ be named as acceleration of $S_{D_+}(x)$. We assume as follows:

Assumption 2.4. *It assumes that potential $V_{D_+}(x,k)$ is defined the inverse of generalized partial entropy $S_{D_+}(x)$. Therefore, acceleration $V'_{D_+}(x,k)$ is defined as the first derivative of $V_{D_+}(x,k)$.* □

In the next chapter, we will describe applications of $V_{D_+}(x,k)$ and $V'_{D_+}(x,k)$ to gravity.

3. Application of Potentials $V_{D_+}(x,k)$ to Gravity

The constants, variables, and functions in the above equations(25) and (26) can be chosen arbitrarily within the range of conditions. Therefore, we attempt to interpret these constants, variables and functions as gravity. Namely, we attempt to interpret $V_{D_+}(x,k)$ as the gravitational potential and $V'_{D_+}(x,k)$ as the gravitational acceleration.

3.1. Interpretation to $V_{D_+}(R,G)$

We consider the interpretation of equation $V_{D_+}(x,k)$ as follows:

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$$
\begin{aligned}
x &:= R \geq 0, && R \textit{ is distance},\\
\frac{1}{\lambda_2} &:= m \gtrapprox 0, && m \textit{ is mass within } R,\\
k &:= G, && G \textit{ is the gravitational constant},\\
\xi &:= \xi^g, && \xi^g \textit{ is a constant},\\
\mu(x) &:= \mu_2^g \gtrapprox 0, && \mu_2^g \textit{ is a positive real constant},\\
\mu_1 &:= \mu_1^g \geq 0, && \mu_1^g \textit{ is a real constant},\\
\lambda_1 &:= \lambda_1^g \geq 0, && \lambda_1^g \textit{ is a real constant},
\end{aligned}
\tag{27}
$$

where the symbol g in the upper right corner of the alphabet means g of gravity. We assume as follows: The direction under smaller R is defined as the central direction, that is, the direction towards the center is negative. The gravitational potential increases away from the center and decreases toward the center. However, when $R = 0$ becomes $V_{D_+}(R, G) = 0$. Moreover, it assumes the constant $1/\lambda_2$ is equal to mass m within R.

Assumption 3.1. *It assumes the constant $1/\lambda_2$ is equal to mass m within R. Assume that 2-times the inverse of entropy acceleration, $2/S''_{D_+}(x,k)$, that is, $1/\lambda_2$ is equal to mass m within R. In other words, mass m within R is defined as the inverse of second-order term of $S_{D_+}(x,k)$, that is, $1/\lambda_2$.* □

According assumption 3.1, if entropy acceleration $S''_{D_+}(x,k)$ is large, mass m becomes small, and if the entropy acceleration $S''_{D_+}(x,k)$ is small, mass m becomes large. Doesn't this relationship between entropy acceleration and mass seem intuitive?

We define $V_{D_+}(R, G)$ as the gravitational potential of G as follows:

$$
\begin{aligned}
V_{D_+}(R,G) &= -\frac{Gm}{R \pm \lambda_1^g m}\left(1 - \exp\left(-\xi^g \mu_2^g \int dR + \mu_1^g\right)\right)\\
&= -\frac{Gm}{R \pm \lambda_1^g m}\left(1 - \exp(-\xi^g \mu_2^g R + \mu_1^g)\right).
\end{aligned}
\tag{28}
$$

The first derivative $V'_{D_+}(R, G)$ of $V_{D_+}(R, G)$ is satisfied as follows:

$$
V'_{D_+}(R,G) = \frac{Gm}{(R \pm \lambda_1^g m)^2}\left(1 - \exp(-\xi^g \mu_2^g R + \mu_1^g)\right) - \frac{Gm\xi^g \mu_2^g}{R \pm \lambda_1^g m}\exp(-\xi^g \mu_2^g R + \mu_1^g), \tag{29}
$$

where $\xi^g \gtrapprox 0$, $\mu_2^g \gtrapprox 0$, $\mu_1^g \geq 0$ and $\lambda_1^g \geq 0$.

Definition 3.2. *The Planck type adjusted gravitational acceleration $\bar{g}_\pm = g(R, G)$ is defined as $-V'_{D_+}(R, G)$. Namely, it is satisfied as follows:*

$$
\bar{g}_\pm = g(R,G) = -V'_{D_+}(R,G). \tag{30}
$$

□

The above equation(29) becomes an expansion of the gravitational acceleration. The first term in brackets of equation(28) becomes like the Yukawa potential (see [2],[21],[22],[23]). The differences between the Planck type and the Yukawa type are explained on section4 later.

(Note): Let M be mass located in range R of mass m. Potential energies $U_\pm(R, G)$ of potentials $V_{D_\pm}(R, G)$ is represented as follows:

$$
U_\pm(R,G) = V_{D_\pm}(R,G)M, \tag{31}
$$

and forces $F_\pm(R, G)$ of accelerations $V'_{D_\pm}(R, G)$ is represented as follows:

$$
F_+(R,G) = \bar{g}_\pm M = -V'_{D_+}(R,G)M, \tag{32}
$$

$$
F_-(R,G) = \hat{g}_\pm M = -V'_{D_-}(R,G)M. \tag{33}
$$

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Therefore, it treats force as acceleration in the same way. The gravitational acceleration $\hat{g}_{\pm}$ is define later on subsection 4.4. Moreover, $Q_{D_+}(R)$ becomes like spectra within R that is independent of G and depends on ξ, μ_2^g, μ_1^g and R, and can be represented as follows:

$$Q_{D_+}(R) = \frac{\xi}{\exp(-\xi\mu_2^g R + \mu_1^g) - 1}. \tag{34}$$

Furthermore, the equation $\bar{g}_{\pm}$, which contains $Q_{D_+}(R)$, becomes itself an equation regarding as describe certain wave distribution. If the definition (assumption) of $\bar{g}_{\pm}$ is valid, isn't it possible to think that the equation $\bar{g}_{\pm}$ itself represents the distribution of the gravitational waves? (End of Note)

The solution of equation(29) for μ_1^g is satisfied as follows:

$$\mu_1^g = \log(\frac{1}{1 + (R \pm \lambda_1^g m)\xi^g \mu_2^g}) + \xi^g \mu_2^g R, \quad \mu_1^g \geq 0. \tag{35}$$

Because, the following conditions are needed to satisfied:

$$\exp(-\xi^g \mu_2^g R + \mu_1^g) = \frac{1}{1 + (R \pm \lambda_1^g m)\xi^g \mu_2^g} \geq 0, \tag{36}$$

hence, it is satisfied as follows:

$$1 + (R \pm \lambda_1^g m)\xi^g \mu_2^g \geq 0. \tag{37}$$

Therefore, we propose as follows:

Suggestion 3.3. *The classification of $V'_{D_+}(R, G)$. According to the values $\xi^g \gtrapprox 0$, $\mu_2^g \gtrapprox 0$, $\lambda_1^g \geq 0$ and $\mu_1^g \geq 0$, the equation(29) can be classified as follows:*

1) *If the constant μ_1^g is satisfied as follows:*

$$\mu_1^g > \log(\frac{1}{1 + (R \pm \lambda_1^g m)\xi^g \mu_2^g}) + \xi^g \mu_2^g R, \tag{38}$$

then the above equation (29) becomes negative, that is, it is satisfied as follows:

$$V'_{D_+}(R, G) < 0. \tag{39}$$

(Note): The right side of inequality(38) can become positive or negative.(End of Note)

2) *If the constant μ_1^g is satisfied as follows:*

$$\mu_1^g < \log(\frac{1}{1 + (R \pm \lambda_1^g m)\xi^g \mu_2^g}) + \xi^g \mu_2^g R, \tag{40}$$

then the above equation(29) becomes positive, that is, it is satisfied as follows:

$$V'_{D_+}(R, G) \geq 0. \tag{41}$$

3) *If the constant $\mu_1^g \to 0$, then the following equation is satisfied:*

$$V'_{D_+}(R, G) = \frac{Gm}{(R \pm \lambda_1^g m)^2}(1 - \exp(-\xi^g \mu_2^g R)) - \frac{Gm\xi^g \mu_2^g}{R \pm \lambda_1^g m}\exp(-\xi^g \mu_2^g R). \tag{42}$$

□

3.2. When Distance R Is Small Enough

If distance R is small enough, that is, because distance R approaches 0, hence the value $\exp(-\xi^g \mu_2^g R)$ approaches 1 infinitely. Therefore, the equation(29) is satisfied as follows:

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$$V'_{D_+}(R,G) \simeq \frac{G}{(\pm\lambda_1^g)^2 m}(1-\exp(\mu_1^g)) - \frac{G\xi^g\mu_2^g}{\pm\lambda_1^g}\exp(\mu_1^g), \quad (\because R \to 0,\ \exp(-\xi^g\mu_2^g R) \to 1). \tag{43}$$

If distance $R \neq 0$ and $\lambda_1^g = 0$, then it is satisfied as follows:

$$V'_{D_+}(R,G) = \frac{Gm}{R^2}(1-\exp(-\xi^g\mu_2^g R + \mu_1^g)) - \frac{Gm\xi^g\mu_2^g}{R}\exp(-\xi^g\mu_2^g R + \mu_1^g). \tag{44}$$

The case of the above equation(44), if $R \to 0$, then it becomes $V'_{D_+}(R,G) \to \infty$. Hence, we consider that it makes $\lambda_1^g \neq 0$ and R is small enough. Later we consider the case $\lambda_1^g = 0$. Therefore, if distance R is small enough, then acceleration $V'_{D_+}(R,G)$ approximate by constant values. The following representation is satisfied:

Suggestion 3.4. *Acceleration $V'_{D_+}(R,G)$ becomes a constant value under small enough R. Let m be a positive real number (mass). For sufficiently small distance $R > 0$, the following equation is satisfied: Acceleration $V'_{D_+}(R,G)$ becomes a constant value, that is,*

$$V'_{D_+}(R,G) \simeq \frac{G}{(\pm\lambda_1^g)^2 m}(1-\exp(\mu_1^g)) - \frac{G\xi^g\mu_2^g}{\pm\lambda_1^g}\exp(\mu_1^g), \tag{45}$$

where $\xi^g \gneq 0, \mu_2^g \gneq 0, \lambda_1^g \geq 0$ and $\mu_1^g \geq 0$. □

The solution of equation(45) for μ_1^g is satisfied as follows:

$$\mu_1^g = \log\left(\frac{1}{1 \pm \lambda_1^g\mu_2^g\xi^g m}\right), \qquad \mu_1^g \geq 0. \tag{46}$$

Because, the following conditions are needed to satisfied:

$$\exp(\mu_1^g) = \frac{1}{1 \pm \lambda_1^g\mu_2^g\xi^g m} \geq 0, \tag{47}$$

Because $\xi^g \gneq 0, \mu_2^g \gneq 0$, $m \gneq 0$ and $\lambda_1^g \geq 0$, it is satisfied as follows:

$$1 \pm \lambda_1^g\mu_2^g\xi^g m > 0. \tag{48}$$

Therefore, it is satisfied as follows:

$$m < \frac{1}{\lambda_1^g\mu_2^g\xi^g} \quad \left(and \quad \frac{-1}{\lambda_1^g\mu_2^g\xi^g} < m\right). \tag{49}$$

