

Article

Not peer-reviewed version

---

# Deterministic Quantum Mechanics – Part I – Conceptual Framework

---

[Peter Vadasz](#) \*

Posted Date: 20 May 2024

doi: 10.20944/preprints202405.1244.v1

Keywords: deterministic quantum mechanics; statistical quantum mechanics; intrinsic spin; quantum jumps



Preprints.org is a free multidiscipline platform providing preprint service that is dedicated to making early versions of research outputs permanently available and citable. Preprints posted at Preprints.org appear in Web of Science, Crossref, Google Scholar, Scilit, Europe PMC.

Copyright: This is an open access article distributed under the Creative Commons Attribution License which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

## Article

# Deterministic Quantum Mechanics – Part I – Conceptual Framework

Peter Vadasz

Department of Mechanical Engineering, Northern Arizona University, Flagstaff, AZ, USA;  
peter.vadasz@nau.edu

**Abstract:** A deterministic quantum mechanics theory is presented in a series of papers starting with the present one. The proposed theory is shown to be consistent with the current mainstream statistical quantum theory as well as with classical physics. It produces solutions, which demonstrate that causality, physical reality, and determinism are restored and can explain in simple form concerns that are raised by results from the current mainstream statistical quantum theory. The meaning of particle-wave duality and complementarity, the possibility of a particle, like the electron, to cross through the nucleus as it does when the angular momentum of the electron is zero at the ground state of the hydrogen atom, the possibility of a point-size particle to have an “intrinsic spin”, the possibility of “quantum jumps” as the electron transitions instantaneously from one stable orbital to another without passing through the space in between the orbitals and does that at irregular time intervals, and the natural collapse of the wave function as part of the solution are some of the results that emerge from the proposed deterministic quantum mechanics theory. The phenomenon of entanglement is shown to be consistent with the proposed deterministic quantum mechanics theory. The Bohr-Schrödinger *energy levels* leading to the experimentally confirmed *spectral lines*, as well as the *fine structure constant* emerge directly from the solution of the equations governing the proposed deterministic quantum mechanics presented in Part II of these series of papers.

**Keywords:** deterministic quantum mechanics; statistical quantum mechanics; intrinsic spin; quantum jumps

## 1. Introduction

This paper is the first in a series of papers introducing a deterministic quantum mechanics theory that is shown to be consistent with the current mainstream probabilistic quantum theory. The slight differences are being addressed and the equivalence between them is being demonstrated. The proposed deterministic quantum mechanics removes the difficulties arising from the interpretation, relationship to classical physics, link to physical reality, and lack of causality of the current mainstream probabilistic quantum theory. Simple, clear and obvious interpretations emerge directly from the proposed theory. All classical principles are being restored. Physical reality as well as causality, are being restored too. The phenomenon of entanglement is shown in section 3.5 to be consistent with the proposed deterministic quantum mechanics theory, and consistent with Bell [1] as well as Kochen-Specker [2] theorems. The importance of the proposed deterministic quantum mechanics theory can be emphasized by the following ten reasons:

- (i) the proposed deterministic theory is consistent with the current mainstream quantum theory and at the same time fully consistent with all classical physics principles,
- (ii) the proposed deterministic theory provides more detail,
- (iii) the proposed deterministic theory can predict the dynamics of transitions between stationary states,
- (iv) the proposed deterministic theory removes difficulties in visualizing the results using known classical means and clear and obvious interpretations,
- (v) the proposed deterministic theory restores reality to atomic and sub-atomic phenomena,

- (vi) the proposed deterministic theory unifies classical macro, meso, micro, atomic, and sub-atomic scales,
- (vii) the proposed deterministic theory is more predictive than descriptive as distinct from the current mainstream statistical quantum theory,
- (viii) the proposed deterministic theory does not have a measurement problem. Its results can be presented with almost arbitrary accuracy (limited not by principles but by available technology),
- (ix) by using the proposed deterministic quantum mechanics it becomes possible to develop new measurement methods, such as using the pressure,
- (x) the proposed deterministic theory provides the only theoretical justification of the validity of the Schrödinger equation and the conditions for its validity.

The proposed quantum mechanics theory does not invalidate or replace the current mainstream probabilistic quantum theory. While it does produce more detail and many other advantages as described in the ten reasons listed above, it also requires the solution of a more elaborate system of equations. The current mainstream quantum theory uses a much simpler method to produce useful though limited and sometimes opaque results. Demonstration of deterministic effects at subatomic quantum level have already been proposed such as 't Hooft [3,4,5,6,7] and Bohm [8]. 't Hooft [3] has shown that several quantum mechanical models are equivalent to certain deterministic systems because "a basis can be found in terms of which the wave function does not spread." 't Hooft [5] introduced a mathematical theory for deterministic quantum mechanics by using the mathematical formalism used in quantum mechanics. The "de Broglie-Bohm pilot wave theory" (Bohm [8]) is also a deterministic theory. However, these theories do not propose interpretations that are explicit enough in order to demonstrate how in reality such systems are generally applied, i.e. removing all concerns and contradictions between the quantum systems and classical concepts applicable to physical reality.

*The deterministic quantum mechanics theory proposed in the present paper introduces only one postulate stating that the electron and most likely other subatomic "particles" are compressible fluids. (The definition of a compressible fluid comes to distinguish it from the "incompressible" fluid, the latter being defined as the fluid of which mass density variation is insignificantly small and can be neglected, leading to an approximately constant density problem. For a compressible fluid the mass density cannot be assumed to be constant.)*

This introduction starts by presenting the fundamentals of the current mainstream quantum theory with the historical context in order to highlight the conceptual concerns related to physical reality, the loss of determinism, the loss of causality, the loss of consistency with classical principles, and the arbitrary choice of which of the classical principles to retain and which to discard. There is no need of a definition of what "physical reality" means as the concerns and their resolution associated with this physical reality are obvious, such as the following two relevant examples. One example of such physical reality is the fact that *"no two physical objects can occupy the same region in space at the same time, nor parts of them can overlap the same region in space at the same time"*. The second example of physical reality is the fact that *"no physical object can "jump" through space from one space location to another instantaneously, and without passing through the space between the two locations"*. Yet the proposed deterministic quantum mechanics theory can show in classical terms how two or more particles can occupy the same region in space at the same time (sections 2.1 and 3.1), and how "quantum jumps" are possible in the classical sense (section 3.2), or how a wave can be a particle at the same time (sections 2.1 and 3.1). Note the distinction between "particle" and "physical object". Once the presentation of the fundamentals of the current mainstream quantum theory with its historical context is completed, the next section will introduce the conceptual framework, the governing equations, the first evidence of consistency between the proposed deterministic quantum mechanics theory and the current mainstream quantum theory and the slight differences that are being addressed.

Planck [9] succeeded to derive the correct formula for the black body radiation that fitted experimental data. In order to fit the experimental data Planck [9] had to assume that the energy is

distributed in quanta rather than continuously. By using classical thermodynamics and Boltzmann kinetic theory (statistical thermodynamics) he obtained a curve for the radiation spectral energy density that fitted the experimental data only if the energy is divided in elements equal to  $h\nu$ , where  $\nu$  is the frequency of radiation in units of Hz, and the quantum constant  $h = 6.62607 \cdot 10^{-34} [\text{J} \cdot \text{s}]$  was eventually named Planck constant. While Planck used theoretical derivations, the result is to be considered an experimental one as it was obtained by fitting experimental data. This in no way reduces the significance of this outstanding discovery. It is stated only to emphasize the lack of theoretical derivation of this constant. Independent theoretical derivation of Planck constant is not available since. Einstein [10,11] used Planck constant to explain the photoelectric effect.

The first, partially, but extremely successful derivation of a model of the atom was introduced by Bohr [12,13,14]. Bohr model was extended into the Schrödinger equation (Schrödinger [15,16,17,18,19]) to form the fundamental of the current mainstream quantum theory. Bohr [12,13,14] assumed that the electron is a particle orbiting the nucleus in a circular path and eventually adding an elliptical correction, a model suggested by Rutherford and named the “planetary model” because of its resemblance to planetary motion. Balancing the centripetal acceleration to the electrostatic force following Coulomb law, Bohr succeeded to obtain the orbital radii, and energy levels subject to his postulate that the angular momentum of the electron on a circular orbit equals an integer multiple of the reduced Planck constant  $\hbar = h/2\pi$ , i.e.  $L = n\hbar$ , leading to the electron orbit radius  $r_n = n^2 (4\pi\epsilon_0) \hbar / m_e e^2$ , where  $e = -1.6022 \cdot 10^{-19} [\text{C}]$  is the electron charge,  $m_e = 9.11 \cdot 10^{-31} [\text{kg}]$  is the electron mass,  $\epsilon_0 = 8.854 \cdot 10^{-12} [\text{F/m}]$  is the permittivity of vacuum, and  $n = 1, 2, 3, \dots$ . This allowed the evaluation of the total energy of an electron in an orbit that yields eventually a formula to predict the spectral lines of radiation from atoms by assuming discrete energy spectra, i.e.  $\Delta E = h\nu$ . These results from this simple model that predicted correctly the spectral emission lines (Bohr [20,21]) with the exception of the fine structure were evidence of a significant success. However, from the very beginning there were a few reservations some of them carried over to the current mainstream quantum theory among others via the Schrödinger equation. *The first reservation* that persists in the current mainstream quantum theory is the fact that Bohr model does not account for the electron radiation emitted by any accelerating charge according to the electromagnetic theory and confirmed by experiment. Emission of radiation is linked to a loss of energy, and consequently the electron is expected to spiral as it falls and collides into the nucleus. However, since atoms are quite stable and such a collision with the nucleus was not observed, Bohr enforced a second postulate, i.e. that the electron radiates only as it moves from one stable orbit to another stable orbit, but does not do so when it moves along the same orbit. This postulate is the first one to violate classical mechanics. *In the proposed deterministic quantum mechanics presented in this paper it is shown how an electron-particle can cross the nucleus without conflict with classical mechanics, and in reality it does so, although a bit differently than presented by Bohr* (sections 2.1 and 3.1). *A second reservation* is that although Bohr model predicts correctly the energy levels and the orbital radii, it predicts an incorrect value for the “ground state” orbital angular momentum. The orbital angular momentum in the true ground state is known to be zero, i.e.  $L = \sqrt{l(l+1)} \hbar = 0$  since  $l = 0$  when  $n = 1$ , and not  $L = n\hbar = \hbar$  when  $n = 1$  as Bohr model postulated. Eiseberg and Resnick [22] indicate that “if the Bohr model were modified in a way that would allow for zero angular momentum states, the orbit of such a state would be a radial oscillation in which the electron passes directly through the nucleus, ...”, yet another motion through the nucleus, i.e. having two objects occupying the same space at the same time, conflicting with physical reality. Bohm [23] considered the zero orbital angular momentum as “absurd”. *In the proposed deterministic quantum mechanics presented in this paper it is shown how an electron-particle can cross the nucleus without conflict with classical mechanics, and in reality it does so* (sections 2.1 and 3.1). *A third reservation* that persists in the current mainstream quantum theory is the “quantum jumps” i.e. another postulate by Bohr indicating that the electron radiates only as it moves from one stable orbit to another stable orbit but can never pass through the space between these orbits during these “jumps”. Schrödinger objected to such “quantum jumps”



