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Article

Temperature Dependence of the Thermo-Optic Coefficient of GeO₂-Doped Silica Glass Fiber

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Abstract: In this paper we derived an expression that allows the determination of the thermo-optic coefficient of weakly-guiding germanium-doped silica fibers, based on the thermal behavior of optical fiber devices, such as, fiber Bragg gratings (FBGs). The calculations rely on the full knowledge of the fiber parameters and on the temperature sensitivity of FBGs. In order to validate the results, we estimated the thermo-optic coefficient of bulk GeO₂ glass at 293 K and 1.55 μm to be $18.3 \times 10^{-6} \text{ K}^{-1}$. The determination of this value required to calculate a correction factor which is based on the knowledge of the thermal expansion coefficient of the fiber core, the Pockels' coefficients ($p_{11}=0.125$, $p_{12}=0.258$ and $p_{44}=-0.0662$) and the Poisson ratio ($\nu=0.161$) of the SMF-28 fiber. To achieve that goal, we estimated the temperature dependence of the thermal expansion coefficient of GeO₂ and we discussed the dispersion and temperature dependence of Pockels' coefficients. We have presented expressions for the dependence of the longitudinal and transverse acoustic velocities on the GeO₂ concentration used to calculate the Poisson ratio. We have also discussed the dispersion of the photoelastic constant. An estimate for the temperature dependence of the thermo-optic coefficient of bulk GeO₂ glass is presented for the 200-300 K temperature range.

Keywords: silica glass; refractive index; material dispersion; thermo-optic coefficient; cryogenic temperatures

1. Introduction

Modeling the thermal behavior of fiber gratings in extreme conditions such as at cryogenic temperatures is very important in order to assess the performance of sensor devices. Recently [1], we have discussed the expected values for the thermo-optic coefficient, dn/dT , of silica glass, the main constituent of an optical fiber. In order to obtain dn/dT for the fiber core, we need to know its composition and bear in mind that during fiber drawing, due to the different viscosities of core and cladding materials, stresses will be induced and, therefore, the final refractive index will also be affected. For the sake of simplicity and to control the dopants impact we shall focus on a well-known weakly-guiding fiber, the SMF-28 from Corning which have a GeO₂ dopant concentration less than 4 mol%. Fiber Bragg gratings (FBGs) imprinted in the SMF-28 fiber will be used to extract the temperature behavior of the effective refractive index that will be afterwards related to dn/dT of the fiber core through a set of equations derived for weakly-guiding fibers. Finally, we will use the additivity model to determine the thermo-optic coefficient of bulk GeO₂ glass. Its temperature dependence will be also discussed through the knowledge of FBGs and the thermal expansion coefficient. To validate the obtained results, we will use data from other optical devices such as interferometers.

The paper is organized in three sections. The first is related to weakly-guiding fibers: its full characterization and the derivation of the equation that enables the determination of the core thermo-optic coefficient through the use of the additivity model. The second to fiber Bragg gratings: its effective refractive index, the temperature sensitivity and the determination of the correction factor. The third section is dedicated to the calculation of the correction factor for the SMF-28 fiber as a function of the Poisson ratio and Pockels' coefficients. The analysis requires the correlation of data obtained by using the stimulated Brillouin gain spectrum, the acoustic velocities, whispering gallery modes, the elastic properties of the fiber, the photoelastic coefficient and the thermal expansion

coefficient of the fiber. Finally, we validate the values obtained for the thermo-optic coefficient by comparing with published results achieved by using Fabry-Perot interferometers.

2. Ge-Doped Silica Glass Fiber

In order to properly model fiber gratings we need to characterize the host fiber, namely to have knowledge of the core diameter and of the refractive index difference between the core and cladding regions. The SMF-28 fiber from Corning is one of the most studied standard fibers and the reasons have been pointed out by researchers as being reliable with well-defined parameters and, therefore, having a wide application in optical communications and sensing [2–4]. However, different values can be found in the literature and even the datasheet should be used only as a reference [5–8]. Along the past 40 years several metrological standards have been used to measure the fiber parameters [9–13]. The best practice may recommend to collect the maximum information possible on the fiber and use waveguide equations to correlate the obtained parameters. For instance, the mode field diameter (MFD) at a particular wavelength and the cut-off wavelength (λ_c) can be used to estimate the core radius through the Marcuse's empirical formula [14–17]:

$$MFD = D_{co} \left[0.65 + 0.434 \left(\frac{\lambda}{\lambda_c} \right)^{1.5} + 0.015 \left(\frac{\lambda}{\lambda_c} \right)^6 \right], \quad (1)$$

and afterwards the numerical aperture (NA) can be determined by using the normalized frequency V expression:

$$V = \frac{\pi D_{co}}{\lambda} \sqrt{n_{co}^2 - n_{cl}^2}, \quad (2)$$

by putting $V=2.405$ and $\lambda=\lambda_c$ to yield:

$$NA = \sqrt{n_{co}^2 - n_{cl}^2} = \frac{2.405 \cdot \lambda_c}{\pi D_{co}}. \quad (3)$$

On the other hand, one may think that the refractive index profile (RIP) would clarify any potential ambiguity on the fiber parameters determination, but different techniques such as refracted near field (RNF), transmitted near field, transverse interferometric, quantitative phase imaging, reconstruction through tomographic stress measurement profiles, or using atomic force microscopy results in different values [6,18–22]. In fact, considering the most common technique (RNF), and by sweeping the fiber end-face at 0° and 90° may result in different values for n and D_{co} , being the difference larger for the latter. We should recall that fiber cleaving changes the stress distribution which also affects the refractive index [23] and the determination of its absolute value also requires calibration with a fluid of known refractive index. In general for the SMF-28 fiber, n ranges from $4.4 \cdot 10^{-3}$ and D_{co} from $8.0 \cdot 10^{-3}$ to $8.8 \cdot 10^{-3}$ μm and the most common values are: $n=5.2 \cdot 10^{-3}$ and $D_{co}=8.2 \cdot 10^{-3}$ μm . Based on our measurements [24], we will consider $n \sim 5.4 \cdot 10^{-3}$ and $D_{co} \sim 8.6 \cdot 10^{-3}$ μm which matches the values presented in [4]. Furthermore, the writing of weak FBGs in the SMF-28 fiber allows one to determine its effective refractive index (n_{eff}) [25] and the obtained results are also consistent with the assumed fiber parameters. The SMF-28 is a weakly-guiding fiber [26] for which the normalized propagation constant can be written as:

$$b = \frac{n_{eff} - n_{cl}}{n_{co} - n_{cl}}, \quad (4)$$

thus

$$n_{eff} = n_{cl} + b \Delta n, \quad (5)$$

or be expressed as a function of the normalized frequency as [27]:

$$n_{eff} = n_{cl} + \left(1.1428 - \frac{0.996}{V} \right)^2 \Delta n. \quad (6)$$

