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Abstract: The effect of the heterogeneity in sex factor is considered in the Markovian manpower modeling model. The estimation of the transition probability matrix of the staff flow was done using the maximum likelihood method. The expected future durations of the individuals in the manpower system were obtained under the aggregated and disaggregated (male and female) staff flow as well as their variances. Test for the heterogeneity of the staff based on sex factor was performed and the result of the heterogeneity test shows that there is difference between the disaggregated staff flows. The computed entrant probabilities also indicate that the male group has higher probabilities of attaining higher grades than the female group.

Keywords: markovian model; manpower planning; sex heterogeneity; time-homogeneity

1. Introduction

Manpower modeling is a very important methodology in human resource management for businesses and industries [1–3]. Its general aim is to best match future manpower needs and resources to satisfy the organizational objectives[4]. However, manpower modeling involves three critical flows which includes; flow into the system through personnel recruitment, internal personnel flows across categories (via promotion), and the wastage flows by mean of retirement or resignation, death etc[5]. There are factors which are inherent in any manpower system [6] and must be put into consideration while modeling a manpower system. Apart from the factors associated to human behavior which exhibits high degree of variability in manpower system [1,6], there are other factors which are crucial in modeling manpower systems which includes grade (sub-class of individuals in form of salary bands or work function), duration variables (including age and length of stay in the system), gender, motivation, performance or commitment[2,6,7]. All these factors constitute heterogeneity in manpower system, and can be a subject of classifying members of manpower system into homogeneous groups [5]. However, [6,8] categorized these factors into observable and non-observable(latent) factors.

There are many approaches and consideration while modeling the heterogeneity in manpower system; see [1,3,8] and references therein. Guerry[9] considered the non-observable factors; the mover-stayer, to determine the personnel subgroups that have a higher propensity for homogeneity with respect to transition probabilities using markov-switching model. In the same vain, [8]extended the non-observable factors 'mover-stayer' to mover-mediocre-stayer and used the hidden markov

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model. A multinomial markov-switching model approach was used to investigate the non-observable factor (mover-mediocre-stayer) in intra-departmental and interdepartmental transition in departmentalized manpower system [6].

The assumption of homogeneity is usually central in the modeling manpower system irrespective of the approach used[4,9]. In the markovian model, the challenge of heterogeneity of units in a given state was examined by [10] and suggested the disaggregation of the state into homogeneous sub-groups or states. De Feyter [2] proposed a general framework of getting more homogeneous subgroups using Markov chain theory in manpower planning. However, [11] noted that the heterogeneity or otherwise the homogeneity of the individuals in a manpower system is evident on the time-heterogeneity of the transition probability of the Markov chain. Sales [12] defines the test statistic for testing the time-homogeneity of the transition probability matrix of a Markovian manpower model. There is no doubt that the disaggregation of units into homogeneous sub-groups in situations where heterogeneity exist within units is a way forward in having a reliable statistical inference in manpower modeling. In relation to manpower modeling, effort should be made to examine if there exist heterogeneity in the assumed classified (homogeneous) group. And as pointed out by [10], if a group is non-homogeneous and interest is to disaggregate the group into sub-homogeneous groups, to what point can the subgroups be reasonably homogeneous. Furthermore, [9] also noted that the increase in sub-homogeneous groups result to a greater number of parameters to be estimated thereby affecting the goodness-of-fit of the model.

In this paper, we examined the effect of heterogeneity in the sex group of the academic personnel system of the Ebonyi State University under the assumption of time-homogeneous Markov chain model. The remainder of the paper is organized as follows: Section 2 considered the Markovian manpower model including the maximum likelihood estimation of the transition probability matrix, and the expected future duration in the manpower system is considered in Section 3. The Predication of the future cohort size is considered in Section 4, while Section 5 considers the modeling of heterogeneity in sex factor. Finally, Sections 5 and 6 considered the real-life application and conclusion, respectively.