although he could not present an alternative possibility that retains “quantum” and is consistent with experiments that confirmed these “quantum jumps” that occur at irregular time intervals (Baggot [24]). *In the proposed deterministic quantum mechanics presented in this paper it is shown how such “quantum jumps” can occur without conflict with classical mechanics, and being consistent with more recent experimental results* (section 3.2). Schrödinger [15,16,17,18,19] introduced his wave equation that treats the electron as a wave instead of a particle and produced solutions consistent with the successes of Bohr model but where quantization emerged naturally rather than being imposed as a postulate. Schrödinger equation that will be introduced in the next section when applied to the hydrogen atom, still using the balance between the Coulomb force and an apparent centrifugal term (Griffiths [25] p.141), it leads eventually to the zero angular momentum at the ground state, and predicts the correct values of the orbital radii, as well as their energies. However, the wave function  $\psi$  that Schrödinger introduced via his equation had no physical interpretation. Born [26] suggested an interpretation that relates the wave function to the probability of finding the electron (or any other subatomic particle) in a specified location. The quantitative form will be presented in the next section. The statistical interpretation suggested by Born [26] was adopted as mainstream quantum mechanics, to Schrödinger’s disapproval. Schrödinger believed that the wave function is somehow related to the electric charge density, but he could not explain how then the wave function of a free electron (represented by a wave-packet concentrated in a small region of space such that it might resemble a particle) spreads indefinitely [24], as the solution to the Schrödinger equation indicates. Since the deterministic quantum mechanics theory proposed in this paper uses the postulate that the electron (and other subatomic particles) are in reality compressible fluids an identical challenge could be made. The answer to this challenge rests in the fact that pressure distribution as well as magnetic effects will be shown (in section 3.4) to prevent indefinite spreading of the electron-fluid, a result that Schrödinger equation could not provide as it does not include such effects, but the complete set of governing equations used in the currently proposed deterministic quantum mechanics theory does account for.

*An immediate reservation* that emerges from the current mainstream quantum theory that uses the Schrödinger equation is that the solution to the eigenvalue-eigenfunction problem is in fact a sequence of infinite functions  $\psi_n(t, \mathbf{x})$  for  $n = 1, 2, 3, \dots$  and the equation being linear it immediately imposes the approach of building the general solution as a superposition of the individual eigenmodes, i.e.  $\psi(t, \mathbf{x}) = \sum_{n=1}^{\infty} C_n \psi_n(t, \mathbf{x})$ . For evaluating the constants,  $C_n$ ’s, a normalization condition (to be discussed) was imposed. However, when an experimental result is obtained only one single mode out of these infinite possibilities emerges, not a superposition of all modes. Then another postulate was proposed and accepted, i.e. that upon observation (measurement) the wave function collapses from a superposition of infinite modes to one single mode representing the eigenstate that was observed. This collapse of the wave function is not obtained as a solution to the problem but rather imposed in order to comply with experimental results. *In the proposed deterministic quantum mechanics presented in this paper it is shown how the collapse of the wave function occurs naturally without it being imposed* (section 2.4.2). Schrödinger equation was not derived from any first principles and cannot be derived by using any known classical principles. *The proposed deterministic quantum mechanics does in fact provide such a derivation from first classical principles* (section 2.3). At this point the electron that is a wave according to Schrödinger equation is also a particle according to Born [26] interpretation. How can a wave be also a particle (wave-particle duality) was accepted as the principle of complementarity by the Copenhagen interpretation that expands on Born suggestion. However no physical description on how this can happen was provided. *The proposed deterministic quantum mechanics does provide such an explanation and visual description in classical terms* (section 3.1). As the research in quantum mechanics effects evolved, an additional phenomenon was discovered and linked to magnetic fields. It turns out that the electron-particle has in addition to orbital angular momentum also a “spin” (Griffiths [25]). Attempting to treat the electron particle as a rigid object did not provide acceptable results and therefore associating a spin to a point-particle violates again basic common sense of reality. Here yet again the *proposed deterministic quantum*

mechanics does provide a simple explanation of how such an electron-particle can have a spin (or intrinsic angular momentum) (sections 2.1 and 3.3). Schrödinger equation also cannot produce solutions that include the spin unless Pauli spinors and Pauli spin matrices are being introduced (Griffiths [25]). Dirac [27] by using the theory of relativity developed an equation carrying his name that does reproduce the phenomenon of particle-spin and provides additional insights, such as the existence of the antimatter, that were confirmed a-posteriori.

The equations to be used (Maxwell and inviscid Navier-Stokes equations or Euler equations) are local. However, the proposed deterministic quantum mechanics is not only non-local but also global in the sense that it involves properties that are affected by the bulk of the corresponding volume and not only by its local values. For example, one may evaluate the local linear momentum of the electron-fluid  $\mathbf{p}_f(\mathbf{x}, t)$  (linear momentum per unit volume). However, in order to indicate the value of the total linear momentum of the electron-particle and allocate it to the center of mass one needs to evaluate the integral of  $\mathbf{p}_f(\mathbf{x}, t)$  over the whole volume, i.e.  $\mathbf{p} = \int_{\tilde{V}_o} \mathbf{p}_f d\tilde{V}$ , making the latter a global property of the electron-fluid. In the present context the definition of “global” implies “over a finite volume  $\tilde{V}_o > 0$ ” and is different than just “non-local” as the latter includes any other location away from the present one but still a pointwise location. The position of the center of mass itself  $\mathbf{x}_{cm}$  is also a global property. An important characteristic of a global property is the fact that its value changes instantaneously as the value of the corresponding local property changes at any point in space within the corresponding volume. For example, a change in the value of the mass density  $\rho(\mathbf{x}, t)$  at any far away location  $\mathbf{x}$  will instantaneously affect the value of the center of mass  $\mathbf{x}_{cm}$ , due to the very definition of  $\mathbf{x}_{cm}$  (see accurate definitions in the next section), which does not include any delay. Similarly, the latter applies for all other global properties that emerged by integration over space of a corresponding local property. In particular this applies to the spin of an electron-particle, the latter being associated to the center of mass and being evaluated relative to the center of mass. The spin, which is a global angular momentum, is related to the vorticity of the electron fluid evaluated relative to the center of mass and integrated over the respective volume. It is therefore also a global property of the system. Another example of a global property of the system at the macro-level is the (hydro-) static pressure of a fluid measured locally. Since the (hydro-) static pressure at any depth is an accumulated result of the weight of the whole fluid column above the respective depth (i.e. local), the latter is a global property associated with the whole volume of fluid above that selected depth. However, in this case, affecting this global property by changing the surface pressure is delayed as the signal of the surface pressure change propagates at the speed of sound, according to the Navier-Stokes equations. This is an example demonstrating that one can measure locally values of a global property, but also that not all global properties are instantaneously affected by changes in their respective local counterpart. This is also consistent with observed evidence of entanglement based on Bell’s theorem [1]. The latter is discussed separately in section 3.5 providing further explanations and link them to the EPR (Einstein, Podolsky, and Rosen [28]) paper.

## 2. Conceptual Framework of the Deterministic Quantum Mechanics and Governing Equations

### 2.1. Schrödinger Equation and Interpretations

Schrödinger equation (Griffiths [25]) is presented in the form (explicitly, instead of using the Hamiltonian, on purpose for reasons that will become clear later)

$$i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m} \nabla^2 \psi + V(\mathbf{x})\psi \quad (1)$$

where  $i = \sqrt{-1}$ ,  $\psi(\mathbf{x}, t)$  is the complex wave function as discussed in the previous section,  $t$  is time,  $\mathbf{x}$  is the position vector representing the independent space variables,  $\hbar = h/2\pi$  is the

reduced Planck constant,  $m$  is the mass of the subatomic particle, and  $V(\mathbf{x})$  is the potential energy due to externally imposed fields. Born [26] interpretation for  $\psi$  as being the probability density function in the sense that the probability of finding the particle (in one dimension) in the interval between  $a$  and  $b$  is  $P_r[a \leq x \leq b] = \int_a^b \psi(x) \psi^*(x) dx$ , where  $\psi^*(x)$  is the complex conjugate of the wave function  $\psi(x)$ , for this special one dimensional case. Then representing the wave function in the general form

$$\psi = \rho^{1/2} e^{iS(\mathbf{x},t)/\hbar}; \quad \psi^* = \rho^{1/2} e^{-iS(\mathbf{x},t)/\hbar} \quad (2)$$

yields the probability density function  $\rho(\mathbf{x},t)$  as

$$\rho(\mathbf{x},t) = |\psi(\mathbf{x},t)|^2 = \psi \psi^* \quad (3)$$

Generalizing to three space dimensions, the expectation of finding the electron (or any subatomic particle) at a position  $\mathbf{x}$  within a volume  $\tilde{V}_o$  (for example in Cartesian coordinates  $\mathbf{x} = x\hat{e}_x + y\hat{e}_y + z\hat{e}_z$ , where  $\hat{e}_x, \hat{e}_y, \hat{e}_z$  are unit vectors in the  $x, y, z$  directions, respectively) is

$$\langle \mathbf{x} \rangle = \frac{\int_{\tilde{V}_o} \psi \mathbf{x} \psi^* d\tilde{V}}{\int_{\tilde{V}_o} \psi \psi^* d\tilde{V}} \quad (4)$$

where  $\tilde{V}$  represents the volume. Substituting Equation (3) into (4) produces

$$\langle \mathbf{x} \rangle = \frac{\int_{\tilde{V}_o} \psi \mathbf{x} \psi^* d\tilde{V}}{\int_{\tilde{V}_o} \rho(\mathbf{x},t) d\tilde{V}} \stackrel{(3)}{=} \frac{\int_{\tilde{V}_o} \rho(\mathbf{x},t) \mathbf{x} d\tilde{V}}{\int_{\tilde{V}_o} \rho(\mathbf{x},t) d\tilde{V}} \quad (5)$$

At this point Born interpretation is applied by introducing the normalization condition

$$\int_{\tilde{V}_o} \psi \psi^* d\tilde{V} \stackrel{(3)}{=} \int_{\tilde{V}_o} \rho(\mathbf{x},t) d\tilde{V} = 1 \quad (6)$$

as the electron is certainly to be found somewhere in the volume  $\tilde{V}_o$  especially if  $\tilde{V}_o \rightarrow \infty$  leading to probability 1, and consequently substituting the normalization condition (6) into (5) yields

$$\langle \mathbf{x} \rangle = \int_{\tilde{V}_o} \psi \mathbf{x} \psi^* d\tilde{V} = \int_{\tilde{V}_o} \rho(\mathbf{x},t) \mathbf{x} d\tilde{V} \quad (7)$$