Through the derivative in order to temperature results:

$$\frac{dn_{co}}{dT} = \frac{\frac{dn_{eff}}{dT} + 2\left(1.1428 - \frac{0.996}{V}\right) \frac{0.996 d\lambda}{V\lambda dT} \Delta n + \frac{dn_{cl}}{dT} \left[\left(1.1428 - \frac{0.996}{V}\right)^2 - 1 + \frac{2 \cdot 0.996 n_{cl} \left(1.1428 - \frac{0.996}{V}\right)}{V(n_{co} + n_{cl})} \right]}{\left[\left(1.1428 - \frac{0.996}{V}\right)^2 + \frac{2 \cdot 0.996 n_{co} \left(1.1428 - \frac{0.996}{V}\right)}{V(n_{co} + n_{cl})} \right]}, \quad (7)$$

From the previous equation it can be concluded that one can determine the thermo-optic coefficient of the fiber core by knowing the fiber parameters, the thermo-optic coefficient of the fiber cladding (typically, pure-silica glass) and the temperature dependence of a fiber device, such as, a FBG. On the other hand, the thermo-optic coefficient of the fiber core is related to the thermo-optic coefficients of SiO₂ and GeO₂ by knowing the fractional volume of glass occupied by GeO₂ (m) and using the additivity model [28]:

$$n_{co} = (1 - m)n_{SiO_2} + mn_{GeO_2}, \quad (8)$$

$$\frac{dn_{co}}{dT} = (1 - m) \frac{dn_{SiO_2}}{dT} + m \frac{dn_{GeO_2}}{dT}, \quad (9)$$

being m defined as:

$$m = \frac{\frac{M_{GeO_2}}{M_{SiO_2}} \frac{\rho_{SiO_2}}{\rho_{GeO_2}} x}{1 + x \left[\frac{M_{GeO_2}}{M_{SiO_2}} \frac{\rho_{SiO_2}}{\rho_{GeO_2}} - 1 \right]}, \quad (10)$$

and x is the molar fraction of GeO₂ dopant concentration.

On contrary to pure silica, published values in the literature for the thermo-optic coefficient of pure GeO₂ [29,30] is scarce and, therefore, this set of equations enables to determine its value, say at room temperature and for a particular wavelength (for instance, 293 K and 1.55 μ m). At this point, we shall recall that the refractive index of the core and cladding materials may differ for the preform and for the optical fiber. The differences arise from mechanical and thermal stresses due to the different viscosity and thermal expansion coefficients of core and cladding materials and also from viscoelasticity, due to its time dependence induced during fiber drawing. Typically, the pure silica cladding bears the applied force and it has not enough time to reach thermodynamic equilibrium. The elastic stresses affect mainly the core, that is compressed by the cladding, while viscoelasticity affects mainly the pure silica cladding. Both contributes to a decrease of the refractive index of core and cladding materials in the fiber in comparison to the preform. Taking into account the value of 4.7 MPa [20], for the mean axial stress measured in the SMF-28, the cladding refractive index decrease due to frozen-in viscoelastic stress is calculated to be -3×10^{-5} [31] and can be, therefore, neglected. Furthermore, the residual elastic stresses contribute to a decrease in the cladding refractive index of about -2×10^{-5} and to an increase in the core of about 4×10^{-5} [32,33]. The overall contribution to Δn is of the order of 1×10^{-4} . On the other hand, it has been claimed that the refractive index of the cladding can be several parts in the 4th decimal place higher than that of annealed silica and that is attributed to quenching of the fiber during its production [9,13]. The reference value for annealed silica is the one obtained by Malitson [34] and it is known that the value obtained by Fleming for a quenched glass is about 3×10^{-4} higher [35]. However, as discussed in our previous paper [1], the values obtained by Leviton *et al.* for four samples of annealed silica glass are even higher than for quenched silica (for instance, the values for Corning 7980 silica sample are about 1×10^{-4} above) [36]. Gathering all the information concerning the fiber fabrication and the errors associated to the measurement of the refractive index profile, it is not possible to clearly state that the refractive index of the silica cladding is higher than for annealed bulk samples. Moreover, it is also known that values of the order of 300 g for the drawing tension can reduce the refractive index of the cladding by about 2×10^{-4} [37]. Returning to the SMF-28 fiber, for which only ~12.5 g (peak stress 10 MPa) are used for the drawing tension, we do not expect considerable changes in the cladding refractive index [23,38,39]. Furthermore, those changes are within the uncertainty of the measurements and in fact, more important than knowing the absolute value of the cladding refractive index is to know the index difference, Δn .

As far as the core is concerned, each 1 mol% GeO₂ accounts for 0.1% in Δn [40] and therefore for $\Delta n = 5.4 \times 10^{-3}$ we expect ~3.7 mol% GeO₂. Also, for the core we can question if we should use annealed or quenched GeO₂. Calculations using the Sellmeier's coefficients for pure GeO₂ (quenched and annealed samples) presented in [41] and for silica, the ones obtained by Leviton and Frey for Corning

7980 [36], show that the difference in values for n is of about 1×10^{-4} being the mol% of GeO_2 3.70 ± 0.04 . Therefore, we will also use the GeO_2 annealed sample, resulting in $n = 1.464 \times 10^{-3} \times [\text{GeO}_2(\text{mol}\%)]$ at $1.55 \mu\text{m}$ (valid for small concentrations since the dependence is in fact quadratic, Figure 1).

