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Calculation of the Number of Transactions for Collatz Conjecture

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Abstract: In this article we have constructed a basic methodology from which important perspectives for both the Collatz conjecture and Beal's conjecture can be derived. We falsified the conjecture we previously developed on Beal's conjecture and made the falsified conjecture usable for Collatz conjecture. In this regard, we have clearly expressed the numbers that can reach 1 in the Collatz conjecture. Then, using this, we created a limit for the numbers that should be considered for the Collatz conjecture. Thanks to the analysis of this limit, we have clearly expressed the recursive Collatz function in an equation. Using this structure, we wrote a function that indicates how many times the Collatz function repeats certain numbers to reach 1.

Keywords: collatz conjecture; beal's conjecture; composite numbers; recursive structure; number of transactions

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1. Introduction

In this article, the Beal's conjecture-Collatz conjecture Bridge Methodology, briefly referred to as the (B-C)c Bridge Methodology, aims to present methodologies developed for use in articles concerning the resolution of the mentioned conjectures, which are believed to provide significant insights for both Beal's conjecture and Collatz conjecture. Then, the developed methodology will be used for Collatz conjecture in this article. Beal's conjecture is left for another future article.

Before presenting the methodology, it is useful to clearly define Beal's conjecture and Collatz conjecture.

Let $A, B, C \in \mathbb{N}^+; x, y, z \in N^+ > 2$. Beal's conjecture is conjectured that [1]:
When $A^x + B^y = C^z$, do A, B, C have to have a common prime divisor?

Let $x \in N^+$. Collatz conjecture is conjectured that [2]:

If x is even, divide it by 2. If x is odd, multiply it by 3 and add 1. Repeat the process again and again. The Collatz conjecture is that no matter what the number (i.e., x) is taken, the process will always eventually reach 1. Let's express it mathematically:

$$f^{[g]}(n) = \begin{cases} n/2, & \text{n even} \\ 3n+1, & \text{n odd} \end{cases} \tag{1.1}$$

Definitions, Lemmas and Conjecture

Definition 1. *According to fundamental theorem of arithmetic [3]:*
Composite numbers are numbers that are greater than 1 and have factors other than themselves and 1.

Definition 2. *According to fundamental theorem of arithmetic [3].*
Prime numbers are numbers greater than 1 that do not have any factors other than themselves and 1.

Definition 3. *According to fundamental theorem of arithmetic [3].*
An integer must be either a prime number or a composite number.

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Lemma 1. *According to Aysun and Gocgen [4]:*
$np + p$ gives all composite numbers where n is a positive natural numbers and p is a prime number.

Proof. $np + p = p(n + 1)$. Then, according to fundamental theorem of arithmetic:

$$(n + 1 \in \mathbb{C}) \oplus (n + 1 \in \mathbb{P})$$

Let $n + 1 \in \mathbb{C}$:

$$n + 1 = p_m \times \cdots \times p_{m+k}$$

Then,

$$p(n + 1) = p \times (p_m \times \cdots \times p_{m+k})$$

Let $n + 1 \in \mathbb{P}$:

$$n + 1 = p_m$$

Then,

$$p(n + 1) = p \times p_m$$

□

Lemma 2. *According to Aysun and Gocgen [4]:*
$2np + p$ gives all odd composite numbers where n is a positive natural numbers and p is an odd prime numbers.

Proof. $np + p$ gives odd composite numbers where p is a odd number and n is a even number. Then as already proved $np + p$ gives all composite numbers where n is a positive natural number and p is an prime number. Only possibility for odd composite just specified. Therefore, $np + p$ gives all odd composite numbers where p is a odd number and n is a even number. This equal to: $2np + p$ gives all odd composite numbers where n is a positive natural numbers and p is an odd prime numbers. □

Conjecture 1. *According to Gocgen [5]:*
To determine the value of n in the expression $2np + p$ for powers of p:

$$n = f(x) = \begin{cases} m - 1, & m \leq 2 \\ 1 + \sum_{k=1}^{m-2} p^k, & m > 2 \end{cases} \tag{1.2}$$

2. Methodology

In both conjectures, the methods to be used in future articles are based on methods to be developed to determine the powers of certain numbers, especially prime numbers, from the formulas that produce composite numbers. Let's start working on improving the method in question:

The $np + p$ formula produces both even and odd composite numbers by *Lemma 1*. Therefore, it will be possible to examine the powers of the number 2 with this formula. Since we want to examine the powers of the number 2, $p = 2$ should be in the formula $np + p$.

