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Posted Date: 5 July 2024

doi: 10.20944/preprints202407.0539.v1

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Article

Global Exponential Stability of Impulsive Delayed Neural Networks with Parameter Uncertainties and Reaction-Diffusion Terms

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Abstract: In this paper we study a class of impulsive delayed neural networks with parameter uncertainties and reaction-diffusion terms. Using the properties of M-matrices and some inequality techniques, some sufficient conditions are derived to guarantee the global exponential stability of the given system. The results obtained are presented by simple algebraic inequalities, which are certainly more concise than previous methods. Finally, the validity and usefulness of our results are illustrated by arithmetic examples.

Keywords: reaction diffusion terms; uncertainty parameters; impulse; time-varying delays; global exponential stability

MSC: 34D06;34A37

1. Introduction

Cellular neural networks have been widely studied and applied in signal recognition, image processing, pattern classification and other fields since they were put forward in the 20th century [1]. It is well known that the global dynamic stable neural networks are the prerequisite for solving practical problems. Therefore, the studies on the stability of neural networks have received a lot of attention [2–5].

as we all know, time delays are unavoidable due to the limitations of signal transmission and limited switching speeds of amplifiers and the communication time. In addition, time delays are applied extensively in many aspects, such as biology and engineering [6–8]. In engineering applications, the system is suddenly affected by external factors, and the state of the system will suddenly change, that is impulsive effects. Although the duration of the impulses is very short, it can cause a sudden change for the state of the system, so it is harmful. At present, a quite hot topic is to stabilize the system with impulses. Therefore, it is particularly important to ensure the stability and good performance of the systems in the presence of time delays and impulses. This works on the stability of neural networks with time delays and impulses have been studied extensively, for example [9–18]. Among them, in [15], a novel exponential stability criterion in a form of linear matrix inequalities is firstly established. By applying the properties of M-matrices, Li et al. [16] investigated global exponential stability conditions for discrete-time BAM neural networks affected by impulses and time-varying delays. In [17], Rajchakit et al. investigated the global asymptotic stability problem for Clifford-valued neural network models with time-varying delays and impulsive effects. An auxiliary model-based nonsingular M-matrix approach is used to study the global exponential stability of neural networks with time-varying delays, connection weights and impulses in [18].

In particular, the neural networks generates diffusion phenomenon when electrons move in an asymmetric electromagnetic field, which means that the state of neurons changes not only with time, but also with space. This phenomenon can be modeled by introducing the diffusion term into the traditional neural network models. In order to better reflect the actual states of the neural networks, it is necessary to describe the reaction-diffusion of neurons in space by partial differential equations. The relevant investigations of stability of neural networks with diffusion terms expressed by partial differential equations are presented in [12,19–31] and the reference therein. Among them, global

exponential stability of reaction-diffusion neural networks with time-varying was investigated in [12,19,20,22,27–29]. In [21], the stability of delay fuzzy cellular neural networks with reaction-diffusion was investigated by some new conditions. This conditions do not depend on the delay. Lu [24] studied a model with delayed reaction-diffusion and distributed delays and further discussed the robust global exponential stability and periodicity.

Moreover, parameter uncertainties are often encountered in engineering and biology systems due to the changes in the environment. In recent years, in order to solve the problems caused by parameter uncertainties, many scholars have done a lot of works on different uncertainties problems in this area and have obtained great progress, for example see [9,30,32–36]. In [32,33], the authors investigated the global exponential stability of neural networks by using the linear matrix inequality (LMI) methods. Here, time-delays correlation criterion was proposed to ensure robust stability of uncertain time-delays cellular neural networks. Hua et al. [34] investigated a uncertain stochastic neural network model and obtained a new results on robust exponential stability. Robust exponential stability of uncertain neural networks with time-varying was investigated by Balasubramaniam and Syed Ali in [35] and Zhang in [9]. An uncertain fuzzy cellular neural networks with time-varying delays and reaction diffusion terms was discussed in [30,31].

Despite the widespread works on stability analysis of neural networks, few attentions have been paid to the cases that the uncertain systems contains many complex terms, for example impulsive effect, time-varying and reaction terms. Motivated by the works above and the need for practical problems, we aim to study the global exponential stability of impulsive delayed neural networks with parameter uncertainties and reaction-diffusion terms.

Many authors have studied the global asymptotic stability of different types of uncertainty neural networks by using the linear matrix inequality (LMI) technique. It requires cumbersome conditions and yields tedious results. In this paper, a new differential inequality is established by using the properties of M-matrix and the technique of simple inequalities, which guarantees the global exponential stability of uncertain systems with time-varying delays and reaction-diffusion terms. All the results obtained are presented in a form of simple algebraic inequalities.

Our presentation is organized as follows. In Section 2, we describe the mathematical model of the system and give some preliminaries. The stability analysis of the given system is discussed in Section 3. In Section 4, two examples are given to verify the validity of the obtained results.

2. Model Description and Some Preliminaries

In this paper, we consider the following impulsive delayed neural network model with reaction-diffusion terms and parameter uncertainties:

$$\left\{ \begin{array}{l} \frac{\partial u_p(t, x)}{\partial t} = \sum_{k=1}^m \frac{\partial}{\partial x_k} \left(D_{pk} \frac{\partial u_p(t, x)}{\partial x_k} \right) - (a_p + \Delta a_p(t)) u_p(t, x) + \sum_{q=1}^n (b_{pq} + \Delta b_{pq}(t)) f_q(u_q(t, x)) \\ \quad + \sum_{q=1}^n (c_{pq} + \Delta c_{pq}(t)) g_q(u_q(t - \tau_{pq}(t), x)) \\ \quad + J_p, \quad t \neq t_l, \quad p = 1, 2, \dots, n, \quad x \in \Xi, \\ u_p(t_l^+, x) = I_{pl}(u_p(t_l^-, x)), \quad p = 1, 2, \dots, n, \quad x \in \Xi, \quad l \in N \triangleq \{0, 1, 2, \dots\}, \\ u_p(t, x) = 0, \quad t \geq 0, \quad x \in \partial \Xi, \\ u_p(s, x) = \mu_p(s, x), \quad s \in [-\tau, 0], \quad p = 1, 2, \dots, n, \end{array} \right. \quad (2.1)$$