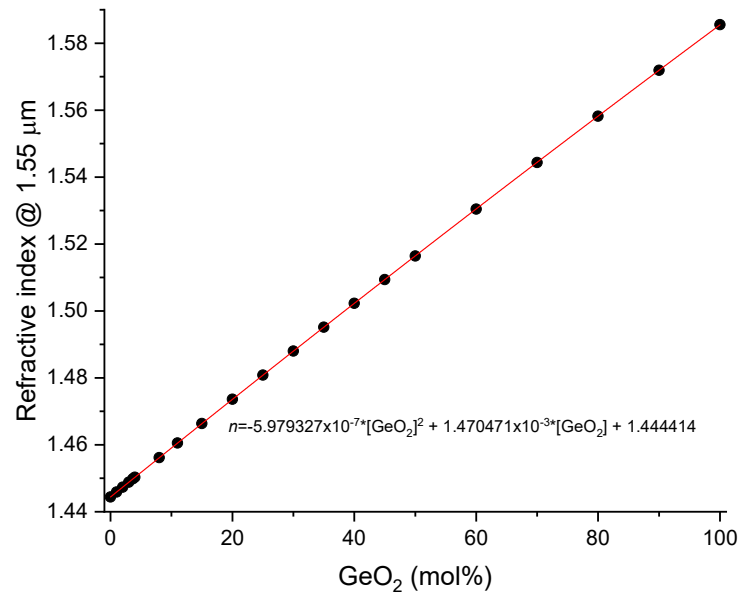


Figure 1. Refractive index of binary SiO_2 - GeO_2 glasses at room temperature and $1.55 \mu\text{m}$.

3. Fiber Bragg Gratings

The determination of the core thermo-optic coefficient requires the knowledge of the value of the effective thermo-optic coefficient. That, can be accomplished by following the thermal behavior of fiber Bragg gratings by using the temperature dependence of the Bragg wavelength, λ_B :

$$\lambda_B = n_{eff} \Lambda, \quad (11)$$

where n_{eff} is the effective refractive index and Λ the pitch of the phase-mask (which is twice that of the grating period).

The derivative of the grating resonance condition yields:

$$\frac{dn_{eff}}{dT} = n_{eff} \left(\frac{1}{\lambda_B} \frac{d\lambda_B}{dT} - \alpha_{cl} \right), \quad (12)$$

where α_{cl} represents the thermal expansion coefficient of the cladding material. It should be noted that being the cladding much larger than the core and since the thermal expansion coefficient of the core is larger than that of the cladding, it is the latter that defines the expansion of the grating period. On the other hand, the core is not free to expand and thus a compressive stress is induced in the core region during fiber heating leading to an increase of the refractive index. Therefore, the effective refractive index should be corrected by using the following expressions [42–44]:

$$\frac{dn_{eff}}{dT}_{corr} = \frac{n_{eff}^3}{2} (2\varepsilon_{oc_{eff}} + \sigma_{oc_{eff}})(\alpha_{co} - \alpha_{cl}), \quad (13)$$

where α_{co} represents the thermal expansion coefficient of the core material, that for the SMF-28 fiber can be calculated using the additivity model resulting in the following expression:

$$\alpha_{SMF28} = \frac{(1-m) \rho_{SiO_2} \alpha_{SiO_2} + m \rho_{GeO_2} \alpha_{GeO_2}}{\rho_{SMF28}}, \quad (14)$$

and ρ_{SMF28} is the density of the fiber core determined as:

$$\rho_{SMF28} = \rho_{SiO_2}(1-m) + \rho_{GeO_2}m. \quad (15)$$

The $\varepsilon_{oc_{eff}}$ and $\sigma_{oc_{eff}}$ represent the strain and stress-optic effective coefficients and are expressed as:

$$\varepsilon oc_{eff} = (xn_{GeO_2}^3 \varepsilon oc_{GeO_2} + (1-x)n_{SiO_2}^3 \varepsilon oc_{SiO_2}) \frac{1}{n_{eff}^3}, \quad (16)$$

$$\sigma oc_{eff} = (xn_{GeO_2}^3 \sigma oc_{GeO_2} + (1-x)n_{SiO_2}^3 \sigma oc_{SiO_2}) \frac{1}{n_{eff}^3}, \quad (17)$$

where n , εoc and σoc represent the values of refractive index, strain and stress-optic coefficients of the bulk materials:

$$\varepsilon oc = p_{12} - \nu(p_{11} + p_{12}), \quad (18)$$

$$\sigma oc = p_{11} - 2\nu p_{12}. \quad (19)$$

The values of molar mass (M), density, thermal expansion coefficient [1], Pockels' photoelastic coefficients (p_{11} , p_{12}) and Poisson ratio (ν) of germanium-doped silica glass are, respectively, presented in Table 1 [45].

Table 1. Physical parameters.

Material	M (g/mol)	ρ (kg/m ³)	α (x10 ⁻⁶ K ⁻¹)	P_{11}	P_{12}	ν
SiO ₂	60.08	2200	0.45	0.121	0.270	0.170
GeO ₂	104.64	3650	7.7	0.130	0.288	0.212

There is a final issue requiring attention as a result of the fact that during the grating inscription a δn_{co} is induced in the fiber core (averaged over the grating length being half of the amplitude modulation for a grating with a duty-cycle of 0.5) which is larger for strong gratings as the reflectivity approaches 1. Consequently, the effective refractive index will also change. In this context, the induced effective refractive index δn_{eff} can be determined from the grating spectrum by knowing the Bragg wavelength, λ_B the grating length, L and the reflectivity, R and, therefore, δn_{co} can be afterwards obtained by using the confinement factor, η [46,47].

$$\delta n_{eff} = \frac{\lambda_B}{2\pi L} \tanh^{-1} \sqrt{R}, \quad (20)$$

$$\delta n_{co} = \frac{\delta n_{eff}}{\eta}, \quad (21)$$

$$\eta = 1 - \frac{1}{\nu^2}. \quad (22)$$

For the calculations we have assumed the values discussed in the previous section, namely, $D_{co}=8.6 \mu m$ and $n=5.4 \times 10^{-3}$ and for the cladding refractive index at $1.55 \mu m$ the value obtained for Corning 7980 (1.4444) [1], by using Eq. (2) and Eq. (6) we determined the effective refractive index for the SMF-28 fiber. Afterwards, by considering a moderate grating ($R \sim 24\%$) [48], and by replacing the values in Eq. (20) – Eq. (22) and Eq. (11), we estimated δn_{eff} , δn_{co} and η to be respectively, 4.64×10^{-5} , 5.96×10^{-5} and $1.0729 \mu m$. Then, Eq. (6) – Eq. (19) yields the following values for the thermo-optic coefficients (corrected effective, core and bulk GeO₂): 8.45 , 8.55 and $18.3 \times 10^{-6} K^{-1}$. In order to validate

