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Posted Date: 12 July 2024

doi: 10.20944/preprints202407.1008.v1

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Evaluating Order Allocation Sustainability Using a Novel Z Number ITARA-CoCoSo Approach

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Abstract: The United Nations' sustainable development goals have underscored the importance of enhancing supply chain sustainability while appropriately distributing orders. This study proposes a novel framework involving Z-number, game theory, indifference threshold-based attribute ratio analysis (ITARA), and combined compromise solution method (CoCoSo) to evaluate the sustainability of suppliers and order allocations. To better reflect the decision makers' current choices for the sustainability of assessed suppliers and order allocations; enhance the comprehensiveness of decision-making, the importance parameter of the supplier is obtained through game theory objectively for transforming supplier performance into order allocation performance. The Z-numbers are involved in ITARA (so-called ZITARA) and CoCoSo (so-called ZCoCoSo) to overcome the issue of information uncertainty in the process of expert evaluation. ZITARA and ZCoCoSo are applied to assess the objective weight of criteria and the ranking of assessed order allocations, respectively. Afterwards, a case study of China's company is illustrated the utility of the proposed framework and inform their decision-making process regarding the orders to be assigned to the assessed suppliers.

Keywords: sustainability; order allocation; Z-number; indifference threshold-based attribute ratio analysis (IATARA); combined compromise solution method (CoCoSo)

1. Introduction

In the face of mounting pressures from resource consumption, environmental degradation, and raw material scarcity, driven by rapid industrialization, population growth, and intensified business competition [1,2], companies and organizations are feeling the need to adopt more sustainable and environmentally friendly practices. Manufacturing industries must implement targeted measures to minimize the environmental impact of their products [3,4]. Consequently, organizations are progressively integrating eco-friendly practices into their manufacturing operations, with the dual objective of improving their economic and environmental performance while also meeting customer demands and assigning the orders [5]. Therefore, the main objective of this study is to provide decision-makers with evaluation framework that allow them to select sustainable supplier and order allocation in comprehensive analysis.

In regard to the subject of supplier sustainability, two distinct concepts have been proposed and are regarded as important indicators for evaluating whether a company is sustainable: Environmental, Social and Governance (ESG) and triple-bottom-line [6]. In addition to the aforementioned concepts, the following areas are also relevant: environmental protection and greenhouse gas emissions reduction, energy efficiency, human rights, workplace health and safety,

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expectations between different stakeholders, contribution to society, and so on. Furthermore, in the context of ESGs, supply chain management is a crucial aspect, necessitating that when a company selects suppliers, it must also adhere to the principles of sustainability.

In recent years, prior research has employed a range of methodologies to address the issue of sustainable supplier selection and order allocation. These methodologies generally fall into three categories: multi-criteria decision-making (MCDM) methods, mathematical optimization techniques, and artificial intelligent methods. MCDM techniques are the most popular among all categories, with the weighted aggregated sum product assessment [7], the best-worst method [8], and the analytical hierarchy process [9] being the most prevalent. In contrast, mathematical optimization techniques encompass multi-objective programming [10], mixed-integer programming [11], goal programming [12], and other related methodologies. In the third category, the primary techniques are neural networks [13], support vector regression [14], and expert systems [15]. However, the majority of the literature has employed a crisp value to assess both qualitative and quantitative criteria in addressing sustainable supplier selection and order allocation problems simultaneously. In addressing the uncertainty inherent in the expert evaluation process, the most commonly employed approaches are those based on grey theory [16] and fuzzy sets, including the triangular fuzzy set [17] and the trapezoidal fuzzy set [18]. However, these methods are limited in that they address only the uncertainty inherent in the expert evaluation. In this study, the Z-number [19] is employed to address both qualitative and quantitative criteria, taking into account both the uncertainty inherent in expert evaluations and the confidence with which these evaluations are made.

In the context of sustainability, suppliers play a pivotal role in the advancement of sustainable development. This study evaluates the sustainability of suppliers and reflects its sustainability on order allocation, thereby conducting a sustainability evaluation of order allocation through a novel conversion process. The supplier sustainability is converted to order allocation evaluation. Then, the Z-number is incorporated into the indifference threshold-based attribute ratio analysis [20] (called ZITARA) and the combined compromise solution method (CoCoSo) method [21] (called ZCoCoSo) to resolve the uncertainty of the expert evaluation process. The weight of sustainable indicators is obtained using ZITARA; the supplier and order allocation sustainability is evaluated using ZCoCoSo.

The remainder of this study is organized as follows: The concept of trapezoidal fuzzy numbers and calculation logic of Z-number are introduced in Section 2. The sustainable order allocation evaluation framework is proposed in Section 3. In Section 4, a case of China's company is adopted to demonstrate the applicability. Finally, the conclusion is summarized in Section 5.

2. Preliminaries

In this section, the basic concepts of trapezoidal fuzzy numbers and the calculation logic of Z-number are introduced.

2.1. The Concept of Trapezoidal Fuzzy Number

In practical situations, many qualitative evaluations are inherently ambiguous and uncertain due to the presence of a significant amount of unidentified and unknown information during the decision-making process. In a context of uncertainty, ambiguity and subjective judgment exert a significant influence on the decision-making process. In order to express the uncertainty of information encountered in decision-making, Bellman and Zadeh [22] have proposed the use of fuzzy theory. Linguistic variables are frequently employed to convey information about an expert's evaluation, which represents a convenient and human-friendly approach to expressing evaluation ideas. And also an effective means of converting qualitative content into a form of quantitative data that is less precise and more ambiguous [23]. A considerable number of studies employ trapezoidal fuzzy numbers for the purpose of modeling fuzzy information, which may be either symmetric or asymmetric. In comparison to the conventional triangular fuzzy number, trapezoidal fuzzy numbers offer a more comprehensive representation of uncertainty [24].

A fuzzy set $\tilde{Q}$ on a universe discourse X can be written as a pair of (x, μ_Q), where μ_Q: $x \in [0, 1]$ is the membership function. The fuzzy number $\tilde{Q}$ can be defined as a trapezoidal fuzzy number and can be denoted as $\tilde{Q} = (q_1, q_2, q_3, q_4)$, where $q_1 < q_2 < q_3 < q_4$. The membership function $\mu_{\tilde{Q}}(x)$ is shown in Equation (1) by Huang and Lo [25]:

$$\mu_{\tilde{Q}}(x) = \begin{cases} 0 & x < q_1, \\ \frac{(x-q_1)}{(q_2-q_1)} & q_1 \le x < q_2, \\ 1 & q_2 \le x < q_3, \\ \frac{(q_4-x)}{(q_4-q_3)} & q_3 \le x \le q_4, \\ 0 & x > q_4. \end{cases} \quad (1)$$

The linguistic terms utilized in this study for the assessment of experts are referenced to the research of Pribićević, et al. [26], as illustrated in Table 1.

Table 1. Linguistic terms and trapezoidal fuzzy numbers [25].