where $u_p(t, x)$ is the state of the p th neuron in space x and time t . $x = (x_1, x_2, \dots, x_m)^T \in \Xi \subset R^m$. $\Xi = \{x = (x_1, x_2, \dots, x_m)^T | |x_k| < L_k, k = 1, 2, \dots, m\}$ is a bounded compact set with smooth boundary $\partial \Xi$ and $\text{mes } \Xi > 0$ in space R^m (L_k is a positive constant). $u(t, x) = (u_1(t, x), u_2(t, x), \dots, u_n(t, x))^T \in R^n$. $D_{pk} > 0$ denotes diffusion coefficient. $a_p > 0$ denotes the intensity of the p th element. b_{pq} and c_{pq} represent are elements of feedback and feedforward template. $\Delta a_p(t), \Delta b_{pq}(t), \Delta c_{pq}(t)$ represent bounded unstructured perturbations. $\tau_{pq}(t) \in [0, \tau]$ represents the transmission delays from the p th

element to the q th element; $f_q(\cdot) \in C(R, R)$ and $g_q(\cdot) \in C(R, R)$ represent the activation function with no delays and delays, respectively. J_p represents the external input. I_{pk} shows impulsive perturbation of the p th neuron.

Let $\omega(s^+) = \omega(s)$ and $\omega(s^+, x) = \omega(s, x)$ for all $s \in [-\tau, 0]$, $\omega(s^-) = \omega(s)$ and $\omega(s^-, x) = \omega(s, x)$ for all but a finite number of points $s \in [-\tau, 0]$. Define $PC \triangleq \{\omega : [-\tau, 0] \rightarrow R^n \mid \omega(s)$ and $PC(\Xi) \triangleq \{\omega : [-\tau, 0] \times \Xi \rightarrow R^n \mid \omega(s, x)\}$. It is easy to see that PC and $PC(\Xi)$ are bounded on $[-\tau, 0] \times \Xi$.

For any $\omega(s, x) = (\omega_1(s, x), \omega_2(s, x), \dots, \omega_n(s, x))^T \in PC(\Xi)$, the norm on $PC(\Xi)$ is defined by

$$\|\omega\| = \sup_{-\tau \leq s \leq 0} \sum_{p=1}^n \|\omega_p(s, x)\|_2 \quad \text{and} \quad \|u_p(t, x)\|_2 = \left[\int_{\Xi} |u_p(t, x)|^2 dx \right]^{\frac{1}{2}}, p = 1, 2, \dots, n.$$

For $C, D \in R^{m \times n}$, we define the Schur product by $C \otimes D = (c_{pq}d_{pq})_{m \times n}$, in which $C \geq D$ ($C > D$) means that the inequality $c_{pq} \geq b_{pq}$ ($c_{pq} > d_{pq}$) holds. $\mathbf{e} = (1, 1, \dots, 1)^T \in R^n$ and E is a n -dimensional identity matrix.

To complete the goal of this paper, we make the following assumptions:

(H1) There exist positive diagonal matrices $F = \text{diag}(F_1, F_2, \dots, F_n)$ and $G = \text{diag}(G_1, G_2, \dots, G_n)$ such that

$$F_p = \sup_{u \neq w} \left| \frac{f_p(u) - f_p(w)}{u - w} \right|, \quad G_p = \sup_{u \neq w} \left| \frac{g_p(u) - g_p(w)}{u - w} \right|,$$

for all $u, w \in R$ ($u \neq w$), $p = 1, 2, \dots, n$.

(H2) There exists a positive diagonal matrix $\mathfrak{J}_l = \text{diag}(I_{1l}, \dots, I_{nl})$ such that

$$|I_{pl}(u) - I_{pl}(v)| \leq I_{pl}|u - v|$$

for all $u, v \in R$ ($u \neq v$), $p \in \{1, 2, \dots, n\}$, $l \in N$.

(H3) There exist some positive numbers δ_{p1}, δ_{p2} and δ_{p3} , $p = 1, 2, \dots, n$, such that $\max |\Delta a_p(t)| \leq \delta_{p1}$, $\max |\Delta b_{pq}(t)| \leq \delta_{p2}$ and $\max |\Delta c_{pq}(t)| \leq \delta_{p3}$, $p, q = 1, 2, \dots, n$, $\forall t \geq 0$.

Remark 2.1. In [9,32,33], the unstructured perturbations were dealt with through a relatively complex process. In real life, many perturbations are micro-perturbations, so we only assume that the unstructured perturbations are bounded shown in assumption (H3).

Definition 2.1. If a function $u(t, x) : R \times \Xi \rightarrow R^n$ satisfies the following conditions:

- (1) $u(t, x)$ is a piecewise continuous function with the first kind of discontinuity point at t_l for all $l \in N$ and $u(t, x)$ is right-continuous at each discontinuity point t_l ;
- (2) $u(t, x)$ satisfies system (2.1) for all $t \geq 0$, and $u(t, x) = \mu(t, x)$ ($t \in [-\tau, 0]$);

then $u(t, x)$ is a particular solution that satisfies the initial conditions for system (2.1), and $u(t, \mu, x)$ represents the special solution of system (2.1) under initial condition $\mu \in PC(\Xi)$.

Definition 2.2. $u(t, \omega, x)$ is a particular solution of the system (2.1) and satisfies the initial conditions $\omega \in PC(\Xi)$. In the initial condition of $\omega \in PC(\Xi)$, $u(t, \omega, x)$ is any solution of the system (2.1). If there is a positive number $\lambda > 0$ and $M \geq 1$ such that

$$\|u(t, \omega, x) - u(t, \bar{\omega}, x)\| \leq M \|\omega - \bar{\omega}\| e^{-\lambda t} \quad \text{for all } t \geq 0$$

then the system (2.1) is said to be globally exponentially stable.

