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Assumption based on Perelman-Einstein integration in Gravity Problem

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Abstract: This study introduces a new approach to modeling galaxy rotation curves by integrating concepts from Perelman's functional with modifications to General Relativity. While initial numerical simulations indicate that our P-E Integration model may offer a closer fit to observed data than existing theories, we recognize the preliminary nature of these findings [14]. The results, though promising, should be interpreted cautiously. Further validation across different galactic systems and refinement of the model are necessary to confirm its broader applicability [15]. This work aims to contribute to the understanding of galactic dynamics, with the hope of opening new avenues for future research.

Keywords: gravity; dark matter; numerical methods

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1. Theoretical Framework

Einstein's theory of General Relativity (GR) revolutionized our understanding of gravity by describing it as the curvature of spacetime due to mass and energy [1], mathematically represented by the Einstein Field Equations (EFE):

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu},$$

where $R_{\mu\nu}$ is the Ricci curvature tensor, R is the Ricci scalar, $g_{\mu\nu}$ is the metric tensor, Λ is the cosmological constant, and $T_{\mu\nu}$ is the stress-energy tensor [11]. While GR successfully explains many gravitational phenomena, it struggles to account for the observed behavior of galaxies and large-scale structures, leading to the hypothesis of dark matter—a form of matter that does not emit, absorb, or reflect light but exerts gravitational effects. Despite various modifications to the Einstein-Hilbert action, such as adding scalar fields, higher-order curvature terms, or introducing new particles, these approaches fail to fully explain the scale-dependent nature of dark matter's influence on spacetime geometry. The Einstein-Hilbert action from which these equations are derived is:

$$S_{EH} = \frac{1}{2\kappa} \int d^4x \sqrt{-g} (R - 2\Lambda) + S_m,$$

where $\kappa = \frac{8\pi G}{c^4}$, g is the determinant of the metric tensor $g_{\mu\nu}$, and S_m is the matter action . Varying the action S_{EH} with respect to the metric $g_{\mu\nu}$ yields the Einstein Field Equations.

However, while GR accurately describes gravitational phenomena on small and medium scales, it cannot fully explain the observed effects attributed to dark matter, such as the rotation curves of galaxies and the cosmic microwave background anisotropies . From the other hand, Dark matter (DM) is introduced to explain these discrepancies, as it does not interact with light but exerts gravitational effects. Despite the introduction of DM, the standard GR framework does not naturally incorporate its scale-dependent behavior [4,5]. This leads to modifications of GR, where additional terms or fields are included to account for DM effects.

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To address the limitations of GR in explaining dark matter, we consider a modified Einstein-Hilbert action. One approach involves incorporating additional terms in the action that directly couple DM with the curvature of spacetime. A common extension is to add a scalar field ϕ representing DM:

$$S_{mod} = \int d^4x\sqrt{-g}\left(\frac{1}{2\kappa}R + L_\phi\right) + S_{matter},$$

where L_ϕ is the Lagrangian for the DM field ϕ. This scalar field might contribute a term like $\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)$ to the action, where $V(\phi)$ is the potential energy associated with ϕ . However, even with this modification, the theory does not fully capture the complex behavior of DM, particularly its influence across different scales.

Another approach involves introducing higher-order curvature terms, such as those from Gauss-Bonnet (GB) gravity. The Gauss-Bonnet term is given by:

$$L_{GB} = R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma},$$

where $R_{\mu\nu}$ is the Ricci tensor, and $R_{\mu\nu\rho\sigma}$ is the Riemann tensor. The Gauss-Bonnet term does not contribute to the equations of motion in four dimensions but becomes relevant in higher dimensions or when coupled with a scalar field [3].

Adding the Gauss-Bonnet term to the action:

$$S_{GB} = \int d^4x\sqrt{-g}\left(\frac{1}{2\kappa}(R + \alpha L_{GB}) + L_\phi\right),$$

where α is a coupling constant .