2. Materials and Methods

Let $\langle X_t, t \geq 0\rangle$ denotes a stochastic process. Then, if X_t assumes a positive integer valued number and $P(X_t = j|X_0, X_1, X_2, \ldots, X_t = i) = P(X_{t+1} = j|X_{t-1} = i)$, X_t is referred to as a time-homogeneous Markov chain. Say, for $i, j = 1,2,3, \ldots, H$, $P = (P_{i,j})$ is the transition probability matrix (TPM) of the time-homogeneous Markov chain and H represents the number of states in the Markov chain.

Let H denotes the number of grades in a manpower system and the grades are mutually exclusive and collectively exhaustive; t denotes the time horizon of the manpower system (t is in year), and $n_i(t)$ and $n_{ij}(t)$ are the number of individuals in the i^{th} grade at time t, and at time interval $(t-1, t)$, respectively. Individual in grade, say, 1 may move to grade 2 by promotion and may leave the system at the end of the year. The number of individuals leaving the system at time t is denoted by $W_i(t)$. The probability that an individual in grade i moves to grade j at time interval $(t-1,\ t)$ is represented by p_{ij} and $p_{i0}(t)$ represents the probability of leaving the system. Let N denotes the matrix of flow indicating $n_{ij}(t)$, then N is defined as

$$N = \begin{bmatrix} n_{11}(t) & \cdots & n_{1H}(t) \\ \vdots & \ddots & \vdots \\ n_{H1}(t) & \cdots & n_{HH}(t) \end{bmatrix}; \quad (1)$$

and the TPM, P, is given by

$$P = \begin{bmatrix} p_{11} & \cdots & p_{1H} \\ \vdots & \ddots & \vdots \\ p_{H1} & \cdots & p_{HH} \end{bmatrix} = (P_{ij}); \quad (2)$$

where $p_{ij} \geq 0$. Define $n_i(t) = \sum_H n_{ij}(t)$, the maximum likelihood estimate, $\hat{P}$, of P is given by

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$$\hat{P} = \begin{matrix} 1 \\ \vdots \\ h \end{matrix} \begin{bmatrix} \hat{p}_{11} & \cdots & \hat{p}_{1H} \\ \vdots & \ddots & \vdots \\ \hat{p}_{H1} & \cdots & \hat{p}_{HH} \end{bmatrix} = (\hat{P}_{ij}); \tag{3}$$

where $\hat{p}_{ij} = \frac{n_{ij}(t)}{n_i(t)}$. However, for over the time period $= 1,2, \dots, T$, $\hat{p}_{ij} = \frac{\sum_{t=1}^{T} n_{ij}(t)}{\sum_{t=1}^{T} n_i(t)}$.

Let $R_j(t)$ denote the number of new individuals in grade j at time t, respectively. Then p_{0j} is the associated probability of entering the j^{th} grades. Then according to [13], the total number of individuals in the manpower system can be obtained using

$$N_j(t+1) = \sum_{i=1}^{H} P_{ij}(t) N_i(t) + R_i(t) \; ; \; j = 1,2, \dots, H \tag{2}$$

2.1. *Expected future duration in a manpower system*

It is very critical in manpower planning to determine in advance how long an employee is expected to last while in the system. According to[5], it serves as a measure of the career prospect of an employee. The expected future duration, μ, can be obtained as

$$\mu = (1 - P)^{-1}; \tag{3}$$

where $\mu = \mu_{ij}$ is a matrix of the same dimension with the TPM, P.