Hence:

$np + p = 2n + 2$

We can define powers of 2 as 2^m, where $m \in \mathbb{N}^+$. We will examine the forces in three situations:

1) $m = 1$

2) $m = 2$

3) $m > 2$

Let's consider each situation one by one:

First of all, let $m = 1$. In this situation:

$2n + 2 = 2^1$

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Thus:
Since $2n = 0$, $n = 0$ must be.
Now let $m = 2$. In this situation:
$2n + 2 = 2^2$
Since $2n = 2$, $n = 1$ must be.
Finally, let $m > 2$:
$2n + 2 = 2^m$
We can write this expression as follows:
$2(n + 1) = 2^m$
Hence:
$n + 1 = 2^{m-1}$. Thence:
$n = 2^{m-1} - 1$

Therefore, with the aim of determining the powers of 2 in the expression $2n + 2$, the following expression can be written to determine the n values:

$$n = f(m) = \begin{cases} 0, & m = 1 \\ 1, & m = 2 \\ 2^{m-1} - 1 & m > 2 \end{cases} \tag{2.1}$$

This expression; since $n = m - 1$ for $m \leq 2$, can be written as follows:

$$n = f(m) = \begin{cases} m - 1 & m \leq 2 \\ 2^{m-1} - 1 & m > 2 \end{cases} \tag{2.2}$$

Since $m - 1 = 2^{m-1} - 1$ for $m = 1$ and $m - 1 = 2^{m-1} - 1$ for $m = 2$, we can briefly write the expression as follows:

$$n = f(m) = 2^{m-1} - 1 \tag{2.3}$$

Moreover, this expression can be easily deduced from the equation $2n + 2 = 2^m$ for any m value.
So in summary, for powers of 2 in the formula $np + p$:
The expression $2(2^{m-1}) + 2$ can be written.
Likewise, when this expression is edited:
It is clear that we will encounter the expression $2^m - 2 + 2 = 2^m$.

It is clear that there is no need to examine even numbers when examining the powers of prime numbers other than 2. Therefore, not the $np + p$ formula, but the $2np + p$ formula, which gives only odd composite numbers, should be used for this examination by *Lemma 2*.

Let's examine it again in three cases:
1) $m = 1$
2) $m = 2$
3) $m > 2$
First, let $m = 1$:
$2np + p = p^m$
Let's arrange:
$p(2n + 1) = p^1$
Thus:
$2n + 1 = 1$
Hence:
$2n = 0$
As a result:
$n = 0$
Now, let $m = 2$:
$2np + p = p^2$

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Let's arrange:
$p(2n+1) = p^2$
Thence:
$n = \frac{p-1}{2}$.
Finally, let $m > 2$:
$2np + p = p^m$
Hence:
$n = \frac{p^{m-1}-1}{2}$
Therefore, in order to write the powers of primes other than 2 in the formula $2np + p$, the following expression can be written to determine the value of n:

$$n = f(m) = \begin{cases} 0, & m = 1 \\ \frac{p-1}{2}, & m = 2 \\ \frac{p^{m-1}-1}{2} & m > 2 \end{cases} \tag{2.4}$$

Since $\frac{p^{m-1}-1}{2} = \frac{p-1}{2}$ for $m = 2$, we can rearrange the expression as follows:

$$n = f(m) = \begin{cases} 0, & m = 1 \\ \frac{p^{m-1}-1}{2} & m \geq 2 \end{cases} \tag{2.5}$$

Also since $\frac{p^{m-1}-1}{2} = \frac{0}{2} = 0$ for $m = 1$:

$$n = f(m) = \frac{p^{m-1}-1}{2}. \tag{2.6}$$

Now that we have developed the methodology, let's examine previously developed but unproven constructs:
Let's assume *Conjecture 1* is true and say $p = 5$:
In this case it will be $2np + p = 10n + 5$. Let's apply the mentioned conjecture to determine the n value in order to find powers of 5:
Let $m = 3$:
$n = 1 + \sum_{k=1}^{m-2} 5^k$
Thus:
$10(1 + \sum_{k=1}^{m-2} 5^k) + 5 = 5^3$
If we arrange this equation:
$10(1 + 5^1) + 5 = 5^3$.
In conclusion:
Since $65 \neq 125$, *Conjecture 1* is falsified.
Let's revise *Conjecture 1*, which was developed to be used for Beal's conjecture and which we have falsified, to be used in Collatz conjecture in this articles. This revision process involves drawing the claim to the ground $p = 2$:

Conjecture 2. *To determine the value of n in the expression $2n + 2$ for powers of 2:*

$$n = f(x) = \begin{cases} m - 1, & m \leq 2 \\ 1 + \sum_{k=1}^{m-2} 2^k, & m > 2 \end{cases} \tag{2.7}$$

Let $m \leq 2$:
If $m = 1$:
$2(1-1) + 2 = 2^1$.
Thence:
$2 = 2$.
If $m = 2$:
$2(2-1) + 2 = 2^2$
Then:
$4 = 4$.
We can say that the equality $2(m-1) + 2 = 2^m$, where $m \leq 2$, is satisfied.

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Let $m > 2$*:*
$2(1 + \sum_{k=1}^{m-2} 2^k) + 2 = 2^m$.
For example, let $m = 4$*:*
$2(1 + 2^1 + 2^2) + 2$.
Then:
$2^2 + 2^2 + 2^3 = 2^4$.
Herein:
It is necessary to understand that the expression $2^{m-1} + 2^{m-1} = 2^m$ *is satisfied.*
We can write the expression $2^{m-1} + 2^{m-1} = 2^m$ *as follows:*
$2(2^{m-1}) = 2^m$
Thus situated:
Since $2^m = 2^m$*, the expression* $2^{m-1} + 2^{m-1} = 2^m$ *is explicitly satisfied.*
If we generalize the expression in the example we gave:
$2(1 + \cdots + 2^{m-2}) + 2 = 2^m$
Let's arrange:
$2^2 + \cdots + 2^{m-1} = 2^m$
We know that the powers of 2 will be written sequentially on the left-hand side of this equation. In that case:
Based on the equation $2^{m-1} + 2^{m-1} = 2^m$*:*
The equation $2^2 + \cdots + 2^{m-1} = 2^m$ *will be satisfied.*
As a result:
We can say that the equality $2(1 + \sum_{k=1}^{m-2} 2^k) + 2 = 2^m$*, where* $m > 2$*, is satisfied.*
Therefore Conjecture 2 is satisfied.

3. Collatz Conjecture

Since 2^m is even and $\frac{2^m}{2} = 2^{m-1}$ is also even, 2^m as $\frac{2^m}{2^m} = 1$ It can be clearly seen that the values that can be written in will be equal to 1 after m operations.

Using these values, we can also determine all the values that can be expressed as 2^m when the $3x + 1$ operation is applied only once:

$$\frac{2^m - 1}{3} \tag{3.1}$$

It is obvious that the above operation will not always equal the integer. Therefore, in order to offer a different perspective and to prepare a suitable basis for the analysis we will present in the rest of the paper, at this point we will look at how the numbers that can be written in 2^m can be expressed with the formula $np + p$.

It is clear that *Conjecture 2* will be applied for this.

With this structure, it becomes easier to examine the numbers that will be generated by the $\frac{2^m-1}{3}$ operation.

We know that there are two situations:
1) $m \leq 2$
2) $m > 2$
Let's consider the first case first:
In the first case, there are two values that m can take: 1 and 2.
Let's examine both cases respectively:
First of all, it can be $m = 1$. In this case, the result will be 2.
Since $\frac{2-1}{3} \notin \mathbb{N}$, it does not provide the situation we want.
The other and last possibility is the one with $m = 2$. In this case, the result will be 4.
Since $\frac{4-1}{3} \in \mathbb{N}$, it provides the situation we want.
We can write the following expression for $\frac{2^m-1}{3}$, where $m > 2$:
$\frac{2(1+2^1+\cdots+2^{m-2})+2-1}{3}$
Here is a simple observation on this statement:

Conjecture 3. *When m is even:*
We can see that $\frac{2(1+2^1+\cdots+2^{m-2})+2-1}{3} \in \mathbb{N}$
When m is odd:

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We can see that $\frac{2(1+2^1+\cdots+2^{m-2})+2-1}{3} \notin \mathbb{N}$.