our results we have also used a strong grating [49], although in this case we knew the pitch of the phase mask (1.070 μm) but we had to estimate the grating length since it was inscribed on the splice region of two dissimilar fibers being one of them the Corning SMF-28. Based on the knowledge that we had on the impact of the arc discharge on the fiber's stress annealing (a region of about 1 mm) [50] and also on the separation of the peaks obtained in the Fabry-Perot spectrum ($\phi\lambda = \lambda^2/2n_{\text{co}}L = 1 \text{ nm}$) [51] we estimated the grating length to be of $\sim 4.6 \text{ mm}$ (the length of the phase mask was 10 mm). Since we had the phase mask pitch we could obtain directly the effective refractive index and apply an iterative method to optimize the value obtained for the induced δn_{co} . In this case the values obtained for the thermo-optic coefficients (corrected effective, core and bulk GeO_2) were 8.48, 8.59 and $19.5 \times 10^{-6} \text{ K}^{-1}$. As can be observed the values are very close to the ones obtained previously for the moderate grating. It should be stressed that the grating temperature sensitivity ($d\lambda/dT$) depends on the fiber (with or without coating and its type), on the wavelength, and on temperature [52]. The values used (9.45 and $9.46 \text{ pm}/^\circ\text{C}$) were obtained for FBGs inscribed in the SMF-28 fiber without coating at $\sim 1.55 \mu\text{m}$ and at 20°C . Care should be taken since the temperature sensitivity depends quadratically on temperature, a fact that sometimes seems to be ignored. We have also tested a strong FBG with a wavelength of 1608.5 nm exhibiting a sensitivity of $9.85 \text{ pm}/^\circ\text{C}$ (quadratic fitting between 10 and 50°C) [53] and the results obtained were 8.50, 8.62 and $20.1 \times 10^{-6} \text{ K}^{-1}$. The temperature gauge factor $K_T = 1/\lambda_B \cdot d\lambda_B/dT$ of the above gratings increased respectively, from $6.09 \times 10^{-6} \text{ K}^{-1}$ to $6.12 \times 10^{-6} \text{ K}^{-1}$, revealing the strong impact of this parameter. Note also that the reference value of $19.4 \times 10^{-6} \text{ K}^{-1}$ was obtained at room temperature in the visible region thus, assuming a similar dispersion relation for GeO_2 , as for SiO_2 , it is expected a value 6% lower at $1.5 \mu\text{m}$. Therefore, it is instructive to measure the impact of the different parameters accuracy on the estimation of the bulk GeO_2 thermo-optic coefficient. Starting from typical fiber parameters: $D_{\text{co}} = 5.2\text{--}5.4 \mu\text{m}$ and $\phi n = 5.2\text{--}5.4 \times 10^{-3}$, differences of the order of $\sim 4\%$ results in relative errors of $\sim 1\%$. Regarding the thermal expansion coefficient of SiO_2 it is known that it depends on the fictive temperature and on the OH^- content [54,55]. Nevertheless, typical values at room temperature range from $0.40\text{--}0.55 \times 10^{-6} \text{ K}^{-1}$ [43,56–58]. Common values for type III silica glass at 20°C can be considered to be $(0.47 \pm 0.04) \times 10^{-6} \text{ K}^{-1}$ [59–62]. For GeO_2 at room temperature we shall consider $6.9 \times 10^{-6} \text{ K}^{-1}$ [63] (typical average value from 25°C up to 300°C : $7.5 \times 10^{-6} \text{ K}^{-1}$ [64–68]). Considering the uncertainty of 8.5% in the thermal expansion coefficient of silica glass, it impacts $\sim 11\%$ the value of the thermo-optic coefficient of GeO_2 glass through Eq. (12)–(13). On the other hand, the difference in determining the grating temperature sensitivity at 20°C through a linear or quadratic fitting, results in an uncertainty of 6.3% that leads to a 100% variation in the value of the thermo-optic coefficient of GeO_2 glass. Being aware of that fact, a $\sim 0.45 \text{ mm}$ weak-FBG ($R < 0.1\%$) was inscribed in the SMF-28 fiber, where a single pulse of 3 mJ at 248 nm was used through a phase mask of 1065.39 nm. The FBG has a resonance wavelength of 1541.58 nm having, therefore, an effective refractive index of 1.446964. The thermal behavior of the FBG was studied from 5°C up to 95°C and after fitting with a second order polynomial we obtained a value of $9.454 \text{ pm}/^\circ\text{C}$ at 20°C . Figure 2 shows the temperature dependence of K_T for this grating. For this weak-FBG, the former values of the thermal expansion coefficients would lead to thermo-optic coefficients (corrected effective, core and bulk GeO_2): 8.52, 8.63 and $20.4 \times 10^{-6} \text{ K}^{-1}$, while the new values ($0.47 \times 10^{-6} \text{ K}^{-1}$ and $6.9 \times 10^{-6} \text{ K}^{-1}$) lead to 8.46, 8.55 and $18.3 \times 10^{-6} \text{ K}^{-1}$. The latter corresponds to a 6% reduction going from visible to the infrared, as observed for bulk SiO_2 . The value of the effective dn/dT (without correction) is $8.20 \times 10^{-6} \text{ K}^{-1}$. Applying to the temperature dependence of the Bragg wavelength a similar analysis as the one presented in [69], that is, considering that the period of the phase mask increases linearly with temperature and the refractive index has a quadratic behavior (the reference is 20°C), yields a value of $8.25 \times 10^{-6} \text{ K}^{-1}$ (0.6% higher). The core's thermo-optic coefficient increases linearly with GeO_2 concentration (mol%) at a ratio of ~ 0.106 . Recently [53], it was suggested that the cladding of the SMF-28 fiber would have similar thermo-optic coefficients as the Suprasil glass. However, based on our results for Suprasil 3001 [1], this would lead to a thermo-optic coefficient of bulk GeO_2 of $11.56 \times 10^{-6} \text{ K}^{-1}$, which is not correct. Therefore, the reason for the discrepancy lays in the higher values obtained for K_T as a consequence of the linear fitting applied to the Bragg wavelength.