The proposed deterministic quantum mechanics uses only one postulate, i.e. that the electron and most likely other subatomic particles are in reality compressible fluids. They possess mass density  $\rho(\mathbf{x},t) [\text{kg}/\text{m}^3]$ , and if they are charged particles like the electron, they possess electric charge density too  $\rho_e(\mathbf{x},t) [\text{C}/\text{m}^3]$ , both allowed to vary in space as well as in time. As the mass density is space dependent it becomes appealing to define the center of mass  $\mathbf{x}_{cm}$  in the same form as applied to rigid bodies

$$\mathbf{x}_{cm} = \frac{\int_{\tilde{V}_o} \rho(\mathbf{x},t) \mathbf{x} d\tilde{V}}{\int_{\tilde{V}_o} \rho(\mathbf{x},t) d\tilde{V}} \quad (8)$$

where the electron-mass contained in the volume  $\tilde{V}_o$  is the denominator in Equation (8) and it is therefore constant. For one electron system this is

$$m_e = \int_{\tilde{V}_o} \rho(\mathbf{x}, t) d\tilde{V} = \text{const.} \quad (9)$$

where  $m_e$  is the mass of the electron. Equation (8) emerging from defining the subatomic particles as compressible fluids is identical to Equation (5) that emerged from the Schrödinger equation and Born statistical interpretation, i.e.

$$\mathbf{x}_{cm} = \langle \mathbf{x} \rangle \quad (10)$$

Defining the average mass density as

$$\rho_{av} = \frac{m_e}{\tilde{V}_o} = \frac{\int_{\tilde{V}_o} \rho(\mathbf{x}, t) d\tilde{V}}{\tilde{V}_o} \quad (11)$$

can be used to convert the mass density  $\rho$  into a dimensionless form, i.e.  $\rho_* = \rho / \rho_{av}$  where the subscript  $*$  represents dimensionless quantities, and eventually normalizing the equation for the center of mass of the electron-fluid. Converting the volume into a dimensionless form, i.e.  $\tilde{V}_* = \tilde{V} / \tilde{V}_o$  produces  $\tilde{V}_{o*} = \tilde{V}_o / \tilde{V}_o = 1$ . Then Equation (8) becomes

$$\mathbf{x}_{cm*} = \int_{\tilde{V}_{o*}} \rho_*(\mathbf{x}_*, t_*) \mathbf{x}_* d\tilde{V}_* \quad (12)$$

The normalized Equation (12) emerging from defining the subatomic particles as compressible fluids is identical to the normalized Equation (7) that emerged from the Schrödinger equation and Born statistical interpretation, i.e.  $\mathbf{x}_{cm*} = \langle \mathbf{x} \rangle$ . The only difference is that the normalization constant (the denominator in Equation (8)), which emerged when converting Equation (8) into the normalized form (12) is equal to  $m_e$  according to Equation (9), while the normalization constant (the denominator in Equation (5)) that emerged from the Schrödinger equation and Born statistical interpretation is 1.

The results presented so far are an early indication of what is to follow, as more and more identical results will emerge. At this point it becomes appealing to *define the center of mass of the electron-fluid as the electron-particle and any future reference to electron-particle in this paper will refer to the center of mass of the electron-fluid*. With this definition and interpretation of electron-particle it becomes already clear how two particles can be found in the same region of space at the same time without having any parts of the matter (physical objects) even contacting each other. This is just the same as a small solid sphere concentrically located in the center of a larger solid torus. No material from the sphere is touching the torus but their centers of mass are in the same location in space at all times. For rigid bodies the center of mass plays an extremely important role as one may assume that all the mass is concentrated at the center of mass (in one point) defining the particle used in dynamics to describe the motion of a rigid body as long as rotation is not involved. Defining the electron-fluid center of mass (the concept can be extended to other subatomic particles) as the *electron-particle* not only removes the problem of what is the *wave-particle duality*, the latter becoming obvious by the very definition and physical description provided, but it now shows how an “intrinsic spin” can be associated with the electron-particle. While the electron-fluid can move in an orbital producing orbital angular momentum, it can also produce a spinning motion around its center of mass, which does not violate any physical reality. This is similar to how at macro-level hurricanes and tornadoes can move in space while spinning around their axes. It occurs the same way as a rigid body can have an angular momentum by orbiting around a certain point in space and at the same time spinning around its center of mass too. Actually, for a compressible fluid there are more degrees of freedom as the compressible fluid can expand/contract in each direction in addition to the spinning, the expansion/contraction not being degrees of freedom available for rigid bodies. Rigid bodies have six degrees of freedom (3 directions of motion of the center of mass and 3 rotation directions around the center of mass) while compressible fluids when treated as “particles” as presented in this paper can



have nine degrees of freedom (3 directions of motion of the center of mass, 3 rotation directions around the center of mass, and 3 directions of expansion/contraction relative to the center of mass).

## 2.2. Electron-Fluid (Subatomic-Fluid) Governing Equations

Since the electron-particle definition was introduced, it is appropriate to present the governing equations for the electron-fluid. The equations will be presented for application to the one-electron one-proton hydrogen atom. The generalization for multi-electrons multi-protons atoms as well as to molecules is straight-forward and will be presented at the end.

The equations governing the flow and electro-magnetic effects due to the motion of the electron-fluid are the inviscid Navier-Stokes equations (Euler equations) (Landau and Lifshitz [29]) from fluid dynamics and Maxwell equations (Griffiths [30], Jackson [31]) from electro-magnetism. The inviscid Navier-Stokes equations represent conservation of mass and linear momentum per unit volume of fluid. It is assumed that there are no shear stresses present, the latter causing dissipation effects and we lack evidence of such effects being present at the subatomic level at leading order. Also the electron-fluid occupies the empty space and therefore using Maxwell equations for empty space is appropriate. The electric current density (charge flux)  $\mathbf{J}_e$  [A/m<sup>2</sup>] in Maxwell equations is identical to the electric charge density multiplied by the electron-fluid velocity, i.e.  $\mathbf{J}_e = \rho_e \mathbf{v}$ , where  $\mathbf{v}(\mathbf{x}, t)$  [m/s] is the electron-fluid velocity. Consequently we have the following governing equations

(i) *(Equation of Mass Continuity (Conservation of Mass)):*

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0 \quad (13)$$

(ii) *Equation of Electric Charge Continuity (Conservation of Electric Charge):*

$$\frac{\partial \rho_e}{\partial t} + \nabla \cdot (\rho_e \mathbf{v}) = 0 \quad (14)$$

(iii) *Momentum Equation (Conservation of Linear Momentum)*

$$\rho \left[ \frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{v} \right] = -\nabla P + \rho_e \mathbf{E}_e + \rho_e \mathbf{E}_p + \rho_e (\mathbf{v} \times \mathbf{B}) \quad (15)$$

where  $\mathbf{E}_e(\mathbf{x}, t)$  [N/C] is the dependent variable representing the intrinsic electrostatic field due to forces that differential electron-fluid elements impress on each other,  $\mathbf{E}_p$  [N/C] is the electrostatic field impressed on an electron-fluid position by the nucleus's proton,  $\mathbf{B}(\mathbf{x}, t)$  [T] is the dependent variable representing the magnetic flux density, and  $P(\mathbf{x}, t)$  [Pa] is the pressure resulting from the normal (diagonal) components in the stress tensor. The combination of the terms  $\rho_e \mathbf{E}_e + \rho_e \mathbf{E}_p + \rho_e (\mathbf{v} \times \mathbf{B})$  represents the Lorentz force per unit volume. The magnetic and pressure terms play an extremely important role in preventing the indefinite spread of the wave function (or of the electron-fluid) for a free electron. The left-hand-side of Equation (15) represents the mass times acceleration per unit volume of the electron-fluid (or the material derivative of the linear momentum per unit volume). Gravitational effects due to attraction of masses following Newton law of universal gravitation are neglected, as these effects are extremely small in comparison to the electro-magnetic ones in such cases. When dealing with uncharged subatomic-particles (subatomic-fluids) gravitational effects might need to be included. In such cases the electro-magnetic terms including the Lorentz force per unit volume are to be excluded, a gravitational term  $\rho \mathbf{g}(\mathbf{x}, t)$  is to be added to Equation (15), and the equations  $\nabla \times \mathbf{g} = 0$ ,  $\nabla \cdot \mathbf{g} = -4\pi G \rho$  are to be added,

where  $G$  is the universal gravitational constant, and  $\mathbf{g}(\mathbf{x}, t)$  is the gravitational field vector. For particles moving at the speed of light, like the photons, or close to the speed of light the Navier-Stokes/Euler equations need to be amended to correctly represent relativistic effects. These cases and their corresponding equations are discussed in more detail in the Appendix A.

(iv) *Coulomb law in field form*

$$\nabla \cdot \mathbf{E}_e = \frac{1}{\epsilon_o} \rho_e \quad (16a)$$

$$\nabla \cdot \mathbf{E}_p = \frac{1}{\epsilon_o} \rho_p \quad (16b)$$

where the proton electric charge density is

$$\rho_p = \begin{cases} \frac{3q_p}{4\pi r_N^3} = \frac{3|e|}{4\pi r_N^3} & \text{for } r \leq r_N \\ 0 & \text{for } r > r_N \end{cases} \quad (17)$$

and where  $q_p = |e| = 1.6022 \cdot 10^{-19} [\text{C}]$  is the electric charge of the proton (assumed in the first instance to be homogeneously distributed within the nucleus), and  $r_N = 8.783 \cdot 10^{-16} [\text{m}]$  is the radius of the nucleus. Solving the proton electric field equation for  $\mathbf{E}_p$  in spherical coordinates, i.e.

$(1/r^2) [d(r^2 E_r)/dr] = \rho_p / \epsilon_o$  yields the familiar form of Coulomb law

$$\mathbf{E}_p = \frac{|e|}{4\pi\epsilon_o} \frac{1}{r^2} \hat{\mathbf{e}}_r \quad (18)$$

where  $\hat{\mathbf{e}}_r$  is a unit vector in the radial direction,

(v) *Ampere law*

$$\nabla \times \mathbf{B} = \mu_o \rho_e \mathbf{v} + \frac{1}{c_o^2} \frac{\partial \mathbf{E}_e}{\partial t} \quad (19)$$

where we removed the  $\partial \mathbf{E}_p / \partial t$  because it vanishes identically when using (18).

(vi) *Faraday law of induction*

$$\nabla \times \mathbf{E}_e = -\frac{\partial \mathbf{B}}{\partial t} \quad (20)$$

where the term including  $\mathbf{E}_p(r)$  was removed because  $\nabla \times \mathbf{E}_p(r) = 0$  identically when using (18).