Definition 2.3. [37] A real matrix $D = (d_{pq})_{n \times n}$ is said to be a nonsingular M-matrix if $d_{pq} \leq 0$, $p, q = 1, 2, \dots, n$, $p \neq q$, and all successive principal minors of D are positive.

Lemma 2.1. [37] Let $D = (d_{pq})_{n \times n}$ with $d_{pq} \leq 0$ ($p \neq q$), then D is a nonsingular M -matrix if and only if the diagonal elements of D are all positive and there exists a positive vector d such that $Dd > 0$ or $D^T d > 0$.

Lemma 2.2. [31] Let Ξ be a cube $|x_k| < L_k$ ($k = 1, 2, \dots, m$) and $h(x)$ be a real-valued function belonging to $C^1(\Xi)$ which vanish on the boundary of Ξ , i.e. $h(x)|_{\partial\Xi} = 0$, then

$$\int_{\Xi} h^2(x) dx \leq L_k^2 \int_{\Xi} \left| \frac{\partial h}{\partial x_k} \right|^2 dx.$$

Lemma 2.3. Let $\tau > 0, a < b \leq +\infty$, suppose that $U(t) = (U_1(t), U_2(t), \dots, U_n(t))^T \in C[[a, b], R^n]$ satisfies the following differential inequality:

$$\begin{cases} D^+U(t) \leq PU(t) + (Q \otimes \bar{U}(t))\mathbf{e}, & a \leq t < b, \\ U(a+s) \in PC, & -\tau \leq s < 0, \end{cases}$$

where $P = (\tilde{p}_{pq})_{n \times n}$, $\tilde{p}_{pq} \geq 0$ ($p \neq q$), $Q = (\tilde{q}_{pq})_{n \times n}$, $\tilde{q}_{pq} \geq 0$, $\bar{U}(t) = (U(t - \tau_{pq}(t)))_{n \times n}$. If the initial condition satisfies

$$U(t) \leq \kappa \xi e^{-\lambda(t-a)}, \quad \kappa \geq 0, \quad \xi = (\xi_1, \xi_2, \dots, \xi_n)^T > 0, \quad t \in [a - \tau, a],$$

where the value of λ is determined by the following inequality:

$$[\lambda E + P + Q \otimes \zeta(\lambda)]\zeta < 0, \quad \zeta(\lambda) = (e^{\lambda \tau_{pq}(t)})_{n \times n},$$

then $U(t) \leq \kappa \xi e^{-\lambda(t-a)}$ for $t \in [a, b]$.

Proof. For $p \in \{1, 2, \dots, n\}$ and arbitrary $\varepsilon > 0$, define $w_p(t) \triangleq (\kappa + \varepsilon)\xi_p e^{-\lambda(t-a)}$. We prove that

$$U_p(t) \leq w_p(t) = (\kappa + \varepsilon)\xi_p e^{-\lambda(t-a)}, \quad t \in [a, b], \quad p = 1, 2, \dots, n. \quad (2.2)$$

If this is not true, then there exist a number $t^* \in [a, b]$ and some integer r such that

$$U_r(t^*) = w_r(t^*), \quad D^+U_r(t^*) \geq \dot{w}_r(t^*), \quad U_p(t) \leq w_p(t), \quad t \in [a, t^*], \quad p = 1, 2, \dots, n. \quad (2.3)$$

From Lemma 2.3 and (2.2), we have

$$\begin{aligned} D^+U_r(t^*) &\leq \sum_{q=1}^n [\tilde{p}_{rq}U_q(t^*) + \tilde{q}_{rq}U_q(t^* - \tau_{rq}(t^*))] \\ &\leq \sum_{q=1}^n [\tilde{p}_{rq}(\kappa + \varepsilon)\xi_q e^{-\lambda(t^*-a)} + \tilde{q}_{rq}(\kappa + \varepsilon)\xi_q e^{-\lambda(t^* - \tau_{rq}(t^*) - a)}] \\ &= \sum_{q=1}^n [\tilde{p}_{rq}(\kappa + \varepsilon)\xi_q e^{-\lambda(t^*-a)} + \tilde{q}_{rq}e^{\lambda \tau_{rq}(t^*)}(\kappa + \varepsilon)\xi_q e^{-\lambda(t^*-a)}] \\ &\leq \sum_{q=1}^n [\tilde{p}_{rq} + \tilde{q}_{rq}e^{\lambda \tau_{rq}(t^*)}] (\kappa + \varepsilon)\xi_q e^{-\lambda(t^*-a)}. \end{aligned} \quad (2.4)$$

Since $[\lambda E + P + Q \otimes \zeta(\lambda)]\zeta < 0$, $\tilde{p}_{pq} \geq 0$, $\tilde{q}_{pq} \geq 0$, ($p \neq q$) and $e^{\lambda \tau_{pq}} > 0$, it follows that

$$\sum_{q=1}^n [\tilde{p}_{rq} + \tilde{q}_{rq}e^{\lambda \tau_{rq}(t^*)}] \xi_q < -\lambda \xi_r < 0,$$

From (2.4), one can obtain

$$D^+U_r(t^*) < -\lambda\zeta_r(\kappa + \varepsilon)e^{-\lambda(t^*-l)} = \dot{w}_r(t^*).$$

It is obvious that the result obtained is in contradiction with equation (2.2), so the inequality (7) holds. Let $\varepsilon \rightarrow 0$, one has

$$U_p(t) \leq \kappa\zeta_p e^{-\lambda(t-a)}, \quad p = 1, 2, \dots, n, \quad t \in [a, b].$$

The proof is completed. \square

3. Global Exponential Stability Criteria

In this section, we obtain sufficient conditions for the global exponential stability of system (2.1).