To see this claim more clearly, we can create a pyramid for n values:

$$2^2$$

$$2^2+2^3$$
$$2^2+2^3+2^4$$
$$2^2+2^3+2^4+2^5$$
$$2^2+2^3+2^4+2^5+2^6$$
$$2^2+2^3+2^4+2^5+2^6+2^7$$
$$2^2+2^3+2^4+2^5+2^6+2^7+2^8$$
$$2^2+2^3+2^4+2^5+2^6+2^7+2^8+2^9$$
$$2^2+2^3+2^4+2^5+2^6+2^7+2^8+2^9+2^{10}$$
$$2^2+2^3+2^4+2^5+2^6+2^7+2^8+2^9+2^{10}+2^{11}$$
$$2^2+2^3+2^4+2^5+2^6+2^7+2^8+2^9+2^{10}+2^{11}+2^{12}$$
$$2^2+2^3+2^4+2^5+2^6+2^7+2^8+2^9+2^{10}+2^{11}+2^{12}+2^{13}$$
$$2^2+2^3+2^4+2^5+2^6+2^7+2^8+2^9+2^{10}+2^{11}+2^{12}+2^{13}+2^{14}$$

When we look at the pyramid, we see that the expression $(1+2^1)$ at the beginning of the n values is not included. We removed the expression $(1+2^1)$ because it did not affect the divisibility of the result by 3 and would only create a burden in the calculations in this analysis, which about *Conjecture 3*.

When we look at this pyramid, we see a reflection of our claim.

What is meant is this:

We see that for the n values, i.e., each row in the pyramid, if the last value in that row is even, it is not divisible by 3; if it is odd, it is divisible by 3.

Let's examine it line by line:

It will be seen that it follows a pattern such as one divisible by 3 and one not divisible by 3, sequentially from top to bottom.

As a result of these observations:

Conjecture 4. *If the value n is divisible by 3 (its last value is odd), the result of the operation in Conjecture 3 is not an integer. If the value of n is not divisible by 3 (its last value is even), the result of the operation in Conjecture 3 is an integer.*

Since *Conjecture 4* gives us a different perspective on *Conjecture 3*, let's first prove *Conjecture 4* and then *Conjecture 3*:

Our expression is $2^n + 2^{n+1} + \cdots + 2^{n+k}$ (here n is meant as an independent variable from that in the $np+p$ formula). First possibility here: n is even and k is odd. When we divide this expression by 3, we will try to find the remainder of the result.

First, we can use the geometric series to express this type of sum. The general formula for the geometric series is:

$$a + ar + ar^2 + \cdots + ar^n = a\frac{r^{n+1}-1}{r-1} \tag{3.2}$$

For our series $(a = 2^n)$, $(r = 2)$ and the number of terms is $(k+1)$ (since there are total $(k+1)$ terms including (n)).

In this case, the sum of the series is:

$$2^n + 2^{n+1} + 2^{n+2} + \cdots + 2^{n+k} = 2^n\left(\frac{2^{k+1}-1}{2-1}\right) = 2^n(2^{k+1}-1) \tag{3.3}$$

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We will divide this total by 3. First, let's rewrite the expression:

$$\frac{2^n(2^{k+1}-1)}{3} \tag{3.4}$$

Now let's simplify this expression using modular arithmetic.
Since n is even, we must find $(2^n \mod 3)$.
$2^n = 2^{2k} = (2^2)^k = 4^k$
The value of 4 mod 3 is 1 because:

$$4 \mod 3 = 1 \tag{3.5}$$

Thus:

$$4^k \mod 3 = 1^k = 1 \tag{3.6}$$

Therefore, $2^n \mod 3$ is always 1 when n is even.
Since k is odd, $(k+1)$ will be even. In this situation:
Since (2^{k+1}) is an even power, $(2^{k+1} \equiv 1 \pmod 3)$.
In this case $(2^{k+1} - 1 \equiv 1 - 1 \equiv 0 \pmod 3)$.
As a result:

$$2^n(2^{k+1}-1) \equiv 1 \cdot 0 \equiv 0 \pmod 3 \tag{3.7}$$

Therefore, the expression $(\frac{2^n(2^{k+1}-1)}{3})$ is divisible and the result is an integer.