The corresponding variance of the expected future duration, according to [14], is given by

$$\sigma_{ij}^2 = \begin{cases} 2\mu_{ij}\mu_{jj} - \mu_{ij} - \mu_{ij}^2 & i < j \\ \mu_{ii}^2 - \mu_{ii} & i = j \end{cases} \tag{4}$$

Furthermore, the probability of an entrant in grade i attaining higher grade, denoted by λ_{ij}, is given by

$$\lambda_{ij} = \begin{cases} \frac{\mu_{ij}}{\mu_{jj}}; & i < j \\ \frac{\mu_{ii}}{\mu_{ii}}; & i = j \end{cases} \tag{5}$$

2.2. *Prediction of future cohort sizes*

Planning for future recruitment and the determination the size of the recruit are important aspects of manpower system[2,15,16]. Let $n(t)$ be a vector comprising elements $n_i(t)$, then the predicted future cohort sizes denoted by $n(t+1)$ is defined as

$$n(t+1) = Q\Upsilon(\mathrm{t}); \tag{6}$$

where $Q = p^0 P$, and $\Upsilon(\mathrm{t}) = \sum_{i=1}^{H} n_i(t)$. The parameter P is the TPM and
$p^0 = n_i(t)/\Upsilon(t)$ is the probability vector of the based year.

2.3. *Modeling the heterogeneity*

We consider the number of individuals in the manpower system consists of male and female individuals which constitute heterogeneity. Accordingly, the number of individuals in the manpower system is disaggregated into two homogeneous groups with the corresponding matrix of flows, respectively, denoted as N_1 and N_2. Similarly, the estimated transition probability matrix, the expected future duration, the variance of the expected future duration, and the probability of an entrant in grade i attaining higher grade for the two groups, respectively, denoted as $\hat{P}_1$ and $\hat{P}_2$, μ_1 and μ_2, σ_1^2 and σ_2^2, and λ_{1ij} and λ_{2ij}.

Considering the existence of heterogeneity in the sex category of the manpower population under the time-homogeneous Markov model, a test to validate the assumption is $H_0: \sigma_1^{*2} =$

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σ_2^{*2} vs $H_1: \sigma_1^{*2} \neq \sigma_2^{*2}$. The corresponding test statistic is $F = \sigma_2^{*2}/\sigma_1^{*2}$; where $\sigma_i^{*2}, i = 1{,}2$ is the mean variance of the expected future duration.

3. Results

In order to illustrate the effect of heterogeneity in the sex category under time-homogeneous Markovian manpower model, data from the manpower system of the Ebonyi state University, Nigeria were used. The data set covers a period between 2013/2014 to 2016/2017 academic session. Tables 1–12 show the aggregated and disaggregated staff matrix of flow of the period considered.

Table 1. Aggregated($N(t)$) staff matrix of flow for 2013/2014 session.

	1	2	3	4	5	6	7	W_i	$n_i(t)$
1	51	0	0	0	0	0	0	0	51
2	0	95	3	0	0	0	0	1	99
3	0	0	138	3	0	0	0	2	143
4	0	0	0	135	9	0	0	2	146
5	0	0	0	0	140	4	0	1	145
6	0	0	0	0	0	51	3	2	56
7	0	0	0	0	0	0	38	2	40
R_j	11	11	29	15	8	0	1		75

Table 2. Disaggregated ($N_1(t)$) staff matrix of flow for 2013/2014 session.

	1	2	3	4	5	6	7	W_i	$n_i(t)$
1	31	0	0	0	0	0	0	0	31
2	0	67	2	0	0	0	0	1	70
3	0	0	110	2	0	0	0	2	114
4	0	0	0	81	5	0	0	1	87
5	0	0	0	0	98	3	0	1	102
6	0	0	0	0	0	41	2	2	45
7	0	0	0	0	0	0	28	1	29
R_{1j}	9	9	23	12	6	0	1		60

Table 3. Disaggregated ($N_2(t)$) staff matrix of flow for 2013/2014 session.

	1	2	3	4	5	6	7	W_i	$n_i(t)$
1	20	0	0	0	0	0	0	0	20
2	0	28	1	0	0	0	0	0	29
3	0	0	28	1	0	0	0	0	29
4	0	0	0	54	4	0	0	1	59
5	0	0	0	0	42	1	0	0	43
6	0	0	0	0	0	10	1	0	11
7	0	0	0	0	0	0	10	1	11
R_{2j}	2	2	6	3	2	0	0		15

Table 4. Aggregated ($N(t)$) staff matrix of flow for 2014/2015 session.