Theorem 3.1. Under assumption (H1)–(H3), then system (2.1) is globally exponentially stable if the following two conditions (C1) and (C2) hold:

(C1) There exist a vector $\xi = (\xi_1, \xi_2, \dots, \xi_n)^T > 0$ and a constant $\lambda > 0$ such that

$$[\lambda E - D + BF + CG \otimes \zeta(\lambda)]\xi < 0, \quad t \geq 0, \quad \zeta(\lambda) = e^{\lambda\tau_{pq}(t)},$$

where $D = \text{diag}(d_1, d_2, \dots, d_n)$ with $d_p = a_p - \delta_{p1} + \sum_{k=1}^m \frac{D_{pk}}{I_k^2}$, $\zeta(\lambda) = (e^{\lambda\tau_{pq}(t)})_{n \times n}$, $B = (|b_{pq} + \delta_{p2}|)_{n \times n}$, $C = (|c_{pq} + \delta_{p3}|)_{n \times n}$, $F = \text{diag}(F_1, F_2, \dots, F_n)$, $G = \text{diag}(G_1, G_2, \dots, G_n)$, E is a n -dimensional identity matrix.

(C2) There is a positive number $\gamma > 0$ such that

$$\sup_{l \in N} \left\{ \frac{\ln \gamma_l}{t_l - t_{l-1}} \right\} \leq \gamma < \lambda,$$

where $\gamma_l = \max_{1 \leq p \leq n} \{1, I_{pl}\}$, $l \in N$.

Proof. For $\omega, \varpi \in PC(\Xi)$, the solutions of system (2.1) through $(0, \omega)$ and $(0, \varpi)$ are denoted respectively as follows:

$$\begin{aligned} u(t, \omega, x) &= (u_1(t, \omega, x), u_2(t, \omega, x), \dots, u_n(t, \omega, x))^T, \\ u(t, \varpi, x) &= (u_1(t, \varpi, x), u_2(t, \varpi, x), \dots, u_n(t, \varpi, x))^T. \end{aligned}$$

Define $u_t(\omega, x) = u(t + s, \omega, x)$, $u_t(\varpi, x) = u(t + s, \varpi, x)$, $-\tau < s \leq 0$, $t \geq 0$, then $u_t(\omega, x), u_t(\varpi, x) \in PC(\Xi)$ for $t \geq 0$. Let $U_p(t, \psi, x) = u_p(t, \omega, x) - u_p(t, \varpi, x)$, $p = 1, 2, \dots, n$, $\psi = \omega - \varpi$, we have

$$\begin{aligned} \frac{\partial U_p(t, \psi, x)}{\partial t} &= \sum_{k=1}^m \frac{\partial}{\partial x_k} \left(D_{pk} \frac{\partial U_p(t, \psi, x)}{\partial x_k} \right) - (a_p + \Delta a_p(t)) U_p(t, \psi, x) \\ &\quad - \sum_{q=1}^n (b_{pq} + \Delta b_{pq}(t)) [f_q(u_q(t, \omega, x)) - f_q(u_q(t, \varpi, x))] \\ &\quad - \sum_{q=1}^n (c_{pq} + \Delta c_{pq}(t)) [g_q(u_q(t - \tau_{pq}(t), \omega, x)) - g_q(u_q(t - \tau_{pq}(t), \varpi, x))] \quad (3.5) \end{aligned}$$

for $t \neq t_l$, $x \in \Xi$, $p = 1, 2, \dots, n$.

Multiplying both sides of (3.5) by $U_p(t, \psi, x)$ and integrate them, we have

$$\frac{d}{dt} \int_{\Xi} (U_p(t, \psi, x))^2 dx = 2 \int_{\Xi} U_p(t, \psi, x) \sum_{k=1}^m \frac{\partial}{\partial x_k} \left(D_{pk} \frac{\partial U_p(t, \psi, x)}{\partial x_k} \right) dx - g_q(u_q(t - \tau_{pq}(t), \varpi, x))] dx$$

$$\begin{aligned}
& +2 \sum_{q=1}^n (b_{pq} + \Delta b_{pq}(t)) \int_{\Xi} U_p(t, \psi, x) [f_q(u_q(t, \omega, x)) - f_q(u_q(t, \varpi, x))] dx \\
& +2 \sum_{q=1}^n (c_{pq} + \Delta c_{pq}(t)) \int_{\Xi} U_p(t, \psi, x) [g_q(u_q(t - \tau_{pq}(t), \omega, x)) \\
& -2 \int_{\Xi} (a_p + \Delta a_p(t)) U_p^2(t, \psi, x) dx.
\end{aligned} \tag{3.6}$$

Applying the Dirichlet boundary conditions and Green's formula, we can get

$$\int_{\Xi} U_p(t, \psi, x) \sum_{k=1}^m \frac{\partial}{\partial x_k} \left(D_{pk} \frac{\partial U_p(t, \psi, x)}{\partial x_k} \right) dx = - \sum_{k=1}^m \int_{\Xi} D_{pk} \left(\frac{\partial (U_p(t, \psi, x))}{\partial x_k} \right)^2 dx.$$

By Lemma 2.2, we have

$$\begin{aligned}
\int_{\Xi} U_p(t, \psi, x) \sum_{k=1}^m \frac{\partial}{\partial x_k} \left(D_{pk} \frac{\partial U_p(t, \psi, x)}{\partial x_k} \right) dx & \leq - \sum_{k=1}^m \frac{D_{pk}}{L_k^2} \int_{\Xi} (U_p(t, \psi, x))^2 dx \\
& = - \sum_{k=1}^m \frac{D_{pk}}{L_k^2} \|U_p(t, \psi, x)\|_2^2.
\end{aligned} \tag{3.7}$$

Applying the assumptions (H1)-(H3) and Holder inequality, it yields

$$\begin{aligned}
& \sum_{q=1}^n (b_{pq} + \Delta b_{pq}(t)) \int_{\Xi} U_p(t, \psi, x) [f_q(u_q(t, \omega, x)) - f_q(u_q(t, \varpi, x))] dx \\
& \leq \sum_{q=1}^n (b_{pq} + \Delta b_{pq}(t)) \int_{\Xi} |U_p(t, \psi, x)| |f_q(u_q(t, \omega, x)) - f_q(u_q(t, \varpi, x))| dx \\
& \leq \sum_{q=1}^n (b_{pq} + \delta_{p2}) \int_{\Xi} |U_p(t, \psi, x)| |U_q(t, \psi, x)| F_q dx \\
& \leq \sum_{q=1}^n |b_{pq} + \delta_{p2}| \|U_p(t, \psi, x)\|_2 F_q \|U_q(t, \psi, x)\|_2.
\end{aligned} \tag{3.8}$$