(vii) *Gauss law for the magnetic field*

$$\nabla \cdot \mathbf{B} = 0 \quad (21)$$

Not all the equations need to be solved, as it is simple to show that satisfying Equations (16a) and (19) leads to identical satisfaction of Equation (14) as they are equivalent. Also the following additional assumption is being made

$$\beta_e = \frac{\rho}{|\rho_e|} = \frac{m_e}{|e|} = 5.685 \cdot 10^{-12} [\text{kg/C}] \quad (22)$$

Equation (22) implies that the ratio between the mass density and electric charge density is constant and equals to the ratio between the electron mass and the electron charge. The justification for this assumption lies in the fact that it is difficult to imagine the electric charge moving independently of the mass sustaining it. One cannot have an electric charge in the electromagnetic sense without a mass carrying it. If a different constitutive relationship exists, then Equation (22) will represent a first order Taylor expansion of such a relationship. A relaxation of this assumption might be needed in due course. An additional approximation is needed to reflect the constitutive relationship between the mass density and pressure assuming a barotropic compressible fluid (i.e. mass density depends on pressure only). For macro-level fluids such relationships have been established experimentally. For the electron-fluid this relationship is unknown and therefore a linear approximation is adopted in the form

$$\rho = \rho_o [1 + \beta_p (P - P_o)] \quad (23)$$

where  $P_o$  and  $\rho_o$  are reference values of pressure and mass density such that  $\rho = \rho_o$  when  $P = P_o$  and

$$\beta_p = \frac{1}{\rho_o} \frac{\partial \rho}{\partial P} = \frac{1}{\rho_o v_o^2} = \text{const.} [\text{Pa}^{-1}] \quad (24)$$

is the compression coefficient which is constant, and  $v_o = \sqrt{\partial P / \partial \rho}$  [m/s] is the constant “speed of propagation of the electron-fluid pressure wave” in analogy to compressible fluids, assuming isentropic wave propagation as customary in compressible fluid dynamics. The inverse relationship needed for substitution into the momentum Equation (15) is obtained by using Equations (23) and (24) in the form

$$P = P_o + \frac{1}{\rho_o \beta_p} (\rho - \rho_o) = P_o + v_o^2 (\rho - \rho_o) \quad (25)$$

Taking the gradient of Equation (25) leads to

$$\nabla P = \frac{1}{\rho_o \beta_p} \nabla \rho = v_o^2 \nabla \rho \quad (26)$$

Substituting (26) into the momentum Equation (15) produces the following equation

$$\rho \left[ \frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{v} \right] = -v_o^2 \nabla \rho + \rho_e \mathbf{E}_e + \rho_e \mathbf{E}_p + \rho_e (\mathbf{v} \times \mathbf{B}) \quad (27)$$

Dividing Equation (15) by  $\rho$  and using Equation (22) yields

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{v} = -\frac{\nabla P}{\rho} - \frac{1}{\beta_e} \mathbf{E}_e - \frac{1}{\beta_e} \mathbf{E}_p - \frac{1}{\beta_e} (\mathbf{v} \times \mathbf{B}) \quad (28)$$

and dividing Equation (27) by  $\rho$  and using Equation (22) yields

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{v} = -v_o^2 \frac{\nabla \rho}{\rho} - \frac{1}{\beta_e} \mathbf{E}_e - \frac{1}{\beta_e} \mathbf{E}_p - \frac{1}{\beta_e} (\mathbf{v} \times \mathbf{B}) \quad (29)$$

where

$$v_o^2 = \frac{1}{\rho_o \beta_p} \quad (30)$$

Substituting Equation (22) into Equation (29) leads to

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{v} = -v_o^2 \frac{\nabla \rho_e}{\rho_e} - \frac{1}{\beta_e} \mathbf{E}_e - \frac{1}{\beta_e} \mathbf{E}_p - \frac{1}{\beta_e} (\mathbf{v} \times \mathbf{B}) \quad (31)$$

The momentum equation in the form presented in (31) can be used with Equations (16a), (19) and (20) to find the solution of the electric field  $\mathbf{E}_e(\mathbf{x}, t)$ , charge density  $\rho_e(\mathbf{x}, t)$ , electron-fluid velocity  $\mathbf{v}(\mathbf{x}, t)$ , and the magnetic field  $\mathbf{B}(\mathbf{x}, t)$ . Then the mass density  $\rho(\mathbf{x}, t)$  can be evaluated by using Equation (22).

### 2.3. Equivalence between the Electron-Fluid Navier-Stokes/Euler Equations and the Schrödinger Equation, and the Resulting Natural Collapse of the Wave Function

Proceeding now to demonstrate the equivalence between the momentum equation in the form expressed by (28) and Schrödinger Equation (1) and introduce an extension to the Schrödinger equation that accounts for the pressure term as a potential is the next step. First, we already know that Schrödinger Equation (1) does not include magnetic effects and therefore we do not expect recovering the term  $(\mathbf{v} \times \mathbf{B})/\beta_e$ .

The complex wave function  $\psi(\mathbf{x}, t)$  can be expressed without loss of generality in the following way

$$\psi = R e^{iS/\hbar} \quad (32)$$

where  $R(\mathbf{x}, t)$ , and  $S(\mathbf{x}, t)$  are real functions. By introducing the notation

$$\rho = R^2 = \psi \psi^* \Rightarrow R = \rho^{1/2} ; \quad m \mathbf{v} = \nabla S \Rightarrow \mathbf{v} = \frac{1}{m} \nabla S \quad (33)$$

and substituting (32) and (33) into the Schrödinger Equation (1) leads to the following set of equations, derived originally by Madelung [32]

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0 \quad (34)$$

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{v} = -\frac{1}{m} \nabla V - \frac{1}{m} \nabla U \quad (35)$$

where the potential terms in Equation (35) are related to the Schrödinger Equation (1) as  $V$  is the same potential energy as it appears in Equation (1) and  $U$  was named a quantum potential energy ( $U/m$  is the quantum potential) as it did not relate to any other physical variables. Its definition emerged from the derivations via a relationship to  $\rho$  in the form

$$U = -\frac{\hbar^2}{2m} \frac{\nabla^2 \rho^{1/2}}{\rho^{1/2}} \quad (36)$$

Equation (34) is identical to the mass continuity (conservation of mass) Equation (13) for the electron-fluid. Equation (35) is very similar to the momentum Equation (28) for the electron-fluid. Since Equations (34) and (35) resemble very closely the Navier-Stokes/Euler Equations (13) and (28) Madelung [32] attempted to link the  $\nabla U$  term to the pressure gradient term  $-\nabla P/\rho$  in Equation (28) but was not successful in proving such a link. Bohm [8] used Madelung [32] equations and provided a different interpretation that evolved into the “de Broglie-Bohm pilot-wave theory”. Still the precise explicit meaning of the  $\nabla U$  term was not established. In comparing Equation (35) to

Equation (28) it becomes appealing to identify the latter term with the electron-fluid electrostatic field term  $\mathbf{E}_e/\beta_e$ . Consequently, by using (22)

$$\mathbf{E}_e = \frac{1}{|e|} \nabla U = -\frac{\hbar^2}{2m|e|} \nabla \left[ \frac{\nabla^2 \rho^{1/2}}{\rho^{1/2}} \right] \quad (37)$$

The relationship between  $\mathbf{E}_e$  and  $\rho$  provided by (37) can be checked a-posteriori once accurate solutions for  $\mathbf{E}_e$  and  $\rho$  are obtained for specific problems. However, it is evident from (37) that  $\nabla \times \mathbf{E}_e = 0$  a fact, which is consistent with the governing Equation (20) for the present case when  $\mathbf{B} = 0$ . When dealing with uncharged subatomic particles (subatomic fluids) the gravitational field vector  $\mathbf{g}(\mathbf{x}, t)$  should replace the electrostatic field vector  $\mathbf{E}_e$  in the definition of the quantum potential energy, i.e.  $\mathbf{g} = -\nabla U/m = (\hbar^2/2m^2) \nabla [\nabla^2 \rho^{1/2}/\rho^{1/2}]$ . By considering the case of the electron in the hydrogen atom one can use the electric field created by the proton as one of the potential terms in Equation (35) as  $\nabla V_1/m$ , where

$$V_1 = -\frac{e^2}{4\pi\epsilon_0} \frac{1}{r} \quad (38)$$

is the potential energy due to Coulomb attraction from the proton obtained from (18). For Equation (35) to be identical (excluding the magnetic term) to the momentum Equation (28) we can use Equation (26) to evaluate  $-\nabla P/\rho = -v_o^2 \nabla \rho/\rho = -\nabla (v_o^2 \ln \rho)$  and to define a new potential term  $\nabla V_2/m$  in Equation (35). The latter defines the new energy linked to the potential  $V_2/m = v_o^2 \ln \rho = v_o^2 \ln(\psi\psi^*)$  in the form

$$V_2 = mv_o^2 \ln \rho \underset{(33)}{=} mv_o^2 \ln(\psi\psi^*) \quad (39)$$

Since  $V = V_1 + V_2$  one can return to the Schrödinger Equation (1) and substitute these explicit potential energy functions (38) and (39) to obtain the following amended Schrödinger equation

$$i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m} \nabla^2 \psi - \frac{e^2}{4\pi\epsilon_0} \frac{\psi}{r} + mv_o^2 \psi \ln(\psi\psi^*) \quad (40)$$

that can be presented by using a characteristic length scale (such as the nucleus radius  $r_N$ ), where  $r_* = r/r_N$ , in the following additional form

$$i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m} \nabla^2 \psi + \frac{e^2}{\epsilon_0 r_N} \psi \left[ -\frac{1}{4\pi r_*} + \frac{1}{Ma^2} \ln(\psi\psi^*) \right] \quad (41)$$

where a dimensionless group emerged in the form of an electron-fluid Mach number  $Ma$ , defined as

$$Ma = \frac{v_c}{v_o} = \frac{(e^2/m\epsilon_0 r_N)^{1/2}}{v_o} \quad (42)$$

The term on the numerator of (42) is a scaling velocity  $v_c$ , and  $v_o$  was already introduced in Equations (24) and (30) as the “speed of propagation of the electron fluid pressure wave”. It is now obvious to note that when the Mach number is much larger than 1

( $Ma \gg 1$ ) the additional term introduced in this amendment to the Schrödinger equation becomes very small and can be neglected, therefore explaining how the original Schrödinger Equation (1) produces results that fit very well experimental data. However, Equation (41) is the more general



one, and this is a nonlinear equation due to the nonlinear energy  $V_2 = mv_o^2 \ln(\psi\psi^*)$  resulting from the pressure term. The magnetic term  $(\mathbf{v} \times \mathbf{B})$  from Equation (29) cannot be included in the Schrödinger equation via a scalar potential because the field that this term represents is not conservative, consequently a scalar potential associated with the term  $(\mathbf{v} \times \mathbf{B})$  does not generally exist.