Linguistic terms	Trapezoidal fuzzy numbers
Extremely good (EG)	(8, 9, 10, 10)
Very good (VG)	(7, 8, 9, 10)
Good (G)	(6, 7, 8, 9)
Medium good (MG)	(5, 6, 7, 8)
Fair (F)	(4, 5, 6, 7)
Medium poor (MP)	(3, 4, 5, 6)
Poor (P)	(2, 3, 4, 5)
Very poor (VP)	(1, 2, 3, 4)
Extremely poor (EP)	(0, 1, 2, 3)

2.2. The Concept and Calculation of Z-Number

Zadeh [19] put forth a variation of fuzzy numbers, designated as the Z-number, which incorporates the degree of confidence associated with the expert's judgment as a parameter in fuzzy operations. The Z-number is noted as $Z = (\tilde{Q}, \tilde{E})$. The symbols $\tilde{Q}$ and $\tilde{E}$ are respectively the trapezoidal fuzzy number for the judgment value and the triangular fuzzy number fir the measure of the confidence. They can be expressed as $\tilde{Q} = (x, \mu_{\tilde{Q}}) \mid x \in [0, 1]$ and $\tilde{E} = (x, \mu_{\tilde{E}}) \mid x \in [0, 1]$. The confidence of the Z-number $\tilde{E}$ can be transformed into a confidence weight α by the following equation:

$$\alpha = \frac{\int x\mu_{\tilde{E}}(x)dx}{\int \mu_{\tilde{E}}(x)dx}. \quad (2)$$

The weighted Z-number Z^α is further obtained by the following equation:

$$Z^\alpha = \left\{(x, \mu_{\tilde{Q}^\alpha}) \mid \mu_{\tilde{Q}^\alpha}(x) = \alpha\mu_{\tilde{E}}(x), x \in \sqrt{\alpha}x\right\}. \quad (3)$$

A simple instance to illustrate the procedure of Z-number calculation is described as follows. There is a Z-number $Z = (\tilde{Q}, \tilde{E})$ with the judgment value $\tilde{Q} = (1, 2, 3, 4)$ and the confidence $\tilde{E} = (0.3, 0.5, 0.7)$. The confidence weight α is

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$$\alpha = \frac{\int x\mu_{\tilde{E}}(x)dx}{\int \mu_{\tilde{E}}(x)dx} = \frac{\int_{0.3}^{0.5} x(x-0.3/0.5-0.3)dx + \int_{0.5}^{0.7} x(0.7-x/0.7-0.5)dx}{\int_{0.3}^{0.5} (x-0.3/0.5-0.3)dx + \int_{0.5}^{0.7} (0.7-x/0.7-0.5)dx} = 0.4998. \tag{4}$$

After that, the weighted Z-number Z^{α} is

$$\begin{aligned} Z^{\alpha} &= \{(1,\ 2,\ 3,\ 4) \mid \alpha = 0.4998\} \\ &= (\sqrt{0.4998}\times 1, \sqrt{0.4998}\times 2, \sqrt{0.4998}\times 3, \sqrt{0.4998}\times 4) \\ &= (0.707, 1.414, 2.121, 2.828) \end{aligned} \tag{5}$$

In this study, the linguistic terms used to assess the confidence in experts' judgments are based on the research of Hu and Lin [27], as shown in Table 2. According to Tables 1 and 2, the linguistics of weighted Z-number are summarized in Table 3.

Table 2. The linguistic terms of the confidence in experts' judgment [27].

Linguistic term	Triangular fuzzy number
Very high (VH)	(0.7, 1, 1)
High (H)	(0.5, 0.7, 0.9)
Medium (M)	(0.3, 0.5, 0.7)
Low (L)	(0.1, 0.3, 0.5)
Very low (VL)	(0, 0, 0.3)

Table 3. The linguistic terms of weighted Z-number.

	Confidence of judgment				
Assessment	VL	L	M	H	VH
EP	(0, 0.316, 0.632, 0.949)	(0, 0.548, 1.096, 1.644)	(0, 0.707, 1.414, 2.121)	(0, 0.837, 1.673, 2.509)	(0, 1, 2, 3)
VP	(0.316, 0.632, 0.949, 1.265)	(0.548, 1.096, 1.644, 2.192)	(0.707, 1.414, 2.121, 2.828)	(0.837, 1.673, 2.509, 3.347)	(1, 2, 3, 4)
P	(0.632, 0.949, 1.265, 1.581)	(1.096, 1.644, 2.192, 2.739)	(1.414, 2.121, 2.828, 3.535)	(1673, 2.509, 3.347, 4.183)	(2, 3, 4, 5)
MP	(0.949, 1.265, 1.581, 1.897)	(1.644, 2.192, 2.739, 3.288)	(2.121, 2.828, 3.535, 4.242)	(2.509, 3.347, 4.183, 5.019)	(3, 4, 5, 6)
F	(1.265, 1.581, 1.897, 2.214)	(2.192, 2.739, 3.288, 3.836)	(2.828, 3.535, 4.242, 4.949)	(3.347, 4.183, 5.019, 5.857)	(4, 5, 6, 7)
MG	(1.581, 1.897, 2.214, 2.529)	(2.739, 3.288, 3.836, 4.384)	(3.535, 4.242, 4.949, 5.656)	(4.183, 5.019, 5.857, 6.693)	(5, 6, 7, 8)
G	(1.897, 2.214, 2.529, 2.846)	(3.288, 3.836, 4.384, 4.932)	(4.242, 4.949, 5.656, 6.363)	(5.019, 5.857, 6.693, 7.529)	(6, 7, 8, 9)
VG	(2.214, 2.529, 2.846, 3.162)	(3.836, 4.384, 4.932, 5.479)	(4.949, 5.656, 6.363, 7.069)	(5.857, 6.693, 7.529, 8.367)	(7, 8, 9, 10)
EG	(2.529, 2.846, 3.162, 3.162)	(4.384, 4.932, 5.479, 5.479)	(5.656, 6.363, 7.069, 7.069)	(6.693, 7.529, 8.367, 8.367)	(8, 9, 10, 10)

3. Evaluating Sustainable Order Allocation with MCDM Techniques

In this section, order allocations are generated to meet the demand initially, and then Z-numbers are incorporated into two MCDM techniques: ITARA and CoCoSo (referred to as ZIARA and ZCoCoSo, respectively) to evaluate the sustainability of these order allocations.

