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Posted Date: 15 July 2024

doi: 10.20944/preprints202407.1130.v1

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Article

Formation of Vibration Fields for a Mechatronic Platform Driven by Dual Asynchronous Motors

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Abstract: This paper investigates the formation of vibration fields in a mechatronic setup driven by dual induction motors, relying on the controlled synchronization of unbalanced rotors. The proposed algorithm enables precise control over rotor speeds and phase shifts. Experimental results from a multi-resonance vibration laboratory setup demonstrate the approach's ability to form the vibration fields. The ability to control these fields is crucial for applications such as vibratory transportation and the mixing of bulk materials. The experimental data highlights the practical advantages and potential improvements in efficiency and reliability offered by this method.

Keywords: vibration machines; phase shift; vibration field; synchronization

1. Introduction

Vibration machines and technologies play a crucial role across various industries, including manufacturing and agriculture. It's challenging to envision conducting tasks such as material separation, vibratory transportation, grinding, rolling, mixing, compaction, pile driving, and more without the aid of vibration machines and associated devices [1]. Their utilization not only yields significant technical and economic benefits but also contributes to enhancing working conditions.

Among the primary sources for inducing oscillations in vibration machines, unbalanced actuators stand out as the most commonly employed [1–5]. These actuators find application in various technologies such as vibratory conveyors, lifters, screens, and rammers, where circular, elliptical, or directional vibrations are necessary. Circular vibrations are typically produced by an unbalanced rotor, or two unbalanced rotors, revolving in phase in the same direction, or anti-phase – in the opposite directions. When there exists a phase discrepancy between the driving forces of two-shaft vibration actuators, elliptical oscillations manifest themselves.

In the advancement of vibratory machines for tasks like screening, crushing, and bulk material transportation, enhancing productivity often hinges on maintaining a stable synchronous rotation mode of unbalanced rotors (vibration actuators). The foundational theories of vibration machine synchronization were established in the 1960s by I.I. Blekhman [3,6–10]. These theories laid the groundwork for synchronization mechanisms in traditional rigid transmissions, or flexible transmissions, gradually giving way to vibration synchronization equipment powered by two or more actuators. Such systems enable the vibrating system to produce various trajectories through a synchronization phenomenon, see [5]. When developing a control system for a vibratory machine (VM), all these considerations come into play, see [4,11–16]. The versatility of multiple synchronization modes is rooted in the phenomenon of self-synchronization, wherein vibration actuators rotate at average angular velocities that are multiples of each other. This mode facilitates more efficient vibration displacement, especially in challenging tasks such as transporting dusty, sticky, or wet goods.

The investigation into the flow field variation in twin-roll strip casting processes induced by oscillating rollers has been addressed in [17]. This study emphasizes achieving a homogeneous distribution of strip impurities, reducing grain size, and thereby enhancing strip quality through the adoption of vibrating casting technology proposed therein.

Electro-hydraulic vibration equipment finds extensive application in various vibration environment simulation tests. Liu *et al.* [18] addresses challenges pertaining to bandwidth, waveform distortion control, stability, offset control, and complex waveform generation in high-frequency vibration conditions.

In a different vein, Shi *et al.* [19] concentrated on the vibration synchronization of mechanical structures propelled by two vibrators. It introduces a synchronization mechanism combining space and plane patterns driven by two vibrators through the extension of vibrator installation to arbitrary directions in spatial axes. Gouskov *et al.* [20], Panovko *et al.* [21] examined variations in the arrangement of technological loads on the machine's working body. It elucidates how shifts in load center from the machine's constructive axis of symmetry lead to changes in resonant frequencies and mutual phasing of unbalances, discussing the feasibility of controlling resonant vibration machines by adjusting vibro-actuator power frequencies to compensate for uncontrolled load shifts.

In the realm of dual-mass vibrating systems, Liu *et al.* [22] investigated vibratory synchronization phenomena driven by two actuators. It employs averaging methods to deduce synchronization and stability criteria in synchronous states, quantitatively analyzing parameters such as actuator rotational speeds, phase differences, and responses in sub-resonant and super-resonant states, accompanied by an engineering example. Using asymptotic and average methods, Zhang *et al.* [23], derived theoretical conditions for implementing multiple synchronization and stability, with particular focus on analyzing synchronization for four actuators. Leniowska and Sierżę [24] studied the controlled vibration damping of a round plate utilizing a controller with phase shift adjustment in the feedback loop. A novel controller is proposed, combining regulator structures with positive position feedback and strain rate feedback. Experimental studies involve measuring plate vibrations using a laser vibrometer, with a control signal applied to the plate via a Macro Fiber Composite (MFC) disk affixed to its center. The findings demonstrate the feasibility of employing this solution to mitigate plate vibration effectively.

The phenomenon of transporting solid and granular bodies over oscillating rough surfaces has found numerous technical applications. Vibrational movement arises from an average directed motion of material particles relative to horizontally oscillating uniformly rough surfaces, stemming from the asymmetry of surface oscillation shapes, as expressed in the inequality of time intervals between consecutive extrema of surface acceleration oscillations [25–27]. Theoretical investigations by Blekhman *et al.* [27] focused on the oscillatory transportation of solid particles over flat surfaces under non-translational vibrations. The study reveals that the existing theory, with additional parameters introduced, can effectively determine the speed of vibrational transportation, with experimental validation conducted on a vibration platform. While previous works primarily analyze particle behavior during transportation, advancements towards intelligent technological systems involve employing mechatronics principles, particularly computer-controlled vibration setups with feedback mechanisms [27].