In summary, when n is even and k is odd, the expression $\frac{2^n+2^{n+1}+\dots+2^{n+k}}{3}$ is divisible and the result is an integer.

Here is the second and last possibility: n and k are both even.
Since k is even, $k+1$ will be odd. In this situation:
Since 2^{k+1} is the odd power, $2^{k+1} \equiv 2 \pmod 3$.
In this case $2^{k+1} - 1 \equiv 2 - 1 \equiv 1 \pmod 3$.
As a result:

$$2^n(2^{k+1}-1) \equiv 1 \cdot 1 \equiv 1 \pmod 3 \tag{3.8}$$

Therefore, the result of $\frac{2^n(2^{k+1}-1)}{3}$ is:

$$\frac{1}{3} \pmod 3 \tag{3.9}$$

Since this expression is not an integer, we cannot write the result as modular arithmetic. In this case, we can conclude that the result of the expression is not an integer and cannot be divided in general form.

In summary, when n is even and k is even, $\frac{2^n+2^{n+1}+\dots+2^{n+k}}{3}$ The expression is not divisible and the result is not an integer.

Therefore, *Conjecture 4* has been satisfied.

So let's get to work proving *Conjecture 3*:

First, let's revisit the expression $S = 2^n + 2^{n+1} + \dots + 2^{n+k}$. This is a geometric series and we will use the sum formula of geometric series to find the sum of the series. Here $a = 2^n$, $r = 2$ and $m = k+1$. Let's find the sum of our series using this formula:

$$S = 2^n\frac{2^{k+1}-1}{2-1} = 2^n(2^{k+1}-1) = 2^{n+k+1} - 2^n \tag{3.10}$$

Multiplying the sum of this series by 2:

$$2S = 2 \cdot (2^{n+k+1} - 2^n) = 2^{n+k+2} - 2^{n+1} \tag{3.11}$$

Now let's add 1 to this:

$$2S + 1 = 2^{n+k+2} - 2^{n+1} + 1 \tag{3.12}$$

Now let's divide this expression by 3. We will divide the results one by one and perform operations modulo 3.

First, let's divide 2^{n+k+2} by 3. It would be useful to know the modulo 3 values of 2^m. The relationship of 2^m to modulo 3 is as follows:

$$2^1 \equiv 2 \mod 3$$
$$2^2 \equiv 1 \mod 3$$
$$2^3 \equiv 2 \mod 3$$
$$2^4 \equiv 1 \mod 3$$
$$2^5 \equiv 2 \mod 3$$
$$2^6 \equiv 1 \mod 3$$
$$\vdots$$

That is, the values $2^m \mod 3$ alternate between 2 and 1.

We have already proven the $m =$ even part of this periodic understanding. To fully prove the periodic understanding, it is necessary to prove that mod 3 is equivalent to 2 when $m =$ odd.

If m is odd, it can be written as $m = 2k + 1$ (k here is meant as a variable independent of the k values mentioned so far).

Now, let's write the expression 2^m as $m = 2k + 1$: $2^m = 2^{2k+1}$.

This expression can also be written as $2^{2k+1} = 2 \cdot 2^{2k}$.

Let's examine the expression 2^{2k} with mod 3: $2^{2k} = (2^2)^k = 1^k = 1$ (since $2^2 \equiv 1 \mod 3$).

In this case $2^{2k+1} \equiv 2 \cdot 1 \equiv 2 \mod 3$.

Consequently, as long as m is odd, $2^m \mod 3 = 2$.

Therefore, periodic progression is proven.

Using this, if we divide 2^{n+k+2} by 3, we need to check if $n + k + 2$ is even. Since n is even and k is odd, $n + k$ will be odd and therefore $n + k + 2$ will also be odd. In this situation:

$$2^{n+k+2} \equiv 2 \mod 3 \tag{3.13}$$

Similarly, let's evaluate 2^{n+1}. Since n is even, $n + 1$ is odd. In this situation:

$$2^{n+1} \equiv 2 \mod 3 \tag{3.14}$$

Now, let's find $2S + 1 \mod 3$:

$$2S + 1 = 2^{n+k+2} - 2^{n+1} + 1 \equiv 2 - 2 + 1 \equiv 1 \mod 3 \tag{3.15}$$

As a result, we see that the remainder of $\frac{2S+1}{3}$ is 1.