3.1. The Order Allocation Evaluation Matrix

Given the differing quality of products from various suppliers and the increasing focus on sustainability, it is crucial to evaluate a supplier's sustainability through multiple criteria. In recent years, the trend of using supplier sustainability as a comprehensive evaluation criterion has increased, aiding in making more informed decisions when allocating orders and identifying appropriate suppliers for procurement [28]. Khemiri, et al. [29] demonstrated that the relationship between supplier evaluation and order allocation evaluation can be interpreted through pessimistic, optimistic, or moderate strategies. In this study, the game theory by Zhu, et al. [30] is involved in interpreting the relationship between supplier evaluation and order allocation evaluation objectively. The relationship is expressed in the following way:

Step 1. Constructing the supplier assessment matrix $\otimes\boldsymbol{X}$

Suppose that there are m assessed suppliers A_i, $i = 1, 2, \ldots, m$, assessed through n criteria C_j, $j = 1, 2, \ldots, n$, from which to construct the supplier assessment matrix $\otimes X$ as follows.

$$\otimes\boldsymbol{X} = [\otimes x_{ij}]_{m\times n},\ i = 1, 2, \ldots, m,\ j = 1, 2, \ldots, n, \tag{6}$$

where $\otimes x_{ij} = \left(x_{ij}^{l}, x_{ij}^{m1}, x_{ij}^{m2}, x_{ij}^{u}\right)$ is a weighted Z-number which is converted from linguistic terms, as shown in Table 3, and $\otimes y_{ij}$ represents the performance of the ith assessed supplier under the jth criterion.

Step 2. Constructing the order allocation matrix $\boldsymbol{O}$

Suppose that there are k order allocations that satisfy the demand D and the order allocation matrix $\boldsymbol{O}$ is further constructed as follows,

$$\sum_{i}^{m} o_{ki} = D,\ k = 1, 2, \ldots, v, \tag{7}$$

$$\boldsymbol{O} = [o_{ki}]_{v\times m},\ k = 1, 2, \ldots, v,\ i = 1, 2, \ldots, m. \tag{8}$$

The element o_{ki} represents the quantity of goods supplied by the ith supplier within the kth order allocation.

Step 3. Converting to the order allocation assessment matrix $\otimes\boldsymbol{Y}$

According to the elements in the matrix $\otimes\boldsymbol{X}$ from Step 1, the importance parameter γ_i of the supplier, $i = 1, 2, \ldots, m$ ($0 \le \gamma_i \le 1$ and $\sum_{i=1}^{m}\gamma_i = 1$) is determined by the game theory [30]. Then the order allocation assessment matrix $\otimes Y$ is shown as follows,

$$\otimes y_{kj} = \sum_{i=1}^{m}(\gamma_i \cdot o_{ki} \cdot \otimes x_{ij}), \tag{9}$$

$$\otimes\boldsymbol{Y} = [\otimes y_{kj}]_{v\times n},\ k = 1, 2, \ldots, v,\ j = 1, 2, \ldots, n. \tag{10}$$

The element $\otimes y_{kj} = \left(y_{kj}^{l}, y_{kj}^{m1}, y_{kj}^{m2}, y_{kj}^{u}\right)$ is also a weighted Z-number and represents the performance value of jth criterion in the kth order allocation.

To better reflect the decision makers' current choices for the sustainability of assessed suppliers and enhance the comprehensiveness of decision-making, the importance parameter α_i of the supplier is set up accordingly. In contrast to the three strategies proposed by Khemiri, et al. [29] for transforming supplier performance into order allocation performance, this study utilizes the parameter α_i to achieve this conversion and obtain an objectivity parameter setting.

3.2. ZIARA

A relatively novel approach, ITARA [20], is a method for deriving objective criterion weights that utilizes the performance data of assessed alternatives. The discriminative power of the criteria is determined by the two important parameters, namely the indifference threshold and dispersion logic. Generally, when the performance range of all evaluated alternatives for a criterion is wide, the assigned weight will also be relatively substantial. Conversely, the dispersion logic is a threshold value indicative of the degree of allowable dispersion among the assessed alternatives. ITARA has been used to address weighting issues for sustainability criteria [31], risk indicators [28], and material physical parameters [32],.

To overcome the issue of information uncertainty in the process of expert evaluation, the Z-number is incorporated into the ITARA (referred to as ZITARA) to reflect the uncertainty and ambiguity inherent in the assessment. The detailed steps of the ZITARA are shown below.

Step 1. Constructing the defuzzification assessment matrix ***F***

The supplier assessment matrix $\otimes \boldsymbol{X}$ is defuzzified by the traditional center of gravity defuzzification to defuzzify Z-numbers to form a crisp value f_{ij}, as shown in **Equation (11)**. Following the completion of the defuzzification procedure, the defuzzification assessment matrix ***F*** is obtained as follows.

$$f_{ij} = \frac{x_{ij}^{l} + 2 \cdot x_{ij}^{m1} + 2 \cdot x_{ij}^{m2} + x_{ij}^{u}}{6}, \tag{11}$$

$$\boldsymbol{F} = [f_{ij}]_{m\times n}, \quad \forall i, j. \tag{12}$$

Step 2. Determining the indifference threshold I_j

It is the responsibility of the decision-making group to determine an appropriate I_j for the criteria, $j = 1, 2, \ldots, n$.

Step 3. Generating the normalized matrix ***B***

In this step, the normalized matrix ***B*** is obtained by **Equation (13)**. Similarly, I_j is used to obtain the NI_j, as shown in **Equation (15)**.

$$b_{ij} = \frac{f_{ij}}{\sum_{i=1}^{m} f_{ij}}, \tag{13}$$

$$\boldsymbol{B} = [b_{ij}]_{m\times n}, \quad \forall i, j, \tag{14}$$

$$NI_j = \frac{I_j}{\sum_{i=1}^{m} f_{ij}}, \quad \forall j. \tag{15}$$

Step 4. Sorting the assessed order allocations in ascending order under each criterion and determining the dispersion degree of adjacent assessed order allocations

The elements in the matrix ***B*** are sorted in ascending order based on each criterion as following matrix $\varphi = [\varphi_{ij} \mid i = 1, 2, \ldots, m, j = 1, 2, \ldots, n]$. A smaller value will be to the top, for example, $\varphi_{11} \prec \varphi_{21} \prec \cdots \prec \varphi_{v1}$. The symbol β_{ij} is denoted as the degree of dispersion between two adjacent assessed order allocations and is determined by **Equation (16)**.

$$\beta_{ij} = \varphi_{i+1,j} - \varphi_{ij}. \tag{16}$$

Step 5. Determining the distance between β_{ij} and NI_j

The symbol δ_{ij} is represented as the distance between β_{ij} and NI_j, as shown in **Equation (17)**. If the value of β_{ij} is greater than the value of NI_j, it implies that the degree of dispersion between two adjacent assessed objects is beyond the acceptable range.

$$\delta_{ij} = \begin{cases} \beta_{ij} - NI_j, & \beta_{ij} > NI_j \\ 0, & \beta_{ij} \le NI_j \end{cases}. \tag{17}$$

Step 6. Assigning objective weights to the criteria

The objective weight w_j of each criterion is computed as the following equation,

$$w_j = \frac{v_j}{\sum_{j=1}^{n} v_j}, \quad \forall j, \tag{18}$$

where $v_j = \left(\sum_{i}^{m} \delta_{ij}^2\right)^{\frac{1}{2}}$. Next, the objective weights w_j of each criterion assigned by ZITARA are used to the ZCoCoSo.