According to the sign plus or minus of λ_1^g, the value of equation(45) and its solution for μ_1^g can be classified as finite as follows:

1) if $V'_{D_+}(R,G) \neq 0$;

$$V'_{D_+}(R,G) \simeq \frac{G}{(\lambda_1^g)^2 m}(1-\exp(\mu_1^g)) - \frac{G\xi^g\mu_2^g}{\pm\lambda_1^g}\exp(\mu_1^g), \tag{50}$$

2) if $V'_{D_+}(R,G) \neq 0$ and $\mu_1^g \to 0$;

$$V'_{D_+}(R,G) \simeq -\frac{G\xi^g\mu_2^g}{\pm\lambda_1^g}, \qquad (\because \exp(\mu_1^g) \to 1), \tag{51}$$

3) if $\mu_1^g = \log(\frac{1}{1\pm\lambda_1^g\mu_2^g\xi^g m})$;

$$V'_{D_+}(R,G) = 0. \tag{52}$$

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Therefore, we propose as follows:

Suggestion 3.5. *The classified $V'_{D_+}(R,G)$ under small enough R. According to the values $\xi^g \gtrless 0, \mu_2^g \gtrless 0$ $\lambda_1^g \geq 0$ and $\mu_1^g \geq 0$, the equation(45) can be classified as follows:*

1) *If the constant μ_1^g is satisfied as follows:*

$$\mu_1^g > \log(\frac{1}{1 \pm \lambda_1^g \mu_2^g \xi^g m}), \tag{53}$$

then the above equation(45) becomes negative, that is, it is satisfied as follows:

$$V'_{D_+}(R,G) < 0. \tag{54}$$

(Note): The right side of inequality(53) can become positive or negative. (End of Note)

2) *If the constant μ_1^g is satisfied as follows:*

$$\mu_1^g \leq \log(\frac{1}{1 \pm \lambda_1^g \mu_2^g \xi^g m}), \tag{55}$$

then the above equation(45) becomes positive, that is, it is satisfied as follows:

$$V'_{D_+}(R,G) \geq 0. \tag{56}$$

3) *If the constant $\mu_1^g \to 0$, then the following representation is satisfied:*

$$V'_{D_+}(R,G) \simeq -\frac{G\xi^g \mu_2^g}{\pm\lambda_1^g}, \qquad (\because \exp(\mu_1^g) \to 1). \tag{57}$$

□

3.2.1. Summarize the Gravitational Acceleration for Small Enough R

We summarize as the gravitational acceleration $\bar{g}_\pm = -V'_{D_+}(R,G)$. According to the symbol of plus or minus rule for values λ_1^g, it defines $\bar{g}_\pm$ as the adjusted gravitational acceleration with ξ^g and μ_2^g, that is, $\bar{g}_{\pm\lambda_1^g \pm \mu_1^g}$ become as follows:

$$\bar{g}_{\pm\lambda_1^g + \mu_1^g} = -\frac{Gm}{(R \pm \lambda_1^g m)^2}(1 - \exp(-\xi^g \mu_2^g R + \mu_1^g)) + \frac{Gm\xi\mu_2^g}{R \pm \lambda_1^g m}\exp(-\xi^g \mu_2^g R + \mu_1^g), \tag{58}$$

If the constant $\lambda_1^g = 0$, then the adjusted gravitational acceleration with ξ^g, μ_2^g and $\lambda_1^g = 0$ become as follows:

$$\bar{g}_{\pm 0 + \mu_1^g} = -\frac{Gm}{R^2}(1 - \exp(-\xi^g \mu_2^g R + \mu_1^g)) + \frac{Gm\xi^g \mu_2^g}{R}\exp(-\xi^g \mu_2^g R + \mu_1^g), \tag{59}$$

If distance $R \to 0$, then the adjusted gravitational acceleration with ξ^g and $R \to 0$ become as follows:

$$\bar{g}_{\pm\lambda_1^g + \mu_1^g} = -\frac{G}{(\pm\lambda_1^g)^2 m}(1 - \exp(\mu_1^g)) + \frac{G\xi^g \mu_2^g}{\pm\lambda_1^g}\exp(\mu_1^g), \tag{60}$$

Therefore, if distance $R \to 0$, then it is satisfied as follows:

$$
\begin{aligned}
&\bar{g}_{\pm\lambda_1^g+\mu_1^g} = \lim_{R\to 0} \bar{g}_{\lambda_1^g+\mu_1^g \pm}, \\
&\lim_{\mu_1^g\to 0} \bar{g}_{\pm\lambda_1^g+\mu_1^g} = \frac{G\xi^g\mu_2^g}{\pm\lambda_1^g}, \\
&\lim_{\mu_1^g\to 0,\lambda_1^g\to 0} \bar{g}_{\pm\lambda_1^g+\mu_1^g} = \pm\infty, \\
&\lim_{\mu_1^g\to 0,\lambda_1^g\to \infty} \bar{g}_{\pm\lambda_1^g+\mu_1^g} = 0, \\
&\lim_{\mu_1^g\to \infty} \bar{g}_{+\lambda_1^g+\mu_1^g} = \infty, \quad \because)\ \frac{-1}{\pm\lambda_1^g m} + \mu_2^g\xi^g > 0, \\
&\lim_{\mu_1^g\to \infty} \bar{g}_{-\lambda_1^g+\mu_1^g} = -\infty \quad \because)\ \frac{-1}{\pm\lambda_1^g m} + \mu_2^g\xi^g < 0.
\end{aligned}
\tag{61}
$$

From the above, we can suppose as follows:

Suggestion 3.6. *Within distance R is small enough, gravity has 5-states. Within distance R is small enough, it is possible that gravity $\bar{g}$ has 5-states such that finite 2-states $\frac{G\xi^g\mu_2^g}{\pm\lambda_1^g}$, that infinite 2-states $\pm\infty$, and that zero 1-state* 0. □

The values $-G\xi^g\mu_2^g/\lambda_1^g$ and $\bar{g}_\pm = -V'_{D_+}(R,G) < 0$ have the same direction as Newton's gravity, where the direction towards the center is negative. However the values $G\xi^g\mu_2^g/\lambda_1^g$ and $\bar{g}_\pm = -V'_{D_+}(R,G) > 0$ have the opposite direction as Newton's gravity. This means that it may represent the existence of anti-gravity. If distance R is small enough, hence the acceleration $V'_{D_+}(R,G)$ becomes certain finite constant depend on constants $\xi^g, \lambda_1^g, \mu_1^g$ and μ_2^g, not infinite. However, if the constant λ_1^g or μ_1^g approach 0 or ∞, then the acceleration $V'_{D_+}(R,G)$ becomes ∞ or 0. Depending on the value of μ_1 and λ_1^g, the value $V'_{D_+}(R,G)$ can be positive or negative. When the value $V'_{D_+}(R,G)$ is negative, deceleration acts toward the center. These constants is depended on generalized entropy and the part of generalized partial entropy. Namely, acceleration depends on generalized entropy. Therefore, there exists 5-states within distance R is small. The constant λ_1^g is the coefficients of approximated generalized entropy $\lambda_2^g x^2 + \lambda_1^g x$, and a constant μ_1^g is an integral constant obtained by integrating the parts of partial entropy $S''_{D_+}(x)$. In other words, acceleration moving away from the center is changed by these constant, that is, simply acceleration depends on entropy. The above description can be applied to Coulomb's law (electric field). By adjusting the values $\mu_1, \mu_2, \lambda_1, m = 1/\lambda_2$ and ξ, it may be possible to make the argument by replacing the gravitational constant G to Coulomb's constant k_e. (In this paper, Coulomb's constant is defined as k_e.) We will describe this possibility next.
(Note): The center direction is defined as the positive direction. (End of Note)

3.2.2. Compare $V_{D_+}(R,G)$ and $V_{D_+}(R,k_e)$ for Small R

We attempt to compare $V_{D_+}(R,G)$ and $V_{D_+}(R,k_e)$. Similarly, the gravitational potential $V_{D_+}(R,G)$, we define the Coulomb potential $V_{D_+}(R,k_e)$ as follows:

$$
V_{D_+}(R,k_e) = -\frac{k_e e_q}{R \pm \lambda_1^c e_q}(1 - \exp(-\xi^c\mu_2^c R + \mu_1^c)),
\tag{62}
$$

where $e_q > 0$ is the elementary charge, and $\xi^c \gneqq 0$, $\mu_2^c \gneqq 0$, $\mu_1^c \geq 0$ and $\lambda_1^c \geq 0$, and the symbol c in the upper right corner of the alphabet means c of Coulomb. For example, we set the constants as follows:

$$
\begin{aligned}
&G := 6.674E\text{-}11, && G \text{ is the gravitational constant},\\
&\xi^g = \xi^c := h = 6.626E\text{-}34, && h \text{ is Planck's constant},\\
&k_e := 8.987E\text{+}9, && k_e \text{ is Coulomb's constant},\\
&e_q := 1.604E\text{-}19, && e_q \text{ is the elementary charge},\\
&m := m_p = 2.176E\text{-}8, && m_p \text{ is the Planck mass}(kg),\\
&\mu_2^g := 1, && \mu_2^g \text{ is a real constant},\\
&\mu_2^c := 1, && \mu_2^c \text{ is a real constant},\\
&\lambda_1^g := 1, && \lambda_1^g \text{ is a real constant},\\
&\lambda_1^c := 1, && \lambda_1^c \text{ is a real constant}.
\end{aligned} \tag{63}
$$

Using the above constants, the gravitational potential $V_{D_+}(R,G)$ is satisfied as follows:

$$V_{D_+}(R,G) = 2.442E\text{-}12, \quad if\ \mu_1^g = 1, \tag{64}$$
$$V_{D_+}(R,G) = 2.480E\text{-}3, \quad if\ \mu_1^g = 21.28, \tag{65}$$

where $R := 1.000E$-6 meter and the Planck mass m_p is used instead of mass m. Let the sign of μ_1^g and λ_1^g be plus such as $+\mu_1^g$ and $+\lambda_1^g$. Similarly, the Coulomb potential $V_{D_+}(R,k_e)$ is satisfied as follows:

$$V_{D_+}(R,k_e) = 2.474E\text{-}3, \quad if\ \mu_1^c = 1, \tag{66}$$
$$V_{D_+}(R,k_e) = 2.512E\text{+}6, \quad if\ \mu_1^c = 21.28, \tag{67}$$

where $R := 1.000E$-6 meter, the elementary charge e_q is used instead of mass m and Coulomb's constant k_e is used instead of G. The signs of μ_1^c and λ_1^c are $+\mu_1^c$ and $+\lambda_1^c$. The values $V_{D_+}(R,G)$ and $V_{D_+}(R,k_e)$ change depending on how the constants μ_1^g and $\mu 1^c$ are selected. The above values (65) and (66) is close. Therefore, for small distance R, $\mu_1^g = 21.28$ and $\mu_1^c = 1$, it is satisfied $V_{D_+}(R,G) \simeq V_{D_+}(R,k_e)$. In consequence, it is satisfied as follows:

Suggestion 3.7. *Let m_p be the Planck mass, e_q be the elementary charge, G be the gravitational constant and k_e be Coulomb's constant. For small distance $R > 0$, there exist certain constants ξ^g, ξ^c, μ_2^g, μ_2^c, μ_1^g, μ_1^c, λ_1^g and λ_1^c such that the following equation is satisfied:*

$$V_{D_+}(R,G) \simeq V_{D_+}(R,k_e), \tag{68}$$

where $\xi^g, \xi^c, \mu_2^g, \mu_2^c \gtrsim 0$ and $\lambda_1^g, \lambda_1^c, \mu_1^g, \mu_1^c \geq 0$. □

Because, if it is satisfied as follows:

$$
\begin{aligned}
V_{D_+}(R,G) &= -\frac{Gm_p}{R \pm \lambda_1^g m_p}(1 - \exp(-\xi^g \mu_2^g R + \mu_1^g))\\
&= -\frac{k_e e_q}{R \pm \lambda_1^c e_q}(1 - \exp(-\xi^c \mu_2^c R + \mu_1^c))\\
&= V_{D_+}(R,k_e),
\end{aligned} \tag{69}
$$

then transforming the above equation, it becomes as follows:

$$\exp(-\xi^g \mu_2^g R + \mu_1^g) = 1 + \frac{R \pm \lambda_1^g m_p}{Gm_p} \frac{k_e e_q}{R \pm \lambda_1^c e_q}(\exp(-\xi^c \mu_2^c R + \mu_1^c) - 1). \tag{70}$$

Therefore, if the value μ_2^c is given, the value μ_2^g can be found as follows:

$$\mu_2^g = \frac{1}{\xi^g R}\left[\mu_1^g - \log\left(1 + (\frac{R \pm \lambda_1^g m_p}{R \pm \lambda_1^c e_q})(\frac{k_e e_q}{Gm_p})(\exp(-\xi^c \mu_2^c R + \mu_1^c) - 1)\right)\right]. \tag{71}$$

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Namely, using the equation for potential derived from entropy, within small distance, it may be possible to treat the gravitational potential and the Coulomb potential in the same way by appropriately choosing certain constants. In same way, applying the gravitational acceleration $V'_{D_+}(R,G)$ and Coulomb's law (electric field) $V'_{D_+}(R,k_e)$, we can obtain a suggestion as follows:

Suggestion 3.8. *Let m_p be the Planck mass, e_q be the elementary charge, G be the gravitational constant and k_e be Coulomb's constant. For small distance $R > 0$, there exist certain constants ξ^g, ξ^c, μ_2^g μ_2^c, μ_1^g μ_1^c λ_1^g and λ_1^c such that the following equation is satisfied:*

$$V'_{D_+}(R,G) \simeq V'_{D_+}(R,k_e), \tag{72}$$

where $\xi^g, \xi^c, \mu_2^g, \mu_2^c \gneq 0$ and $\lambda_1^g, \lambda_1^c, \mu_1^g, \mu_1^c \geq 0$. □

3.3. When Distance R Is Large, However ξ Is Small Enough

Assuming distance R is large and the constant ξ^g is small like Planck's constant, that is, $\xi^g \sim h$. The constant h is Planck's constant, 6.626E-34$J \cdot s$ and the constant $\mu_2^g = 1$. Assume that R is the radius of the universe within 46.5 billion light years (4.65E+10). Because, one light year is approximately 9.461E+15 meter, we assume that the radius of the universe $R \simeq 4.399E+26$ meter. Therefore, the following condition is satisfied:

$$\xi^g \mu_2^g R \simeq 2.915E\text{-}7 \ll 1. \tag{73}$$

The function $\exp(-\xi^g \mu_2^g R)$ is approximately equal to 1, that is, it is satisfied as follows:

$$\exp(-\xi^g \mu_2^g R) \simeq 1. \tag{74}$$

Therefore, the following representations are satisfied:

$$\begin{aligned} \bar{g}_\pm &= -\frac{Gm}{(R \pm \lambda_1^g m)^2}(1-\exp(-\xi^g \mu_2^g R+\mu_1^g))+\frac{Gm\xi^g \mu_2^g}{(R \pm \lambda_1^g m)}\exp(-\xi^g \mu_2^g R+\mu_1^g) \\ &\simeq -\frac{Gm}{(R \pm \lambda_1^g m)^2}(1-\exp(\mu_1^g))+\frac{Gm\xi^g \mu_2^g}{(R \pm \lambda_1^g m)}\exp(\mu_1^g), \ (\because \exp(-\xi^g \mu_2^g R) \simeq 1). \end{aligned} \tag{75}$$

When the condition $\xi^g \mu_2^g R \ll 1$ is satisfied, applying to mass M in circular orbit around mass m, the following equation is satisfied:

$$-\frac{GmM}{(R \pm \lambda_1^g m)^2}(1-\exp(\mu_1^g)) + \frac{GmM\xi^g \mu_2^g}{(R \pm \lambda_1^g m)}\exp(\mu_1^g) = M\frac{v^2}{R}, \tag{76}$$

where m is mass within radius R and v is the rotation speed of mass M on radius R. The right side of equation(76) becomes centrifugal acceleration of mass M. Hence the following representations satisfied:

$$\begin{aligned} v &= \sqrt{\frac{-GmR}{(R \pm \lambda_1^g m)^2}(1-\exp(\mu_1^g)) + \frac{Gm\xi^g \mu_2^g}{(1 \pm \frac{\lambda_1^g m}{R})}\exp(\mu_1^g)} \\ &= \sqrt{\frac{-Gm}{(R \pm \lambda_1^g m)(1 \pm \frac{\lambda_1^g m}{R})}(1-\exp(\mu_1^g)) + \frac{Gm\xi^g \mu_2^g}{(1 \pm \frac{\lambda_1^g m}{R})}\exp(\mu_1^g)} \\ &\simeq \sqrt{Gm\xi^g \mu_2^g \exp(\mu_1^g)}, \quad (\because R \text{ is large enough and } (1 + \frac{\lambda_1^g m}{R}) \to 1). \end{aligned} \tag{77}$$

Therefore, we propose that the following is satisfied:

Suggestion 3.9. *Let $m > 0$ (mass) and v (the speed of rotation) be a positive real numbers. For large distance $R > 0$ within 4.399E+26, the following condition is satisfied:*

$$v \simeq \sqrt{Gm\xi^g \mu_2^g \exp(\mu_1^g)}, \tag{78}$$

where $\xi^g, \mu_2^g \gneq 0$ *and* $\lambda_1^g, \mu_1^g \geq 0$*. As a results, the speed of rotation* v *at radius* R *approximate by a constant values* $\sqrt{Gm\xi^g \mu_2^g \exp(\mu_1^g)}$*, not depend on radius* R*.* □

Therefore, the speed of rotation v is depended on constants G, m, ξ^g, μ_1^g and μ_2^g, not depend on radius R. It is noticed that these constants is decided by generalized entropy $S_{D_+}(x,k)$ and the distribution function $Q_{D_+}(x)$. According the suggestion 3.9, let m be equal to mass of the Milky Way Galaxy, that is, $m \simeq 1.989E{+}30 \times 2.0E{+}12$ kg, where mass of the sun is $1.989E{+}30$ and the sun count in the Milky Way Galaxy is $2.0E{+}12$. Therefore, if setting $\xi = 1E\text{-}34 \sim h$ (Planck's constant) and $\mu_2^g = 1$, then the speed of rotation is satisfied depending the constant μ_1^g as follows:

$$v \simeq 4.194E\text{-}1\sqrt{\exp(\mu_1^g)}\ m/s. \tag{79}$$

For example, let $\mu_1^g = 26.36$, the speed of rotation v became as follows:

$$v \simeq 2.222E{+}5\ m/s. \tag{80}$$

In this case, the speed of (80) is close to the rotation speed of the Milky Way Galaxy, that is, approximately $2.200E{+}5 \sim 2.400E{+}5$ m/s. Even without assuming dark matter, the galaxy rotation problem can be explained by the concept of entropy. This does not mean denying dark matter. New constants μ_1^g and μ_2^g may represent some kind of dark or virtual mass. Besides, the suggestion 3.9 may consider to apply to velocities over short distances such as electrons in atomic nuclei. Similar results are obtained for $\hat{g}_\pm$ of the Yukawa type potential and acceleration discussed later on subsection 4.4.

3.4. When Distance R Is Large Enough

If distance R is large enough, the equation(29) is satisfied as follows:

$$\tilde{g}_\pm = -V'_{D_+}(R,G) = -\frac{Gm}{(R \pm \lambda_1^g m)^2}, \quad (\because \exp(-\xi^g \mu_2^g R + \mu_1^g) \to 0). \tag{81}$$

Therefore, the following conditions are satisfied:

$$-\frac{Gm}{(R-\lambda_1^g m)^2} \lesssim -\frac{Gm}{R^2} \lesssim -\frac{Gm}{(R+\lambda_1^g m)^2}. \tag{82}$$

If distance R is large enough and the constant λ_1^g is small enough, that is, $\lambda_1^g \to 0$, then the gravitational acceleration $V'_{D_+}(R,G)$ becomes Newton's gravity.

3.4.1. Summarize the Gravitational Acceleration for Large Distance R

We summarize the above gravitational acceleration $V'_D(R,G)$ as follows:

1. The adjusted gravitational acceleration, R is large enough:

$$\tilde{g}_\pm = -\frac{Gm}{(R \pm \lambda_1^g m)^2}, \tag{83}$$

2. The original gravitational acceleration, R is large enough and $\lambda_1^g \to 0$:

$$g = -\frac{Gm}{R^2}, \tag{84}$$

where R is large enough. Newton's gravity is satisfied when R is large enough and $\lambda_1^g \to 0$. According to g of (84) and $\tilde{g}_\pm$ of (83) in the above equation, we propose as follows:

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Suggestion 3.10. *Gravity changes depending on the value λ_1^g of generalized entropy coefficient. Let $m > 0$ be a real number (mass). For large $R > 1$, the following conditions are satisfied:*

$$\tilde{g}_- = -\frac{Gm}{(R-\lambda_1^g m)^2} \lesssim g \lesssim -\frac{Gm}{(R+\lambda_1^g m)^2} = \tilde{g}_+, \tag{85}$$

where $\lambda_1^g \geq 0$ is a real constant. □

The suggestion above is an expansion of Newton's gravity. For large distance R, it is possible that the adjusted gravity $\tilde{g}_\pm$ is smaller or larger towards the center than Newton's gravity g. In other words, the gravitational acceleration towards the center of a rotating object can change slightly with sufficient large distance. The gravitational acceleration moving away from the center is changed by the constant λ_1^g. The constant λ_1^g is the coefficients of degree one of approximate generalized entropy $\lambda_2^g x^2 + \lambda_1^g x$. In other words, the gravitational acceleration moving away from the center is changed by the coefficients of approximated generalized entropy, that is, the gravitational acceleration is considered to depend on entropy.