As a second and final possibility, since n and k are even, $n + k$ is also even, and therefore $n + k + 2$ is also even. In this situation:

$$2^{n+k+2} \equiv 1 \mod 3 \tag{3.16}$$

Similarly, let's evaluate 2^{n+1}. Since n is even, $n + 1$ is odd. In this situation:

$$2^{n+1} \equiv 2 \mod 3 \tag{3.17}$$

Now, let's find $2S + 1 \mod 3$:

$$2S + 1 = 2^{n+k+2} - 2^{n+1} + 1 \equiv 1 - 2 + 1 \equiv 0 \mod 3 \tag{3.18}$$

As a result, we see that $\frac{2S+1}{3}$ is an integer, 0 modulo 3.

When n is even, $m \in \mathbb{N}+$ and $m > 1$:

$\frac{2(1+2^1+\cdots+2^n)+2-1}{3}$ + [4] and do expressions other than the 2^m expression have to turn into 1 after the corresponding transactions?

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Basis 1. *If we call expressions other than the mentioned expression x, the expression $3x+1$ will give an even number that is not a power of 2.*

Then, we can say that Collatz conjecture is satisfied if even numbers that are not powers of 2 must be converted to 1 after the relevant operations.

$2^m + 2k \leq 2 \cdot 2^m - 2$ where k, n is a positive natural number and $n > 1$:

$2^m + 2k$ returns even numbers that are not powers of two.

At this point, we can express the Collatz conjecture as follows:

$2^m + 2k \leq 2 \cdot 2^m - 2$ where k, n is a positive natural number and $n > 1$:

Do the values produced by the expression $2^m + 2k$ have to be converted to a power of two (and therefore to 1) after the relevant operations?

The expression $2^m + 2k$ is even. Therefore, according to the Collatz function, it must first be divided by 2. Let's assume that this expression will be subjected to the $3x+1$ operation after being divided by 2, and from now on, it will first be divided by 2 and then subjected to the $3x+1$ operation in a continuous order. A process diagram that proceeds in this way can be expressed as $2^m + 2k$ as follows:

$$\left(\frac{3^{i+1} \cdot 2^{m-1}}{2^i} + \frac{3^{i+1}k}{2^i} + \sum_{j=0}^{i} \frac{3^j}{2^j}\right) + 1 = 2^l \tag{3.19}$$

Let's isolate m and k in the given equation, respectively. First, let's manipulate the terms to isolate m:

$$\frac{3^{i+1} \cdot 2^{m-1}}{2^i} = 3^{i+1} \cdot 2^{m-1-i}$$

$$\frac{3^{i+1}k}{2^i} = 3^{i+1}k \cdot 2^{-i}$$

$$\sum_{j=0}^{i} \frac{3^j}{2^j} = \sum_{j=0}^{i} \left(\frac{3}{2}\right)^j$$

Let's rewrite the equation in these terms:

$$3^{i+1} \cdot 2^{m-1-i} + 3^{i+1}k \cdot 2^{-i} + \sum_{j=0}^{i} \left(\frac{3}{2}\right)^j + 1 = 2^l$$

Now, let's arrange these terms. To make the expression simpler, let's separate the terms that are common in the denominators:

$$3^{i+1} \cdot 2^{m-1-i} + 3^{i+1} \cdot k \cdot 2^{-i} + \sum_{j=0}^{i} \left(\frac{3}{2}\right)^j + 1 = 2^l$$

Let's add terms with common denominators to the total:

$$3^{i+1}(2^{m-1-i} + k \cdot 2^{-i}) + \sum_{j=0}^{i} \left(\frac{3}{2}\right)^j + 1 = 2^l$$

Hence:

$$3^{i+1}(2^{m-1-i} + k \cdot 2^{-i}) = 2^l - \sum_{j=0}^{i} \left(\frac{3}{2}\right)^j - 1$$

Thence:

$$2^{m-1-i} + k \cdot 2^{-i} = \frac{2^l - \sum_{j=0}^{i} \left(\frac{3}{2}\right)^j - 1}{3^{i+1}}$$