3.3. ZCoCoSo

The Combined Compromise Solution (CoCoSo) technique, introduced by Yazdani, et al. [21], offers a compromise hybrid solution for evaluating alternatives. It integrates the Weighted Sum Model (WSM) and the Weighted Product Model (WPM), allowing for flexible decision-making based on expert opinions. This technique prioritizes higher values for positive aspects and lower values for

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negative aspects [33]. To enhance the CoCoSo technique and address uncertainties and confidence levels in the information, the Z-number is also involved in the CoCoSo technique (so-called ZCoCoSo). The steps of the ZCoCoSo technique are as follows:

Step 1. Obtaining the normalized matrix $\otimes G$ for ZCoCoSo

For obtaining normalized matrix $\otimes G$ for ZCoCoSo, the order allocation assessment matrix $\otimes Y$ is normalized as follows **Equation (19)**. In this regard, the following equation is used to normalize criteria with benefit and cost aspects.

$$\otimes g_{kj} = \begin{cases} \dfrac{\otimes g_{kj}}{\text{Max}_k(g^u_{kj})} & \text{for benefit criterion} \\ \dfrac{\text{Min}_k(g^l_{kj})}{\otimes g_{kj}} & \text{for cost criterion} \end{cases} \tag{19}$$

$$\otimes G = [\otimes g_{kj}]_{v\times n}, \quad \forall k, j . \tag{20}$$

The element $\otimes g_{kj} = \left(g^l_{kj},\ g^{m1}_{kj},\ g^{m2}_{kj},\ g^u_{kj}\right)$ is a weighted Z-number and represents the normalized value of jth criterion in the kth order allocation.

Step 2. Computing the score of each assessed order allocation by WSM and WPM

For each assessed order allocation, the combined sum of the weighted comparability sequence and the total power weight of comparability sequences are represented as $\otimes S_k$ and $\otimes P_k$, respectively. $\otimes S_k$ is to calculate the total score of each option by performing a weighted sum of the scores of each option across different criteria, is as follows **Equation (21)**. $\otimes P_k$ is to calculate the total score of each option by performing a weighted product of the scores of each option across different criteria, is as follows **Equation (22)**.

$$\otimes S_k = \sum_{j=1}^{n} \otimes g_{kj} \cdot w_j, \forall k, \tag{21}$$

$$\otimes P_k = \sum_{j=1}^{n} (\otimes g_{kj})^{w_j}, \forall k. \tag{22}$$

Step 3. Evaluating the integrated scores based on three strategies

To integrate the total scores of $\otimes S_k$ and $\otimes P_k$, the three strategies are used to determine. The first strategy uses the mean of $\otimes S_k$ and $\otimes P_k$ as shown in **Equation (23)**; the second strategy computes the sum of $\otimes S_k$ and $\otimes P_k$ as shown in **Equation (24)**, and the third strategy quantifies the balanced compromise between $\otimes S_k$ and $\otimes P_k$ as shown in **Equation (25)**.

$$\otimes M_k = \frac{\otimes S_k + \otimes P_k}{\sum_{k=1}^{v}(\otimes S_k + \otimes P_k)}, \ \forall k, \tag{23}$$

$$\otimes U_k = \frac{\otimes S_k}{\text{Min}_k \otimes S_k} + \frac{\otimes P_k}{\text{Min}_k \otimes P_k}, \ \forall k, \tag{24}$$

$$\otimes Q_k = \frac{\lambda \cdot \otimes S_k + (1-\lambda)\cdot \otimes P_k}{\lambda \cdot \text{Max}_k \otimes S_k + (1-\lambda)\cdot \text{Max}_k \otimes P_k}, \ 0 \le \lambda \le 1, \ \forall k. \tag{25}$$

In this regard, the value of λ is determined by experts. Typically, λ is set to 0.5, reflecting a balanced perspective.

Step 4. Calculate the final ranking of assessed order allocation

In this step, the final scores of the assessed order allocation are computed according to **Equations (26) and (27)**, and the options are rated in descending order in sequence.

$$\otimes H_k = (\otimes M_k \cdot \otimes U_k \cdot \otimes Q_k)^{\frac{1}{3}} + \frac{1}{3}(\otimes M_k \cdot \otimes U_k \cdot \otimes Q_k), \tag{26}$$

$$R(\otimes H_k) = \frac{h^l_k + 2\cdot(h^{m1}_k) + 2\cdot(h^{m2}_k) + h^u_k}{6}. \tag{27}$$

4. Case study

The case company is a manufacturer with a production facility in China. Given the numerous orders that must be distributed among four suppliers and the company's commitment to sustainable

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practices, it is necessary for management to identify order allocations that are more environmentally responsible. To achieve this goal, a decision-making team comprising 12 managers from various departments conducted a comprehensive evaluation. All team members held a master's degree or higher and had at least ten years of industry experience. There are sixteen criteria for sustainability, determined by reviewing academic articles, and are listed in Table 4. The assessment content for the first expert is represented in linguistic terms (Table 3), as shown in Table 5. The evaluation of the first expert is shown in Table 5. For example, the evaluated value of C_1 in A_1 is (EP, H) which can be transformed into (0, 0.837, 1.673, 2.509).

Table 4. The criteria in this case.

Dimension	Criteria	References
Social	Information sharing (C_1)	[34–36]
	Worker education, safety and health (C_2)	[34,35,37–39]
	Social feedback (C_3)	[31,34,38,40]
	Rights protection of stakeholder (C_4)	[36,38,39]
	Local employment opportunities (C_5)	[36,38,41]
Environmental	Pollution control capability (C_6)	[34,36,37,42]
	Green design (C_7)	[34,37,39,42]
	Environmental certification (C_8)	[37,39,42]
	Product recyclability (C_9)	[34,35,42]
	Renewable energy utilization (C_{10})	[36,43]
Economic	Enterprise size (C_{11})	[31,38,40]
	Product quality (C_{12})	[34,37,39]
	Delivery accuracy (C_{13})	[34,35,37,39,44]
	R&D flexibility and coordination (C_{14})	[34,35,43]
	Material price (C_{15})	[34,35,37,44]
	Product technology and patents (C_{16})	[35,37]

Table 5. The ratings for the 4 suppliers by the first expert.