The introduction of feedback control in vibration field manipulation has been addressed by Tomchina [28], Fradkov *et al.* [29], initially focusing on simple synchronization cases. However, the advent of multiple synchronous modes introduces asymmetry, enhancing vibration movement efficacy by creating complex trajectories for rotors and carrier bodies. Nevertheless, the stability of multiple synchronous operation remains a challenge, necessitating further exploration into controlling vibration fields under conditions of multiple synchronization of vibroactuators.

The chaotization of platform vibrations by means of the phase-shift control was recently demonstrated in [14]. Coupling effects of support stiffness on geometric scaling factor powers of rotor-bearing systems and vibration characteristics are studied in [30].

This paper investigates the vibrational system controlled by the method proposed by Andrievsky and Boikov [31] from the perspective of creating the various vibration fields for the operational platform, driven by unbalanced rotors. Experimental validation conducted at the Multi-resonance Mechatronic Laboratory Setup confirms the approach's efficacy, demonstrating its practical applicability in controlling the type of vibration fields of the working platform, crucial for bulk material transportation processes.

The subsequent sections are structured as follows: Section 2 briefly outlines the Mechatronic Setup SV-2M utilized for experiments. Section 3 presents the bidirectional control law for multiple synchronization of the unbalanced rotors. The experimental results along with the data processing algorithm are presented in Section 4. Concluding remarks and future work intentions are outlined in Section 5.

2. Description of Experimental Mechatronic Setup

The Multiresonance Mechatronic Laboratory Setup (MMLS) SV-2M of the IPME RAS, used in this work for experimental investigations has broad research capabilities, see [14,29,32–34] for details.

A vibration complex SV-2M is a nonlinear electromechanical system equipped by two induction motors (IM) with unbalanced rotors [32,33]. The self-synchronization mode is not always sufficiently stable due to random variations in motor and system parameters, and structural oscillations. Instability in the self-synchronization mode can lead to significant deviations in the phase difference between the rotors from the desired values. The desired rotations phasing can also become unstable. This raises the challenge of controlling the synchronization of the electric motors, which requires measuring the rotor rotation angles and acting on the relative phase shift.

Optical motion sensors *DFRobot Smart Grayscale Sensors* are installed for obtaining information about the linear and angular coordinates of 6 degrees of freedom platform. The location of linear displacement sensors on the main platform is shown schematically in Figure 1. Sensors S_1 – S_3 measure the position of the main platform in the vertical plane, and sensors S_4 – S_6 measure its position in the horizontal plane; sensors S_4 and S_5 measure displacement along the X axis, sensor S_6 measures displacement along the Y axis. In Figure 1, the numbers #1 ... #4 indicate the points of interest for the following study of the platform vibrations. Optical motion sensors *DFRobot Smart Grayscale Sensors* are installed for obtaining information about the linear and angular coordinates of 6 degrees of freedom platform. The location of linear displacement sensors on the main platform is shown schematically in Figure 1. Sensors S_1 – S_3 measure the position of the main platform in the vertical plane, and sensors S_4 – S_6 measure its position in the horizontal plane; sensors S_4 and S_5 measure displacement along the X axis, sensor S_6 measures displacement along the Y axis. In Figure 1, numbers #1 ... #4 indicate the points of interest for the succeeding study of the platform vibrations.

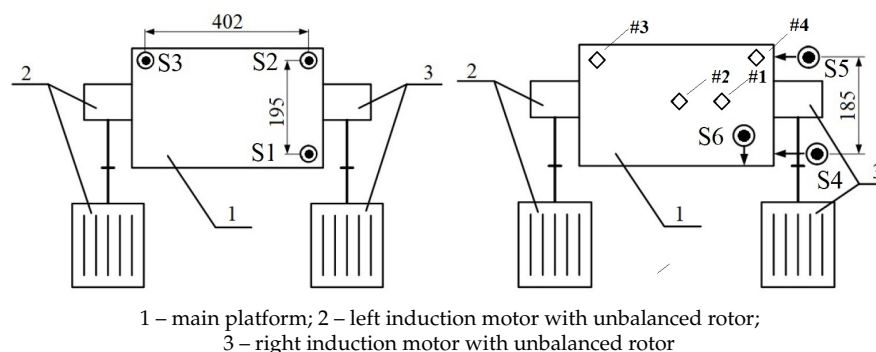


Figure 1. Schematic diagram of the setup (top view) and location of sensors S_1 – S_6 on the main platform

3. Bidirectional Control Law for Multiple Synchronization of Unbalanced Rotors

In [29,31,34] the problem of controlled synchronization for unbalanced rotors phase shift was considered. The control law peoposed in [31] for multiple synchronization of unbalanced rotors has the following form:

$$\omega_r^* = \kappa^{-1} \omega_l^*, \quad (1)$$

$$e_{\omega_l} = \omega_l^* - \omega_l, \quad e_{\omega_r} = \omega_r^* - \omega_r, \quad (2)$$

$$\dot{\sigma}_{\omega_l} = e_{\omega_l}, \quad u_{\omega_l} = K_{i\omega_l} \sigma_{\omega_l} + K_{p\omega_l} e_{\omega_l}, \quad (3)$$

$$\dot{\sigma}_{\omega_r} = e_{\omega_r}, \quad u_{\omega_r} = K_{i\omega_r} \sigma_{\omega_r} + K_{p\omega_r} e_{\omega_r}, \quad (4)$$