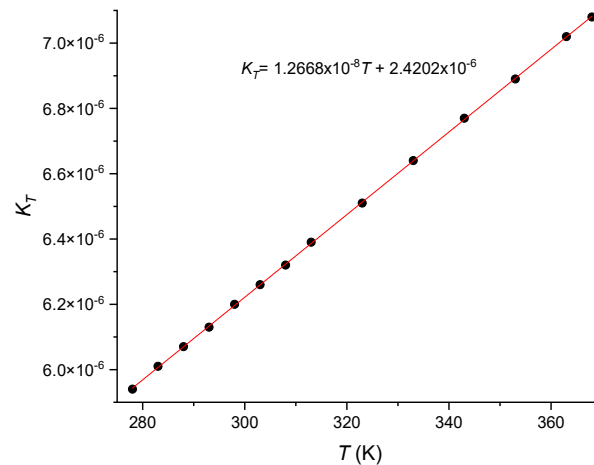


Figure 2. Temperature gauge factor of a weak-FBG.

As a final remark, note that different values can be found in the literature for the parameters ν , p_{11} and p_{12} since they may depend on temperature, wavelength and if it is a bulk or fiber glass. Therefore, in the next section we will present another approach to obtain the correction factor, $\frac{dn_{eff}}{dT_{corr}}$.

4. Effective Parameters ν , p_{11} and p_{12} for the SMF-28 Fiber

The correction factor will be determined by using Eq. (13) through the parameters for the SMF-28 fiber. The Poisson ratio, for the fiber cladding is obtained by applying the following expression:

$$\nu = \frac{1 - 2\left(\frac{v_s}{v_L}\right)^2}{2\left(1 - \left(\frac{v_s}{v_L}\right)^2\right)}, \quad (23)$$

where v_s e v_L are the transverse and longitudinal acoustic velocities and can be determined by knowing the cladding radius [70]:

$$\frac{v_s}{R_{cl}} = 59.345 \pm 0.009 \text{ m/(s.}\mu\text{m)}, \quad (24)$$

$$\frac{v_L}{R_{cl}} = 94.463 \pm 0.006 \text{ m/(s.}\mu\text{m)}. \quad (25)$$

Thus, ν for SiO₂ cladding is obtained by the ratio of the acoustic velocities yielding 0.1740 ± 0.0002 at 20 °C. To determine Pockels' coefficients for silica cladding we followed the procedure presented in [32], that relies on the strain dependence of TE and TM polarized whispering gallery modes (WGM) resonances. The Pockels' coefficients were determined for two wavelengths, 1.064 μm and 1.55 μm . When using the obtained values for the calculation of the photoelastic constant, C , we found that it would be larger at the longer wavelength, what is not correct (to be discussed below) [71–73]. Thus, by careful analysis of the figures in [32] and [74], we realized that the slopes in those figures were incidentally interchanged. Therefore, the correct values at 1.064 μm and 1.531 μm are: $p_{11}=0.113$, $p_{12}=0.250$ and $p_{44}=-0.0685$ and $p_{11}=0.130$, $p_{12}=0.265$ and $p_{44}=-0.0676$, respectively. It is interesting to note that the coefficients are essentially wavelength independent in the 3rd telecommunication window: $dp_{11}/d\lambda=3.66 \times 10^{-5} \text{ nm}^{-1}$, $dp_{12}/d\lambda=3.28 \times 10^{-5} \text{ nm}^{-1}$ and $dp_{44}/d\lambda=1.93 \times 10^{-6} \text{ nm}^{-1}$. The signs of the wavelength dependence of Pockels' coefficients compare fairly well in the visible and near infrared range [72,73]. For the sake of further comparison (we will use instead the stress-optic rotation coefficient, g defined below by Eq. 32), the latter value corresponds to $dg/d\lambda=-0.069g/\lambda \text{ nm}^{-1}$, which is in excellent agreement with the value of $-0.069g/\lambda$ obtained for a silica fiber core-doped with 3.4 mol% GeO₂ and B₂O₃ co-doped cladding, in the 1.064-1.3 μm wavelength range [75]. On the other hand, for a pure

silica-core fibers and B₂O₃ co-doped cladding a value of $-0.056\text{g}/\lambda\text{ nm}^{-1}$ was obtained in the 630-880 nm [76] and for dispersion-shifted fibers (DSF) $\text{dg}/\text{d}\lambda = -0.090\text{g}/\lambda\text{ nm}^{-1}$ at $1.55\text{ }\mu\text{m}$ [77].

As far as the core is concerned, a more laborious path is required. First, it should be mentioned that the cladding diameter was not measured and the specifications of Fibercore SM1500 4.2/125 states that the cladding as a $2\text{ }\mu\text{m}$ uncertainty. Therefore, using the nominal radius of $62.5\text{ }\mu\text{m}$ and Eq. (24)-(25) yields values of $v_L = 5940 \pm 95\text{ m/s}$ and $v_s = 3709 \pm 60\text{ m/s}$, respectively. On the other hand, for the same fiber, the echo of the longitudinal and transverse acoustic waves reflecting at the cladding/coating interface repeats at a periodicity of $\sim 21\text{ ns}$ and $\sim 33\text{ ns}$ (with a 0.1 ns resolution) [78], leading to velocity values around 5952 m/s and 3788 m/s . Due to discrepancy, we will proceed through the analysis of the stimulated Brillouin gain spectrum (SBS). The longitudinal acoustic velocity, v_L can be related to the Brillouin frequency shift, f_B through the following equation [79]:

$$f_B = \frac{2n_{\text{eff}}v_L}{\lambda_p}, \quad (26)$$

where n_{eff} is the effective refractive index and λ_p the pump wavelength. We have used data corresponding to three germanium-doped silica fibers (3.65 and 8 mol% GeO₂) [79], being one the SMF-28 fiber (3.67 mol% GeO₂) [80–82]. We have also corrected the effect of the drawing tension on the Brillouin frequency shift ($-42\text{ MHz}/100\text{g}$) considering a drawing tension similar to the one used in the SMF-28 fiber [83]. Table 2 summarizes the results at 20°C .

Table 2. SBS and acoustic velocity.