	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8
A_1	(EP, H)	(EP, H)	(EP, VH)	(VP, M)	(MP, M)	(P, H)	(EP, H)	(MP, VH)
A_2	(MP, H)	(VP, H)	(MP, VH)	(MP, M)	(F, M)	(EP, H)	(G, H)	(P, VH)
A_3	(EG, H)	(EG, H)	(EG, VH)	(EG, M)	(EP, M)	(EG, H)	(MG, H)	(EG, VH)
A_4	(MG, H)	(VG, H)	(EG, VH)	(VP, M)	(MP, M)	(EG, H)	(MP, H)	(MG, VH)
	C_9	C_{10}	C_{11}	C_{12}	C_{13}	C_{14}	C_{15}	C_{16}
A_1	(MP, M)	(MG, M)	(VP, M)	(MP, M)	(VP, H)	(MP, VH)	(MP, H)	(F, M)
A_2	(MP, M)	(VP, M)	(F, M)	(MG, M)	(VP, H)	(EP, VH)	(MP, H)	(MP, M)
A_3	(MG, M)	(EG, M)	(EG, M)	(EG, M)	(EG, H)	(MG, VH)	(VG, H)	(EG, M)
A_4	(MG, M)	(MG, M)	(VG, M)	(MP, M)	(VP, H)	(VG, VH)	(P, VH)	(EG, M)

4.1. Generating the Order Allocation Evaluation Matrix

After integrating the opinions of the decision-making team, the linguistic terms are converted into the weighted Z-numbers to obtain the supplier assessment matrix $\otimes\boldsymbol{X}$, as shown in Table 5, according to Table 3. Each complete performance questionnaire can be used to form a 4×16 matrix (4 assessed suppliers and 16 criteria).

Table 6. The supplier assessment matrix $\otimes\boldsymbol{X}$.

	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8
A_1	(1.6910, 2.3503,	(2.3150, 3.0875,	(1.1255, 1.8970,	(3.5915, 4.5095,	(2.315, 3.0875,	(2.4745, 3.1815,	(1.5435, 2.3150,	(0.5000, 1.4185,

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	3.0110, 3.6708)	3.8590, 4.6305)	2.6685, 3.4410)	5.4285, 6.3465)	3.859, 4.6305)	3.8885, 4.5955)	3.0875, 3.8590)	2.3365, 3.2545)
A_2	(2.9133, 3.5735, 4.2333, 4.8935)	(3.8590, 4.6305, 5.4030, 6.1745)	(0.7720, 1.5435, 2.3150, 3.0875)	(0.4185, 1.3365, 2.2545, 3.1735)	(2.6685, 3.441, 4.2125, 4.984)	(2.4745, 3.1815, 3.8885, 4.5955)	(0.3535, 1.1255, 1.8970, 2.6685)	(4.0095, 4.9285, 5.8465, 6.7645)
A_3	(4.5868, 5.2470, 5.9070, 6.3890)	(4.9195, 5.6910, 6.4630, 6.881)	(5.8210, 6.5925, 7.3650, 7.718)	(5.0095, 5.9285, 6.8465, 7.7645)	(5.0495, 5.821, 6.5925, 7.365)	(5.3025, 6.0095, 6.7160, 7.0690)	(6.1745, 6.9460, 7.7180, 7.7180)	(5.5915, 6.5095, 7.4285, 8.3465)
A_4	(4.8935, 5.5533, 6.2125, 6.5985)	(2.3150, 3.0875, 3.8590, 4.6305)	(1.1255, 1.8970, 2.6685, 3.4410)	(6.8465, 7.7645, 8.6835, 9.1835)	(1.897, 2.6685, 3.441, 4.2125)	(4.9490, 5.6560, 6.3625, 6.7160)	(6.1745, 6.9460, 7.7180, 7.7180)	(3.7545, 4.6735, 5.5915, 6.5095)

	C_9	C_{10}	C_{11}	C_{12}	C_{13}	C_{14}	C_{15}	C_{16}
A_1	(2.9140, 3.7675, 4.6210, 5.4745)	(3.8590, 4.6305, 5.4030, 6.1745)	(2.7340, 3.5055, 4.2770, 5.0495)	(0.4185, 1.2550, 2.0910, 2.9280)	(0.5480, 1.2405, 1.9325, 2.6240)	(0.7070, 1.5605, 2.4140, 3.2675)	(0.7720, 1.5435, 2.3150, 3.0875)	(2.5605, 3.4140, 4.2675, 5.1210)
A_2	(2.7675, 3.6210, 4.4745, 5.3280)	(0.7720, 1.5435, 2.3150, 3.0875)	(2.3150, 3.0875, 3.8590, 4.6305)	(2.9280, 3.7650, 4.6010, 5.4380)	(1.5145, 2.2060, 2.8985, 3.5915)	(2.9140, 3.7675, 4.6210, 5.4745)	(2.3150, 3.0875, 3.8590, 4.6305)	(3.4140, 4.2675, 5.1210, 5.9745)
A_3	(6.4745, 7.3280, 8.1815, 8.5345)	(5.3375, 6.1100, 6.8810, 7.299)	(3.8590, 4.6305, 5.4030, 6.1745)	(6.2750, 7.1110, 7.9480, 8.3670)	(5.5385, 6.2305, 6.9230, 6.9230)	(6.8280, 7.6815, 8.5345, 8.5345)	(5.3375, 6.1100, 6.8810, 7.2990)	(0.0000, 0.8535, 1.7070, 2.5605)
A_4	(4.6210, 5.4745, 6.3280, 7.1815)	(3.4410, 4.2125, 4.9840, 5.7565)	(4.277, 5.0495, 5.8210, 6.5925)	(4.6010, 5.4380, 6.2750, 7.1110)	(4.8465, 5.5385, 6.2305, 6.9230)	(6.4745, 7.3280, 8.1815, 8.5345)	(1.1900, 1.9615, 2.7340, 3.5055)	(1.5605, 2.4140, 3.2675, 4.1210)

According to the demand of 3 units, the order allocation matrix $\boldsymbol{O}$ is obtained through **Equation (7)**. The result of the order allocation matrix $\boldsymbol{O}$ is shown in Table 7. Next, the importance parameter γ_i of the supplier, i = 1, 2, …, 4 is determined as 0.19, 0.2, 0.36, and 0.25 through the game theory. Based on **Equation (9)**, the order allocation evaluation matrix $\otimes\boldsymbol{Y}$ is further obtained as shown in Table 8. For example, the first element (0.964, 1.34, 1.716, 2.092) in matrix $\otimes\boldsymbol{Y}$ is computed by (3×1.691×0.19, 3×2.35025×0.19, 3×3.011×0.19, 3×3.67075×0.19).

Table 7. The order allocation matrix $\boldsymbol{O}$.

	Order allocation																			
A_1	3	0	0	0	2	2	2	1	1	1	0	0	0	0	0	0	1	1	1	0
A_2	0	3	0	0	1	0	0	2	0	0	2	2	1	1	0	0	1	1	0	1
A_3	0	0	3	0	0	1	0	0	2	0	1	0	2	0	2	1	1	0	1	1

A_4	0	0	0	3	0	0	1	0	0	2	0	1	0	2	1	2	0	1	1	1

Table 8. The order allocation evaluation matrix ⊗***Y***.