Let's arrange:

$$2^{m-1-i} = \frac{2^l - \sum_{j=0}^{i} \left(\frac{3}{2}\right)^j - 1}{3^{i+1}} - k \cdot 2^{-i}$$

$$2^{m-1-i} = \frac{2^l - \sum_{j=0}^{i}\left(\frac{3}{2}\right)^j - 1 - k \cdot 2^{-i} \cdot 3^{i+1}}{3^{i+1}}$$

$$m - 1 - i = \log_2\left(\frac{2^l - \sum_{j=0}^{i}\left(\frac{3}{2}\right)^j - 1 - k \cdot 2^{-i} \cdot 3^{i+1}}{3^{i+1}}\right)$$

Thus:

$$m = \log_2\left(\frac{2^l - \sum_{j=0}^{i}\left(\frac{3}{2}\right)^j - 1 - k \cdot 2^{-i} \cdot 3^{i+1}}{3^{i+1}}\right) + 1 + i \tag{3.20}$$

Now let's start isolating k:

$$m - 1 - i = \log_2\left(\frac{2^l - \sum_{j=0}^{i}\left(\frac{3}{2}\right)^j - 1 - k \cdot 2^{-i} \cdot 3^{i+1}}{3^{i+1}}\right)$$

Let's get rid of the logarithm expression:

$$2^{m-1-i} = \frac{2^l - \sum_{j=0}^{i}\left(\frac{3}{2}\right)^j - 1 - k \cdot 2^{-i} \cdot 3^{i+1}}{3^{i+1}}$$

Let's get rid of the fraction:

$$2^{m-1-i} \cdot 3^{i+1} = 2^l - \sum_{j=0}^{i}\left(\frac{3}{2}\right)^j - 1 - k \cdot 2^{-i} \cdot 3^{i+1}$$

Let's arrange:

$$2^{m-1-i} \cdot 3^{i+1} + k \cdot 2^{-i} \cdot 3^{i+1} = 2^l - \sum_{j=0}^{i}\left(\frac{3}{2}\right)^j - 1$$

Let's make the k term more specific:

$$k \cdot 2^{-i} \cdot 3^{i+1} = 2^l - \sum_{j=0}^{i}\left(\frac{3}{2}\right)^j - 1 - 2^{m-1-i} \cdot 3^{i+1}$$

Let's isolate k term.

$$k = \frac{2^l - \sum_{j=0}^{i}\left(\frac{3}{2}\right)^j - 1 - 2^{m-1-i} \cdot 3^{i+1}}{2^{-i} \cdot 3^{i+1}}$$

We can write this expression more simply:

$$k = \frac{2^l - \sum_{j=0}^{i}\left(\frac{3}{2}\right)^j - 1}{2^{-i} \cdot 3^{i+1}} - \frac{2^{m-1-i} \cdot 3^{i+1}}{2^{-i} \cdot 3^{i+1}}$$

$$k = \frac{2^l - \sum_{j=0}^{i}\left(\frac{3}{2}\right)^j - 1}{2^{-i} \cdot 3^{i+1}} - 2^{m-1-i+i}$$

In its final form, k is expressed as:

$$k = \frac{2^l - \sum_{j=0}^{i}\left(\frac{3}{2}\right)^j - 1}{2^{-i} \cdot 3^{i+1}} - 2^{m-1} \tag{3.21}$$

We know that this equation is based on an assumption and that not all the values we are interested in will satisfy this assumption. We know that for values that do not satisfy the relevant assumption,

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there will only be values that can be divided by two more than one consecutive time. Therefore, we can update the operation to include all the values we want as follows:

$$\left(\frac{3^{i+1}\cdot 2^{m-1}}{2^{i+q}}+\frac{3^{i+1}k}{2^{i+q}}+\sum_{j=0}^{i}\frac{3^j}{2^{j+q}}\right)+1=2^l \tag{3.22}$$

Let's isolate m and then k again, respectively. Let's gather the terms in parentheses into a common denominator:

$$\frac{3^{i+1}\cdot 2^{m-1}}{2^{i+q}}+\frac{3^{i+1}k}{2^{i+q}}=\frac{3^{i+1}\cdot 2^{m-1}+3^{i+1}k}{2^{i+q}}$$

Let's simplify the equation:

$$\frac{3^{i+1}\cdot(2^{m-1}+k)}{2^{i+q}}+\sum_{j=0}^{i}\frac{3^j}{2^{j+q}}+1=2^l$$