$$\psi = \kappa \varphi_r - \varphi_l, \quad (5)$$

$$e_\psi = \psi^* - \psi, \quad (6)$$

$$\dot{\sigma}_\psi = e_\psi, \quad u_\psi = -K_{i,\psi} \sigma_\psi + K_{p,\psi} e_\psi, \quad (7)$$

$$u_l = u_{\omega_l} + u_\psi, \quad u_r = u_{\omega_r} - u_\psi, \quad (8)$$

where ω_r^*, ω_l^* are the reference motors' velocities, κ is the normalization coefficient; $e_{\omega_l}, e_{\omega_r}$ are the motor velocity errors; PI-controllers for velocities of the left and right motors are described by (3), (4), respectively, where $K_{i\omega_l}, K_{i\omega_r}$ are for the integral, and $K_{p\omega_l}, K_{p\omega_r}$ are for the proportional controller gains; in (5), φ_r and φ_l are the phase angles of the rotors, ψ denotes the normalized phase shift between the rotors; the phase shift error is denoted by e_ψ , where ψ^* is the prescribed phase shift between the rotors; u_l and u_r in (8) denote the control signals applied to the left and right drive systems produced by the corresponding PI-controllers.

4. Experimental Results

The experimental study of the multiple synchronization by means of the control law (1)–(8) has been performed on the MMLS SV-2M and the results for various frequencies ω_l^* and of normalization coefficient κ and demanded normalized phase shift ψ^* are presented in [31].

In the present paper these results are extended to studying the corresponding vibration platform deflections and summarized in the form of the spectral dencetients of selected points of platform deflections and the “vibration fields” of the platform.

The main aim of this study is to demonstrate the vibration fields by the example of moving the platform sections #1–#4 (see Figure 1).

4.1. Data Processing Algorithm and its Software Implementation

The following processing algorithm is implemented to obtain the type of vibration fields based on experimental data.

Platform position sensor data are acquired at a selected sampling interval T_s and saved to a file for further offline processing. Given the range of platform motion frequencies, an interval of $T_s = 0.002$ s seconds was chosen, corresponding to a Nyquist frequency $\omega_N \approx 3100$ rad/s. At the end of the experiment, the data array was saved to the hard disk drive (HDD) for subsequent processing.

Data acquisition from the sensors is performed using Simulink Desktop Real-Time toolbox, which includes Simulink I/O driver blocks enabling closed-loop control of physical systems from a desktop computer, allowing connections to sensors, actuators, and other devices. Data Type Conversion blocks are used to convert the measurements to MATLAB *double* type data for further processing. Simulink blocks “To Workspace” are used to save the data to the workspace. After the experiment, using the MATLAB save routine, the data obtained are saved on the HDD in the form of a MATLAB mat-file.

Since the measurement characteristics of optical sensors $S_1, \dots, S_6$, given by $L_i = f_i(l_i)$, $i = 1, \dots, 6$, are close to the inversely proportional mappings presented in Figure 2 (cf. [34]), the displacements $L_i(t_k)$, at times $t_k = kT_s$, $k = 1, 2, \dots$ of the corresponding platform points $S_1, \dots, S_6$ are restored based on the sensor readings. Before the experiments, the position optical sensors had been calibrated

to obtain their real measurement characteristics and the results were tabulated in 20 points inside the working range. Sensor measurements sometimes include random spikes, which are erroneous readings outside the normal range. Therefore, these anomalies were removed before further processing. For this purpose, each sensor's entire array of measurements was subjected to statistical analysis using the MATLAB procedure *histcounts* with $M = 1000$ bins. Next, values that occur in the measurement array less than the specified threshold value $n_{\min}$ are selected ($n_{\min} = 10$ was set) and are replaced by the arithmetic mean of the measurement array. To recover displacement values $L_i(t_k)$ from sensor signals $l_i(t_k)$, a standard MATLAB procedure *interp1* in the option 'spline' was used on the every data processing step $k = 0, 1, \dots$

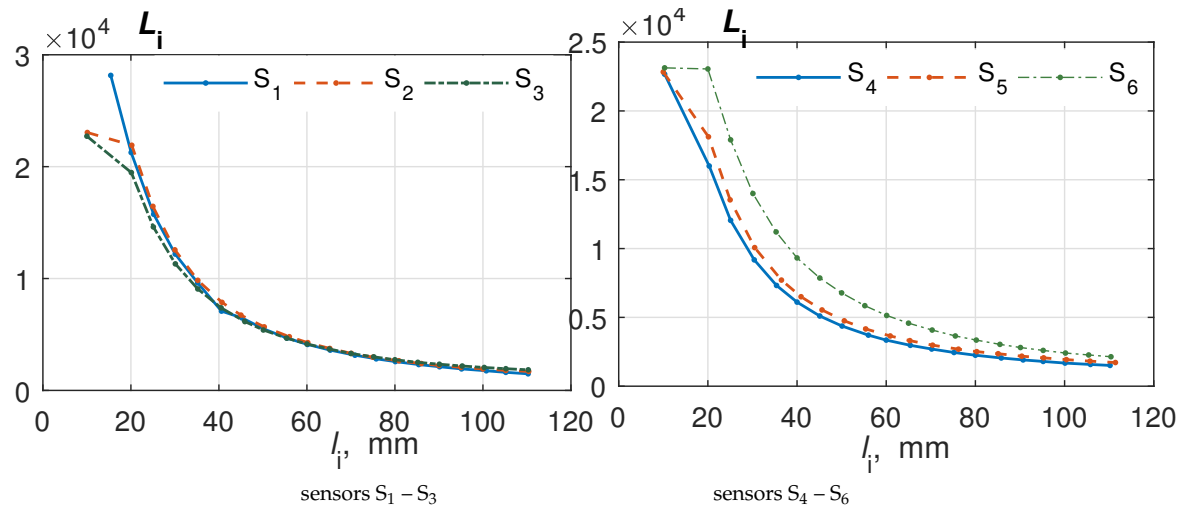


Figure 2. Measuring characteristics of sensors $S_1 - S_6$ (as in [34])