Similarly, we can get

$$\begin{aligned}
& \sum_{q=1}^n (c_{pq} + \Delta c_{pq}(t)) \int_{\Xi} U_p(t, \psi, x) [g_q(u_q(t - \tau_{pq}(t), \mu, x)) - g_q(u_q(t - \tau_{pq}(t), \varpi, x))] dx \\
& \leq \sum_{q=1}^n |c_{pq} + \delta_{p3}| \|U_q(t - \tau_{pq}(t), \psi, x)\|_2 G_q \|U_p(t, \psi, x)\|_2.
\end{aligned} \tag{3.9}$$

Applying (3.6)–(3.9) to (3.5), we can obtain

$$\begin{aligned}
\frac{1}{2} \frac{d}{dt} \|U_p(t, \psi, x)\|_2^2 & \leq - \left(a_p - \delta_{p1} + \sum_{k=1}^m \frac{D_{pk}}{L_k^2} \right) \|U_p(t, \psi, x)\|_2^2 \\
& + \sum_{q=1}^n |c_{pq} + \delta_{p3}| \|U_q(t - \tau_{pq}(t), \psi, x)\|_2 G_q \|U_p(t, \psi, x)\|_2 \\
& + \sum_{q=1}^n |b_{pq} + \delta_{p2}| \|U_p(t, \psi, x)\|_2 F_q \|U_q(t, \psi, x)\|_2,
\end{aligned}$$

i.e.,

$$\begin{aligned} D^+ \|U_p(t, \psi, x)\|_2 &\leq -\left(a_p - \delta_{p1} + \sum_{k=1}^m \frac{D_{pk}}{L_k^2}\right) \|U_p(t, \psi, x)\|_2 + \sum_{q=1}^n |b_{pq} + \delta_{p2}| F_q \|U_q(t, \vartheta, x)\|_2 \\ &\quad + \sum_{q=1}^n |c_{pq} + \delta_{p3}| \|U_q(t - \tau_{pq}(t), \psi, x)\|_2 G_q. \end{aligned} \quad (3.10)$$

Let $U_p(t) = \|U_p(t, \psi, x)\|_2$, $U(t) = (U_1(t), U_2(t), \dots, U_n(t))^T$, $\bar{U}(t) = (U_p(t - \tau_{pq}(t)))_{n \times n}$, $d_p = a_p - \delta_{p1} + \sum_{k=1}^m (D_{pk}/L_k^2)$, $p = 1, 2, \dots, n$, $D = \text{diag}(d_1, d_2, \dots, d_n)$, $B = (|b_{pq} + \delta_{p2}|)_{n \times n}$, $C = (|c_{pq} + \delta_{p3}|)_{n \times n}$, $P = -D + BF$, $Q = CG$. Then, (3.10) can be reduced to the following form:

$$D^+ U(t) \leq PU(t) + (Q \otimes \bar{U}(t))e. \quad (3.11)$$

It means that there exist a vector $\xi = (\xi_1, \xi_2, \dots, \xi_n)^T > 0$ and a constant $\lambda > 0$ such that

$$[\lambda E - D + BF + CG \otimes \xi(\lambda)]\xi < 0. \quad (3.12)$$

Take $\kappa = \|\psi\| / \min_{1 \leq p \leq n} \{\xi_p\}$, it is easy to get

$$U(t) \leq \kappa \xi e^{-\lambda t}, \quad t \in [-\tau, t_0], \quad t_0 = 0. \quad (3.13)$$

By Lemma 2.3, we have

$$U(t) \leq \kappa \xi e^{-\lambda t}, \quad t > 0.$$

If $m \leq l$, the following inequality holds:

$$U(t) \leq \kappa \gamma_0 \cdots \gamma_{m-1} \xi e^{-\lambda t}, \quad t \in [t_{m-1}, t_m), \quad \gamma_0 = 1. \quad (3.14)$$

When $m = l + 1$, we get

$$\begin{aligned} U(t_l) &= \|u(t_l, \omega, x) - u(t_l, \omega, x)\|_2 \\ &= \|I_l(u(t_l^-, \omega, x)) - I_l(u(t_l^-, \omega, x))\|_2 \\ &\leq \mathfrak{I}_l \|U(t_l^-, \psi, x)\|_2 \\ &= \mathfrak{I}_l U(t_l^-) \\ &\leq \kappa \gamma_0 \cdots \gamma_{l-1} \mathfrak{I}_l \xi \lim_{t \rightarrow t_l^-} e^{-\lambda t} \\ &\leq \kappa \gamma_0 \cdots \gamma_{l-1} \gamma_l \xi e^{-\lambda t_l}. \end{aligned} \quad (3.15)$$

Combining (3.14), (3.15) and $\gamma_l \geq 1$, we have

$$U(t) \leq \kappa \gamma_0 \cdots \gamma_{l-1} \gamma_l \xi e^{-\lambda t}, \quad t_l - \tau \leq t \leq t_l. \quad (3.16)$$

By (3.12), (3.13), (3.16) and Lemma 2.3, we can obtain

$$U(t) \leq \kappa \gamma_0 \cdots \gamma_{l-1} \gamma_l \xi e^{-\lambda t}, \quad t_l \leq t < t_{l+1}. \quad (3.17)$$

By mathematical induction, we can get

$$U(t) \leq \kappa \gamma_0 \cdots \gamma_{l-1} \xi e^{-\lambda t}, \quad t_{l-1} \leq t < t_l, \quad l \in N. \quad (3.18)$$