Order allocation	C_1				C_2				…	C_{16}			
	y^l_{k1}	y^{m1}_{k1}	y^{m2}_{k1}	y^u_{k1}	y^l_{k2}	y^{m1}_{k2}	y^{m2}_{k2}	y^u_{k2}	…	y^l_{k16}	y^{m1}_{k16}	y^{m2}_{k16}	y^u_{k16}
(3, 0, 0, 0)	0.964	1.340	1.716	2.092	1.320	1.760	2.200	2.639	…	1.459	1.946	2.432	2.919
(0, 3, 0, 0)	1.712	2.100	2.488	2.875	2.268	2.721	3.175	3.628	…	2.006	2.508	3.009	3.511
(0, 0, 3, 0)	4.966	5.681	6.395	6.917	5.326	6.162	6.998	7.450	…	0.000	0.924	1.848	2.772
(0, 0, 0, 3)	3.724	4.226	4.728	5.022	1.762	2.350	2.937	3.524	…	1.188	1.837	2.487	3.136
(2, 1, 0, 0)	1.213	1.593	1.973	2.353	1.636	2.080	2.525	2.969	…	1.642	2.133	2.625	3.116
(2, 0, 1, 0)	2.298	2.787	3.276	3.701	2.655	3.227	3.799	4.243	…	0.973	1.605	2.238	2.870
(2, 0, 0, 1)	1.884	2.302	2.720	3.069	1.467	1.956	2.445	2.934	…	1.369	1.910	2.451	2.991
(1, 2, 0, 0)	1.463	1.846	2.230	2.614	1.952	2.401	2.850	3.299	…	1.824	2.320	2.817	3.313
(1, 0, 2, 0)	3.632	4.234	4.835	5.309	3.991	4.694	5.398	5.847	…	0.486	1.265	2.043	2.821
(1, 0, 0, 2)	2.804	3.264	3.724	4.045	1.614	2.153	2.691	3.229	…	1.278	1.873	2.469	3.064
(0, 2, 1, 0)	2.797	3.294	3.790	4.223	3.287	3.868	4.449	4.902	…	1.337	1.980	2.622	3.265
(0, 2, 0, 1)	2.383	2.809	3.234	3.591	2.099	2.597	3.096	3.593	…	1.733	2.284	2.835	3.386
(0, 1, 2, 0)	3.881	4.487	5.093	5.570	4.307	5.015	5.723	6.176	…	0.669	1.452	2.235	3.018
(0, 1, 0, 2)	3.053	3.517	3.981	4.306	1.930	2.473	3.016	3.559	…	1.460	2.061	2.661	3.261
(0, 0, 2, 1)	4.552	5.196	5.839	6.285	4.138	4.891	5.644	6.141	…	0.396	1.228	2.061	2.894
(0, 0, 1, 2)	4.138	4.711	5.284	5.654	2.950	3.620	4.290	4.833	…	0.792	1.533	2.274	3.015
(1, 1, 1, 0)	2.547	3.040	3.533	3.962	2.971	3.548	4.124	4.573	…	1.155	1.793	2.430	3.067
(1, 1, 0, 1)	2.133	2.555	2.977	3.330	1.783	2.277	2.770	3.264	…	1.551	2.097	2.643	3.189
(1, 0, 1, 1)	3.218	3.749	4.280	4.677	2.803	3.424	4.045	4.538	…	0.882	1.569	2.256	2.942
(0, 1, 1, 1)	3.467	4.002	4.537	4.938	3.119	3.744	4.370	4.867	…	1.065	1.756	2.448	3.140

4.2. Appling ZITARA to Assign the Weight of the Criterion

The defuzzification assessment matrix ***F*** is obtained through the defuzzification procedure by **Equation (11)** for the supplier assessment matrix ⊗***X*** is shown in Table 9. In addition, the indifference threshold I_j of each criterion is determined as 0.05 by the decision-making team. Next, the normalized matrix ***B*** and normalized indifference threshold NI_j are constructed using **Equations (13)** and **(15)**, as shown in Table 10.

Table 9. The defuzzification assessment matrix ***F***.

	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9	C_{10}	C_{11}	C_{12}	C_{13}	C_{14}	C_{15}	C_{16}
A_1	2.6807	3.4731	2.2829	4.9690	3.4731	3.5350	2.7013	1.8774	4.1943	5.0168	3.8914	1.6731	1.5863	1.9873	1.9294	3.8408
A_2	3.9034	5.0168	1.9294	1.7957	3.8266	3.5350	1.5112	5.3873	4.0478	1.9294	3.4731	4.1830	2.5525	4.1943	3.4731	4.6943
A_3	5.5471	6.0181	6.9090	6.3873	6.2069	6.3038	7.2034	6.9690	7.6713	6.4364	5.0168	7.4600	6.4614	7.9658	6.4364	1.2803
A_4	5.8373	3.4731	2.2829	8.1543	3.0548	5.9503	7.2034	5.1323	5.9013	4.5984	5.4351	5.8563	5.8846	7.6713	2.3478	2.8408

Table 10. The normalized matrix ***B*** and normalized indifference threshold.

	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9	C_{10}	C_{11}	C_{12}	C_{13}	C_{14}	C_{15}	C_{16}
A_1	0.1492	0.1932	0.1703	0.2332	0.2097	0.1829	0.1451	0.0969	0.1923	0.2790	0.2184	0.0873	0.0962	0.0911	0.1360	0.3035
A_2	0.2172	0.2790	0.1439	0.0843	0.2311	0.1829	0.0812	0.2782	0.1856	0.1073	0.1949	0.2182	0.1548	0.1922	0.2448	0.3709
A_3	0.3087	0.3347	0.5154	0.2998	0.3748	0.3262	0.3869	0.3599	0.3517	0.3580	0.2816	0.3891	0.3920	0.3651	0.4537	0.1012
A_4	0.3249	0.1932	0.1703	0.3827	0.1845	0.3079	0.3869	0.2650	0.2705	0.2557	0.3051	0.3055	0.3570	0.3516	0.1655	0.2245
NI_j	0.0028	0.0028	0.0037	0.0023	0.0030	0.0026	0.0027	0.0026	0.0023	0.0028	0.0028	0.0026	0.0030	0.0023	0.0035	0.0040

The subsequent steps, as detailed in Subsection 3.2 (Equations (16)–(18)), involve the calculation of the objective weights of the criteria and the presentation of the results in Table 11. The criterion

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with most discriminative power is C_3, which is assigned a weight of 0.126. The top five criteria are social feedback (C_3), green design (C_7), material price (C_{15}), delivery accuracy (C_{13}), and R&D flexibility and coordination (C_{14}). Each of these criteria is distributed between three perspectives, indicating that these three perspectives simultaneously reflect the importance of sustainability. Although other criteria are not included in the top five, they nevertheless contribute to the overall assessment results.

Table 11. The objective weights and rankings of the criteria.