For the obtained data arrays, spectrograms of the displacement processes of the sensor positions are constructed and displayed to the user. To do this, using the *mean* function, their time-average values are found, and the Discrete Fourier Transform (DFT), implemented by the *fft* MATLAB procedure, is applied to the resulting centered processes $\tilde{L}_i(k)$. The resulting complex-valued array of images $\mathcal{Y}$ is converted into the array of spectral densities $S_i(\omega)$ by calculating the squared $\mathcal{Y}$ modulus, normalized by the number of points. Sensor measurements were cleaned using the DFT concerning $\tilde{L}_i(k)$. The resulting arrays $\mathcal{Y}_i(\omega)$ were processed as follows. Based on the obtained spectral densities $S_i(\omega)$, frequencies $\omega_j, j = 1, 2, 3$, were selected where $S_i(\omega)$ bursts were observed. Next, the set of frequencies Ω in the vicinities of ω_j were found as: $\Omega = \bigcup_{j=1}^3 (|\omega - \omega_j| \leq \Delta\omega)$, where $\Delta\omega > 0$ is a certain bandwidth

relative to the peak frequencies ω_j . New complex-valued arrays $\tilde{\mathcal{Y}}_i(\omega)$ were formed by zeroing the values of $\mathcal{Y}_i(\omega)$ except those that correspond to the frequencies from Ω , which values are copied from $\mathcal{Y}_i(\omega)$, and their spectral densities $\tilde{S}_i(\omega)$ were found. The Inverse Discrete Fourier Transform (IDFT), implemented by MATLAB routine *ifft* is applied to $\tilde{\mathcal{Y}}_i(\omega)$ and, finally, the filtered centered platform displacements $\tilde{D}_i(k)$ were calculated as $\tilde{L}_i(k) = \text{real}(\mathcal{L}_i(k)) \|S_i(\omega)\| / \|\tilde{S}_i(\omega)\|$, where $\mathcal{L}_i(k)$ denote complex-valued inverse Fourier transforms to $\tilde{\mathcal{Y}}_i(\omega)$, and $\text{real}(\cdot)$ denotes the real part of $\mathcal{L}_i(k)$ (standard MATLAB routine *real*); $\|\cdot\|$ stands for the Euclidean norm of the complex-valued vector argument (routine *norm*).

As an example of the application of the described procedure, consider the case of filtering data of $L_2(t), L_5(t)$, obtained for $\omega_l = 80$ rad/s, $\omega_r = 40$ rad/s, $\psi = \pi/2$, taking $\omega_j \in \{20.5, 40, 80\}$ rad/s, $\Delta\omega = 5$ rad/s. The resulting time histories for $L_i(t_k), i \in \{2, 5\}$ along with $\tilde{L}_i(t_k)$ are depicted in Figure 3.