From condition (C2) and (3.18), we have

$$U(t) \leq \kappa e^{\gamma t_1} e^{\gamma(t_2-t_1)} \dots e^{\gamma(t_{l-1}-t_{l-2})} \zeta e^{-\lambda t} \leq \kappa \zeta e^{\gamma t} e^{-\lambda t} = \kappa \zeta e^{-(\lambda-\gamma)t} \quad (3.19)$$

for all $l \in N$, $t_{l-1} \leq t < t_l$. It shows

$$\begin{aligned} \|u(t, \omega, x) - u(t, \varpi, x)\| &= \sum_{p=1}^n \|u_p(t, \omega, x) - u_p(t, \varpi, x)\|_2 \\ &= \sum_{p=1}^n U_p(t) \\ &\leq \sum_{p=1}^n \kappa \zeta_p e^{-\lambda t} \\ &= \frac{\sum_{p=1}^n \zeta_p}{\min_{1 \leq p \leq n} \{\zeta_p\}} \|\omega - \varpi\| e^{-\lambda t}. \end{aligned}$$

That is

$$\|u_t(\omega, x) - u_t(\varpi, x)\| \leq M \|\omega - \varpi\| e^{-\lambda t}, \quad t \geq 0,$$

where $M = \left(\sum_{p=1}^n \zeta_p / \min_{1 \leq p \leq n} \{\zeta_p\} \right)$. The proof is completed. \square

Remark 3.1. In Theorem 1, the condition (C1) ensures the global exponential stability of the uncertainty system (2.1), and the value of λ is taken with respect to the system parameters, the diffusion coefficient and the time delays. Condition (C2) describes the effect of impulse intensity and impulse spacing on the global exponential stability of system (2.1).

If impulsive operator $I_{pl}(u_p) = 0, p = 1, 2, \dots, n, l \in N$, system (1) will become a model of the following form:

$$\begin{cases} \frac{\partial u_p(t, x)}{\partial t} = \sum_{k=1}^m \frac{\partial}{\partial x_k} \left(D_{pk} \frac{\partial u_p(t, x)}{\partial x_k} \right) - (a_p + \Delta a_p(t)) u_p(t, x) + \sum_{q=1}^n (b_{pq} + \Delta b_{pq}(t)) f_q(u_q(t, x)) \\ \quad + \sum_{q=1}^n (c_{pq} + \Delta c_{pq}(t)) g_q(u_q(t - \tau_{pq}(t), x)) \\ \quad + J_p, \quad t \neq t_l, \quad p = 1, 2, \dots, n, \quad x \in \Xi, \\ u_p(t, x) = 0, \quad t \geq 0, \quad x \in \partial \Xi, \\ u_p(s, x) = \mu_p(s, x), \quad s \in [-\tau, 0], \quad p = 1, 2, \dots, n, \end{cases} \quad (3.20)$$

Theorem 3.2. System (3.20) is globally exponentially stable if it satisfies the assumptions (H1) and (H3) and the following condition:

$$\left(a_p - \delta_{p1} + \sum_{k=1}^m \left(\frac{D_{pk}}{L_k^2} \right) \right) > \sum_{q=1}^n [|b_{pq} + \delta_{p2}| F_q + |c_{pq} + \delta_{p3}| G_q], \quad p, q = 1, 2, \dots, n. \quad (3.21)$$

Proof. Let $\Theta = D - BF - CG$, then we know from condition (3.21) that all successive principal minors of Θ are positive because of diagonally dominant principle. It shows Θ is a nonsingular M -matrix by definition 2.3. From Lemma 2.1, there exists $\zeta = (\zeta_1, \zeta_2, \dots, \zeta_n)^T > 0$ such that

$$[D - BF - CG \otimes \zeta(0)] \zeta > 0, \quad \zeta(0) = 1.$$

Since the continuity, there exists $\lambda > 0$ satisfies:

$$[\lambda E - D + BF + CG \otimes \zeta(\lambda)]\xi < 0, \quad \zeta(\lambda) = e^{\lambda \tau_{pq}(t)}, \quad (3.22)$$

where $D = \text{diag}(d_1, d_2, \dots, d_n)$, $d_p = a_p - \delta_{p1} + \sum_{k=1}^m \frac{D_{pk}}{L_k^2}$, $\zeta(\lambda) = (e^{\lambda \tau_{pq}(t)})_{n \times n}$, $B = (|b_{pq} + \delta_{p2}|)_{n \times n}$, $C = (|c_{pq} + \delta_{p3}|)_{n \times n}$, $F = \text{diag}(F_1, F_2, \dots, F_n)$, $G = \text{diag}(G_1, G_2, \dots, G_n)$. This shows that condition (C1) holds. \square

Remark 3.2. In Theorem 3.2, delays-independent stability criterion have been obtained when delays $\tau_{pq}(t)$ satisfies $0 \leq \tau_{pq}(t) \leq \tau$, $p, q = 1, 2, \dots, n$.

Remark 3.3. It is worth noting that the existing works on some uncertain neural networks become a special case of the system (2.1) constructed in this paper, for example [32,33]. In their works, the results obtained are independent of the impulse and reaction-diffusion. Compared to [32], it is easy to see that our standards have improved. However, it must be noted that comparing to LMI method in [33], none of our criteria is independent of the time delays when impulses and diffusion are not considered.

Remark 3.4. Although the model considered in this paper is different from one in [30], the criteria we adopt have certain commonalities. And it is easy to see that all our criteria are presented in the form of simple algebraic inequalities, the obtained results are more concise.

Corollary 3.1. If $\tau_{pq}(t) = \tau$, $p, q = 1, 2, \dots, n$, τ is a positive constant and system (3.20) satisfies Assumptions (H1), (H3) and (3.21), then system (3.20) is globally exponentially stable.