	1	2	3	4	5	6	7	W_i	$n_i(t)$
1	44	1	0	0	0	0	0	2	47
2	0	85	5	0	0	0	0	2	92
3	0	0	111	6	0	0	0	3	120
4	0	0	0	156	7	0	0	1	164
5	0	0	0	0	132	5	0	2	139

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6	0	0	0	0	0	48	2	2	52
7	0	0	0	0	0	0	39	3	42
R_j	6	7	6	3	1	1	0		24

Table 5. Disggregated ($N_1(t)$) staff matrix of flow for 2014/2015 session.

	1	2	3	4	5	6	7	W_i	$n_i(t)$
1	26	1	0	0	0	0	0	1	28
2	0	60	4	0	0	0	0	0	64
3	0	0	89	5	0	0	0	2	96
4	0	0	0	94	4	0	0	1	99
5	0	0	0	0	92	4	0	1	97
6	0	0	0	0	0	38	1	1	40
7	0	0	0	0	0	0	31	2	33
$R_{\mathbf{1}jj}$	5	6	5	3	1	0	0		20

Table 6. Disaggregated ($N_2(t)$) staff matrix of flow for 2014/2015 session.

	1	2	3	4	5	6	7	W_i	$n_i(t)$
1	18	0	0	0	0	0	0	1	19
2	0	25	1	0	0	0	0	2	28
3	0	0	22	1	0	0	0	1	24
4	0	0	0	62	3	0	0	0	65
5	0	0	0	0	40	1	0	1	42
6	0	0	0	0	0	10	1	1	12
7	0	0	0	0	0	0	8	1	9
R_{2j}	1	1	1	0	0	1	0		4

Table 7. Aggregated ($N(t)$) staff matrix of flow for 2015/2016 session.

	1	2	3	4	5	6	7	W_i	$n_i(t)$
1	59	0	0	0	0	0	0	2	61
2	0	89	5	0	0	0	0	2	96
3	0	0	126	8	0	0	0	4	138
4	0	0	0	146	13	0	0	3	162
5	0	0	0	0	163	5	0	0	168
6	0	0	0	0	0	63	2	2	67
7	0	0	0	0	0	0	45	3	48
R_j	21	28	11	5	4	2	1		72

Table 8. Disaggregated ($N_1(t)$) staff matrix of flow for 2015/2016 session.

	1	2	3	4	5	6	7	W_i	$n_i(t)$
1	35	0	0	0	0	0	0	1	36
2	0	62	4	0	0	0	0	1	67
3	0	0	101	6	0	0	0	3	110
4	0	0	0	88	10	0	0	2	100
5	0	0	0	0	130	4	0	0	134
6	0	0	0	0	0	50	2	1	53
7	0	0	0	0	0	0	36	2	38
$R_{\mathbf{1}j}$	19	25	10	5	4	2	1		66

Table 9. Disaggregated ($N_2(t)$) staff matrix of flow for 2015/2016 session.

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	1	2	3	4	5	6	7	W_i	$n_i(t)$
1	24	0	0	0	0	0	0	1	25
2	0	27	1	0	0	0	0	1	29
3	0	0	25	2	0	0	0	1	28
4	0	0	0	58	3	0	0	1	62
5	0	0	0	0	33	1	0	0	34
6	0	0	0	0	0	13	0	1	14
7	0	0	0	0	0	0	9	1	10
R_{2j}	2	3	1	0	0	0	0		6

Table 10. Aggregated ($N(t)$) staff matrix of flow for 2016/2017 session.

	1	2	3	4	5	6	7	W_i	$n_i(t)$
1	47	2	0	0	0	0	0	1	50
2	0	76	7	0	0	0	0	1	84
3	0	0	121	5	0	0	0	2	128
4	0	0	0	133	11	0	0	0	144
5	0	0	0	0	127	6	0	2	135
6	0	0	0	0	0	56	2	1	59
7	0	0	0	0	0	0	47	2	49
R_j	1	1	3	2	2	0	0		9

Table 11. Disaggregated ($N_1(t)$) staff matrix of flow for 2016/2017 session.