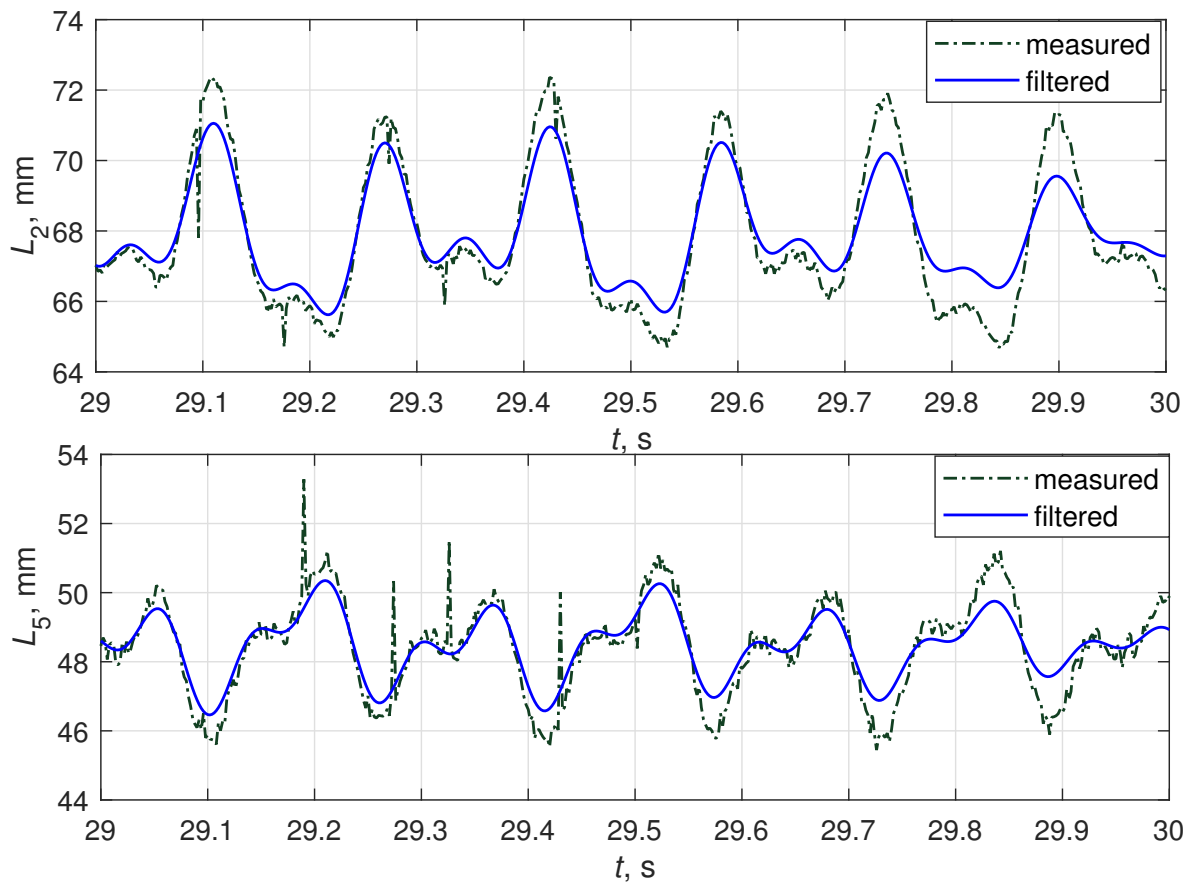


Figure 3. Time histories of measured (dash-dot line) and filtered (solid line) platform positions $L_2(t)$, $L_5(t)$, $t \in [29, 30]$ s, for the case of $\omega_l = 80$ rad/s, $\omega_r = 40$ rad/s, $\psi = \pi/2$; $\omega_j \in \{20.5, 40, 80\}$ rad/s, $\Delta\omega = 5$ rad/s.

After $\tilde{L}_i(t_k)$ are found, the platform points of interest #1–#4 (see Figure 1) displacements Δx_j , Δy_j , $j = 1, \dots, 4$ on the vertical plane OXY are calculated as follows:

$$\begin{aligned}\Delta x_1(t_k) &= (\tilde{L}_4(t_k) + \tilde{L}_5(t_k))/2, & \Delta y_1(t_k) &= (\tilde{L}_1(t_k) + \tilde{L}_2(t_k) + \tilde{L}_3(t_k))/3, \\ \Delta x_2(t_k) &= (\tilde{L}_4(t_k) + \tilde{L}_5(t_k))/2, & \Delta y_2(t_k) &= (\tilde{L}_2(t_k) + \tilde{L}_3(t_k))/2, \\ \Delta x_3(t_k) &= \tilde{L}_5(t_k), & \Delta y_3(t_k) &= \tilde{L}_3(t_k), \\ \Delta x_4(t_k) &= \tilde{L}_5(t_k), & \Delta y_4(t_k) &= \tilde{L}_4(t_k).\end{aligned}$$

The experimental results for various revolving speeds of rotors and phase shifts are presented below. For legibility of the graphs, the trajectories are displayed only in the final section of the experiment $t \in [t_a, t_b]$ with a duration of $t_b - t_a$ about two characteristic periods of oscillation.

4.2. Case of Identical Rotation Velocities

At first, consider the case when both rotors revolve with the same desired frequency. Namely, let $\omega_l^* = \omega_r^* = 80$ rad/s be taken (i.e. in (1) – (8) parameter $\kappa = 1$ be set). The desired phase shift between the rotors ψ^* is set to zero. The spectral densities for displacements of sensors # 2, # 3 are pictured in Figure 4. As is seen from the plot, the “outstanding” peak on frequency close to 20.5 rad/s appears. It corresponds to the low-frequency resonant mode of the platform.

The motion of the various points of the platform on the vertical plane OXY (where OX stands for the horizon axis, OY denotes the vertical one), are plotted in Figure 5. The time interval for plotting the curves is taken as $t \in 27 + [0.1726, 0.6438]$ s. This interval corresponds 6 periods of motion with a frequency 80 rad/s.