When $D_{pk} = 0$ or the reaction diffusion can be neglected, the system (2.1) is rewritten as the following form:

$$\left\{ \begin{array}{l} \frac{\partial u_p(t, x)}{\partial t} = -(a_p + \Delta a_p(t))u_p(t, x) + \sum_{q=1}^n (b_{pq} + \Delta b_{pq}(t))f_q(u_q(t, x)) \\ \quad + \sum_{q=1}^n (c_{pq} + \Delta c_{pq}(t))g_q(u_q(t - \tau_{pq}(t), x)) \\ \quad + J_p, \quad t \neq t_l, \quad p = 1, 2, \dots, n, \quad x \in \Xi, \\ u_p(t_l^+, x) = I_{pl}(u_p(t_l^-, x)), \quad p = 1, 2, \dots, n, \quad x \in \Xi, \quad l \in N \triangleq \{0, 1, 2, \dots\}, \\ u_p(t, x) = 0, \quad t \geq 0, \quad x \in \partial\Xi, \\ u_p(s, x) = \mu_p(s, x), \quad s \in [-\tau, 0], \quad p = 1, 2, \dots, n, \end{array} \right. \quad (3.23)$$

Corollary 3.2. Under assumption (H1)–(H3), then system (2.1) is globally exponentially stable if the following two conditions (D1) and (D2) hold:

(D1) There exist a vector $\xi = (\xi_1, \xi_2, \dots, \xi_n)^T > 0$ and a constant $\lambda > 0$ such that

$$[\lambda E - D^* + BF + CG \otimes \zeta(\lambda)]\xi < 0, \quad t \geq 0, \quad \zeta(\lambda) = e^{\lambda \tau_{pq}(t)},$$

where $D^* = \text{diag}(d_1, d_2, \dots, d_n)$, $d_p = a_p - \delta_{p1}$, $\zeta(\lambda) = (e^{\lambda \tau_{pq}(t)})_{n \times n}$, $B = (|b_{pq} + \delta_{p2}|)_{n \times n}$, $C = (|c_{pq} + \delta_{p3}|)_{n \times n}$, $F = \text{diag}(F_1, F_2, \dots, F_n)$, $G = \text{diag}(G_1, G_2, \dots, G_n)$;

(D2) There is a positive number $\gamma > 0$ such that

$$\sup_{l \in N} \left\{ \frac{\ln \gamma_l}{t_l - t_{l-1}} \right\} \leq \gamma < \lambda,$$

where $\gamma_l = \max_{1 \leq p \leq n} \{1, I_{pl}\}$, $l \in N$.

4. Illustrative Examples

In this section, we will verify the validity of the results obtained by two examples.

Example 4.1 Consider the uncertain impulsive system (2.1) with $n = m = 2$, $I_{pl} = 1.1$, $\Xi = [0, 1]$, $t_l = 0.1\pi l$, $\max |\Delta a_p(t)| \leq \delta_{p1} = 0.5$, $\max |\Delta b_{pq}(t)| \leq \delta_{p2} = 0.2$, $\max |\Delta c_{pq}(t)| \leq \delta_{p3} = 0.1$, where $p, q = 1, 2$. Let

$$f_q(u) = g_q(u) = \frac{|u+1| - |u-1|}{2}.$$

Based on the definition of the above parameters and functions, it is easy to see that the assumptions (H1)–(H3) are satisfied. Let

$$(D_{p1}) = \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \quad A = \begin{pmatrix} 1 - \delta_{11} & 0 \\ 0 & 1 - \delta_{21} \end{pmatrix}, \quad B = \begin{pmatrix} 0.01 + \delta_{12} & |-0.2 + \delta_{12}| \\ 0.3 + \delta_{22} & |-0.01 + \delta_{22}| \end{pmatrix}$$

$$C = \begin{pmatrix} 0.05 + \delta_{13} & |-0.1 + \delta_{13}| \\ |-0.2 + \delta_{23}| & 0.08 + \delta_{23} \end{pmatrix}, \quad (\tau(t))_{2 \times 2} = \begin{pmatrix} |sint| & |sint| \\ |cost| & |cost| \end{pmatrix}.$$

then we get

$$\zeta(\lambda) = \begin{pmatrix} e^{\lambda|sint|} & e^{\lambda|sint|} \\ e^{\lambda|cost|} & e^{\lambda|cost|} \end{pmatrix}, \quad D = \begin{pmatrix} 1.1 & 0 \\ 0 & 1.1 \end{pmatrix}, \quad F = G = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix},$$

$$B = \begin{pmatrix} 0.21 & 0 \\ 0.5 & 0.19 \end{pmatrix}, \quad C = \begin{pmatrix} 0.15 & 0 \\ 0.1 & 0.18 \end{pmatrix},$$

and $\gamma_l = \max\{1, 1.1\} = 1.1 \geq 1$, $\gamma = \sup_{l \in \mathbb{N}} \frac{\ln \gamma_l}{t_l - t_{l-1}} = \frac{\ln 1.1}{0.1\pi} \approx 0.3033$.

Take $\lambda = 0.3946$ and $\xi = (1, 1)$, then the condition (C1) in Theorem 3.1 are satisfied. Combined with the above equation, Theorem 3.1 holds. Therefore the system (2.1) is globally exponentially stable. Under the given parameters conditions above, the numerical simulation of system (2.1) is shown in Figure 1.

Example 4.2 Consider the uncertain impulsive system (2.1) with $n = m = 2$, $I_{pl} = 0$, $\Xi = [0, 1]$, $\max |\Delta a_p(t)| \leq \delta_{p1} = 0.2$, $\max |\Delta b_{pq}(t)| \leq \delta_{p2} = 0.5$, $\max |\Delta c_{pq}(t)| \leq \delta_{p3} = 0.2$, where $p, q = 1, 2$. Let

$$f_q(u) = g_q(u) = \frac{|u+1| - |u-1|}{2}.$$

By selecting the parameters and functions, it is easy to see that assumption (H1)–(H3) are satisfied. Let

$$(D_{p1}) = \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \quad A = \begin{pmatrix} 1 - \delta_{11} & 0 \\ 0 & 1 - \delta_{21} \end{pmatrix}, \quad B = \begin{pmatrix} 0.02 + \delta_{12} & |-0.3 + \delta_{12}| \\ 0.3 + \delta_{22} & 0.02 + \delta_{22} \end{pmatrix}$$