4. The Yukawa Type Potential and Entropy, Comparison of Accelerations

4.1. Relationship with the Yukawa Type Potential and Potential $V_{D_+}(R,G)$

In this section, we consider the Yukawa type potentials, which represent the potential of elementary particles. Potential $V_{D_+}(R,G)$ of (28) contains an equation similar to the Yukawa potential (see [21]). If we omit the first term in the equation(28), we can obtain as follows:

$$V_{D_+}(R,G)_{omit} = \frac{Gm}{R \pm \lambda_1^g m}\exp(-\xi^g \mu_2^g R \pm \mu_1^g). \tag{86}$$

We consider that by substituting the constants as follows:

$$Gm := g_y^2, \quad \xi^g \mu_2^g := \lambda, \quad \mu_1^g := 0, \quad \lambda_1^g := 0, \tag{87}$$

where g_y is Yukawa's constant and $\lambda = mc/\hbar$ (see [21]). Thereby, we can obtain the following the Yukawa potential:

$$V_{yukawa1}(R) = \pm g_y^2 \frac{\exp(-\lambda R)}{R}. \tag{88}$$

Therefore, it may also have applications in particle theory and other potential theory. Moreover, we also consider that by substituting the constants as follows:

$$\xi^g \mu_2^g := \lambda, \quad \exp(\mu_1^g) := \alpha, \quad \lambda_1^g := 0, \tag{89}$$

where $\lambda = mc/\hbar$ (see [21–23]). Thereby, we can obtain the gravitational potential as follows:

$$V_\alpha(R) = -G\frac{m}{R}(1 - \alpha\exp(-\lambda R)). \tag{90}$$

The above equation has a different sign of second term in brackets compared to the following the Yukawa potential proposed as follows (see [21–23]):

$$V_{yukawa2}(R) = -G\frac{m}{R}(1 + \alpha\exp(-\lambda R)). \tag{91}$$

Therefore, the above equation(90) and (91) are incompatible. It seems necessary to consider another way to integrate two equations. Namely, we need to change the definition2.2 and the assumption2.4. These contents are described in the next subsection.

4.2. Generalized Partial Entropy with Negative

Same as section 2, we define generalized entropy(-) $S_{D_-}(x,k)$, generalized partial entropy(-) $S_{D_-}(x)$, and the number of states $W_{D_-}(x)$. Generalized partial entropy(-) $S_{D_-}(x)$ under $W_{D_-}(x)$ is defined by the number of states $W_{D_-}(x)$ (see [31]).

Definition 4.1. *Generalized partial entropy(-) $S_{D_-}(x)$. Let $x>0$ be a real variable and $\xi \gtrsim 0$ be real constants. Let $D_-(x)$ be a negative real valued function that partitioning x. We define that the number of states $W_{D_-}(x)$ that partitioned x by $D_-(x)$, generalized partial entropy(-) $S_{D_-}(x)$ and a function $Q_{D_-}(x)$ as follows:*

$$W_{D_-}(x) = \frac{(D_-(x)+x)^{D_-(x)+x}}{D_-(x)^{D_-(x)}x^x}, \tag{92}$$

$$Q_{D_-}(x) = \frac{-\xi x}{D_-(x)} \gtrsim 0, \tag{93}$$

$$S_{D_-}(x) = \log W_{D_-}(x). \tag{94}$$

If $x=0$, then it defines as $D_-(0)=1$, $W_{D_-}(0)=1$ and $S_{D_-}(0)=0$. For any x, $Q_{D_-}(x)$ is satisfied as follows:

$$Q_{D_-}(x) \gtrsim 0, \quad Q'_{D_-}(x) \gtrsim 0, \quad \xi > Q_{D_-}(x). \tag{95}$$

However, because $D_-(x)$ is a negative real value, hence the part of $D_-(x)^{D_-(x)}$ and $\log$ with a negative argument become a complex number. Therefore, $W_{D_-}(x)$ and $S_{D_-}(x)$ become a complex number. Therefore, it needs to be extended its to complex numbers as follows:

$$D_-(x) = e^{2(n+1)\pi i} e^{\log(-D_-(x))}, \tag{96}$$

$$\begin{aligned} W_{D_-(x)} &= \frac{e^{2(n+1)\pi i(D_-(x)+x)} e^{(D_-(x)+x)\log(-(D_-(x)+x))}}{e^{2(n+1)\pi i D_-(x)} e^{D_-(x)\log(-D_-(x))} x^x} \\ &= e^{2(n+1)\pi i x}(1+\frac{x}{D_-(x)})^{D_-(x)}(1+\frac{D_-(x)}{x})^x, \end{aligned} \tag{97}$$

$$S_{D_-}(x) = (1+\frac{Q_{D_-}(x)}{-\xi})\log(1+\frac{Q_{D_-}(x)}{-\xi}) - \frac{Q_{D_-}(x)}{-\xi}\left(\log(\frac{Q_{D_-}(x)}{\xi}) + \log(-1)\right), \tag{98}$$

where $\log(-1)=2(n+1)\pi i$, $n \geq 0$, and i is an imaginary number. Treating the above equation(98), $S'_{D_-}(x)$ and $S''_{D_-}(x)$ can be obtained as follows:

$$S'_{D_-}(x) = \frac{Q'_{D_-}(x)}{-\xi}\left(\log(1+\frac{Q_{D_-}(x)}{-\xi}) - \log(\frac{Q_{D_-}(x)}{\xi})\right) - \frac{Q'_{D_-}(x)}{-\xi}\log(-1), \tag{99}$$

$$\begin{aligned} S''_{D_-}(x) = &\frac{Q'_{D_-}(x)}{-\xi}\left(\frac{Q'_{D_-}(x)}{-\xi+Q_{D_-}(x)} - \frac{Q'_{D_-}(x)}{Q_{D_-}(x)}\right) \\ &+ \frac{Q''_{D_-}(x)}{-\xi}\left(\log(1+\frac{Q_{D_-}(x)}{-\xi}) - \log(\frac{Q_{D_-}(x)}{\xi})\right) - \frac{Q''_{D_-}(x)}{-\xi}\log(-1). \end{aligned} \tag{100}$$

The real value of generalized partial entropy(-) $Re[S_{D_-}(x)]$ becomes a negative value, where the function $Re[x]$ denotes the real value x for any positive variable $x>0$. □

Namely, this means that the above definition assumes the existence of negative partial entropy.

Definition 4.2. *Generalized Entropy(-) $S_{D_-}(x,k)$. Let $x>0$ be a real variable, $k \gtrsim 0$ and $D_-(x)$ be a negative real valued function that partitioning x. $S_{D_-}(x,k)$ is defined as follows:*

$$S_{D_-}(x,k) = kD_-(x)Re\left[S_{D_-}(x)\right] > 0, \tag{101}$$

It defines an approximation of $S_{D_-}(x,k)$ and obtains $Re[S_{D_-}(x)]$ as follows:

$$S_{D_-}(x,k) = \lambda_2 x^2 \pm \lambda_1 x, \tag{102}$$

$$Re\Big[S_{D_-}(x)\Big] = \frac{1}{k}\frac{Q_{D_-}(x)}{-\xi x}S_{D_-}(x,k) \leq 0, \tag{103}$$

where $Q_{D_-}(x)$ is discussed again in the following subsection 4.3. □

(Note): The parts (that is, generalized partial entropy(-)) are negative, however the whole (that is, generalized entropy(-)) is positive. It can be thought of the partition $D_-(x)$ and the number of states $W_{D_-(x)}$ become a kind of wave functions. Therefore, generalized partial entropy(-) can be thought of as information in the form of the logarithm of a wave $W_{D_-(x)}$, which is the state of complex number. We would further research the meaning of negative partitions. (End of Note).

4.3. The Function $Q_{D_-}(x)$ for the Yukawa Type Potential

We find the function $Q_{D_-}(x)$ using the idea behind Planck's law and the logistic function for dynamical systems. Put the part of partial entropy $S''_{D_-}(x)$ as follows:

$$\frac{Q'_{D_-}(x)}{-\xi}\left(\frac{-1}{\xi - Q_{D_-}(x)} - \frac{1}{Q_{D_-}(x)}\right) = \mu(x) \geq 0, \tag{104}$$

where $\mu(x) > 0$ is a positive real function and $\xi > Q_{D_-}(x)$. The above parts of $\frac{Q'_{D_-}(x)}{-\xi}$ is negative value and the above parts of $(\frac{-1}{\xi - Q_{D_-}(x)} - \frac{1}{Q_{D_-}(x)})$ is also negative. Therefore, the above equation(104) is positive. Transforming according to the equation(10), we can represent as follows:

$$dQ_{D_-}\left(\frac{-1}{\xi - Q_{D_-}(x)} - \frac{1}{Q_{D_-}(x)}\right) = -\xi\mu(x)dx. \tag{105}$$

Integrating both sides gives as follows:

$$\log(\xi - Q_{D_-}(x)) - \log(Q_{D_-}(x)) = -\xi\int \mu(x)dx \pm \mu_1, \tag{106}$$

where $\mu_1 > 0$ and $\xi > 0$. Therefore, the following equation is satisfied:

$$\log(\frac{\xi}{Q_{D_-}(x)} - 1) = -\xi\int \mu(x)dx \pm \mu_1. \tag{107}$$

By transforming the above equation, it is satisfied as follows:

$$\frac{\xi}{Q_{D_-}(x)} - 1 = \exp(-\xi\int \mu(x)dx \pm \mu_1). \tag{108}$$

Because the right side is positive, the left side must also be positive, thus, it needs to be satisfied $\xi > Q_{D_-}(x)$. Therefore, the function $Q_{D_-}(x)$ is represented as follows:

$$Q_{D_-}(x) = \frac{\xi}{\exp(-\xi\int \mu(x)dx \pm \mu_1) + 1}. \tag{109}$$

$Q_{D_-}(x)$ becomes the distribution function of the position which the real value $-\xi x$ partitioned by $D_-(x)$. Let the distribution function $Q_{D_-}(x)$ named the Yukawa type distribution function. Therefore, the above equation can be adopted as the definition of $Q_{D_-}(x)$. the Yukawa type distribution function $Q_{D_-}(R)$ is thought of the expansion of the Fermi-Dirac distribution function and the distribution of the Woods-Saxon potential.