Therefore, the general form of the Schrödinger equation when accounting for pressure gradients is

$$i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m} \nabla^2 \psi + V(\mathbf{x})\psi + mv_o^2 \psi \ln(\psi\psi^*) \quad (43)$$

that can be presented in a quasi-dimensionless form by using  $r_N$  as a length scale,  $(\varepsilon_o r_N \hbar / e^2)$  as a time scale, and  $(e^2 / \varepsilon_o r_N)$  as a potential energy scale leading to

$$i \frac{\partial \psi}{\partial t_*} = -\frac{1}{2} Qu \nabla_*^2 \psi + V_*(\mathbf{x}_*)\psi + \frac{1}{Ma^2} \psi \ln(\psi\psi^*) \quad (44)$$

where a new quantum dimensionless group having an explicit constant value emerged in the form  $Qu = \varepsilon_o \hbar^2 / e^2 m r_N = 4.794 \cdot 10^3$ , and the wave function is still dimensional, however dividing the whole equation by  $\rho_c^{1/2}$  will convert it into a complete dimensionless form. Experimental results [33] capturing results applicable for Mach numbers that are not large are being discussed in section 3.2 while investigating quantum jumps.

The specific detail that the definition of the velocity  $\mathbf{v}$  when converting from the Schrödinger Equation (1) to the Madelung Equation (35) has a potential, expressed by Equation (33), implies that Equation (35) is in fact the Euler equation for potential flow applicable to ideal fluids, a special case of Navier-Stokes equations. There is no need in using this explicit constraint at this stage. If needed it might be used in forthcoming derivations.

## 2.4. Summary of the Results from this Section

### 2.4.1. Equivalence of the Electron-Fluid Governing Equations and the Schrödinger Equation

Equations (40) and (41) demonstrate that *the electron-fluid Navier-Stokes/Euler governing equations and the Schrödinger equation are identical for large Mach numbers and when excluding the magnetic effects. This provides overwhelming evidence that subatomic particles obeying Schrödinger equation are in fact compressible fluids.*

### 2.4.2. Collapse of the Wave Function

Furthermore, since the general equation is non-linear it immediately forbids the application of superposition of individual eigenstates as a general solution. Even if linearization is performed as an approximation, for large Mach numbers for example, superposition of all modes is not possible because the linear solution is expected to match a nonlinear solution as Mach number is reduced gradually. A similar situation is experienced in natural convection where linear stability solutions are quite accurate however no superposition of eigenmodes is acceptable for the same reason despite early attempts in doing that, which were proven incorrect (Newell and Whitehead [34], Chandrasekhar [35], Malkus and Veronis [36], Segel [37], Daniels [38], Koshmieder [39]). Addition of a finite number of linearized solutions is consistent as an approximation. What is not allowed is presenting the “general solution” as a superposition of infinite modes. The postulate in the current mainstream quantum theory states that upon observation (measurement) the wave function collapses from a superposition of infinite modes to one single mode representing the eigenstate that was observed. This collapse of the wave function is not obtained as a solution to the problem but rather

imposed in order to comply with experimental results. Actually, there is no experimental result so far that supports the superposition of eigenstates. Therefore, excluding superposition of eigenmodes is consistent with experimental results. A weak nonlinear solution based on the linear stability presented in Part II of this sequence of papers is also not consistent with such a superposition.

*The fact that only one eigenstate is allowed, and superposition as a general solution is not an option demonstrates the way the wave function collapsed naturally, because of the nonlinear character of the generalized Schrödinger equation.*

### 3. Additional Effects Resulting from the Electron-Fluid Governing Equations – “Quantum Jump” at “Irregular Time Intervals”, “Intrinsic Spin”, and “spread-Less Electron-Fluid” in Free Space

#### 3.1. Properties of the Governing Equations

Having introduced in the previous sections the electron-fluid center of mass and showing that it is identical to the expectation of finding a subatomic particle in a specific location (average position) suggested the interpretation that *the electron or other subatomic “particles” are in reality compressible fluids*, and this interpretation is plausible. The latter also demonstrated the *wave-particle duality* since the electron-fluid center of mass can be regarded as the *electron-particle* and then the electron can be seen as a wave when looking at the dynamics of the *electron-fluid*, and as a particle when looking at the motion of its center of mass. Furthermore, the latter provided a clear classical explanation of how a *zero angular momentum at the ground state* does not conflict with physical reality and *the electron-particle represented by the motion of the center of mass of the electron-fluid can cross the nucleus while all the electron-fluid being outside the nucleus itself*. Having introduced the electron-fluid Navier-Stokes/Euler and Maxwell governing equations, and having provided proof that the Navier-Stokes/Euler and the Schrödinger equations are equivalent for large Mach numbers when excluding magnetic effects, allowed us to *confirm that the electrons (and other subatomic “particles”) do behave like compressible fluids*. The latter also demonstrated how *the wave function collapses naturally as part of the solution*. The additional effects that emerge require the solution of the governing equations of the electron-fluid. The latter is undertaken and presented in the next following parts of this series of papers. However, it is instrumental for now to present how these additional effects can possibly occur from the solution of these equations even before solving them. In doing so the presentation will identify the set of equations that are to be solved as we indicated earlier that there is some equivalence and overlap between some of the governing equations presented in section 2.2. Also, since the governing equations are coupled, i.e. more than one dependent variable appears in most equations it is tempting to attempt some uncoupling for simplicity.

In the first instance the focus is on Equations (16a) and (19). Applying the divergence ( $\nabla \cdot$ ) operator on Equation (19) yields

$$\underbrace{\nabla \cdot (\nabla \times \mathbf{B})}_{=0} = \mu_o \nabla \cdot (\rho_e \mathbf{v}) + \frac{1}{c_o^2} \frac{\partial (\nabla \cdot \mathbf{E}_e)}{\partial t}$$

the left-hand side term being identically zero since the divergence of a curl of any vector is always zero, producing by using the identity  $c_o^2 = 1/\mu_o \epsilon_o$

$$0 = \mu_o \left[ \nabla \cdot (\rho_e \mathbf{v}) + \epsilon_o \frac{\partial (\nabla \cdot \mathbf{E}_e)}{\partial t} \right] \quad (45)$$

Substituting the term  $\nabla \cdot \mathbf{E}_e$  from Equation (16a) into Equation (45) leads to

$$\nabla \cdot (\rho_e \mathbf{v}) + \frac{\partial \rho_e}{\partial t} = 0 \quad (46)$$

Equation (46) is identical to Equation (14) for the conservation of electric charge. Therefore, when using Equations (16a) and (19) there is no need to use Equation (14) as the latter will be identically

satisfied too. Since the adoption of Equation (22) relating the electric charge density to the mass density, i.e.  $\rho = -\beta_e \rho_e$ , where  $\beta_e = m_e/|e| = 5.685 \cdot 10^{-12} [\text{kg/C}]$ , one can evaluate directly the mass density  $\rho$  once the electric charge density solution is established. This also guarantees that Equation (13) for the conservation of mass is satisfied too, a fact revealed by replacing Equation (22) into Equation (14) to obtain Equation (13).

We therefore need to solve Equations (16a), (19), (20), and (31), that represent 10 scalar equations for the unknown dependent variables  $\rho_e(\mathbf{x}, t)$ ,  $\mathbf{v}(\mathbf{x}, t)$ ,  $\mathbf{E}_e(\mathbf{x}, t)$ , and  $\mathbf{B}(\mathbf{x}, t)$ , representing 10 scalar variables. In the absence of a magnetic field, the latter being typically very weak (a second order effect that leads to the “fine structure” [24, 25]) the electron-fluid Equations (16a), (19), (20), and (31) become

$$\nabla \cdot \mathbf{E}_e = \frac{1}{\epsilon_o} \rho_e \quad (47)$$

$$\epsilon_o \frac{\partial \mathbf{E}_e}{\partial t} + \rho_e \mathbf{v} = 0 \quad (48)$$

$$\nabla \times \mathbf{E}_e = 0 \quad (49)$$

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{v} = v_o^2 \nabla (\ln |\rho_e|) - \frac{1}{\beta_e} \mathbf{E}_e - \frac{1}{\beta_e} \mathbf{E}_p \quad (50)$$

where  $\mathbf{E}_p = (|e|/4\pi\epsilon_o)(1/r^2)\hat{\mathbf{e}}_r$  based on (18), where  $\hat{\mathbf{e}}_r$  is a unit vector in the radial direction. Taking the time derivative of Equation (50) and substituting Equation (48) in the result, yields

$$\frac{\partial^2 \mathbf{v}}{\partial t^2} + \frac{\partial}{\partial t} [(\mathbf{v} \cdot \nabla) \mathbf{v}] = v_o^2 \frac{\partial}{\partial t} [\nabla (\ln |\rho_e|)] + \frac{1}{\epsilon_o \beta_e} \rho_e \mathbf{v} \quad (51)$$

Equations (51) and (46) can be solved together to produce solutions for  $\rho_e(\mathbf{x}, t)$  and  $\mathbf{v}(\mathbf{x}, t)$ . The latter can be substituted into Equation (50) to obtain the solution for  $\mathbf{E}_e(\mathbf{x}, t)$ . Equation (51) is a nonlinear wave equation, mathematically of a hyperbolic type.

### 3.2. Quantum Jump as Shock Wave and Experimental Confirmation

Dividing the whole Equation (51) by the square of a scaling velocity  $v_c^2 = (e^2/m_e\epsilon_o r_N)$ , where  $r_N$  is a characteristic length scale such as the nucleus radius, and introducing a characteristic time  $t_c = (m_e\epsilon_o r_N^3/e^2)^{1/2}$  and a characteristic charge density  $\rho_{ec} = |e|/r_N^3$  renders Equation (51) into a dimensionless form

$$\frac{\partial^2 \mathbf{v}_*}{\partial t_*^2} + \frac{\partial}{\partial t_*} [(\mathbf{v}_* \cdot \nabla_*) \mathbf{v}_*] = \frac{1}{Ma^2} \frac{\partial}{\partial t_*} [\nabla_* (\ln |\rho_{e*}|)] + \rho_{e*} \mathbf{v}_* \quad (52)$$

where  $Ma = v_c/v_o = (e^2/m_e\epsilon_o r_N)^{1/2}/v_o$  is the electron-fluid Mach number. Equation (46) converted in a dimensionless form is

$$\nabla_* \cdot (\rho_{e*} \mathbf{v}_*) + \frac{\partial \rho_{e*}}{\partial t_*} = 0 \quad (53)$$

Equation (52) is a nonlinear wave equation for compressible flow, mathematically of a hyperbolic type. It is well known to produce shock waves for certain values of the Mach number  $Ma$ . This occurs in particular when excited by a source term added to the right-hand-side of Equation (52). Such shock waves are the accurate description of a quantum jump, i.e. a sudden transition from one stable orbital to another stable orbital. The electron-fluid experiences a sudden change in the values of the dependent variables  $\rho_e(\mathbf{x}, t)$ ,  $\mathbf{v}(\mathbf{x}, t)$ , and consequently  $\rho(\mathbf{x}, t)$ ,  $E_e(\mathbf{x}, t)$ ,  $P(\mathbf{x}, t)$ , and  $\mathbf{x}_{cm}$ . While the electron does move through the intermediate space in between the orbitals in such a shock wave transition, recent experimental evidence provided by Minev *et al.* [33] reveals the latter by “catching and reversing a quantum jump mid-flight”. They conclude that “the experimental results demonstrate that the evolution of each completed jump is continuous, coherent, and deterministic” over short time scales, while it can be “unpredictable and discrete” over long time-scales. As demonstrated by the properties of Equation (52), shock waves, i.e. continuous transitions over an extremely short period of time that seem discontinuous (singular) over larger time scales can and do occur. The solution to Equation (52) when a shock wave occurs might indeed be discontinuous, however knowing the behavior of shock waves from fluid dynamics permits us to anticipate a continuous transition if a narrow boundary layer is located on both sides of the shock wave. Shock waves can occur from this equation even in the limit of  $Ma \gg 1$ . The equation then is still hyperbolic and is known to produce shocks especially when excited via external forcing. The fact that such transitions occur at irregular time intervals is explained by the fact that the nonlinear Equation (52) may produce chaotic solutions (possibly turbulent) that lead to transitions at irregular time intervals. Since the equation considered for such transitions is excited by including a source term to the right-hand-side of Equation (52), usually a periodic function of time, the latter may produce also non-linear resonances in the solution.