GeO ₂ concentration (mol%)	Brillouin frequency, f_B (GHz)	Longitudinal velocity, v_L (m/s)
0	11.143 (extrapolated)	5986.5 (extrapolated)
3.65	10.872	5819.5
3.67	10.863	5818.5
8.0	10.542	5620.3

It should be mentioned, that, we have limited the maximum value of GeO₂ core-dopant concentration in order to calculate the effective indices through the above equations valid for weakly-guiding fibers. Care should also be taken since the Brillouin frequency shift/velocity depends on several fiber properties [84]. The extrapolated value obtained for silica glass is in good agreement with the $5990 \pm 10\text{ m/s}$ referenced in [28,85] and it also corresponds to the value obtained for the L₀₄ longitudinal acoustic mode at 20°C (5987 m/s) [86]. The value obtained for the SMF-28 fiber is also a common accepted one (5820 m/s) [87]. Following Koyamada *et al.* [88] relation between longitudinal velocity and GeO₂ concentration ($[\text{GeO}_2] < 20\text{ mol\%}$) we obtain:

$$v_L = 5987(1 - 7.7 \times 10^{-3} * C_{\text{GeO}_2}). \quad (27)$$

In which concerns the transverse velocity we determine v_s in silica from Eq. (23) yielding a value of 3761 m/s . Note that a value of 3764 m/s was also obtained through analysis of leaky surface acoustic waves in several Corning silica samples [89]. It is interesting to note that using these values (5987 m/s and 3761 m/s) we find $R_d = 63.4\text{ }\mu\text{m}$ being within the accuracy stated in the fiber's specifications. It should be stressed that although in [88] it was considered the concentration in wt%, in fact it should be mol% [90]. Also, the fibers used as one of the references [91] for the Koyamada's equations contains B₂O₃ in the cladding, with different concentrations, affecting the values obtained for the velocities. As a first guess, we estimated a value of 3673 m/s for v_s in the SMF-28 fiber:

$$v_s = 3761(1 - 6.4 \times 10^{-3} * C_{\text{GeO}_2}), \quad (28)$$

and, therefore, the Poisson ratio for the SMF-28 fiber would be 0.169 .

The elastic properties of materials, longitudinal and shear modulus, M and G , respectively, can be determined directly from the knowledge of the acoustic velocities:

$$M = \rho v_L^2, \quad (29)$$

$$G = \rho v_s^2. \quad (30)$$

On the other hand, the Young's modulus, E , can be related to M through the Poisson's ratio ν :

$$\frac{M}{E} = \frac{1-\nu}{(1+\nu)(1-2\nu)}. \quad (31)$$

Therefore, for SiO₂ we get $M=78,86$ GPa, $E=73,08$ GPa and $G=31,12$ GPa. For the SMF-28 fiber $M=76,36$ GPa and E can be estimated by knowing the effect of GeO₂ concentration on Young's modulus [64]. A decrease of ~ 0.35 GPa/mol% was found for GeO₂ concentrations up to 4 mol%, although the temperature and density should be corrected [92]. On the other hand, from results presented in [93] a value of -0.4 GPa/mol% can be determined. Therefore, we estimate $E=71.61$ GPa for the SMF-28 fiber which is in excellent agreement with the value measured for an SMF without coating, $E=71.63\pm 0.43$ [94]. A lower value was obtained for the SMF-28e (70.05 ± 0.34) [95], however it requires a precise measurement of the fiber cladding diameter which was not performed. Moreover, by applying the additivity model and by using data related to pure bulk SiO₂ and GeO₂ from [96] results a value $E=71.65$ GPa which, once again, validates our result. For the sake of completion, the model can be improved by considering other factors such as the dissociation energy and ionic radius [97–99]. From the values of E and M for the SMF-28 fiber results $\nu=0.1612$, also in accordance to [93] and thus $v_s=3698$ m/s. Therefore, Eq. (28) should be corrected to be

$$v_s = 3761(1 - 4.6 \times 10^{-3} * C_{GeO_2}). \quad (28^*)$$

It should be highlighted that the obtained Poisson's ratio is lower for GeO₂ doped silica glass fibers which also agrees with [91], but it is in contradiction to what is expected by applying the additivity model to bulk glasses [100]. We are aware that the results obtained depend on the initial values, but by following the existing interconnection between several parameters allows us to validate the results. We shall now work, with Pockels' coefficients (p_{11} , p_{12} and p_{44}), stress-optic rotation coefficient (g) and photoelastic constants ($C=C_1-C_2$). The coefficient g can be determined through twist/rotation measurements and is related to p_{44} through the equation:

$$g = -n^2 p_{44}, \quad (32)$$

and

$$C = C_1 - C_2 = \frac{n^3 p_{44}}{2G}, \quad (33)$$

where the photoelastic constants, longitudinal C_1 and transverse C_2 are defined as:

$$C_1 = \frac{n^3(p_{11}-2\nu p_{12})}{2E}, \quad (34)$$

$$C_2 = \frac{n^3(p_{12}-\nu(p_{11}+p_{12}))}{2E}. \quad (35)$$

Note that g is related to C and $p_{44}=(p_{11}-p_{12})/2$. Through the use of whispering gallery modes [32] we achieved a value of $g=0.141$ at $1.55 \mu\text{m}$. In general, values for SMF range from 0.140 to 0.144 [101–103]. We also found a value for the SMF-28 fiber [104] that may be 0.139 ± 0.002 , since the slope taken from Figure 3 of that paper is at least 69.3×10^{-3} and not 63.9×10^{-3} as stated in the document. Thus, by applying Eq. (32) we obtain $p_{44}=-0.0662$ and by using the value of 0.205 ± 0.004 for the effective strain-optic coefficient p_{eff} reported in the strain measurements of FBGs [105,106]:

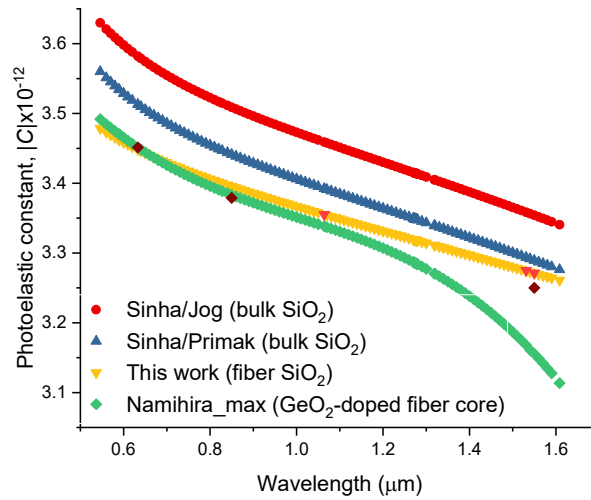


Figure 3. Dispersion of the photoelastic constant for SiO₂ and SiO₂-GeO₂ glasses.

$$p_{eff} = \frac{EC_2}{n} = \frac{n_{eff}^2}{2} (p_{12} - \nu(p_{11} + p_{12})), \quad (36)$$

we found for the SMF-28 fiber, $p_{11}=0.1251$, $p_{12}=0.2575$. Table 3 summarizes the results for SiO₂ and the SMF-28 fiber. Inserting the values in Eq. (13) results a value for the correction factor that differs only in the fourth decimal place when compared to the initial one. Therefore, we conclude that the major factor that impacts the value of the thermo-optic coefficient is the temperature sensitivity of the FBGs.