	1	2	3	4	5	6	7	W_i	$n_i(t)$
1	42	2	0	0	0	0	0	1	45
2	0	68	6	0	0	0	0	1	75
3	0	0	101	4	0	0	0	2	107
4	0	0	0	120	10	0	0	0	130
5	0	0	0	0	114	5	0	1	120
6	0	0	0	0	0	50	2	0	52
7	0	0	0	0	0	0	46	1	47
R_{1j}	1	1	2	2	2	0	0		8

Table 12. Disaggregated ($N_2(t)$) staff matrix of flow for 2016/2017 session.

	1	2	3	4	5	6	7	W_i	$n_i(t)$
1	5	0	0	0	0	0	0	0	5
2	0	8	1	0	0	0	0	0	9
3	0	0	20	1	0	0	0	0	21
4	0	0	0	13	1	0	0	0	14
5	0	0	0	0	13	1	0	1	15
6	0	0	0	0	0	6	0	1	7
7	0	0	0	0	0	0	1	1	2
R_{2j}	0	1	0	0	0	0	0		1

Table 13. Pooled staff matrix of flow from 2013/14 to 2016/17 session.

	1	2	3	4	5	6	7	W_i	$n_i(t)$
1	201	3	0	0	0	0	0	5	209
2	0	345	20	0	0	0	0	6	371
3	0	0	496	22	0	0	0	11	529
4	0	0	0	570	40	0	0	6	616

5	0	0	0	0	562	20	0	5	587
6	0	0	0	0	0	218	9	7	234
7	0	0	0	0	0	0	169	10	179
R_j	39	47	49	25	15	3	2		180

Table 14. Pooled $N_1(t)$ staff matrix of flow from 2013/14 to 2016/17 session.

	1	2	3	4	5	6	7	W_i	$n_i(t)$
1	134	3	0	0	0	0	0	3	140
2	0	257	16	0	0	0	0	3	276
3	0	0	401	17	0	0	0	9	427
4	0	0	0	383	29	0	0	4	416
5	0	0	0	0	434	16	0	3	453
6	0	0	0	0	0	179	7	4	190
7	0	0	0	0	0	0	141	6	147
R_{1j}	34	41	40	22	13	2	2		154

Table 15. Pooled $N_2(t)$ staff matrix of flow from 2013/14 to 2016/17 session.

	1	2	3	4	5	6	7	W_i	$n_i(t)$
1	67	0	0	0	0	0	0	2	69
2	0	88	4	0	0	0	0	3	95
3	0	0	95	5	0	0	0	2	102
4	0	0	0	187	11	0	0	2	200
5	0	0	0	0	128	4	0	2	134
6	0	0	0	0	0	39	2	3	44
7	0	0	0	0	0	0	28	4	32
R_{2j}	5	6	9	3	2	1	0		26

3.1. Estimated TPM

The estimated TPM, $\hat{P}$, was obtained using the maximum likelihood approach with the assumption of time-homogeneity, where $t = 1,2,3,4,$ which represented the number of sessions. Table 16 shows the resultant TPM for the aggregated staff flow from the dataset.

Table 16. The transition Probability Matrix $N(t)$ matrix of flow from 2013/14 to 2016/17 session.

	1	2	3	4	5	6	7	W_i
1	0.9617	0.0144	0	0	0	0	0	0.0239
2	0	0.9299	0.0539	0	0	0	0	0.0162
3	0	0	0.9376	0.0416	0	0	0	0.0208
4	0	0	0	0.9253	0.0649	0	0	0.0097
5	0	0	0	0	0.9574	0.0341	0	0.0085
6	0	0	0	0	0	0.9316	0.0385	0.0299
7	0	0	0	0	0	0	0.9441	0.0559

In the same vain, the TPMs for the disaggregated staff flow which was based on sex factor are respectively, shown in Tables 17 and 18.