Let's arrange:

$$\frac{3^{i+1}\cdot(2^{m-1}+k)}{2^{i+q}}=2^l-1-\sum_{j=0}^{i}\frac{3^j}{2^{j+q}}$$

Let's get rid of the fraction:

$$3^{i+1}\cdot(2^{m-1}+k)=\left(2^l-1-\sum_{j=0}^{i}\frac{3^j}{2^{j+q}}\right)\cdot 2^{i+q}$$

Let's arrange again:

$$2^{m-1}+k=\frac{\left(2^l-1-\sum_{j=0}^{i}\frac{3^j}{2^{j+q}}\right)\cdot 2^{i+q}}{3^{i+1}}$$

$$2^{m-1}=\frac{\left(2^l-1-\sum_{j=0}^{i}\frac{3^j}{2^{j+q}}\right)\cdot 2^{i+q}}{3^{i+1}}-k$$

$$m-1=\log_2\left(\frac{\left(2^l-1-\sum_{j=0}^{i}\frac{3^j}{2^{j+q}}\right)\cdot 2^{i+q}}{3^{i+1}}-k\right)$$

Thus:

$$m=\log_2\left(\frac{\left(2^l-1-\sum_{j=0}^{i}\frac{3^j}{2^{j+q}}\right)\cdot 2^{i+q}}{3^{i+1}}-k\right)+1 \tag{3.23}$$

Now let's isolate k:

$$m-1=\log_2\left(\frac{\left(2^l-1-\sum_{j=0}^{i}\frac{3^j}{2^{j+q}}\right)\cdot 2^{i+q}}{3^{i+1}}-k\right)$$

$$2^{m-1}=\frac{\left(2^l-1-\sum_{j=0}^{i}\frac{3^j}{2^{j+q}}\right)\cdot 2^{i+q}}{3^{i+1}}-k$$

Thence:

$$k=\frac{\left(2^l-1-\sum_{j=0}^{i}\frac{3^j}{2^{j+q}}\right)\cdot 2^{i+q}}{3^{i+1}}-2^{m-1} \tag{3.24}$$

If the other variables satisfy the equation under the specified conditions for all positive natural number values of m and k, we will understand that all even numbers that are not powers of two must be equal to 1 after the relevant operations.

This is why we left the m and k values alone in the relevant equation.

Now, using the structures, understandings and equations built throughout the article so far, we will try to put forward a formula that calculates how many times any number will be equal to 1 after going through the relevant operations:

It is clear that a number of 2^m will equal 1 after m of operations.

It is clear that the numbers $\frac{2(1+2^1+\cdots+2^n)+2-1}{3}+[4]$ will be equal to 1 as a result of $m+1$ operations.

We will use the last equation to calculate how many operations will result in even numbers that are not powers of two equal to 1:

$$\Psi = 2i + 2 + q + l \tag{3.25}$$

Here Ψ gives the number of times that even numbers that are not powers of two will be equal to 1 after going through the relevant operations.

Because the number we are considering is multiplied by 3 and then 1 is added as much as the i+1 expression. It is divided by two as much as i+1. Additionally, it is divided by two again as much as q. The number that comes into the form of 2^l is divided by two as much as l to be equal to 1. Therefore, this expression emerges to number of transactions for even numbers that are not powers of two.

4. Results

In the process of developing a method that is thought to provide insight into Beal's conjecture and Collatz conjecture, methods have been developed to determine the powers of prime numbers using the composite numbers formula. Another method developed for Beal's conjecture in another article has been proven invalid. The method that proved invalid was revised and proven for the Collatz conjecture. Then, values that would be directly equal to 1 were expressed within the framework of the Collatz conjecture. Then, an observation was made to create a limit for the values that should be examined for the Collatz conjecture. Another conjecture emerged as a result of this observation. While working on the new conjecture that emerged, a pyramid structure was created. With the study of the pyramid structure, another conjecture emerged. By proving both conjectures, a clear limit was placed on the values that should be examined. Using this limit, the recursive structure of the Collatz function could be presented compactly in a single equation. It was mentioned why this equation is an important milestone on the way to verifying the Collatz conjecture. Finally, using the relevant equation, a structure was created that gives how many times any number will reach the value 1 by repeating the Collatz function. In future studies, a clear conclusion about Collatz conjecture can be reached by examining the mentioned equation in depth, and Beal's conjecture can be examined using the developed methodology.

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