4.4. The Inverse of $Re[S_{D_-}(x)]$ and Potential $V_{D_-}(x,k)$

We have defined the approximation of generalized entropy(-) $S_{D_-}(x,k)$ by the definition 4.2. Therefore, the inverse of $Re[S_{D_-}(x)]$ is obtained as follows:

$$\frac{1}{Re\left[S_{D\text{-}}(x)\right]} = k\frac{-\xi x}{Q_{D\text{-}}(x)}\frac{1}{\lambda_2 x^2 \pm \lambda_1 x}, \tag{110}$$

where because $Re\left[S_{D\text{-}}(x)\right] \leq 0$ and $Q_{D\text{-}}(x) \geq 0$, then $\lambda_2 x^2 \pm \lambda_1 x > 0$. By the equation(109), we can represent as follows:

$$\frac{1}{Re\left[S_{D\text{-}}(x)\right]} = -k\frac{1}{\lambda_2 x \pm \lambda_1}(\exp(-\xi\int\mu(x)dx \pm \mu_1) + 1). \tag{111}$$

We define the inverse of $Re[S_{D\text{-}}(x)]$ as potential $V_{D\text{-}}(x,k)$:

$$V_{D\text{-}}(x,k) = -k\frac{\frac{1}{\lambda_2}}{x \pm \frac{\lambda_1}{\lambda_2}}(1 + \exp(-\xi\int\mu(x)dx \pm \mu_1)). \tag{112}$$

In other words, the above potential $V_{D\text{-}}(x,k)$ can be defined as the product of a constant k, the partition $Q_{D\text{-}}(x) = -\xi/D_{\text{-}}(x)$, and the inverse of real parts of generalized partial entropy(-) $Re[S_{D\text{-}}(x)] \leq 0$. The first derivative $V'_{D\text{-}}(x,k)$ is satisfied as follows:

$$V'_{D\text{-}}(x,k) = \frac{k\frac{1}{\lambda_2}}{(x \pm \frac{\lambda_1}{\lambda_2})^2}(1+\exp(-\xi\int\mu(x)dx \pm \mu_1)) + \frac{k\frac{1}{\lambda_2}\xi\mu(x)}{x \pm \frac{\lambda_1}{\lambda_2}}\exp(-\xi\int\mu(x)dx \pm \mu_1). \tag{113}$$

Let $V_{D\text{-}}(x,k)$ be named as potential of $S_{D\text{-}}(x)$, and $V'_{D\text{-}}(x,k)$ be named as acceleration of $S_{D\text{-}}(x)$. Same as assumption2.4 and 3.1, the above description assumes the following assumption:

Assumption 4.3. *It assumes that potential $V_{D\text{-}}(x,k)$ is defined the inverse of real parts of generalized partial entropy(-) $Re[S_{D\text{-}}(x)]$. Therefore, acceleration $V'_{D\text{-}}(x,k)$ is defined the first derivative of $V_{D\text{-}}(x,k)$.* □

Assumption 4.4. *It assumes the constant $1/\lambda_2$ is equal to mass m within R. Assume that 2-times the inverse of entropy acceleration, $2/S''_{D\text{-}}(x,k)$, that is, $1/\lambda_2$ is equal to mass m within R. In other words, mass m within R is defined as the inverse of second-order term of $S_{D\text{-}}(x,k)$, that is, $1/\lambda_2$.* □

Therefore, same as the description of section3, we obtain the following results:

$$Q_{D\text{-}}(R) = \frac{\xi^g}{\exp(-\xi^g\mu_2^g R \pm \mu_1^g) + 1}, \tag{114}$$

$$V_{D\text{-}}(R,G) = -\frac{Gm}{R \pm \lambda_1^g m}(1 + \exp(-\xi^g\mu_2^g R \pm \mu_1^g)), \tag{115}$$

$$\begin{aligned} \hat{g}_{\pm} &= -V'_{D\text{-}}(R,G) \\ &= -\frac{Gm}{(R \pm \lambda_1^g m)^2}(1+\exp(-\xi^g\mu_2^g R \pm \mu_1^g)) - \frac{Gm\xi^g\mu_2^g}{R \pm \lambda_1^g m}\exp(-\xi^g\mu_2^g R \pm \mu_1^g), \end{aligned} \tag{116}$$

$$\lim_{\mu_1^g \to 0} \hat{g}_{\pm\lambda_1^g + \mu_1^g} = -\frac{2G}{(\pm\lambda_1^g)^2 m} - \frac{G\xi^g\mu_2^g}{\pm\lambda_1^g}, \quad (\because R \to 0), \tag{117}$$

where $\xi^g \gtrapprox 0$, $\mu_2^g \gtrapprox 0$, $\mu_1^g \geq 0$ and $\lambda_1^g \geq 0$. Let $\hat{g}_{\pm}$ be the Yukawa type adjusted gravitational acceleration($\pm$). The above equation(114) resembles the distribution of nuclei model, and the negative of (114) resembles the Woods-Saxon potential. The above equation(115) and (91) are compatible because the sign of exp is same. Namely, if setting as follows:

$$\exp(\mu_1^g) := \alpha, \quad \xi^g\mu_2^g := \lambda, \quad \lambda_1^g := 0, \tag{118}$$

then the equation(115) becomes the Yukawa potential(91). The second term of (115) is same as negative representation of the Yukawa potential (88). Namely, by introducing partial entropy with negative, that is, generalized partial entropy(-) $Re[S_{D_-}(x)] \leq 0$, the Yukawa type potential can be explained. For comparison, we describe results of section3 as follows:

$$Q_{D_+}(R) = \frac{\xi^g}{\exp(-\xi^g \mu_2^g R + \mu_1^g) - 1}, \tag{119}$$

$$V_{D_+}(R, G) = -\frac{Gm}{R \pm \lambda_1^g m}(1 - \exp(-\xi^g \mu_2^g R \pm \mu_1^g)), \tag{120}$$

$$\begin{aligned} \bar{g}_\pm &= -V'_{D_+}(R, G) \\ &= -\frac{Gm}{(R \pm \lambda_1^g m)^2}(1 - \exp(-\xi^g \mu_2^g R \pm \mu_1^g)) + \frac{Gm\xi^g \mu_2^g}{R \pm \lambda_1^g m}\exp(-\xi^g \mu_2^g R \pm \mu_1^g), \end{aligned} \tag{121}$$

$$\lim_{\mu_1^g \to 0} \bar{g}_{\pm\lambda_1^g + \mu_1^g} = \frac{G\xi^g \mu_2^g}{\pm\lambda_1^g}, \quad (\because R \to 0), \tag{122}$$

where $\xi^g \gneq 0$, $\mu_2^g \gneq 0$, $\mu_1^g \geq 0$ and $\lambda_1^g \geq 0$. The second term of equation(120) is same as the positive representation of the Yukawa type potential(88). Let $\bar{g}_\pm$ be the Planck type adjusted gravitational acceleration($\pm$). As weak proximity acceleration($\pm$), we put the equation(122) as follows:

$$g_\pm^{wp} = \lim_{\mu_1^g \to 0} \bar{g}_{\pm\lambda_1^g + \mu_1^g} = \frac{G\xi^g \mu_2^g}{\pm\lambda_1^g}, \qquad (\because R \to 0), \tag{123}$$

and as strong proximity acceleration($\pm$), we put the equation(117) as follows:

$$g_\pm^{sp} = \lim_{\mu_1^g \to 0} \hat{g}_{\pm\lambda_1^g + \mu_1^g} = -\frac{2G}{(\pm\lambda_1^g)^2 m} - \frac{G\xi^g \mu_2^g}{\pm\lambda_1^g}, \quad (\because R \to 0). \tag{124}$$

Acceleration $g_\pm^{wp}$ has no effect of mass m instead it depends on ξ^g, μ_2^g and λ_1^g. Acceleration $g_\pm^{sp}$ has an effect of mass m. It is considered that there exist 11-types of acceleration including the gravitational acceleration g, such as $\bar{g}_\pm, \hat{g}_\pm, \tilde{g}_\pm, g_\pm^{sp}$ and $g_\pm^{wp}$ related to g.

4.5. Comparing Accelerations $\bar{g}_\pm, \hat{g}_\pm, g_\pm^{sp}$ and $g_\pm^{wp}$

Comparing the above (116), (121), (123) and (124), we obtain the following are relationships:

$$g_+^{sp} < \hat{g}_+ < \tilde{g}_+ < \bar{g}_+ < g_+^{wp}, \tag{125}$$

$$\hat{g}_- < \tilde{g}_- < g_-^{sp} < g_-^{wp} < \bar{g}_-, \quad \because) \quad m < \frac{1}{\lambda_1^g \xi^g \mu_2^g}, \tag{126}$$

$$\bar{g}_- < g_-^{wp} < g_-^{sp} < \tilde{g}_- < \hat{g}_-, \quad \because) \quad \frac{1}{\lambda_1^g \xi^g \mu_2^g} < m. \tag{127}$$

It assumes that $g_-^{wp} < g_-^{sp}$, for all $R > 0$, it is satisfied as follows:

$$\frac{G\xi^g \mu_2^g}{-\lambda_1^g} < -\frac{2G}{(-\lambda_1^g)^2 m} - \frac{G\xi^g \mu_2^g}{-\lambda_1^g}. \tag{128}$$

Therefore, it is satisfied as follows:

$$\frac{1}{\lambda_1^g \xi^g \mu_2^g} < m. \tag{129}$$

It assumes that $g_-^{sp} < \tilde{g}_-$, for all $R > 0$, it is satisfied as follows:

$$R - \lambda_1^g m > \pm\lambda_1^g m \sqrt{\frac{1}{2 - \lambda_1^g m \xi^g \mu_2^g}}, \tag{130}$$

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The above inequality(130) holds true even for small enough R. The following is satisfied:

$$-\lambda_1^g m \geq -\lambda_1^g m\sqrt{\frac{1}{2-\lambda_1^g m\xi^g\mu_2^g}}. \tag{131}$$

Therefore, it is satisfied as follows:

$$\frac{1}{\lambda_1^g\xi^g\mu_2^g} \leq m. \tag{132}$$

Namely, because $R>0$, hence we only consider that inequalities(127) on the condition $\frac{1}{\lambda_1^g\xi^g\mu_2^g}<m$. Summarizing the above inequalities, the following are satisfied:

$$\begin{aligned} &g_+^{sp} < \hat{g}_+ < \bar{g}_- < g_-^{wp} < g_-^{sp} < \tilde{g}_- < g < \tilde{g}_+ < \bar{g}_+ < \hat{g}_- < 0 < g_+^{wp}, \\ &(\because \frac{1}{\lambda_1^g\xi^g\mu_2^g} < m). \end{aligned} \tag{133}$$

(Note): Because the direction toward the center is negative, therefore, the more negative value, that is, smaller, the greater acceleration (force) toward the center. (End of Note)

4.6. One Attempt to Compare the Ratios of 4-Forces

For example, we consider the above inequalities(133). Set constants as follows:

$$\begin{aligned} &G := 6.674E\text{-}11, && G \textit{ is the gravitational constant}, \\ &\xi^g := h = 6.626E\text{-}34, && h \textit{ is Planck's constant}, \\ &m := m_p = 2.176E\text{-}8, && m_p \textit{ is the Planck mass}(kg), \\ &\mu_1^g \gtrsim 0, && \mu_1^g \textit{ is a real constant}, \\ &R \leq 1.305E\text{+}26, && R \textit{ is a radius within the Universe}(meter). \end{aligned} \tag{134}$$

where according to case 2) of suggestion3.3 and of suggestion3.5, inequalities (49) and (132), the values λ_1^g and μ_1^g are satisfied as follows:

$$0 \leq 1 + (R \pm \lambda_1^g m_p)\xi^g\mu_2^g, \quad \frac{-1}{\lambda_1^g\xi^g\mu_2^g} < m_p. \tag{135}$$

(Note): The values μ_2^g in each equations $g_\pm^{sp}$ and g_-^{wp} take on different values $(\mu_2^g)_\pm^{sp}$ and $(\mu_2^g)_-^{wp}$, respectively. (End of Note)

On the above inequalities (133), the following are satisfied:

1. Compare $\hat{g}_-$ and g_-^{wp}; The ratio of the Yukawa type adjusted gravitational$(-)$ $\hat{g}_-$ to weak proximity acceleration$(-)$ g_-^{wp} is obtained as follows:

$$\begin{aligned} \left|\frac{\hat{g}_-}{g_-^{wp}}\right| &\simeq \left|\frac{\frac{2Gm_p}{(R-((\hat{\lambda}_1^g)_-)m_p)^2}}{\frac{G\xi^g(\mu_2^g)_-^{wp}}{(\lambda_1^g)_-^{wp}}} + \frac{\frac{Gm_p\xi^g}{(R-((\hat{\lambda}_1^g)_-)m_p)}}{\frac{G\xi^g(\mu_2^g)_-^{wp}}{(\lambda_1^g)_-^{wp}}}\right| \\ &\simeq \left|\frac{2\cdot 10^{-8}(\lambda_1^g)_-^{wp}}{10^{2A-34}(\mu_2^g)_-^{wp}} + \frac{10^{-8}(\hat{\lambda}_2^g)_-}{10^{A-34}(\mu_2^g)_-^{wp}}\right| \simeq 1E\text{-}34, \end{aligned} \tag{136}$$

where

$$\begin{aligned} &(\mu_2^g)_-^{wp} := 1E\text{+}60, && (\mu_2^g)_-^{wp} \textit{ is the constant } \mu_2^g \textit{ of } g_-^{wp}, \\ &(\lambda_1^g)_-^{wp} := 1E\text{+}2A, && (\lambda_1^g)_-^{wp} \textit{ is the constant } \lambda_1^g \textit{ of } g_-^{wp}, \\ &(\hat{\mu}_2^g)_- := 1E\text{+}A, && (\hat{\mu}_2^g)_- \textit{ is the constant } \mu_2^g \textit{ of } \hat{g}_-, \\ &(\hat{\lambda}_1^g)_- := 1E\text{+}A, && (\hat{\lambda}_1^g)_- \textit{ is the constant } \lambda_1^g \textit{ of } \hat{g}_-, \\ &(R \pm ((\hat{\lambda}_1^g)_-)m_p)^2 \simeq 1E\text{+}2A, && 0 \leq A \leq 26, \; A \textit{ is a constant}. \end{aligned} \tag{137}$$

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$\tilde{g}_+$ and g_-^{wp} can be compared in the same way.
(Note): The following are satisfied:

$$\exp(-\xi^g((\hat{\mu}_2^g)_-)R + \mu_1^g) > \exp(0) = 1, \tag{138}$$

where μ_1^g is satisfied $-\xi^g((\hat{\mu}_2^g)_-)R + \mu_1^g > 0$. Thus, if the case $\hat{g}_-$, then it is satisfied $Q_{D_-}(R) < \frac{1}{2}$. Similarly, if the case $\bar{g}_-$, then it is satisfied $Q_{D_+}(R) > 0$. (End of Note)

2. Compare $\tilde{g}_\pm$ and g_-^{wp}; The ratio of the adjusted gravitational$(\pm)$ $\tilde{g}_\pm$ to weak proximity acceleration$(-)$ g_-^{wp} is obtained as follows:

$$\left|\frac{\tilde{g}_\pm}{g_-^{wp}}\right| = \left|\frac{\frac{Gm_p}{(R\pm((\tilde{\lambda}_1^g)_-)m_p)^2}}{\frac{G\xi^g(\mu_2^g)_-^{wp}}{(\lambda_1^g)_-^{wp}}}\right| = \left|\frac{10^{-8}(\lambda_1^g)_-^{wp}}{10^{2A-34}(\mu_2^g)_-^{wp}}\right| \simeq 1E\text{-}34, \tag{139}$$

where

$$\begin{aligned} &(\mu_2^g)_-^{wp} := 1E{+}60, && (\mu_2^g)_-^{wp} \textit{ is the constant } \mu_2^g \textit{ of } g_-^{wp}, \\ &(\lambda_1^g)_-^{wp} := 1E{+}2A, && (\lambda_1^g)_-^{wp} \textit{ is the constant } \lambda_1^g \textit{ of } g_-^{wp}, \\ &(R \pm ((\tilde{\lambda}_1^g)_-)m_p)^2 \simeq 1E{+}2A, && 0 \le A \le 26,\ A \textit{ is a constant}. \end{aligned} \tag{140}$$

3. Compare g_+^{sp} and g_-^{wp}; The ratio of strong proximity$(+)$ g_+^{sp} to weak proximity acceleration$(-)$ g_-^{wp} is obtained as follows:

$$\left|\frac{g_+^{sp}}{g_-^{wp}}\right| = \left|\frac{-\frac{2G}{(\lambda_1^g)^2 m_p} - \frac{G\xi^g(\mu_2^g)_+^{sp}}{+\lambda_1^g}}{\frac{G\xi^g(\mu_2^g)_-^{wp}}{-\lambda_1^g}}\right| = \left(\frac{2}{\lambda_1^g m_p \xi^g(\mu_2^g)_-^{wp}} + \frac{(\mu_2^g)_+^{sp}}{(\mu_2^g)_-^{wp}}\right) \simeq 1E{+}5, \tag{141}$$

where

$$\begin{aligned} &(\mu_2^g)_+^{sp} := 1E{+}65, && (\mu_2^g)_+^{sp} \textit{ is the constant } \mu_2^g \textit{ of } g_+^{sp}, \\ &(\mu_2^g)_-^{wp} := 1E{+}60, && (\mu_2^g)_-^{wp} \textit{ is the constant } \mu_2^g \textit{ of } g_-^{wp}, \\ &\lambda_1^g := 1E\text{-}20, && \lambda_1^g \textit{ is the constant } \lambda_1^g \textit{ of } g_-^{wp} \textit{ and } g_+^{sp}. \end{aligned} \tag{142}$$

4. Compare g_-^{sp} and g_-^{wp}; The ratio of strong proximity$(-)$ g_-^{sp} to weak proximity acceleration$(-)$ g_-^{wp} is obtained as follows:

$$\left|\frac{g_-^{sp}}{g_-^{wp}}\right| = \left|\frac{-\frac{2G}{(\lambda_1^g)^2 m_p} - \frac{G\xi^g(\mu_2^g)_-^{sp}}{+\lambda_1^g}}{\frac{G\xi^g(\mu_2^g)_-^{wp}}{-\lambda_1^g}}\right| = \left|\frac{2}{\lambda_1^g m_p \xi^g(\mu_2^g)_-^{wp}} - \frac{(\mu_2^g)_-^{sp}}{(\mu_2^g)_-^{wp}}\right| \simeq 1, \tag{143}$$

where

$$\begin{aligned} &(\mu_2^g)_-^{sp} := 1E{+}60, && (\mu_2^g)_-^{sp} \textit{ is the constant } \mu_2^g \textit{ of } g_-^{sp}, \\ &(\mu_2^g)_-^{wp} := 1E{+}60, && (\mu_2^g)_-^{wp} \textit{ is the constant } \mu_2^g \textit{ of } g_-^{wp}, \\ &\lambda_1^g := 1E\text{-}20, && \lambda_1^g \textit{ is the constant } \lambda_1^g \textit{ of } g_-^{wp} \textit{ and} g_-^{sp}. \end{aligned} \tag{144}$$

If the strong force is set to 1, by considering g_+^{sp} as strong force, g_-^{wp} as weak force and $\tilde{g}_\pm$ (or g) as gravity, the ratios of the fundamental 3-forces in nature (strong force, weak force and gravity) is represented as follows:

$$\begin{array}{ccc} strong, & weak, & gravity, \\ 1, & 1E\text{-}5, & 1E\text{-}39, \\ g_+^{sp}, & g_-^{wp}, & \tilde{g}_\pm \\ (\mu_2^g)_+^{sp}, & (\mu_2^g)_-^{wp} \textit{ or } (\mu_2^g)_-^{sp}, & \frac{m_p}{\xi^g}, \\ 1E{+}65, & 1E{+}60, & 1E{+}26. \end{array} \tag{145}$$

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The above ratios correspond to those of $(\mu_2^g)_+^{sp} \simeq 1E+65$, $(\mu_2^g)_-^{wp} \simeq 1E+60$, $(\mu_2^g)_-^{sp} \simeq 1E+60$ and $\frac{m_p}{\xi^g} \simeq 1E+26$. Next, let us consider the relationship with electromagnetic forces. We focus that the relationship between electromagnetic force and gravity are seen as similar forces by suggestion3.8. We apply this suggestion to the equations $\hat{g}_\pm$ and $\hat{g}_\pm$ and set adjusted electromagnetic acceleration(force) as follows:

$$\begin{aligned}\bar{E}_\pm &:= -V'_{D_+}(R,k_e)\\ &= -\frac{k_e e_q}{(R \pm \lambda_1^c e_q)^2}(1-\exp(-\xi^c\mu_2^c R \pm \mu_1^c)) + \frac{k_e e_q \xi^c \mu_2^c}{R \pm \lambda_1^c e_q}\exp(-\xi^c\mu_2^c R \pm \mu_1^c),\end{aligned} \tag{146}$$

$$\begin{aligned}\hat{E}_\pm &:= -V'_{D_-}(R,k_e)\\ &= -\frac{k_e e_q}{(R \pm \lambda_1^c e_q)^2}(1+\exp(-\xi^c\mu_2^c R \pm \mu_1^c)) - \frac{k_e e_q \xi^c \mu_2^c}{R \pm \lambda_1^c e_q}\exp(-\xi^c\mu_2^c R \pm \mu_1^c),\end{aligned} \tag{147}$$

where $\xi^c \gtrapprox 0$, $\mu_2^c \gtrapprox 0$, $\mu_1^c \geq 0$ and $\lambda_1^c \geq 0$. Let $\bar{E}_\pm$ be the Planck type adjusted electromagnetic($\pm$) and $\hat{E}_\pm$ be the Yukawa type adjusted electromagnetic($\pm$). Therefore, we set as follows:

$$\begin{aligned}
&G := 6.674E\text{-}11, && G \textit{ is the gravitational constant},\\
&\xi^g = \xi^c := h = 6.626E\text{-}34, && h \textit{ is Planck's constant},\\
&k_e := 8.987E\text{+}9, && k_e \textit{ is Coulomb's constant},\\
&e_q := 1.604E\text{-}19, && e_q \textit{ is the elementary charge},\\
&m := m_p = 2.176E\text{-}8, && m_p \textit{ is the Planck mass}(kg),\\
&R \textit{ is small enough}, && R \textit{ is distance}(unit : meter).
\end{aligned} \tag{148}$$

where according to case 2) of suggestion 3.5, the values λ_1^g and μ_1^g are satisfied as follows:

$$0 < 1 \pm \lambda_1^g m_p \xi^g \mu_2^g, \quad 0 < 1 \pm \lambda_1^c e_q \xi^c \mu_2^c, \ (R \to 0). \tag{149}$$

(Note): The values μ_2^c in each equations $\hat{E}_+$ and $\bar{E}_-$, take on different the values $\hat{\mu}_2^c$ and $\bar{\mu}_2^c$, respectively. (End of Note)

On the above inequalities(133), the following are satisfied:

1. Compare $\hat{E}_+$ and g_-^{wp}; The ratio of Yukawa type adjusted electromagnetic($+$) $\hat{E}_+$ to weak proximity acceleration($-$) g_-^{wp} is obtained as follows:

$$\left|\frac{\hat{E}_+}{g_-^{wp}}\right| = \left|\frac{-\frac{2k_e}{\lambda_1^{c\,2} e_q}}{\frac{G\xi^g(\mu_2^g)_-^{wp}}{\lambda_1^g}} + \frac{-\frac{k_e\xi^c(\hat{\mu}_2^c)_+}{\lambda_1^c}}{\frac{G\xi^g(\mu_2^g)_-^{wp}}{\lambda_1^g}}\right| \simeq 1.347E\text{+}3 \ \simeq 1E\text{+}3, \tag{150}$$

where

$$\begin{aligned}
&(\hat{\mu}_2^c)_+ := 1E\text{+}43, && (\hat{\mu}_2^c)_+ \textit{ is the constant } \mu_2^c \textit{ of } \hat{E}_+,\\
&(\mu_2^g)_-^{wp} := 1E\text{+}60, && (\mu_2^g)_-^{wp} \textit{ is the constant } \mu_2^g \textit{ of } g_-^{wp},\\
&\lambda_1^g = \lambda_1^c < 1E\text{-}20.
\end{aligned} \tag{151}$$