Table 17. The transition Probability Matrix of the $N_1(t)$ from 2013/14 to 2016/17 session.

	1	2	3	4	5	6	7	W_i
1	0.9571	0.0214	0	0	0	0	0	0.0214
2	0	0.9312	0.0580	0	0	0	0	0.0109

3	0	0	0.9391	0.0398	0	0	0	0.0211
4	0	0	0	0.9207	0.0697	0	0	0.0096
5	0	0	0	0	0.9581	0.0353	0	0.0066
6	0	0	0	0	0	0.9421	0.0368	0.0211
7	0	0	0	0	0	0	0.9592	0.0408

Table 18. The transition Probability Matrix of the $N_2(t)$ from 2013/14 to 2016/17 session.

	1	2	3	4	5	6	7	W_i
1	0.9710	0	0	0	0	0	0	0.0290
2	0	0.9263	0.0421	0	0	0	0	0.0316
3	0	0	0.9314	0.0490	0	0	0	0.0196
4	0	0	0	0.9350	0.0550	0	0	0.0100
5	0	0	0	0	0.9552	0.0299	0	0.0149
6	0	0	0	0	0	0.8864	0.0455	0.0682
7	0	0	0	0	0	0	0.8750	0.1250

3.2. Calculated expected future duration

Calculating the expected future duration in grades 1 to 7 interest is on the corresponding transition probabilities for grades 1 to 7. The expected future duration in grades 1 to 7 for the aggregated and disaggregated staff flow were obtained using eqn. (3). Tables 19–21, respectively, show the expected future duration (in years) for the aggregated and disaggregated staff flow.

Table 19. Expected future duration for aggregated staff flow (N).

	1	2	3	4	5	6	7
1	26.1097	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
2	5.3635	14.2653	0.0000	0.0000	0.0000	0.0000	0.0000
3	4.6329	12.3221	16.0256	0.0000	0.0000	0.0000	0.0000
4	2.5800	6.8621	8.9246	3.3869	0.0000	0.0000	0.0000
5	3.9306	10.4543	13.5964	20.3946	23.4742	0.0000	0.0000
6	1.9595	5.2119	6.7783	10.1675	11.7028	14.6199	0.0000
7	1.3496	3.5896	4.6684	7.0026	8.0600	10.0692	7.8891

Table 20. Expected future duration for disaggregated staff flow (N_1).

	1	2	3	4	5	6	7
1	23.3100	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
2	7.2505	14.5349	0.0000	0.0000	0.0000	0.0000	0.0000
3	6.9052	13.8427	16.4204	0.0000	0.0000	0.0000	0.0000
4	3.4657	6.9476	8.2412	12.6103	0.0000	0.0000	0.0000
5	5.7651	11.5572	13.7092	20.9771	23.8663	0.0000	0.0000
6	3.5148	7.0461	8.3581	12.7892	14.5506	17.2712	0.0000
7	3.1702	6.3553	7.5387	11.5353	13.1241	15.5779	24.5098

Table 21. Expected future duration for disaggregated staff flow (N_2).

	1	2	3	4	5	6	7
1	34.4828	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
2	0.0000	13.5685	0.0000	0.0000	0.0000	0.0000	0.0000
3	0.0000	8.3270	14.5773	0.0000	0.0000	0.0000	0.0000
4	0.0000	6.2773	10.9890	15.3846	0.0000	0.0000	0.0000
5	0.0000	7.7065	13.4910	18.8874	22.3214	0.0000	0.0000.