### 3.3. Intrinsic Spin

The possibility of intrinsic spin in addition to the orbital angular momentum was introduced at the end of section 2.1. This possibility is demonstrated explicitly by considering the velocity of the electron-fluid relative to the center of mass. By using Galilean transformations this motion relative to the center of mass is described as follows in spherical coordinates

$$\mathbf{v}_{e/cm} = \mathbf{v} - \mathbf{v}_{cm} = (\dot{r} - \dot{r}_{cm})\hat{\mathbf{e}}_r + (r\dot{\theta} - r_{cm}\dot{\theta}_{cm})\hat{\mathbf{e}}_\theta + (r\dot{\phi} - r_{cm}\dot{\phi}_{cm})\sin\theta\hat{\mathbf{e}}_\phi \quad (54)$$

where  $\hat{\mathbf{e}}_r, \hat{\mathbf{e}}_\theta$  and  $\hat{\mathbf{e}}_\phi$  are unit vectors in the  $r, \theta$  and  $\phi$  directions, respectively,  $\dot{r}, \dot{\theta}, \dot{\phi}$  are time derivatives of  $r, \theta, \phi$  by using Newton notation and similarly for their center of mass variables,  $\mathbf{v}_{e/cm}$  being the velocity relative to the center of mass,  $\mathbf{v}_{cm}$  being the velocity of the center of mass, and  $\mathbf{v}$  being the velocity of the electron-fluid. Equation (54) shows that it is possible to have a radially pulsating motion of the electron-fluid relative to the orbital motion of the center of mass, in addition to the azimuthal and polar motion of the electron fluid relative to the center of mass. The azimuthal and polar relative motions are identical to the spin of a rigid body, and can be associated with the quantum-electron-spin. By focusing for simplicity on the motion on an  $r - \phi$  plane at  $\theta = \pi/2$  one can present the linear momentum relative to the center of mass in the form

$$\mathbf{p}_{lcm} = p_{r/cm}\hat{\mathbf{e}}_r + p_{\phi/cm}\hat{\mathbf{e}}_\phi \quad (55a)$$

$$p_{r/cm} = \rho(v_r - v_{r,cm}) \quad ; \quad p_{\phi/cm} = \rho(v_\phi - v_{\phi,cm}) = (r\dot{\phi} - r_{cm}\dot{\phi}_{cm}) \quad (55b)$$

and the intrinsic spin, i.e. the angular momentum associated with this motion relative to the center of mass, is

$$\mathbf{L}_s = (\mathbf{r} - \mathbf{r}_{cm}) \times \mathbf{p}_{cm} = (r - r_{cm})\hat{\mathbf{e}}_r \times p_{cm} = (r - r_{cm})(r\dot{\phi} - r_{cm}\dot{\phi}_{cm})\hat{\mathbf{e}}_\theta \quad (56)$$

This can be related to the vorticity of the electron-fluid relative to the center of mass, i.e.  $\zeta = \nabla \times (\mathbf{v} - \mathbf{v}_{cm})$ . When defining the spin then the integral of  $\zeta$  over the corresponding volume is required. The spin direction may then be established by  $\hat{\mathbf{e}}_\zeta = \zeta/|\zeta|$ . How exactly this electron-spin becomes either  $+1/2$  or  $-1/2$  is not possible to explain at this stage. The solution to the problem is anticipated to provide the answer to this question. Examples of spinning fluids at the macro-level are hurricanes and tornadoes that move in space while also rotating around their axes. There, the spin is caused by the Coriolis effect via the Coriolis acceleration term which when moved to the right-hand-side of the momentum equation has the form  $[-2(\mathbf{v} \times \boldsymbol{\Omega})]$ , where  $\boldsymbol{\Omega}$  is the angular velocity of rotation of the reference frame. The electron-fluid spin is triggered by the magnetic field, i.e. explicitly by the term  $[-\beta_e^{-1}(\mathbf{v} \times \mathbf{B})]$ , in complete analogy to the rotating fluid. Therefore, it is now obvious that such an “intrinsic-spin” associated with the electron-fluid and electron-particle (i.e. the center of mass) is possible without violating any classical principles and with a very explicit classical visualization.

### 3.4. Spread-Less Electron-Fluid in Free Space

Placing an electron-fluid in free space implies that no other external forces are impressed upon such an electron-fluid. Then the momentum Equation (31) will take the form

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{v} = -v_o^2 \frac{\nabla \rho_e}{\rho_e} - \frac{1}{\beta_e} \mathbf{E}_e - \frac{1}{\beta_e} (\mathbf{v} \times \mathbf{B}) \quad (57)$$

where the electrostatic field due to the proton was eliminated.

Equation (57) is to be solved together with Equations (16a), (19), and (20). First, it can be established that since there is no possible length scale associated with the free space no discrete eigenvalues can be expected even from a linearized version of this problem. Consequently, no quantization is expected, a result that is identical to current mainstream quantum theory since a free electron can take on a continuous range of energies, and when accelerated can emit radiation with a continuous frequency spectrum (Ballentine [40], p.43). Based on the properties of these equations one can anticipate the following dynamics scenario. First, even without the magnetic effect the pressure distribution represented by the term  $\nabla \rho_e / \rho_e$  can balance the electric field leading to a spread-less stationary state. Then, assuming for simplicity that the free electron-fluid is distributed in a spherically-symmetric way within an initial spherical volume implies that there is no mass density nor any field variations in the  $\phi$  and  $\theta$  directions. Let us assume also for simplicity that the electron-fluid is initially at rest, i.e. the initial velocity of the electron-fluid is zero. As the initial intrinsic electrostatic field is  $\mathbf{E}_e = E_{e,r}(r)\hat{\mathbf{e}}_r$ , the differential fluid elements may start repelling each other causing a radial expansion (electron-fluid spread) with a velocity  $\mathbf{v} = v_r(r,t)\hat{\mathbf{e}}_r$  and with a resulting time dependent charge density  $\rho_e(r,t)$ . This electron-fluid velocity is actually an electric current,  $\mathbf{J}_e = \rho_e \mathbf{v}$  being the corresponding electric current density of the electron-fluid, that varies in time creating an induced magnetic field that builds up, i.e.  $\partial \mathbf{B} / \partial t \neq 0$ . Consequently this leads from Equation (20) to the variation of the initial intrinsic electrostatic field  $\mathbf{E}_e$  inducing an electric field that according to the Lenz law opposes the increase in the current  $\rho_e \mathbf{v}$  (velocity, or charge density, or both). This eventually stops the radial expansion and causes a motion in the opposite direction, i.e. radial compression, moving beyond its initial position due to inertial effects. The recoil force of this magnetic induction according to Lenz law balances and even exceeds the tendency of the electron-fluid electrostatic repulsion. The resulting motion is anticipated to be one of pulsating radial expansion and contraction due to these opposing forces, the intrinsic electrostatic force of the electron-fluid and the recoil force due to Lenz law. This analysis of the governing equations demonstrates how an initial spread of the electron-fluid is reduced to a pulsating (expansion-



compression) motion in finite space due to magnetic effects. To this, one needs to add also the spinning effect that the magnetic field causes as discussed in the previous section. The conclusion is that a free electron-fluid does not spread indefinitely.

### 3.5. Entanglement

In this section the objective is not to describe in detail the entanglement phenomenon. The latter is left for a separate future presentation. The aim here is to show that the information transfer between the entangled “particles” does not occur in a superluminal fashion and is not a “spooky action at a distance” but rather the local measurement of a global property. The latter emerges because the proposed deterministic quantum mechanics is not only non-local but also global in the sense that it involves properties that are affected by the bulk of the corresponding volume and not only by its local values.

The EPR paradox (Einstein, Podolsky, Rosen [28]) was introduced in 1935 as a thought (gedanken) experiment that implied that either information is transferred at superluminal speeds, or the statistical quantum theory is incomplete and some local “hidden variables” exist and need to be discovered. In 1964 Bell [1] introduced and analyzed a system based on measurements on pairs of entangled electrons, photons, or other atomic or subatomic “particles”, that was related to the EPR paradox. His results were expressed in the form of an inequality (Bell’s inequality) that if satisfied will support the EPR view that the statistical quantum theory is incomplete and local “hidden variables” exist. Violation of Bell’s inequality implies that local “hidden variables” do not exist. Bell [41] and separately Kochen and Specker [2] complemented Bell’s theorem [1] by introducing the concept of contextuality, the latter being related to the incompatibility of quantum theory observables. Contextuality means that the outcome of an experiment by observer “A” depends on the decision of observer “B” of what to measure. Freedman and Clauser [42] presented the first test of Bell’s theorem showing the violation of Bell’s inequality. It was followed by Aspect *et al.* [43] that removed some of the loopholes in Freedman and Clauser [42] experimental test and reinforced the latter results violating Bell’s inequality. By 2015 significant loophole-free tests were reported, such as Hensen *et al.* [44], Shalm *et al.* [45], Giustina *et al.* [46], confirming the violation of Bell’s inequality and therefore providing overwhelming evidence that local “hidden variables” do not exist. Clauser *et al.* [47] introduced an inequality on local hidden variables and Gröblacher *et al.* [48] presented an inequality “similar in spirit to the seminal one given by Clauser *et al.* [47], that allowed them “to test an important class of non-local hidden-variable theories.” Their derivation was “based on a recent incompatibility theorem by Leggett [49]”, which was “extended to make it applicable to real experimental situations and also to allow simultaneous tests of all local hidden-variable models”. Then, they “perform an experiment that violates the new inequality and hence excludes for the first time a broad class of non-local hidden-variable theories.”