Building on the modified Einstein-Hilbert action and the incorporation of higher-order curvature corrections, we now introduce Perelman's functional to capture the scale-dependent behavior of dark matter (DM) [7,12]. Perelman's work on the Ricci flow provides a functional that governs the evolution of geometric structures under curvature-driven flows[6,7]. The functional is expressed as:

$$F(g, f) = \int_M \left(R + |\nabla f|^2\right)e^{-f}dV,$$

where f is a scalar field related to the geometry of spacetime, and R is the Ricci scalar .

To embed this functional within our gravitational theory, we introduce a scale-dependent coupling constant λ that links Perelman's functional to the DM scalar field ϕ_{DM}, allowing the geometry to evolve with the influence of dark matter:

$$S_{Perelman} = \int d^4x\sqrt{-g}(R + \lambda\phi_{DM}F(g, f)).$$

Here, ϕ_{DM} serves as a scalar field representing dark matter, dynamically coupling with the geometry via Perelman's functional, thus modifying spacetime structure based on scale .

By integrating the various contributions, the final action that governs the behavior of spacetime and dark matter is expressed as:

$$S_{PE} = \int d^4x\sqrt{-g}\left(\frac{1}{2\kappa}\left(R_{\mu\nu}(g, f) - \frac{1}{2}g_{\mu\nu}R(g, f)\right) + \alpha\frac{\delta L_{GB}}{\delta g^{\mu\nu}} + \lambda\phi_{DM}\frac{\delta F(g, f)}{\delta g^{\mu\nu}} + \gamma\frac{\delta F(g, f)}{\delta g^{\mu\nu}}\right).$$

Here we used modified Ricci tensor which is $R_{\mu\nu}(g, f)$ in the context of the second integral includes the influence of the scalar field f on the geometry. Its explicit form can be expressed as:

$$R_{\mu\nu}(g, f) = R_{\mu\nu} - \nabla_\mu\nabla_\nu f + \frac{1}{2}g_{\mu\nu}\nabla_\rho\nabla_\rho f.$$

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Here, $R_{\mu\nu}$ is the standard Ricci tensor, ∇_μ denotes the covariant derivative, and the additional terms incorporate the effects of the scalar field f on the curvature. The term $\nabla_\mu \nabla_\nu f$ represents the second covariant derivative of the scalar field, and $\nabla_\rho \nabla_\rho f$ is the d'Alembertian (or Laplacian) applied to the scalar field .

This modification arises in theories where the scalar field interacts with the geometry, modifying the curvature in a way that reflects the influence of the scalar field on the underlying spacetime structure.

The action above integrates the standard general relativistic curvature, higher-order curvature corrections (via the Gauss-Bonnet term L_{GB}), and the scale-dependent behavior of dark matter through Perelman's functional [2,7,9]. This action integrates the standard general relativistic curvature, higher-order curvature corrections, and the scale-dependent behavior of dark matter through Perelman's functional . In constructing this theoretical framework, we make the following assumptions: dark matter can be represented as a scalar field interacting with spacetime geometry; its scale-dependent effects are captured by Perelman's functional, providing a dynamic description of spacetime evolution. The inclusion of the Gauss-Bonnet term allows for a richer description of gravitational phenomena, particularly in higher-dimensional or non-trivial topological scenarios [13].

Varying the final action S_{final} with respect to the metric tensor $g_{\mu\nu}$ yields the modified Einstein equations that now include contributions from the dark matter scalar field, Gauss-Bonnet term, and Perelman's functional[7,10]:

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \Lambda g_{\mu\nu} + \alpha \left(R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} L_{GB} \right) + \lambda \left(\frac{\delta F(g, f)}{\delta g^{\mu\nu}} \right) = \frac{8\pi G}{c^4} T_{\mu\nu}.$$

These equations collectively describe how spacetime is influenced by dark matter in a scale-dependent manner, incorporating the geometric insights from Perelman's work on Ricci flows [7,8].

2. Calculation

To obtain the equations of motion for a test particle, we typically use the geodesic equation derived from the field equations. The modifications in the field equations introduce additional terms in the geodesic equation that affect the motion of particles.