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6	0.0000	2.0284	3.5509	4.9712	5.8751	8.8028	0.0000
7	0.0000	0.7383	1.2925	1.8095	2.1385	3.2042	8.0000

3.3. Test for heterogeneity

The variances of the expected future duration for the disaggregated staff flow denoted by s_1^{2*} and s_2^{2*} were obtained using eqn.(4). The computed values of the variances are given by

$$s_1^{2*} = \begin{pmatrix} 520.0461 & 150.9503 & 172.1853 & 71.9303 & 236.1817 & 105.5410 & 142.1816 \\ 0.0000 & 196.7284 & 249.1423 & 120.0059 & 406.5291 & 186.6956 & 264.7891 \\ 0.0000 & 0.0000 & 253.2091 & 131.6894 & 452.7244 & 210.4929 & 305.1734 \\ 0.0000 & 0.0000 & 0.0000 & 146.4094 & 540.2757 & 265.4168 & 420.8573 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 545.7340 & 276.3421 & 457.9720 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 281.0231 & 505.3736 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 576.2205 \end{pmatrix} \quad (7)$$

and

$$s_2^{2*} = \begin{pmatrix} 1154.5807 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 170.5357 & 165.1044 & 147.4657 & 276.9431 & 29.5684 & 17.7169 \\ 0.0000 & 0.0000 & 197.9204 & 206.3756 & 406.7779 & 46.3559 & 17.7169 \\ 0.0000 & 0.0000 & 0.0000 & 221.3013 & 467.5651 & 57.8369 & 23.8682 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 475.9235 & 63.0428 & 27.5043 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 68.6865 & 37.7961 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 56.0000 \end{pmatrix} \quad (8)$$

The mean variances of the lengths of stay, $s_l^2 = \sum_{i,j}^{7} s_{lij}^{2*}/i \times j$, $l = 1,2.$, are computed as 153.2665 and 88.5018, respectively. With the test hypothesis $H_0\colon \delta_1^2 = \delta_2^2 \ vs\ H_1\colon \delta_1^2 \neq \delta_2^2$, the test statistic, $F = s_1^2/s_2^2$ is computed as 1.7318. At $\alpha = 0.05$, the $F_{649,649} = 1$ and the corresponding P-value is 1.8368×10^{-12}. This implies that heterogeneity exists in the aggregated staff flow.

The entrant probabilities of attaining higher grade j from grade i for the disaggregated staff flow N_1 and N_2 were, respectively, obtained using eqn.(5) as follows

$$\lambda_{(1)ij} = \begin{pmatrix} 1.0000 & 0.4988 & 0.4205 & 0.2748 & 0.2416 & 0.2035 & 0.1293 \\ 0.0000 & 1.000 & 0.8430 & 0.5509 & 0.4842 & 0.4080 & 0.2593 \\ 0.0000 & 0.0000 & 1.0000 & 0.6535 & 0.5744 & 0.4839 & 0.3076 \\ 0.0000 & 0.0000 & 0.0000 & 1.0000 & 0.8789 & 0.7405 & 0.4706 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 1.0000 & 0.8425 & 0.5355 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 1.0000 & 0.6356 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 1.0000 \end{pmatrix} \quad (9)$$

and

$$\lambda_{(2)ij} = \begin{pmatrix} 1.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 1.0000 & 0.5712 & 0.4080 & 0.3453 & 0.2304 & 0.0923 \\ 0.0000 & 0.0000 & 1.0000 & 0.7143 & 0.6044 & 0.4034 & 0.1616 \\ 0.0000 & 0.0000 & 0.0000 & 1.0000 & 0.8462 & 0.5647 & 0.2262 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 1.0000 & 0.6674 & 0.2673 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 1.0000 & 0.4005 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 1.0000 \end{pmatrix} \quad (10)$$

4. Discussion

In this paper, the effect of the existence of heterogeneity in the sex factor in the manpower planning assuming time-homogeneous Markov chain was considered. The data used in the study were from Ebonyi State University. The academic staff flow was disaggregated into two homogeneous groups; male and female. A test to determine the existence of heterogeneity was done and the result of the test indicates that differences exist between the groups with P-value of 1.8368×10^{-12}. Furthermore, the probabilities of entrants to higher grades were obtained. The entrant probabilities show that the male staffs have higher probabilities of moving to higher grades except the movement from grade three to grades four and five where the female staffs have higher probabilities of attaining higher grades.

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