2. Compare $\bar{E}_-$ and g_+^{wp}; The ratio of Planck type adjusted electromagnetic($-$) $\bar{E}_-$ to weak proximity acceleration($-$) g_-^{wp} is obtained as follows:

$$\left|\frac{\bar{E}_-}{g_-^{wp}}\right| = \frac{k_e\xi^c(\bar{\mu}_2^c)_-\lambda_1^g}{G\xi^g(\mu_2^g)_-^{wp}\lambda_1^c}\exp(\mu_1^c) \simeq 1.347E\text{+}3 \simeq 1E\text{+}3, \tag{152}$$

where

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$$
\begin{aligned}
&(\bar{\mu}_2^c)_- := 1E{+}43, && (\bar{\mu}_2^c)_+ \text{ is the constant } \mu_2^c \text{ of } \bar{E}_-,\\
&(\mu_2^g)_-^{wp} := 1E{+}60, && (\mu_2^g)_-^{wp} \text{ is the constant } \mu_2^g \text{ of } g_-^{wp},\\
&\mu_1^c \to 0, \quad \lambda_1^g = \lambda_1^c < 1E\text{-}20.
\end{aligned} \tag{153}
$$

where because the above ratio changes depending on R, therefore, for the comparison of the ratios, R is small enough in the near field. We interpret $\hat{E}_+$ and $\bar{E}_-$ as electromagnetic. If the strong force is set to 1, by combine with the table(145), the ratios of the fundamental 4-forces in nature (strong force, electromagnetic force, weak force and gravity) can be expressed as follows:

$$
\begin{array}{cccc}
strong, & electromagnetic, & weak, & gravity,\\
1, & 1E\text{-}2, & 1E\text{-}5, & 1E\text{-}39,\\
g_+^{sp}, & \hat{E}_+ \text{ or } \bar{E}_-, & g_-^{wp}, & \tilde{g}_\pm,\\
(\mu_2^g)_+^{sp}, & \frac{k_e}{G}(\hat{\mu}_2^c)_+ \text{ or } \frac{k_e}{G}(\bar{\mu}_2^c)_-, & (\mu_2^g)_-^{wp}, & \frac{m_p}{\xi^g},\\
1E{+}65, & 1E{+}63, & 1E{+}60, & 1E{+}26,
\end{array} \tag{154}
$$

The above ratios correspond to those of $(\mu_2^g)_+^{sp} \simeq 1E{+}65$, $\frac{k_e}{G}(\hat{\mu}_2^c)_+ \simeq \frac{k_e}{G}(\bar{\mu}_2^c)_- \simeq 1E{+}63$, $(\mu_2^g)_-^{wp} \simeq 1E{+}60$ and $\frac{m_p}{\xi^g} \simeq 1E{+}26$, and depend on the values m_p and ξ^g. Therefore, by considering strong proximity force g_+^{sp} is regarded as strong force, weak proximity force g_-^{wp} as weak force, adjusted gravity $\tilde{g}_\pm$ as gravity and adjusted electromagnetic force $\hat{E}_+$ or $\bar{E}_-$ as electromagnetic force, it is possible to explain the ratios of the fundamental 4-forces in nature.

4.7. Relationship Diagram

The difference between the equations(120) and (115) are whether generalized partial entropy is positive or negative. Under the above definition 4.2, the equation(103) is negative. Namely, it assumes generalized partial entropy(-) with negative. If it assumes generalized partial entropy(-) with negative, we can obtain the Yukawa type gravitational potential. In other words, the existence of Yukawa type equations may indicate the existence of negative partial entropy. Therefore, particle physics can be also considered to be related to entropy.

Relationship diagram:

$$
\begin{array}{ccc}
S_{D_+}(k,x) = k\frac{\xi}{Q_{D_+}(x)} S_{D_+}(x) > 0 & & S_{D_-}(k,x) = -k\frac{\xi}{Q_{D_-}(x)} S_{D_-}(x) > 0\\
S_{D_+}(x) > 0 & \overset{negative}{\longrightarrow} & S_{D_-}(x) < 0\\
Partial\ Entropy & & Negative\ Partial\ Entropy\\
\downarrow & & \downarrow\\
Q_{D_+}(x) = \frac{\xi}{\exp(-\xi\int \mu(x)dx \pm \mu_1) - 1} & & Q_{D_-}(x) = \frac{\xi}{\exp(-\xi\int \mu(x)dx \pm \mu_1) + 1}\\
Planck\ type\ distribution & & Yukawa\ type\ distribution\\
\downarrow & & \downarrow\\
V_{D_+}(R,G) = & & V_{D_-}(R,G) =\\
-\frac{Gm}{R \pm \lambda_1^g m}(1 - \exp(-\xi^g \mu_2^g R \pm \mu_1^g)) & & -\frac{Gm}{R \pm \lambda_1^g m}(1 + \exp(-\xi^g \mu_2^g R \pm \mu_1^g))\\
Planck\ type\ potential & & Yukawa\ type\ potential\\
\begin{matrix}\lambda^g \to 0\\ \xi^g \mu_2^g := \lambda\\ \exp(\mu_1^g) := \alpha\end{matrix}\downarrow & & \begin{matrix}\lambda^g \to 0\\ \xi^g \mu_2^g := \lambda\\ \exp(\mu_1^g) := \alpha\end{matrix}\downarrow\\
V_\alpha(R) = -G\frac{m}{R}(1 - \alpha\exp(-\lambda R)) & & V_{yukawa2}(R) = -G\frac{m}{R}(1 + \alpha\exp(-\lambda R))\\
Planck\ type\ potential & & Yukawa\ type\ potential\\
R \to large \downarrow & & R \to large \downarrow\\
V_\alpha(R) = -G\frac{m}{R} & & V_{yukawa2}(R) = -G\frac{m}{R}\\
\downarrow & & \downarrow
\end{array} \tag{155}
$$

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$$\begin{array}{ccccc} g = -V'_\alpha(R) = -G\frac{m}{R^2} & \rightarrow & g = -G\frac{m}{R^2} & \leftarrow & g = -V'_{yukawa2}(R) = -G\frac{m}{R^2} \\ \textit{Gravitational acceleration} & & & & \textit{Gravitational acceleration} \\ R\to 0 \downarrow & & & & R\to 0 \downarrow \\ \lim_{\mu_1^g\to 0}\bar{g}_{\pm\lambda_1^g+\mu_1^g} = \frac{G\xi^g\mu_2^g}{\pm\lambda_1^g} & & & & \lim_{\mu_1^g\to 0}\bar{g}_{\pm\lambda_1^g+\mu_1^g} = -\frac{2G}{(\pm\lambda_1^g)^2 m} - \frac{G\xi^g\mu_2^g}{\pm\lambda_1^g} \\ \textit{Weak proximity } g_\pm^{wp} & & & & \textit{Strong proximity } g_\pm^{sp} \end{array}$$

The Planck type distribution may be thought to be the Wave distribution type that represents electromagnetic waves, light, thermal radiation, etc. The Yukawa type distribution may be thought to be the Nuclei distribution type that represents the atomic nucleus model, etc. Further research is thought to be needed in the future.

5. Possibility That Mass Generation by Entropy, the Existence of New Forces and Fluctuating of the Constant G

5.1. Possibility That Mass Generation by Entropy

Furthermore, the inverse of the second-order part λ_2^g of the approximation of generalized entropy is regarded as mass m. Leave this the first-order part λ_1^g as it is. The generalized entropy $S_{D_\pm}(R,G)$ is determined by mass m, distance R (the radius of range under consideration) and the correction factor λ_1^g. In other words, mass m is determined by generalized entropy $S_{D_\pm}(R,G)$, distance R and the correction factor λ_1^g. By transforming the equations(17) and (102), mass m can be represented as follows:

$$m = \frac{R^2}{S_{D_\pm}(R,G) \mp \lambda_1^g R}. \tag{156}$$

By transforming the equations(20), (110) and $S_{D_\pm}(R,G) = G\frac{\xi^g R}{Q_{D_\pm}(R)} S_{D_\pm}(R)$, mass m can be represented as follows:

$$m = \frac{R}{(\exp(-\xi^g\mu_2^g R \pm \mu_1^g) \mp 1) G S_{D_\pm}(R) \mp \lambda_1^g}. \tag{157}$$

Namely, mass m can be represented as generalized partial entropy $S_{D_\pm}(R)$, distance R, correction factors λ_1^g, μ_1^g, μ_2^g and constants G, ξ^g. Mass is also considered to depend on entropy. Besides, if we consider $Q_{D_\pm}(R)$ to represent the spectrum (wave) distribution within the range of distance R, we can consider that mass depends on the partial entropy $S_{D_\pm}(R)$, the spectrum distribution $Q_{D_\pm}(R)$ (or division $D_\pm(R)$) within the range R, and the constants G, ξ^g, and λ_1. In other words, mass can be considered to depend on the partial entropy and spectrum (waves). Mass may consider to be generated as depend on entropy and waves.

5.2. Possibility That the Existence of New Forces

By appropriately selecting the constants, the variables and the functions within the range of conditions, $V_{D_\pm}(x,k)$ is interpreted as the gravitational potential and $V'_{D_\pm}(x,k)$ is interpreted as the gravitational acceleration according to gravity theory. However, if we consider carefully, the constants, variables, and functions in the above equations can be arbitrary selected within the range of conditions, so these may be applicable to forces other than gravity and the Coulomb force. By choosing the constants in the equation $V'_{D_\pm}(x,k)$ appropriately, it may be possible to represent weak and strong force. Furthermore, there exists the possibility of expansion and the existence of new forces that are different from the conventional force. Therefore, it is possible that there exist many new forces. Namely, the following suggestion may be considered the possibility that there exist many new potentials and accelerations:

Suggestion 5.1. *Possibility that there exist many new forces of Planck type (1). Let $n \geq 0$ be an integer, m be a weight (mass) and R be a relation (distance). There exist countable numbers of potential $V_{D_+}(R,G_n)$ and an*

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acceleration $V'_{D_+}(R, G_n)$ such that the following conditions are satisfied: There exist constants G_n, ξ_n, μ_{1n}, λ_{1n} and a function $\mu_n(R)$ such that the following equations are satisfied:

$$V_{D_+}(R, G_n) = -\frac{G_n m}{R \pm \lambda_{1n} m}(1 - \exp(-\xi_n \int \mu_n(R) dR \pm \mu_{1n})), \tag{158}$$

$$\begin{aligned} V'_{D_+}(R, G_n) = & \frac{G_n m}{(R \pm \lambda_{1n} m)^2}(1 - \exp(-\xi_n \int \mu_n(R) dR \pm \mu_{1n})) \\ & - \frac{G_n m \xi_n \mu_n(R)}{R \pm \lambda_{1n} m} \exp(-\xi_n \int \mu_n(R) \pm \mu_{1n}), \end{aligned} \tag{159}$$

where $G_n \gneqq 0$, $\xi_n \gneqq 0$, $\mu_{1n} \geq 0$, $\lambda_{1n} \geq 0$, $m > 1$, $R > 1$ and $\mu_n(R) > 0$. Namely, it is possible that there exist many new forces. Note that these description above assumed that assumptions2.4 and 3.1. □