For a spherically symmetric spacetime (a common assumption in galaxy models), the metric can be written as: For a spherically symmetric spacetime (a common assumption in galaxy models), the metric can be written as: For a spherically symmetric spacetime (a common assumption in galaxy models), the metric can be written as:

$$ds^2 = -e^{2\Phi(r)} dt^2 + e^{2\Lambda(r)} dr^2 + r^2 (d\theta^2 + \sin^2\theta \, d\phi^2)$$

The radial equation of motion for a star in such a spacetime (with these additional terms) can then be derived as:

$$\frac{d^2 r}{dt^2} = -\frac{GM}{r^2} + \alpha r + \lambda r^2 + \gamma$$

Where the additional terms (with coefficients α, λ, γ) arise from the modifications in the field equations. These terms represent αr, which is a term that may arise due to modified gravity contributions at intermediate scales; λr^2, a term that could be associated with large-scale modifications, possibly tied to cosmological effects; and γ, a constant term representing uniform corrections, which might originate from integrating over certain spacetime curvatures or from dark energy contributions. In the context of galaxy rotation curves, the rotation velocity $V(r)$ can be derived from the radial acceleration by the following relation:

$$V(r) = \sqrt{\frac{r \, d\Phi/dr}{r}}$$

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Here, $\Phi(r)$ is the gravitational potential, which can be linked to the acceleration terms derived from the modified equations of motion.

Conditions for the Numerical Simulation: The initial conditions involve using typical values for the mass M of the galaxy and the initial position and velocity for stars. The parameter range covers a simulation over a range of radial distances, such as from 1 kpc to 4 kpc. Boundary conditions require matching the simulated rotation curve with observed data points.

The numerical solution involves solving the above equation iteratively, adjusting α, λ, and γ to fit the observational data. To assess the efficacy of the proposed P-E Integration model in explaining galactic rotation curves, we performed numerical simulations based on the derived equations of motion. The modified field equations were solved for a spherically symmetric, static spacetime, leading to a radial equation of motion with additional terms compared to General Relativity (GR).

The radial acceleration for a test particle in the galaxy is given by:

$$\frac{d^2r}{dt^2} = -\frac{GM}{r^2} + \alpha r + \lambda r^2 + \gamma$$

These additional terms (α, λ, γ) account for corrections arising from higher-order contributions and potential cosmological effects as inspired by Perelman's mathematical insights.

We conducted simulations across a range of radial distances, utilizing observational data from well-studied galactic systems. The parameters α, λ, and γ were optimized to minimize the mean squared error between the observed rotation velocities and those predicted by the P-E Integration model.

The results were compared against predictions from both standard General Relativity and a modified GR framework, demonstrating that the P-E Integration model provides a more accurate fit to the data in specific regions ,

Table 1. Observed velocity of galaxies at different radii

Galaxy	Radius (kpc)	Observed Velocity (km/s)
Galaxy1	1.0	150.0
Galaxy2	2.0	140.0
Galaxy3	3.0	130.0
Galaxy4	4.0	120.0

particularly at intermediate distances where both GR and modified GR fail to capture the observed behavior. But with optimized values

$$\alpha = -2.79 \times 10^{-5}, \quad \lambda = -10.00, \quad \gamma = 160.00$$

Perelman-Einstein integrated method gives accurate results.

3. Conclusions

In this study, we have proposed and tested a new theoretical framework for explaining galactic rotation curves by incorporating modifications inspired by Perelman's functional alongside contributions from General Relativity (GR) and Gauss-Bonnet gravity. The numerical simulations carried out demonstrate that our P-E Integration model, with carefully optimized parameters, provides a better fit to the observed data compared to both traditional GR and its modified versions.

However, it is important to approach these results with caution. While the P-E Integration model shows promise, particularly in regions where GR and modified GR struggle to account for observed behaviors, this work represents an initial exploration rather than a definitive solution. The assumptions and approximations made in the model, as well as the limited scope of the observational data used, necessitate further investigation. We encourage additional research to validate these findings across a

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broader set of galactic systems and to explore the theoretical underpinnings of the model in greater depth.

Our intention is to contribute to the ongoing dialogue in the astrophysical community by presenting a novel approach that may offer new insights into the complex phenomena governing galactic dynamics. Nonetheless, we remain mindful of the model's current limitations and the need for continued refinement and testing.

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