Table 3. Physical parameters calculated for SiO₂ and the SMF-28 fiber @1.55 μm .

	SiO ₂	SMF-28 (3.67 mol% GeO ₂)
n_{cl} or n_{eff}	1.444414	1.446973
p_{11}	0.130	0.125
p_{12}	0.266	0.258
p_{44}	-0.0676	-0.0662
g	0.141	0.139
$C_1(\text{Pa}^{-1})$	7.81×10^{-13}	8.90×10^{-13}
$C_2(\text{Pa}^{-1})$	4.06×10^{-12}	4.14×10^{-12}
$C(\text{Pa}^{-1})$	-3.27×10^{-12}	-3.25×10^{-12}
$M(\text{GPa})$	78.86	76.36
$E(\text{GPa})$	73.08	71.63
$G(\text{GPa})$	31.12	30.83
ν	0.174	0.161

It is instructive to note that if we consider for the SMF-28 fiber a value of $g=0.141$ [103], it would result in $p_{44}=-0.0672$ and $C=-3.30 \times 10^{-12} \text{ Pa}^{-1}$. Thus, due to the uncertainty in all calculations, the best we can say is that the photoelastic constant for the SMF-28 fiber is very close to the one obtained for pure silica cladding fiber, which is also in excellent agreement with previous results [107,108]. Sinha [109] proposed an expression for the dispersion of the photoelastic constant of fused silica by fitting data from Jog and Krishnan [71] and from Primak and Post [110].

$$C_\lambda = C_{\lambda_0} \frac{n_{\lambda_0} \lambda^2 (\lambda_0^2 - \lambda_1^2) (\lambda^2 - \lambda_2^2)}{n_\lambda \lambda_0^2 (\lambda^2 - \lambda_1^2) (\lambda_0^2 - \lambda_2^2)}, \quad (37)$$

where $\lambda_1=0.1215 \mu\text{m}$ and $\lambda_2=6.900 \mu\text{m}$ and the normalization was considered at $0.541 \mu\text{m}$ to be $C=3.63$ and 3.56 (absolute value in brewster= 10^{-12} Pa^{-1}) for each data set, respectively. Figure 3 shows the dispersion of the photoelastic constants for bulk fused silica (Jog and Primak), for fiber cladding

(calculated in this work) and for low concentration GeO₂-doped silica fiber [108]. For the latter, another expression was fitted to the experimental (maximum) values:

$$C_\lambda = 3.99565 - 1.72552\lambda + 1.5246\lambda^2 - 0.0708\lambda^3. \quad (38)$$

It can be observed that the photoelastic constants are lower for optical fibers when compared to bulk samples and that the values for the core region (doped with low concentrations of GeO₂) are lower than the ones obtained for the cladding. From the experimental values and due to uncertainty [108] it is not possible to clearly state that GeO₂ increases/decreases the value of the photoelastic constant despite it seems that it affects g [103] and, therefore, p_{44} and ultimately C . Since Eq. (38) deviates from the expected values above 1.1 μm , we estimated the dispersion of C by assuming a linear dependence of p_{44} for the whole spectral range. Considering the uncertainty in values of C , at shorter wavelengths, for the Ge-doped fibers [108], they might be slightly lower than for the silica cladding.

The temperature dependence of Pockels' coefficients can be obtained from the temperature derivative of Eq. (33) and from SBS spectrum [90]:

$$g_0 = \frac{2\pi n^7 p_{12}^2}{c \lambda_p^2 \rho v_L \Delta f}, \quad (39)$$

where c is the light speed, λ_p is the pump wavelength, Δf is the spectral width and g_0 the intensity. Considering that the product intensity-spectral width is temperature independent [90], results:

$$v_L \alpha n^7 p_{12}^2, \quad (40)$$

and therefore,

$$\frac{1}{v_L} \frac{dv_L}{dT} = 7 \frac{1}{n} \frac{dn}{dT} + 2 \frac{1}{p_{12}} \frac{dp_{12}}{dT}. \quad (41)$$

Since the temperature derivative of the acoustic velocity is ~ 0.57 [70] and that the normalized thermo-optic coefficient equals $5.65 \times 10^{-6} \text{ K}^{-1}$ [1], yields $dp_{12}/dT = 0.74 \times 10^{-5} \text{ K}^{-1}$.

From Eq. (30) and Eq. (33), results:

$$\frac{1}{p_{44}} \frac{dp_{44}}{dT} = 2 \frac{1}{v_S} \frac{dv_S}{dT} + \frac{1}{C} \frac{dC}{dT} - 3 \frac{1}{n} \frac{dn}{dT}, \quad (42)$$

taking into consideration that $dv_S/dT = 0.22$ [70] and that $(1/C)dC/dT = 1.34 \times 10^{-4} \text{ K}^{-1}$ [75], yields $dp_{44}/dT = 1.57 \times 10^{-5} \text{ K}^{-1}$. Finally, from the relation between the Pockels' coefficients, $dp_{12}/dT = -2.40 \times 10^{-5} \text{ K}^{-1}$. The temperature dependence of the Poisson ratio is $dv/dT = 3.76 \times 10^{-5} \text{ K}^{-1}$ [70]. Due to the weak dependence on temperature exhibited by the Pockels' coefficients and Poisson ratio, the correction factor is essentially dominated by the difference in the thermal expansion coefficients of the core and cladding. Figure 4 shows the thermal expansion coefficient for pure silica and the SMF-28 fiber. As can be observed the difference between the two curves decreases as the temperature decreases and, consequently, the correction factor also decreases. The thermal expansion coefficient for the SMF-28 fiber was obtained through the use of the additivity model ($<4 \text{ mol\% GeO}_2$) [43,111,112] where for GeO₂ glass [113,114] we have used data from [115,116] but with fix values at very low temperatures [117], 293 K [63] and 473 K [118]. Following this procedure, data was fitted with an equation similar to the one used by Okaji *et al.* [57] for SiO₂ glass being the coefficients 1.27, 82.42, 1.23, 8.85 and 522.8, respectively. As a final remark, it should be mentioned that without the correction factor, the effective thermo-optic coefficient for the core (SMF-28 fiber) and cladding would be essentially the same. Figure 5 was obtained by using Eq. (6), Eq. (7) and Eq. (12), the values adjusted of K_T from [119] and the temperature dependence of the SiO₂ from [1]. An estimative for the temperature dependence of the thermo-optic coefficient is presented in Figure 6, where we have assumed a linear dependence on temperature for Pockels' coefficients and Poisson ratio [70].