Similarly, for potentials $V_{D_-}(R, G_n)$ and accelerations $V'_{D_-}(R, G_n)$, we can describe as follows:

Suggestion 5.2. *Possibility that there exist many new forces of Yukawa type (2). Let $n \geq 0$ be an integer, m be a weight (mass) and R be a relation (distance). There exist countable numbers of potential $V_{D_-}(R, G_n)$ and an acceleration $V'_{D_-}(R, G_n)$ such that the following conditions are satisfied: There exist constants G_n, ξ_n, μ_{1n}, λ_{1n} and a function $\mu_n(R)$ such that the following equations are satisfied:*

$$V_{D_-}(R, G_n) = -\frac{G_n m}{R \pm \lambda_{1n} m}(1 + \exp(-\xi_n \int \mu_n(R) dR \pm \mu_{1n})), \tag{160}$$

$$\begin{aligned} V'_{D_-}(R, G_n) = & \frac{G_n m}{(R \pm \lambda_{1n} m)^2}(1 + \exp(-\xi_n \int \mu_n(R) dR \pm \mu_{1n})) \\ & + \frac{G_n m \xi_n \mu_n(R)}{R \pm \lambda_{1n} m} \exp(-\xi_n \int \mu_n(R) \pm \mu_{1n}), \end{aligned} \tag{161}$$

where $G_n \gneqq 0$, $\xi_n \gneqq 0$, $\mu_{1n} \geq 0$, $\lambda_{1n} \geq 0$, $m > 1$, $R > 1$ and $\mu_n(R) > 0$. Namely, it is possible that there exist many new forces. Note that these description above assumed that the assumptions 4.3 and 4.4. □

The gravitational constant G and Coulomb's constant k_e may simply be coefficients related to forces that humans can currently sense throughout the universe. Instead of asking why there exist 4-forces, it might be better to ask why humans are primarily only able to sense 4-forces.

5.3. Possibility that Fluctuating of the Constant G

As mentioned above, if we consider that there exist many forces, then we can assume that there will be many variations in the constants. If there exists a constant change (difference) in a variable, the constant will appear to be fluctuating. Furthermore, the gravitational constant G can be considered as being determined by generalized entropy $S_{D_\pm}(G, R)$, the partition $D_\pm(R)$ and the partial entropy $S_{D_\pm}(R)$ partitioned by $D_\pm(R)$ (or the distribution $\pm\xi R / Q_{D_\pm}(R)$). Namely, generalized entropy $S_{D_\pm}(G, R)$ is represented as follows:

$$S_{D_\pm}(R, G) = G \cdot D_\pm(R) \cdot S_{D_\pm}(R) = G \frac{\pm\xi R}{Q_{D_\pm}(R)} S_{D_\pm}(R). \tag{162}$$

Therefore, the gravitational constant G is represented as follows:

$$G = \frac{S_\pm(R, G)}{D_\pm(R) \cdot S_{D_\pm}(R)} = \frac{S_{D_\pm}(R, G)}{S_{D_\pm}(R)} \cdot \frac{Q_{D_\pm}(R)}{\pm\xi R}. \tag{163}$$

In other words, it is possible that the gravitational constant G can fluctuate if entropy changes.

6. Discussion and Conclusion

6.1. Possibility That Gravity Depending on Entropy

The ideas behind Planck's law is to apply the number of cases(states) of partition by resonators to entropy. These ideas are similar to the logistic function for dynamical system (see [1,18,31]). Applying these, we have treated the division of entropy as a non-minimal and non-linear function $D_+(x)$, and derived potential $V_{D_+}(x,k)$ and acceleration $V'_{D_+}(x,k)$. Therefore, we have assumed that generalized entropy $D_+(x,k)$ can represent as a second-degree polynomial, and potential $V_{D_+}(x,k)$ is defined as the inverse of $S_{D_+}(x)$. As a result, each variable and constant used in potential $V_{D_+}(x,k)$ and acceleration $V'_{D_+}(x,k)$ are interpreted as the perspective of gravity, and mass is defined as the inverse of the quadratic coefficient term of $S_{D_+}(x,k)$, that is, $1/\lambda_2$. The constant λ_1 is the first-order coefficient of approximated generalized entropy $\lambda_2 x^2 \pm \lambda_1 x$. In other words, the gravitational acceleration changes depending on coefficients of approximated generalized entropy. In addition, the constants μ_2 and μ_1 are defined as coefficients of generalized partial entropy or the distribution function $Q_\pm(x)$. In other words, the inverse of the second-order part λ_2^g of the second-order approximation of generalized entropy is considered to be mass. The first-order part λ_1^g is left unchanged. The generalized entropy $S_{D_+}(R,G)$ is determined using mass m, distance(radius) R of the range under consideration, and the correction factor λ_1^g. In this paper, it has assumed that the assumptions 2.4 and 3.1, and the existence of entropy dependent constants $\xi, \lambda_2, \lambda_1, \mu_2$, and μ_1. We have proposed the following conclusions:

1. If distance R is small enough, then the gravitational acceleration $V'_{D_+}(R,G)$ has 2-states with finite value depend on constants $\xi, \lambda_1^g, \mu_1^g$ and μ_2^g. Depending on the values μ_1^g and λ_1^g, the value $V'_{D_+}(R,G)$ can be positive or negative. If the constant $\lambda_1^g \to 0$, then the gravitational acceleration $V'_{D_+}(R,G)$ becomes $\pm\infty$. If the constant $\lambda_1^g \to \infty$, then the gravitational acceleration $V'_{D_+}(R,G)$ becomes 0. Therefore, it is possible that gravity has 5-states within distance R is small enough. Among the 5-states, there may exist anti-gravity, which is the opposite of Newton's gravity (Possibility existence of anti-gravity). Furthermore, using the equation for potential derived from entropy, within small distance, it is possible to treat the gravitational potential and the Coulomb potential in the same way by appropriately choosing certain constants. Similarly, the same can apply to the gravitational acceleration and Coulomb's law (electric field).
2. At distance large enough to be within the size of the universe, gravity follows the adjusted inverse square law. Within this distance, the rotation speed of the galaxy v follows the gravitational constant G, mass $m = 1/\lambda_2^g$ and constants ξ^g, μ_2^g and μ_1^g which depend on entropy. Besides, the rotation speed of the galaxy v is independent of its radius R, (the galaxy rotation curve problem). Even without assuming dark matter, the problem of the rotation speed of the galaxy may be explained by the concept of entropy. This does not mean denying dark matter. The new constants μ_1^g and μ_2^g proposed in this paper may represent some kind of dark or virtual mass.
3. At large distance, gravity follows adjusted inverse square law. By comparing to conventional gravity g, the gravitational acceleration $\tilde{g}_\pm$ towards the center of rotation becomes slightly weaker or stronger. This means that gravitational acceleration towards the center of a rotating object can change slightly with distance (The Pioneer Anomaly).

From the above description, it is possible there exist certain constants $\xi, \lambda_2, \lambda_1, \mu_2$ and μ_1 which depend on entropy that control gravity and the speed of the galaxy. Besides, it is considered to apply to velocities over short distances, such as electrons in atomic nuclei.

6.2. Interpretation of Yukawa Type Potential by Generalized Partial Entropy(-) with Negative

Same as $V_{D_+}(x,k)$, by defining $V_{D_-}(x,k)$ and acceleration $V'_{D_-}(x,k)$, it is considered to be related to the Yukawa type potential and generalized partial entropy(-) with negative. Namely, this assumes the existence of negative partial entropy, and in fact, negative partial entropy is thought to exist. Besides, the Yukawa type potential is related to particle physics, hence particle physics may be related to entropy. From the discussion so far, it may be suggested that there exist 11-types of acceleration including the gravitational acceleration g, such as $\bar{g}_\pm, \hat{g}_\pm, \tilde{g}_\pm, g_\pm^{sp}$ and $g_\pm^{wp}$ related to g. Using these forces, we attempted to compare that the ratios of the fundamental 4-forces in nature (strong force, electromagnetic force, weak force, and gravity) are 1, 1E-2, 1E-5, and 1E-39, respectively if the strong force is set to 1. We have shown that strong proximity force$(+)$ g_+^{sp} can be regarded as strong force, weak proximity$(-)$ force g_-^{wp} as weak force, adjusted gravity$(\pm)$ $\tilde{g}_\pm$ as gravity and adjusted electromagnetic force $\hat{E}_+$ or $\bar{E}_-$ as electromagnetic force. Moreover, it has described the possibility mass generation by entropy, the existence of new forces, and fluctuating of the gravitational constant G. In consequence, gravity may depend on entropy.

6.3. Integration of Thermodynamics, Quantum, Gravity and Ecology by Entropy

By combining concepts of the logistic function for dynamical system, Boltzmann's entropy and Planck's quantum, we have obtained adjusted gravity. Namely, by developing the concept of the logistic function and combining it with entropy and Planck's ideas, we have derived that potentials $V_{D_\pm}(x,k)$ and accelerations $V'_{D_\pm}(x,k)$. Because the nonlinear behavior obtained from the logistic function is non-Newtonian mechanics, thus Newtonian mechanics, including the theory of gravity, is consider to be included in non-Newtonian mechanics. Moreover, the Planck type distribution function $Q_{D_+}(R)$ is thought of the expansion of the Bose-Einstein distribution function and the Planck distribution function (Planck's law). The Yukawa type distribution function $Q_{D_-}(R)$ is thought of the expansion of the Fermi-Dirac distribution function and the distribution of nuclei model. This suggests that gravity and quantum mechanics may be linked through entropy.

The concept of the logistic function is applied to ecology like population theory and the evolution of life. It is also known that the concept of entropy was established by Clausius and related to quantum theory by Planck (see [1]). Besides, Boltzmann's concept of partition by the number of states can be seen as quantization. Therefore, gravity may be quantized by interpreting it as entropy, and gravity might be thought of as generating entropy. Furthermore, it is considered that entropy is related to the concept of the logistic function (see [31]). The concept of the logistic function is applied to population theory, the evolution of life and ecology. Therefore, it is considered that entropy is related to ecology (see [18,31]). In this paper, we have argued that the concept of entropy is related to gravity theory. These findings suggest that by combining the concepts of entropy and the logistic function, it may be possible to understand the evolution of the universe in the same way as the evolution of life. Entropy can also be seen as information, and so can gravity, quantum, and ecology. Thus, it is considered that thermodynamics, quantum (particle), gravity, electromagnetic and ecology (biology) may unify through entropy.

$$
\begin{array}{ccc}
Thermodynamics & Information & Ecology(Biology) \\
\searrow\nwarrow & \downarrow\uparrow & \swarrow\nearrow \\
 & Entropy & \\
\swarrow\nearrow & \downarrow\uparrow & \searrow\nwarrow \\
(Particle)Quantum & Gravity & Electromagnetic
\end{array}
$$

We hope that entropy explain more and provide new perspectives and insights.

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