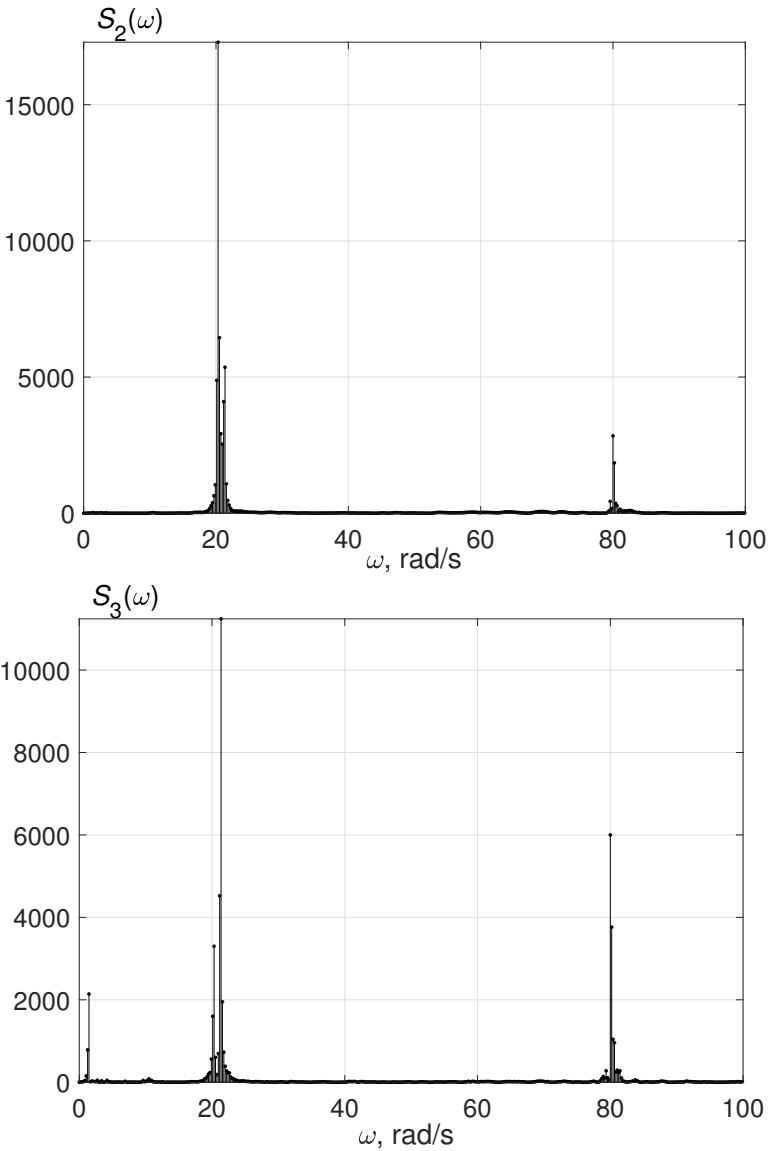


Figure 4. Spectral densities for sensors # 2, 3 displacements (in mm). $\omega_l = \omega_r = 80$ rad/s, $\psi = 0$.

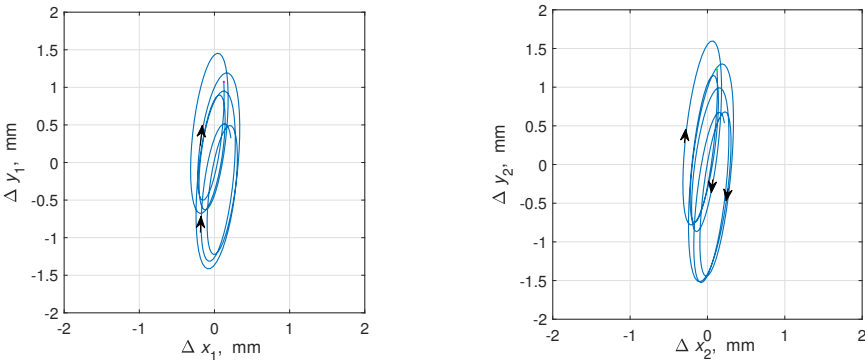


Figure 5. Cont.

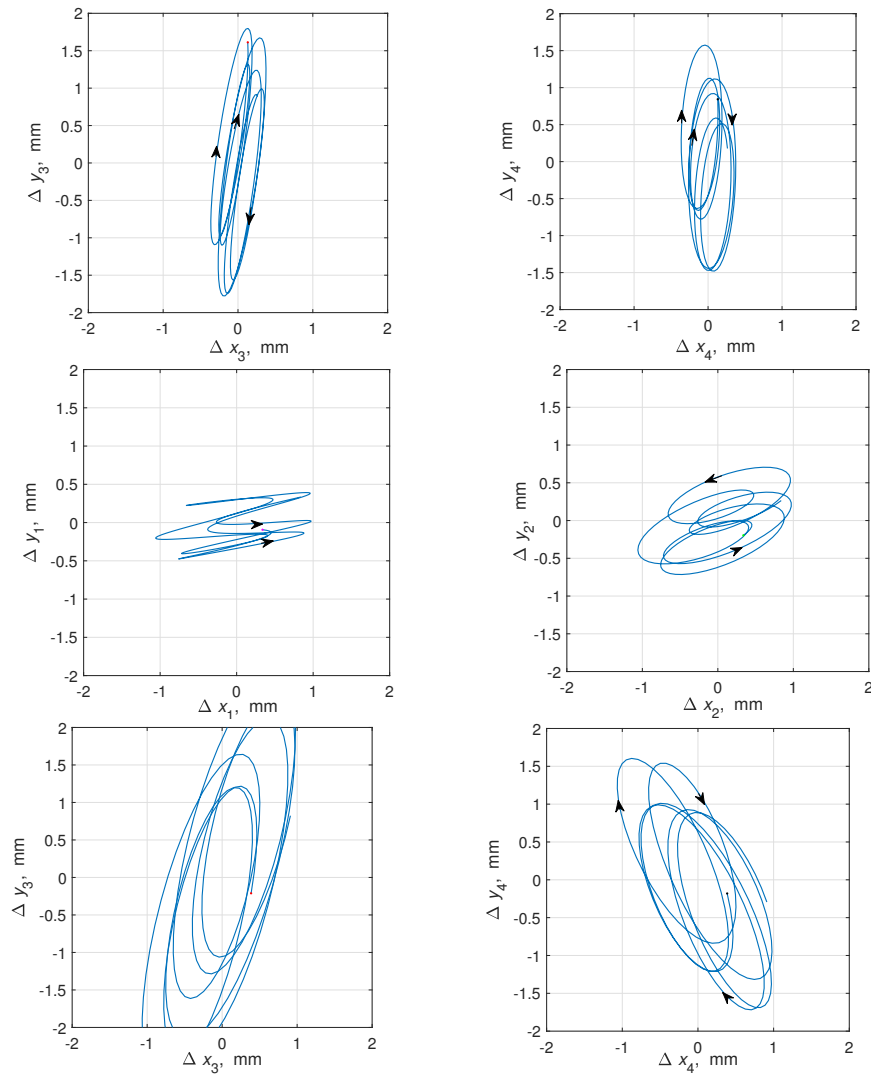


Figure 5. Vibration fields in the platform sections #1–#4. $\omega_l = \omega_r = 80$ rad/s. Upper plot: $\psi = 0$. Lower plot: $\psi = \pi$ rad.