$$C = \begin{pmatrix} 0.05 + \delta_{13} & 0.2 + \delta_{13} \\ |-0.04 + \delta_{23}| & 0.1 + \delta_{23} \end{pmatrix}, \quad (\tau(t))_{2 \times 2} = \begin{pmatrix} |sint| & |sint| \\ |cost| & |cost| \end{pmatrix}.$$

then we get $F_1 = 1, F_2 = 0, G_1 = 0, G_2 = 1$ and

$$1.5 \leq \left(a_p - \delta_{p1} + \sum_{k=1}^1 (D_{pk}/L_k^2) \right),$$

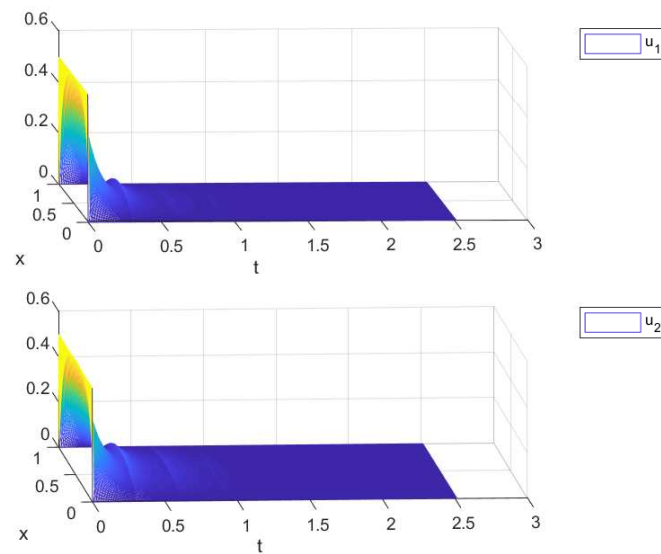


Figure 1. States response curves of the system in Example 1 with $\Delta a_p(t) = 0.5$, $\Delta b_{pq}(t) = 0.2$ and $\Delta c_{pq}(t) = 0.1$, $p, q = 1, 2$.

$$0 \leq \sum_{q=1}^2 [|b_{1q} + \delta_{12}| F_q + |c_{1q} + \delta_{13}| G_q] \leq 0.77,$$

$$0 \leq \sum_{q=1}^2 [|b_{2q} + \delta_{22}| F_q + |c_{2q} + \delta_{23}| G_q] \leq 0.82,$$

for $p, q = 1, 2$. It is obvious that the conditions (3.22) in Theorem 3.2 hold. Then the solution of system (2.1) under the given parameters conditions is globally exponentially stable, and the numerical simulation of system (2.1) is shown in Figure 2.

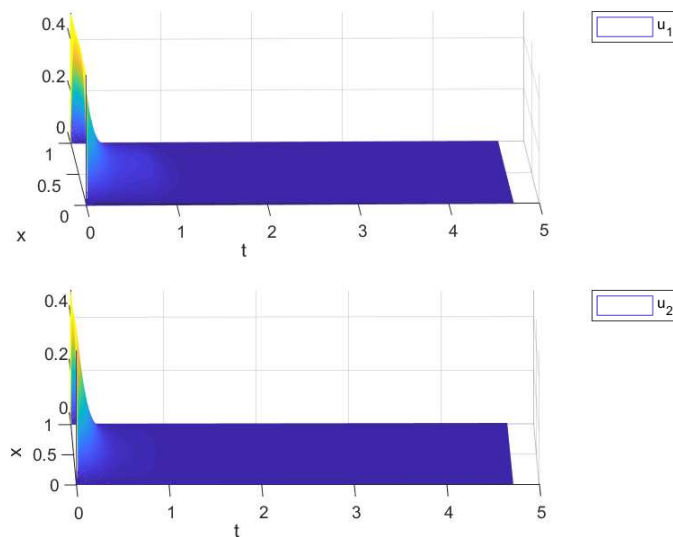


Figure 2. States response curves of the system in Example 2 with $\Delta a_p(t) = 0.2$, $\Delta b_{pq}(t) = 0.5$ and $\Delta c_{pq}(t) = 0.2$, $p, q = 1, 2$.

5. Conclusions

This paper investigates a class of impulsive delayed neural networks with parameter uncertainties and reaction-diffusion terms. By establishing a new differential inequality and combining the properties of M-matrices and some inequality tricks, several sufficient conditions are presented to guarantee the global exponential stability of the impulsive systems. And all results are given by a form of simple algebraic inequalities, which are certainly more concise than previous LMI methods. Finally the validity and generality of our results are illustrated with two arithmetic examples.

To reduce the computational difficulties, we mainly investigate stability of neural networks with impulse, in which the impulsive intensity is constant number. However, in engineering applications, the form of the impulsive intensity is rather complicated. Thus, in our future work, we plan to focus on investigating the stability of neural networks with random impulsive and reaction-diffusion terms. Therefore, it is important and interesting, but much more difficult.

Authorship Contribution: Fei Luo: Investigation, Methodology, Validation, Data curation, Software, Writing – original draft. Weiyi Hu: Conceptualization, Investigation, Methodology, Validation, Data curation, Software, Writing – reviewing and editing. Enli Wu: Conceptualization, Investigation, Methodology, Validation, Data curation, Software. Xiufang Yuan: Conceptualization, Investigation, Methodology, Software.

Funding: This work is supported by the National Natural Science Foundation of Sichuan (Grant No. 2024NSFSC1406), the Opening Project of Sichuan Province University Key Laboratory of Bridge Non-destruction Detecting and Engineering Computing (Grant No.2019QYY03, 2021QYJ03).

Institutional Review Board Statement: Not applicable.

Informed Consent Statement: Not applicable.

Data Availability Statement: All data included in this study are available upon request by contact with the corresponding author.

Acknowledgments: The authors are very grateful to the associate editor and the anonymous referees for their careful reading and valuable suggestions.

Conflicts of Interest: The authors declare that they have no conflicts of interest.

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