Dimension	Social					
Criteria	C_1	C_2	C_3	C_4	C_5	
Weight	0.0409	0.0363	0.1260	0.0660	0.0529	
Rank	13	15	**1**	7	11	
Dimension	Environmental					
Criteria	C_6	C_7	C_8	C_9	C_{10}	
Weight	0.0455	0.0909	0.0677	0.0404	0.0610	
Rank	12	**2**	6	14	9	
Dimension	Economic					
Criteria	C_{11}	C_{12}	C_{13}	C_{14}	C_{15}	C_{16}
Weight	0.0247	0.0640	0.0771	0.0685	0.0812	0.0570
Rank	16	8	**4**	**5**	**3**	10

4.3. Using ZCoCoSo to Determine Order Allocation Sustainability

After completing the calculation of the order allocation assessment matrix $\otimes \boldsymbol{Y}$ and the objective weights w_j of the criteria, the order allocations are evaluated using ZCoCoSo. First, the normalized matrix $\otimes \boldsymbol{G}$ for ZCoCoSo is obtained based on the matrix $\otimes \boldsymbol{Y}$ and **Equation (19)**, as listed in Table 12. Second, the scores of each assessed order allocation by WSM and WPM are computed thought **Equations (21)** and **(22)**, as shown in Table 13.

Table 12. The normalized matrix $\otimes \boldsymbol{G}$ for ZCoCoSo.

Order allocation	C_1				C_2				…	C_{16}			
	g_{k1}^{l}	g_{k1}^{m1}	g_{k1}^{m2}	g_{k1}^{u}	g_{k2}^{l}	g_{k2}^{m1}	g_{k2}^{m2}	g_{k2}^{u}		g_{k16}^{l}	g_{k16}^{m1}	g_{k16}^{m2}	g_{k16}^{u}
(3, 0, 0, 0)	0.139	0.148	0.189	0.231	0.177	0.235	0.293	0.352	⋯	0.416	0.254	0.318	0.381
(0, 3, 0, 0)	0.247	0.232	0.275	0.317	0.304	0.363	0.424	0.484	⋯	0.571	0.328	0.393	0.459
(0, 0, 3, 0)	0.718	0.627	0.706	0.764	0.715	0.822	0.934	0.994	⋯	0.000	0.121	0.241	0.362
(0, 0, 0, 3)	0.538	0.467	0.522	0.554	0.236	0.313	0.392	0.470	⋯	0.338	0.240	0.325	0.410
(2, 1, 0, 0)	0.175	0.176	0.218	0.260	0.220	0.278	0.337	0.396	⋯	0.468	0.279	0.343	0.407
(2, 0, 1, 0)	0.332	0.308	0.362	0.409	0.356	0.431	0.507	0.566	⋯	0.277	0.210	0.292	0.375
(2, 0, 0, 1)	0.272	0.254	0.300	0.339	0.197	0.261	0.326	0.391	⋯	0.390	0.250	0.320	0.391
(1, 2, 0, 0)	0.211	0.204	0.246	0.289	0.262	0.320	0.380	0.440	⋯	0.520	0.303	0.368	0.433
(1, 0, 2, 0)	0.525	0.467	0.534	0.586	0.536	0.626	0.720	0.780	⋯	0.139	0.165	0.267	0.369
(1, 0, 0, 2)	0.405	0.360	0.411	0.447	0.217	0.287	0.359	0.431	⋯	0.364	0.245	0.323	0.400
(0, 2, 1, 0)	0.404	0.364	0.418	0.466	0.441	0.516	0.594	0.654	⋯	0.381	0.259	0.343	0.427
(0, 2, 0, 1)	0.344	0.310	0.357	0.396	0.282	0.346	0.413	0.479	⋯	0.494	0.298	0.370	0.442
(0, 1, 2, 0)	0.561	0.495	0.562	0.615	0.578	0.669	0.764	0.824	⋯	0.190	0.190	0.292	0.394
(0, 1, 0, 2)	0.441	0.388	0.439	0.475	0.259	0.330	0.402	0.475	⋯	0.416	0.269	0.348	0.426
(0, 0, 2, 1)	0.658	0.574	0.645	0.694	0.555	0.653	0.753	0.819	⋯	0.113	0.161	0.269	0.378
(0, 0, 1, 2)	0.598	0.520	0.583	0.624	0.396	0.483	0.572	0.645	⋯	0.226	0.200	0.297	0.394
(1, 1, 1, 0)	0.368	0.336	0.390	0.437	0.399	0.473	0.550	0.610	⋯	0.329	0.234	0.317	0.401
(1, 1, 0, 1)	0.308	0.282	0.329	0.368	0.239	0.304	0.370	0.435	⋯	0.442	0.274	0.345	0.417
(1, 0, 1, 1)	0.465	0.414	0.472	0.516	0.376	0.457	0.540	0.605	⋯	0.251	0.205	0.295	0.384
(0, 1, 1, 1)	0.501	0.442	0.501	0.545	0.419	0.500	0.583	0.649	⋯	0.303	0.229	0.320	0.410

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Table 13. The scores of each assessed order allocation by WSM and WPM.

Order allocation	$\otimes S_k$				$\otimes P_k$			
(3, 0, 0, 0)	0.133	0.192	0.251	0.310	13.885	14.319	14.597	14.808
(0, 3, 0, 0)	0.165	0.222	0.283	0.344	14.067	14.460	14.714	14.909
(0, 0, 3, 0)	0.696	0.853	0.966	1.030	14.714	15.762	15.908	15.982
(0, 0, 0, 3)	0.364	0.446	0.525	0.590	14.903	15.127	15.304	15.432
(2, 1, 0, 0)	0.144	0.202	0.262	0.321	14.074	14.416	14.664	14.860
(2, 0, 1, 0)	0.321	0.412	0.489	0.550	14.899	15.099	15.265	15.377
(2, 0, 0, 1)	0.210	0.277	0.342	0.403	14.436	14.709	14.918	15.078
(1, 2, 0, 0)	0.154	0.212	0.272	0.333	14.129	14.460	14.702	14.893
(1, 0, 2, 0)	0.508	0.633	0.727	0.790	15.308	15.493	15.641	15.726
(1, 0, 0, 2)	0.287	0.361	0.434	0.497	14.709	14.948	15.135	15.275
(0, 2, 1, 0)	0.342	0.432	0.510	0.573	14.958	15.148	15.309	15.419
(0, 2, 0, 1)	0.231	0.296	0.363	0.426	14.538	14.788	14.984	15.136
(0, 1, 2, 0)	0.519	0.643	0.738	0.801	15.337	15.513	15.657	15.742
(0, 1, 0, 2)	0.298	0.371	0.444	0.508	14.755	14.983	15.165	15.301
(0, 0, 2, 1)	0.585	0.717	0.819	0.883	15.426	15.613	15.756	15.837
(0, 0, 1, 2)	0.475	0.582	0.672	0.736	15.237	15.417	15.566	15.662
(1, 1, 1, 0)	0.331	0.422	0.500	0.561	14.936	15.128	15.291	15.401
(1, 1, 0, 1)	0.221	0.287	0.353	0.415	14.505	14.759	14.958	15.112
(1, 0, 1, 1)	0.398	0.497	0.580	0.643	15.091	15.276	15.431	15.533
(0, 1, 1, 1)	0.408	0.507	0.591	0.655	15.122	15.300	15.452	15.552

The ranking index $R(\otimes H_k)$ of the assessed order allocations is calculated using **Equations (23)–(27)**. The analysis results obtained with the ZCoCoSo method are presented in Table 14. Here, $\otimes M_k$, $\otimes U_k$ and $\otimes Q_k$ represent the results of the integrating $\otimes S_k$ and $\otimes P_k$ based on three different strategies, respectively. The order allocation (0, 0, 3, 0) is the best performing order allocation in this case, with a ranking index of 0.877. This indicates that the supplier A_3 is the best choice for the assignment of all orders. According to the top five order allocations, the supplier A_3 plays an important role in this case.