The upper plots series correspond zero phase shift ($\psi^* = 0$) between the rotors. Respectively, the lower plot is related to phase shift $\psi^* = \pi$. It is seen that the vibration field, having, in general, a similar shape that in the case of $\psi^* = 0$, changes its inclination. This property can be used for control of the vibrational transportation speed and direction.

4.3. Case of Different Rotation Velocities

Secondly, the similar experiments were performed for the case of different rotation frequencies of rotors. Namely, $\omega_l^* = 90$ rad/s and $\omega_r^* = 45$ rad/s were taken (i.e. in (1)–(8) parameter $\kappa = 2$ was set). The desired phase shift between the rotors ψ^* is set to zero. The corresponding spectral densities for displacements of sensors # 2, # 3 are pictured in Figure 6. As is seen from the plot, the spectrograms have three peaks: on the frequencies of 45 rad/s, 90 rad/s, corresponding the rotation velocities of the drives, and the peak on 20 rad/s appears related to the natural oscillating of the platform.

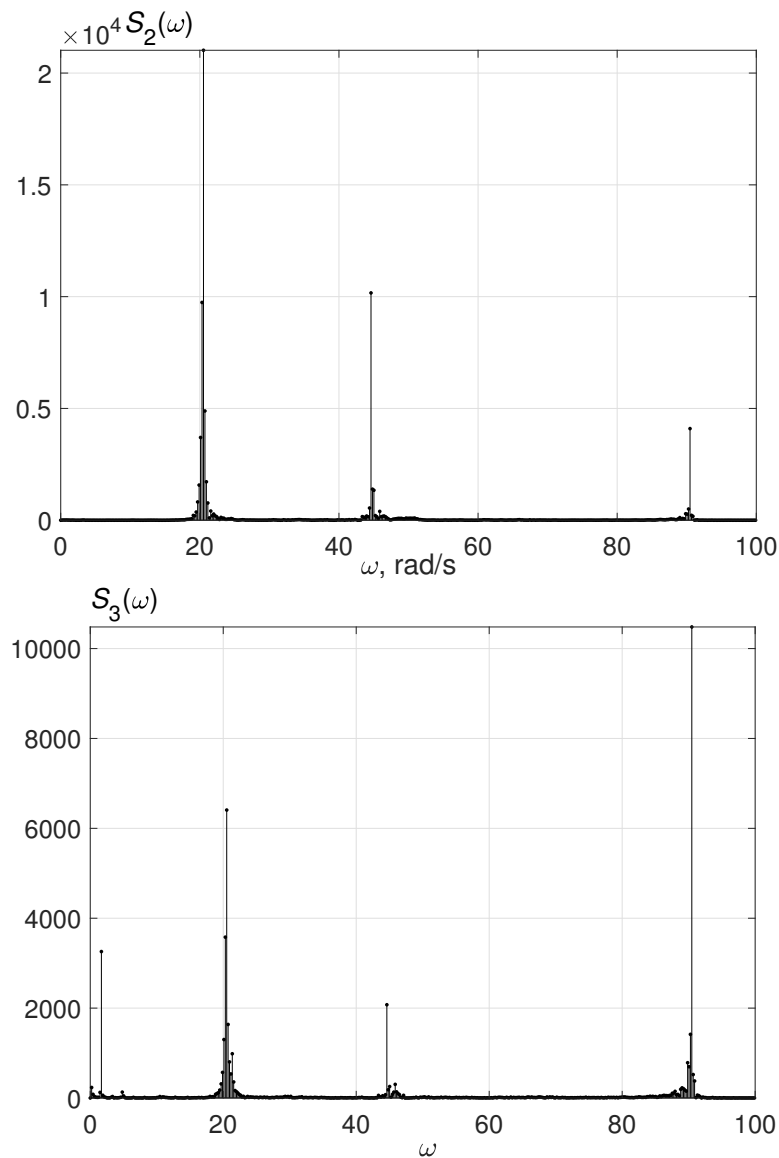


Figure 6. Spectral densities for sensors # 2, 3 displacements (in mm). $\omega_l = 90$ rad/s, $\omega_r = 45$ rad/s, $\psi = 0$.

The corresponding vibration fields of the platform sections #1–#4 on the vertical plane OXY are plotted in Figure 7. The time interval for plotting the curves is taken as $t \in 27 + [0.4867, 0.9056]$ s. This interval corresponds 6 periods of motion with a frequency 90 rad/s. The platform motion is for the considered multiple case in more complex compared with the single-frequency one due to interference both from the slow oscillations with the natural frequency of the platform (20 rad/s) and two rotation speeds of the drives with the different values (90 and 45 rad/s).

The upper plots in Figure 7 correspond zero phase shift ($\psi^* = 0$) between the rotors, while the lower plot is related to phase shift $\psi^* = \pi$. It is seen that the vibration field is more complex than in the single-frequency case and also is varying with the phase shift, set between the rotors. This gives additional flexibility in control of the platform vibration fields and can be used in developing smart vibrational technologies.