As stated already, the proposed deterministic quantum mechanics theory is not only non-local but also global as described above, in the sense that it involves global properties that are defined over a finite volume rather than at a pointwise location. These are properties that are affected by the bulk of the corresponding volume and not only by their local values. Consequently Bell theorem and its variations as well as Leggett [49], Clauser *et al.* [47], and Gröblacher *et al.* [48] theorems do not apply to the *global* results of the proposed deterministic quantum mechanics. Spin of a single subatomic particle was shown in section 3.3 to be a global property of the electron-fluid relative to the center of mass, which will be instantaneously affected by any change occurring within the respective volume. When two such particles are entangled, it would usually be difficult to even distinguish between what region of space belongs to each of the particles if they are regarded as compressible fluids. It would be typically difficult to establish the boundary between them. In the case of the entanglement experiments such a distinction becomes possible because the particles move (waves propagate) in opposite directions. Nevertheless, once the particles are entangled, they share a common center of mass and if their initial spin was in opposite directions, it implies a zero global angular momentum of the entangled pair that is associated with the common center of mass. Any change in the spin of one of the particles should conserve the global angular momentum, unless a torque is applied on one, or the other, or both entangled particles. However, an inviscid barotropic fluid as adopted in the present theory cannot sustain torques and consequently the angular momentum is always conserved. For an electron in an atomic orbital one needs to distinguish between the orbital angular momentum and the one corresponding to the spin. However, in the case of Bell-type entanglement experiments

the particles move (waves propagate) in a straight line, and there is no ambiguity regarding the fact that the angular momentum is associated only with the spin. The global angular momentum is affected instantaneously by any change in the spin of either individual particles due to the very definition of the total angular momentum being the integral of the local angular momentum over the volume occupied by each or both particles. If  $\mathbf{L}_f(\mathbf{x}, t)$  is the local angular momentum of the entangled pair of electrons-fluid, while  $\tilde{V}_A$  and  $\tilde{V}_B$  are the volumes occupied by electrons-fluid “A” and “B”, respectively, then the total angular momenta for each of the electrons-fluid and for the entangled pair are

$$\mathbf{L}_A = \int_{\tilde{V}_A} \mathbf{L}_f d\tilde{V} ; \quad \mathbf{L}_B = \int_{\tilde{V}_B} \mathbf{L}_f d\tilde{V} ; \quad \mathbf{L} = \int_{\tilde{V}} \mathbf{L}_f d\tilde{V} \quad (58)$$

where  $\tilde{V} = \tilde{V}_A + \tilde{V}_B$ . From (58) it follows that

$$\mathbf{L} = \int_{\tilde{V}} \mathbf{L}_f d\tilde{V} = \int_{\tilde{V}_A} \mathbf{L}_f d\tilde{V} + \int_{\tilde{V}_B} \mathbf{L}_f d\tilde{V} \quad (59)$$

and therefore

$$\mathbf{L} = \mathbf{L}_A + \mathbf{L}_B \quad (60)$$

Since the total angular momentum of the entangled pair is conserved  $\mathbf{L} = \text{constant}$ , and therefore

$$\mathbf{L}_B = \mathbf{L} - \mathbf{L}_A \quad (61)$$

implies that any change in  $\mathbf{L}_A$  affects instantaneously the value of  $\mathbf{L}_B$ . In the case that initially  $\mathbf{L}_A$  was spin “up” and  $\mathbf{L}_B$  was spin “down” the total angular momentum  $\mathbf{L}$  of the entangled pair will be zero. Then changing the spin of “A” to “down” (e.g. via a measurement that affects the measured object, or any other way), instantaneously causes the spin of “B” to be “up” in order to comply with conservation of angular momentum. The conclusion is that the reason for the instantaneous “communication” between the two entangled electrons (or photons) is the fact that the measurement of the spin (polarization in the case of photons) consists of measuring locally a global property (just like measuring the global property of hydrostatic pressure of a fluid column at a chosen local point). This also settles the issue of consistency of the proposed deterministic quantum mechanics with the Kochen-Specker theorem [2, 41] as the result of measurement of  $\mathbf{L}_B$  for example depends on the decision of whether to measure  $\mathbf{L}_A$ , or any other observable that does not affect  $\mathbf{L}$  as  $\mathbf{L}_A$  does. This demonstrates how information does not transfer at superluminal speeds and the fact that the measurement of the entangled “particle” shows an instantaneous change is nothing more than a change due to the definition of the property being measured, i.e. a global property. Therefore the result is not “spooky action at a distance”.

How precisely these effects related to entanglement occur in detail, i.e. its mechanism, requires the solution of the governing equations, which will be presented in follow-up papers. This section aimed at demonstrating the consistency of the proposed deterministic quantum mechanics theory with the results of the Bell-type entanglement experiments, and it did so by introducing the distinction between global, local and non-local properties.

### 3. Conclusions

The conceptual framework for a deterministic quantum mechanics theory was introduced in this first paper of the series dealing with the same topic. The only postulate made was the fact that the electron, and most likely other subatomic particles are compressible fluids. The proposed theory was shown to be consistent with the current mainstream statistical quantum theory as the associated governing equations were shown to lead to Schrödinger equation in the limit of high Mach numbers and when magnetic effects were neglected. Since the governing equations were derived from first classical principles the proposed theory is also consistent with classical physics. Causality, physical reality, and determinism were restored and simple explanations of the concerns that were raised by

the current mainstream statistical quantum theory results were provided. The meaning of particle-wave duality and complementarity was addressed by introducing the definition of the electron-fluid center of mass as the electron-particle, while the electron-fluid itself represents the wave behavior. The possibility of a particle, like the electron, to cross through the nucleus as it does when the angular momentum of the electron is zero at the ground state of the hydrogen atom was shown as classically meaningful by using the definition of the electron-fluid center of mass as the electron-particle. How a point-size particle can have an “intrinsic spin” by using the same definition and associating the spin to the angular momentum of the electron-fluid relative to this center of mass was revealed too. The possibility of “quantum jumps” as the electron transitions suddenly from one stable orbital to another was demonstrated as a shock wave that is typical for compressible flows and was shown to be completely consistent with recent experimental results that captured the electron in its transition motion from one stable orbital to another. The fact that the latter occurs at irregular time intervals was associated to chaotic solutions of this nonlinear compressible fluid problem and nonlinear resonances were indicated as possible solutions too. The natural collapse of the wave function was shown to be part of the solution. The phenomenon of entanglement was shown to be consistent with the proposed deterministic quantum mechanics theory. Part II of the series of papers on this topic focuses on the analytical solution to the electron-fluid governing equations (Navier-Stokes/Euler and Maxwell equations) for the hydrogen atom. Extensions and generalizations of the governing equations to apply to multiple-electron multiple-proton systems and to molecules are provided there too.

**Acknowledgement:** The author wishes to thank the National Science Foundation for supporting this research under grant no. CBET-1804251.

## Appendix A

This appendix is focused on discussing additional issues that are related to quantum mechanics effects, which can be assessed in detail by eventually solving the governing equations, but one can indicate at this stage what would be these governing equations and how are they to be solved. In section 2.2 reference was made to cases involving uncharged subatomic particles (subatomic-fluids). In such cases, the governing equations are

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho v) = 0 \quad (\text{A-1})$$

$$\rho \left[ \frac{\partial v}{\partial t} + (v \cdot \nabla) v \right] = -\nabla P + \rho g \quad (\text{A-2})$$

$$\nabla \times g = 0 \quad (\text{A-3})$$

$$\nabla \cdot g = -4\pi G \rho \quad (\text{A-4})$$

where  $g(x, t)$  is the gravitational field vector, and  $G$  is the universal gravitational constant.

$$\nabla \cdot E = 0 \quad (\text{A-5})$$

$$\nabla \times B = \frac{1}{c_o^2} \frac{\partial E}{\partial t} \quad (\text{A-6})$$

$$\nabla \times E = -\frac{\partial B}{\partial t} \quad (\text{A-7})$$

$$\nabla \cdot \mathbf{B} = 0 \quad (\text{A-8})$$

Note that the coupling between electromagnetic and fluid dynamics effects disappeared. Consequently, by applying the curl ( $\nabla \times$ ) operator on Equations (A-6), using the operator identity  $\nabla \times \nabla \times \mathbf{B} = \nabla(\nabla \cdot \mathbf{B}) - \nabla^2 \mathbf{B}$  and Equations (A-8) and (A-7) yields

$$\frac{1}{c_o^2} \frac{\partial^2 \mathbf{B}}{\partial t^2} - \nabla^2 \mathbf{B} = 0 \quad (\text{A-9})$$

and applying the same operator on Equation (A-7) and using Equations (A-5) and (A-6) leads to

$$\frac{1}{c_o^2} \frac{\partial^2 \mathbf{E}}{\partial t^2} - \nabla^2 \mathbf{E} = 0 \quad (\text{A-10})$$

Equations (A-9) and (A-10) are the classical wave equations and indicate that the electromagnetic field are waves that propagate at the speed of light  $c_o$ .

For subatomic particles (subatomic-fluids) that move (propagate) at the speed of light, like photons, or close to the speed of light, the Navier-Stokes/Euler Equations (A-1), (A-2), and (A-3) might need to be amended to correctly represent relativistic effects and become Lorentz invariant. Nevertheless, Vadasz [50] has shown that the inviscid Navier-Stokes/Euler equations subject to a continuous gravitational field are indeed Lorentz invariant under certain conditions. Maxwell equations are already Lorentz invariant so one does not need to amend them.

For dealing with the problem of modeling the creation and annihilation of particles one uses still the same equations as presented in section 2.2 however removing the proton effect from the governing equations and allowing the charge density to be either positive or negative in different regions of space. This way for example, the annihilation of an electron-positron pair can be modeled to produce two photons. An example of how these equations can lead to the charge density to change its sign is presented in part II of these sequence of papers. Since the anticipated speeds in such a problem are expected to be close and equal to the speed of light a conversion of these equations to correctly represent relativistic effects and become Lorentz invariant is needed here too (see Vadasz [50]). Feynman diagrams can be used for such problems with the amendment of replacing the process used in quantum electrodynamics in such a case with the process of applying the Navier-Stokes/Euler and Maxwell set of equations as indicated above. Similar procedures apply for any other subatomic particle interactions. Finally, the specific answer on how precisely these processes occur in detail is to be expected from the solutions to the governing equations that were introduced.