Table 14. ZCoCoSo analysis results.

Order allocation	$\otimes M_k$				$\otimes U_k$				$\otimes Q_k$				$R(\otimes H_k)$	Rank
(3, 0, 0, 0)	0.0464	0.0469	0.0472	0.0475	2.0000	2.4765	2.9420	3.4026	0.8240	0.8530	0.8728	0.8887	0.5140	20
(0, 3, 0, 0)	0.0472	0.0475	0.0477	0.0479	2.2559	2.7120	3.1894	3.6628	0.8366	0.8630	0.8815	0.8966	0.5362	17
(0, 0, 3, 0)	0.0511	0.0537	0.0536	0.0534	6.3001	7.5607	8.4172	8.9081	0.9059	0.9767	0.9919	1.0000	0.8770	**1**
(0, 0, 0, 3)	0.0506	0.0504	0.0503	0.0503	3.8163	4.4461	5.0537	5.5516	0.8974	0.9154	0.9305	0.9418	0.6757	8
(2, 1, 0, 0)	0.0471	0.0473	0.0475	0.0477	2.0946	2.5586	3.0265	3.4906	0.8358	0.8593	0.8774	0.8924	0.5230	19
(2, 0, 1, 0)	0.0504	0.0502	0.0501	0.0500	3.4865	4.1927	4.7837	5.2506	0.8946	0.9118	0.9261	0.9363	0.6567	11
(2, 0, 0, 1)	0.0485	0.0485	0.0485	0.0486	2.6207	3.1417	3.6520	4.1234	0.8609	0.8809	0.8971	0.9101	0.5742	16
(1, 2, 0, 0)	0.0473	0.0474	0.0476	0.0478	2.1794	2.6369	3.1089	3.5773	0.8396	0.8625	0.8802	0.8950	0.5303	18

(1, 0, 2, 0)	0.0524	0.0521	0.0520	0.0519	4.9294	5.8812	6.6044	7.0827	0.9297	0.9479	0.9622	0.9709	0.7753	**4**
(1, 0, 0, 2)	0.0497	0.0495	0.0495	0.0495	3.2214	3.7961	4.3546	4.8389	0.8815	0.8999	0.9152	0.9271	0.6270	13
(0, 2, 1, 0)	0.0507	0.0504	0.0503	0.0502	3.6526	4.3465	4.9462	5.4222	0.8994	0.9158	0.9299	0.9400	0.6687	9
(0, 2, 0, 1)	0.0489	0.0488	0.0488	0.0489	2.7899	3.2976	3.8162	4.2962	0.8682	0.8867	0.9022	0.9148	0.5879	14
(0, 1, 2, 0)	0.0525	0.0522	0.0521	0.0520	5.0125	5.9578	6.6853	7.1681	0.9321	0.9497	0.9638	0.9724	0.7808	**3**
(0, 1, 0, 2)	0.0499	0.0496	0.0496	0.0496	3.3056	3.8737	4.4364	4.9251	0.8849	0.9026	0.9175	0.9293	0.6334	12
(0, 0, 2, 1)	0.0530	0.0528	0.0527	0.0525	5.5189	6.5270	7.2996	7.7921	0.9412	0.9599	0.9743	0.9829	0.8184	**2**
(0, 0, 1, 2)	0.0521	0.0517	0.0516	0.0515	4.6729	5.4899	6.1792	6.6738	0.9236	0.9404	0.9545	0.9639	0.7500	**5**
(1, 1, 1, 0)	0.0506	0.0503	0.0502	0.0501	3.5701	4.2700	4.8652	5.3366	0.8975	0.9141	0.9282	0.9383	0.6629	10
(1, 1, 0, 1)	0.0488	0.0486	0.0487	0.0488	2.7066	3.2204	3.7346	4.2102	0.8656	0.8844	0.9000	0.9127	0.5814	15
(1, 0, 1, 1)	0.0513	0.0510	0.0509	0.0508	4.0813	4.8427	5.4826	5.9631	0.9105	0.9272	0.9412	0.9509	0.7046	7
(0, 1, 1, 1)	0.0515	0.0511	0.0510	0.0509	4.1645	4.9195	5.5638	6.0488	0.9129	0.9292	0.9431	0.9527	0.7105	6

5. Conclusions

In light of the crucial role played by sustainable suppliers in enabling various industries to achieve supply chain sustainability, this study presents a novel framework integrating ZITARA, ZCoCoSo, and game theory to facilitate an objective assessment of sustainable suppliers and order allocations. A case study is presented to illustrate the utility of the proposed framework. In the case study of a company from China, the third supplier (A_3) is the most suitable supplier and therefore the order should be assigned to them firstly. Furthermore, the results of the ZITARA assessment indicate that the top five criteria are social feedback (C_3), green design (C_7), material price (C_{15}), delivery accuracy (C_{13}), and R&D flexibility and coordination (C_{14}). These criteria are distributed across three dimensions: social, environmental, and economic. In order to assess the sustainability of new suppliers, managers may prioritize the use of the five aforementioned criteria. Furthermore, they may also utilize the ranking result of ZCoCoSo to inform their decision-making process regarding the orders to be assigned to the assessed suppliers. The contributions of this study are summarized as follows:

(i). A novel framework integrating ZITARA, ZCoCoSo, and game theory has been developed to facilitate an objective assessment of sustainable suppliers and order allocations.

(ii). Game theory is applied to interpret the relationship between supplier evaluation and order allocation evaluation objectively.

(iii). Z-number is involved into ITARA and CoCoSo to deal with both the qualitative and quantitative criteria, considering not only the uncertainty in the expert's evaluation but also their confidence of judgment in the evaluation process.

Author Contributions: Conceptualization, Y.-K.L. and K.-Y.L.; methodology, K.-Y.L.; software, K.-Y.L.; validation, K.-Y.L.; formal analysis, K.-Y.L.; investigation, K.-Y.L.; resources, Y.-K.L.; data curation, K.-Y.L.; writing—original draft preparation, K.-Y.L.; writing—review and editing, Y.-K.L.; All authors have read and agreed to the published version of the manuscript.

Funding: This research received no external funding.

Data Availability Statement: Data are contained within the article.

Acknowledgments: This research was supported in part by the National Science and Technology Council of Taiwan, ROC under grants 111-2221-E-A49-082-MY3 and 112-2221-E-A49-110-MY3.

Conflicts of Interest: The authors declare no conflicts of interest.

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