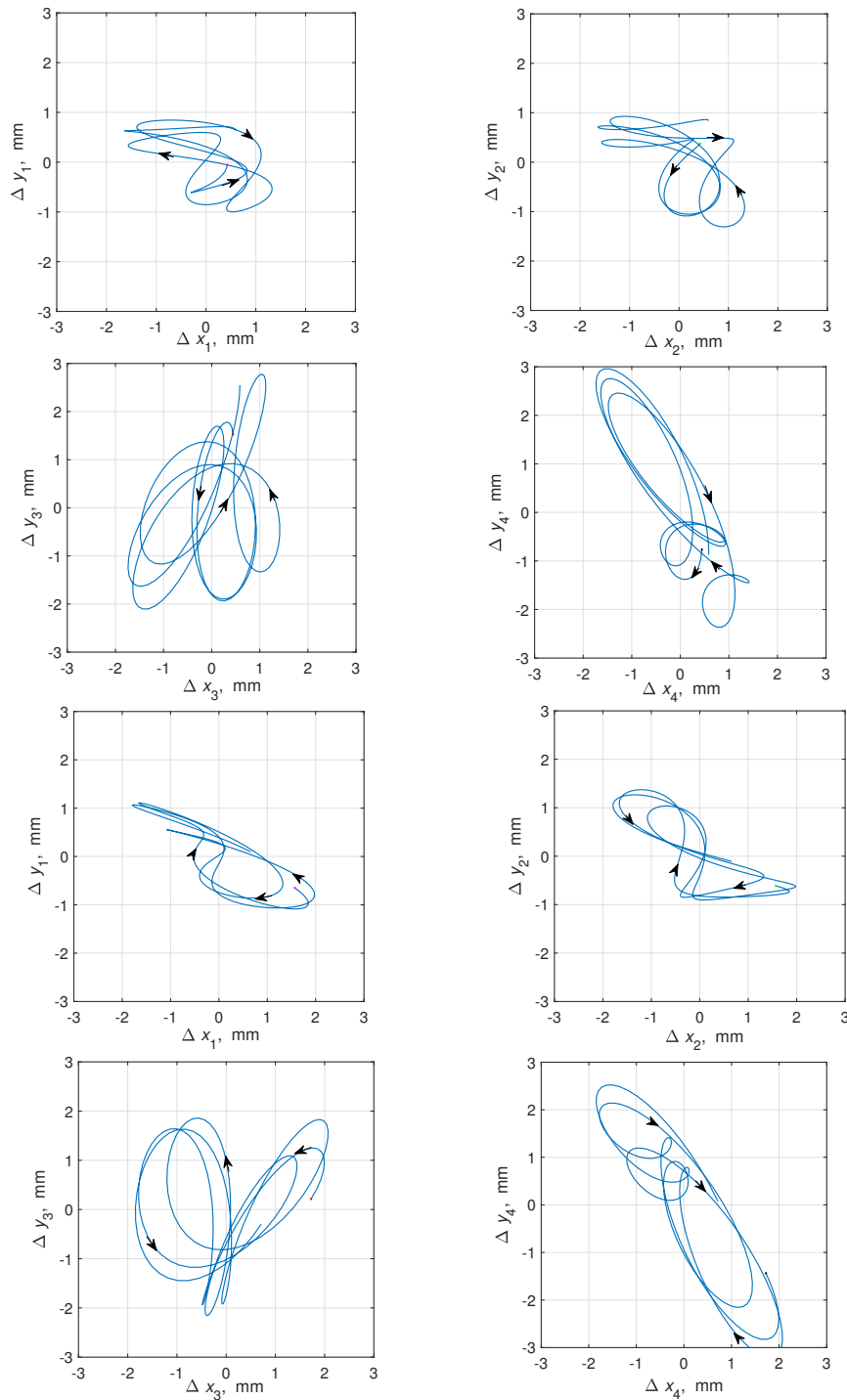


Figure 7. Vibration fields in the platform sections #1–#4. $\omega_l = 90$ rad/s, $\omega_r = 45$ rad/s. Upper plot: $\psi = 0$. Lower plot: $\psi = \pi$ rad.

5. Conclusions

The paper demonstrates the possibility of using controlled multiple synchronization of unbalanced rotors driven by induction motors to form vibration fields in the working platform of a vibration mechatronic setup. The experimental results obtained at the MMLS SV-2M of the IPME RAS are presented, demonstrating the efficacy of the proposed approach.

Expanding the capabilities of controlled synchronization can be gained by employing unidirectional (master-slave synchronization) and alternative control methods. These methods include

adaptive control, variable-structure sliding-mode control, speed gradient techniques, Neural Networks, and nonlinear correction methods, which represent promising areas for further research.

Author Contributions: Conceptualization, O.T.; data curation, I.Z.; formal analysis, A.F. and B.A.; funding acquisition, A.F.; investigation, I.Z.; methodology, B.A.; project administration, A.F.; software, I.Z.; supervision, B.A.; writing—original draft, A.F. and B.A.; writing—review and editing, B.A. and O.T. All authors have read and agreed to the published version of the manuscript.

Funding: The research was supported by the Ministry of Science and Higher Education of the Russian Federation (project no. 124041500008-1).

Data Availability Statement: Not applicable.

Acknowledgments: The Authors are grateful to V. I. Boikov for his invaluable work in creating the electronic and computer facilities of the MMLS SV-2M.

Conflicts of Interest: The authors declare no conflict of interest.

Abbreviations

The following abbreviations are used in this manuscript:

AC	Alternating Current
DFT	Discrete Fourier Transform
DoF	Degrees of Freedom
IDFT	Inverse Discrete Fourier Transform
IM	Induction Motor
HDD	Hard Disk Drive
MFC	Macro Fiber Composite
MMLS	Multiresonance Mechatronic Laboratory Setup
VM	Vibratory Machine

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