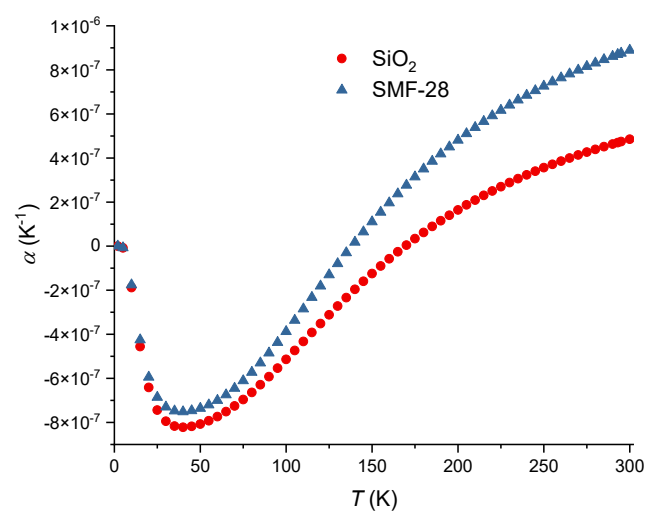


Figure 4. Thermal expansion coefficients for SiO_2 and SMF-28 fiber.

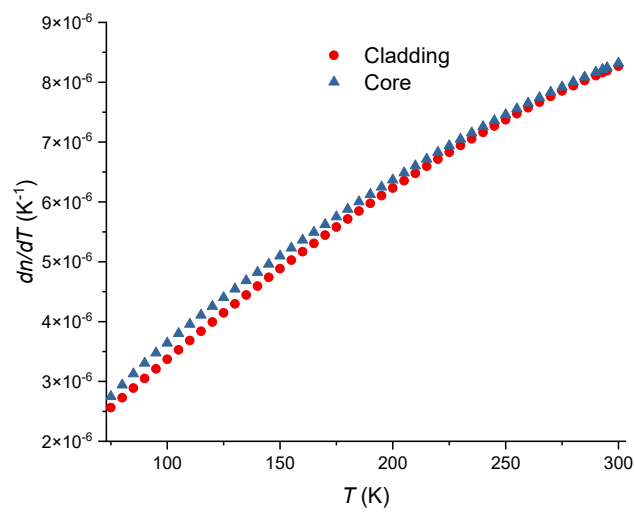


Figure 5. Thermo-optic coefficients for the cladding and core of the SMF-28 fiber.

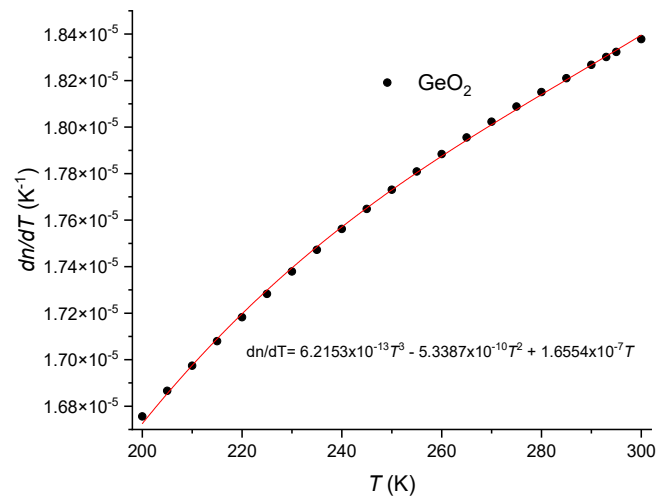


Figure 6. Thermo-optic coefficient for bulk GeO₂ glass.

Other potential techniques to determine the thermo-optic coefficient of an optical fiber are based, for instance, on Fabry-Perot interferometers (FPI) [120–124], Rayleigh backscattering [125–127] and optoelectronic oscillations [128,129]. FPI is the most used approach and thus, for the sake of comparison we estimated from [121] a value of $8.22 \times 10^{-6} \text{ K}^{-1}$ for the effective thermo-optic coefficient of a standard fiber, being therefore in excellent agreement (with the value, without correction, obtained in the previous section). On the other hand, following the procedure presented in [120] the values ranged from $8.10 \times 10^{-6} \text{ K}^{-1}$ (average heating cycles) up to $8.70 \times 10^{-6} \text{ K}^{-1}$ (first heating up). The reasons for the discrepancy are related with two facts: first, the reference temperature, 20°C , is the lower limit of the temperature interval of the experiment causing uncertainties to the derivative of the fitting equation; second, heating successively above 600°C makes irreversible changes to the glass structure which affects the temperature sensitivity and, consequently, the thermo-optic coefficient.

5. Conclusions

We have derived an expression that allows to determine the thermo-optic coefficient of weakly guiding fibers. Our analysis was based on FBG although it can also be used for FPI and Rayleigh scattering. We concluded that the thermo-optic coefficient (effective) of the SMF-28 fiber and of the cladding are essentially the same if one does not consider the correction factor. We estimated the thermo-optic coefficient of the bulk GeO₂ glass from 200 K up to 300 K being $18.3 \times 10^{-6} \text{ K}^{-1}$ at 293 K and $1.55 \mu\text{m}$. We obtained an expression for the temperature dependence of the thermal expansion coefficient of GeO₂-doped silica fibers. Expressions for the transverse and longitudinal acoustic velocities as a function of GeO₂ concentration were also presented. We have determined values for the Poisson ratio, the Pockels' coefficients and photoelastic constant for the SMF-28 fiber. We have also discussed the dispersion and temperature dependence of Pockels' coefficients. We are aware of the uncertainty of some values used, however this paper presents the relations between the different parameters that allow a straightforward correction, if required. Therefore, currently we are investigating the temperature sensitivity of FBGs inscribed, with femtosecond and UV laser radiation, in fibers with different GeO₂ concentration and we are also researching the effect of the Bragg wavelength, hydrogen loading and reflectivity, and the results will be published elsewhere.

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