## References

1. Bell, J.S. "On the Einstein Podolsky Rosen paradox", *Physics*, **Vol. 1** (3), pp. 195-200, 1964.
2. Kochen, S., Specker, E.P., "The problem of hidden variables in quantum mechanics", *Journal of Mathematics and Mechanics*, **Vol. 17** (1), pp. 59-87, 1967.
3. 't Hooft, G., "Equivalence relations between deterministic and quantum mechanical systems", *Journal of Statistical Physics*, **Vol. 53** (1/2), pp. 323-344, 1988.
4. 't Hooft, G., "Quantum gravity as a dissipative system", *Classical and Quantum Gravity*, **Vol. 16** (10), pp. 3263-3279, 1999.
5. 't Hooft, G., "A mathematical theory for deterministic quantum mechanics", *Journal of Physics: Conference Series*, **Vol. 67**, pp. 012015(1-15), 2007.
6. 't Hooft, G., "How a wave function can collapse without violating Schroedinger's equation, and how to understand Born's rule", arXiv:1112.1811v3, 2012.
7. 't Hooft, G., "Deterministic quantum mechanics", In: *The Cellular Automaton Interpretation of Quantum Mechanics. Fundamental Theories of Physics*, **Vol. 185**, Springer, Cham, pp. 49-56, 2016.
8. Bohm, D., "A suggested interpretation of quantum theory in terms of 'hidden variables' I", *Physical Review*, **Vol. 85** (2), pp. 166-179, 1952.
9. Planck, M., "Ueber das Gesetz der Energieverteilung in Normalspectrum" (translation: On the law of distribution of energy in the normal spectrum), *Annalen der Physik*, **Vol. 4** (3), p. 553-563, 1901.



10. Einstein, A., "Über einen die Erzeugung und Verwandlung des Lichtes betreffenden heuristischen Gesichtspunkt", *Annalen der Physik*, **Vol. 322** (6), pp. 132-148, 1905.
11. Einstein, A., "Concerning an Heuristic Point of View Toward the Emission and Transformation of Light"-English translation of *Annalen der Physik*, **Vol. 322** (6), pp. 132-148, 1905, in *American Journal of Physics*, Vol. 33 (5), pp. 367-374, 1965.
12. Bohr, N., "On the constitution of atoms and molecules, Part I", *Philosophical Magazine*, **Vol. 26**, pp. 1-24, 1913.
13. Bohr, N., "On the constitution of atoms and molecules, Part II Systems containing a single nucleus", *Philosophical Magazine*, **Vol. 26**, pp. 476-502, 1913.
14. Bohr, N., "On the constitution of atoms and molecules, Part III Systems containing several nuclei", *Philosophical Magazine*, **Vol. 26**, pp. 857-875, 1913.
15. Schrödinger, E., "Quantisierung als Eigenwertproblem", *Annalen der Physik*, **Vol. 384** (4), pp. 361-376, 1926. (Part I).
16. Schrödinger, E., "Quantisierung als Eigenwertproblem", *Annalen der Physik*, **Vol. 384** (6), pp. 489-527, 1926. (Part II).
17. Schrödinger, E., "Quantisierung als Eigenwertproblem", *Annalen der Physik*, **Vol. 385** (13), pp. 437-490, 1926. (Part III).
18. Schrödinger, E., "Quantisierung als Eigenwertproblem", *Annalen der Physik*, **Vol. 386** (18), pp. 109-139, 1926. (Part IV).
19. Schrödinger, E., "21 Collected Papers on Wave Mechanics", New York: Chelsea Publishing Company, 1982.
20. Bohr, N., "The spectra of helium and hydrogen", *Nature*, **Vol. 92**, pp. 231-232, 1914.
21. Bohr, N., "Atomic structure", *Nature*, **Vol. 106**, pp. 104-107, 1921.
22. Eisberg, R. Resnick, R., "Quantum physics of atoms, molecules, solids, nuclei, and particles", 2<sup>nd</sup> edition, John Wiley & Sons Inc., New York, Chichester, Brisbane, Toronto, Singapore, 1985, p. 254.
23. Bohm, D., "Quantum Theory", Prentice Hall Inc., 1951, (David Bohm 1979, Dover publication 1989, p.44.)
24. Baggot, J., "The meaning of quantum theory", Oxford University Press Inc., New York, 1992 (reprinted 2004).
25. Griffiths, D.J., "Introduction to Quantum Mechanics", Second Edition, Pearson Education Inc., Upper Saddle River, NJ, 2005, p. 145-146.
26. Born, M., "Quantenmechanik der Stoßvorgänge", *Zeitschrift für Physik*, **Vol. 38**, pp. 803-827, 1926.
27. Dirac, P.A.M., "The quantum theory of the electron", *Proceedings of the Royal Society A: Mathematical, Physical, and Engineering Sciences*, **Vol. 117** (778), pp. 610-624, 1928.
28. Einstein, A., Podolsky, B. and Rosen, N., "Can quantum-mechanical description of physical reality be considered complete?", *Physical Review*, **Vol. 47**, pp. 777-780, 1935.
29. Landau, L.D., Lifshitz, E.M., "Fluid Mechanics", translated from Russian by Skyes, J.B., and Reid, W.H., Pergamon Press, Oxford, New York, Toronto, Sydney, Paris, Frankfurt, 1978.
30. Griffiths, D. J., "Introduction to Electrodynamics", Third Edition, Prentice Hall, Inc., Upper Saddle River NJ, 1999.
31. Jackson, J.D., "Classical Electrodynamics", Third Edition, John Wiley & Sons, Inc., 1999.
32. Madelung, E., "Quantum theory in hydrodynamical form", *Zeitschrift für Physik*, **Vol. 40** (3-4), pp. 322-326, 1926-1927, English translation (translated by D.H. Delphenich), pp.863-867, 1926.
33. Mineev, Z.K., Mundhada, S.O., Shankar, S., Reinhold, P., Gutiérrez-Jáuregui, R., Schoelkopf, R.J., Mirrahimi, M., Carmichael, H.J., and Devoret, M.H., "To catch and reverse a quantum jump mid-flight", *Nature*, **Vol. 570**, pp. 200-204, 2019.
34. Newell, A.C., Whitehead, J.A., "Finite bandwidth, finite amplitude convection", *Journal of Fluid Mechanics*, **Vol. 38** (2), pp. 279-303, 1969.
35. Chandrasekhar, S., "Hydrodynamic and Hydromagnetic Stability", Oxford University Press, 1961, Dover edition, 1981.
36. Malkus, W.V.R. and Veronis, G., "Finite amplitude cellular convection", *Journal of Fluid Mechanics*, **Vol. 4** (3), pp. 225-260, 1958.
37. Segel, L.A., "Distant side-walls cause slow amplitude modulation of cellular convection", *Journal of Fluid Mechanics*, **Vol. 38** (1), pp. 203-224, 1969.
38. Daniels, P.G. "Roll pattern evolution in finite amplitude Rayleigh-Benard convection in a two-dimensional fluid layer bounded by distant sidewalls", *Journal of Fluid Mechanics*, **Vol. 143**, pp. 125-152, 1984.
39. Koschmieder, E.L., "Benard Cells and Taylor Vortices", Cambridge University Press, 1993.
40. Ballentine, L.E., "Quantum Mechanics – A Modern Development", World Scientific Publishing Co. Pte. Ltd., Singapore, Hachensack NJ, London, 2008.
41. Bell, S.J., "On the problem of hidden variables in quantum mechanics", *Review of Modern Physics*, **38** (3), pp. 447-452, 1966.
42. Freedman, S.J., Clauser, J.F., "Experimental test of local hidden-variable theories", *Physical Review Letters*, **Vol. 28** (14), pp. 938-941, 1972.

43. Aspect, A., Dalibard, J., Roger, G., "Experimental test of Bells' inequalities using time-varying analyzers", *Physical Review Letters*, **Vol. 49** (25), pp. 1804-1807, 1982.
44. Hensen, B., Bernien, H., Dréau, A.E., Reiserer, A., Kalb, N., Blok, M.S., Ruitenberg, J., Vermeulen, R.F.L., Schouten, R.N., Abellán, C., Amaya, W., Pruneri, V., Mitchell, M.W., Markham, M., Twitchen, D.J., Elkouss, D., Wehner, S., Taminiau, T.H., Hanson, R., "Loophole-free Bell inequality violation using electron spins separated by 1.3 kilometers", *Nature*, **Vol. 526**, pp. 682-286, 2015.
45. Shalm, L.K., Meyer-Scott, E., Christensen, B.G., Bierhorst, P., Wayne, M.A., Stevens, M.J., Gerrits, T., Glancy, S., Hamel, D.R., Allman, M.S., Coakley, K.J., Dyer, S.D., Hodge, C., Lita, A.E., Verma, V.B., Lambrocco, C., Tortorici, E., Migdall, A.L., Zhang, Y., Kumor, D.R., Farr, W.H., Marsili, F., Shaw, M.D., Stern, J.A., Abellán, C., Amaya, W., Pruneri, V., Jennewein, T., Mitchell, M.W., Kwiat, P.G., Bienfang, J.C., Mirin, R.P., Knill, E., Sae Woo Nam, "Strong loophole-free test of local realism", *Physical Review Letters*, **Vol. 115** (25), pp. 250402 (1-10), 2015.
46. Giustina, M., Versteegh, M.A.M., Wengerowsky, S., Handsteiner, J., Hochrainer, A., Phelan, K., Steinlechner, F., Kofler, J., Larsson, J.-Å., Abellán, C., Amaya, W., Pruneri, V., Mitchell, M.W., Beyer, J., Gerrits, T., Lita, A.E., Shalm, L.K., Sae Woo Nam, Scheidl, T., Ursin, R., Wittmann, B., Zeilinger, A., "Significant-loophole-free test of Bell's theorem with entangled photons", *Physical Review Letters*, **Vol. 115** (25), pp. 250401 (1-7), 2015.
47. Clauser, J.F., Horne, M.A., Shimony, A., Holt, R.A., "Proposed experiment to test local hidden-variable theories", *Physical Review Letters*, **Vol. 23** (15), pp. 880-884, 1969.
48. Gröblacher, S., Paterek, T., Kaltenbaek, R., Brukner, C., Żukowski, M., Aspelmeyer, Zeilinger, A., "An experimental test of non-local realism", *Nature*, **Vol. 446** (7138), pp. 871-875, 2007.
49. Leggett, A.J., "Nonlocal hidden-variable theories and quantum mechanics: an incompatibility theorem", *Foundations of Physics*, **Vol. 33** (10), pp. 1469-1493, 2003.
50. Vadasz, P., Newtonian Gravitational Waves from a Continuum, *Proc. Royal Society of London A*, **480** (2289), pp. 20230656 (1-13), <https://doi.org/10.1098/rspa.2023.0656>, 2024.

**Disclaimer/Publisher's Note:** The statements, opinions and data contained in all publications are solely those of the individual author(s) and contributor(s) and not of MDPI and/or the editor(s). MDPI and/or the editor(s) disclaim responsibility for any injury to people or property resulting from any ideas, methods, instructions or products referred to in the content.