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Optimization of a Dynamic Supply Chain Network: Kinetic Modeling of E-Waste Plants

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Abstract: E-waste management (EWM) refers to the operation-management of discarded or unproductive electronic devices and components, a challenge exacerbated due to overindulgent urbanization. This article presents a multidimensional cost-function-based analysis of the EWM framework structured on three modules - environmental, economic, and social uncertainties - which contemplate the 3 pillars of sustainability in an e-waste recycling plant, including the production-delivery-utilization process. The framework incorporates material recovery from a single e-waste facility provisioning for chemical and mechanical recycling. Each module is ranked using independent Machine Learning (ML) protocols: a) Analytical Hierarchical Process (AHP) and b) combined AHP and Principal Component Analysis (PCA). From a long list of possible contributors, the model identifies and ranks two key sustainability contributors to the EWM supply chain: overall energy consumption and volume of carbon dioxide generated. Another key finding is a precise time window for policy resurrection, which for the data considered, happens to be 400-600 days from the start of operation. Another interesting outcome is the quality of prediction using a combination of AHP and PCA, which consistently produced better results than any of these ML methods individually implemented. Model outcomes have been verified using a case study to outline a future E-waste sustained roadmap.

Keywords: supply chain management; E-waste management; cleaner production; multiple criteria analysis; global optimization

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1. Introduction

E-waste management (EWM) is a worldwide challenge. Core issues related to EWM resonate with other general waste management (Oteng-Ababio 2010) coasters with the critical added complexity of a burgeoning production-usage cycle that seems to grow unabated, unlike many other waste management sectors. Several factors such as energy efficiency, carbon footprint, availability of e-waste, stable supply, heterogeneity of e-waste, technology management and overall administration of disposables and reutilization in a feedback loop may drastically affect the supply chain network. To add to the challenge, these factors mostly evolve in an asynchronized aperiodic mode, collective effects which relate to randomized time evolution of the mechanics of EWM, that we mathematically refer to as uncertainties in the relevant supply chain network.

E-waste management is a modern lucrative business sector, particularly given expansive urbanization worldwide. Fast life and transient expectations of outcomes often identify

success as the ability to access or procure expensive electronic devices, markets of which are continuously evolving and catering to ever-increasing demands from customers. Such demands have escalated manifold in the past decade (Debnath 2020). Primarily, the supply-to-demand ratio targeted devices with short innovation cycles, delivered through intelligent marketing strategies and made affordable through lucrative prices with eye candy features that can attract more customers than a competitor. A tacit underlier has been the ever-decreasing life span of electronic items (Debnath 2019), a marketing accessory embedded in the supply-chain structure to prevent market stagnation. This is the primary reason for higher rates of product obsolescence in the electronic industry, as laterally accepted by the United Nations as well, in what is referred to as a 'tsunami of electronic waste' (news.un.org 2015).

As suggested by Kumar et al. (2017), this is the world's fastest growing waste stream (Kumar et al. 2017). Some obvious facts will clarify this. Worldwide generation of e-waste was nearly 53.6 million metric tonnes in 2019 that is expected to reach 74.7 million metric tonnes by 2030, and 120 million metric tonnes by 2050 (Forti et al. 2020; PACE 2019). E-waste is a very heterogeneous material and it contains an assortment of materials including metals, polymers, siliceous materials, including glass (Widemar et al. 2005; Wath et al. 2011). Such discarded e-waste streams thus trap a huge amount of metallic and non-metallic resources that relegates urban mining of e-waste not an option anymore, but rather a necessity ((Debnath et al. 2019). Waste contains a greater volume of metals than natural ores and hence it becomes much lucrative to recover metals from e-waste ((Debnath et al. 2018). The past five years highlight another alarming statistic. Electronic discards are becoming increasingly lighter, making them more carriable, simply because the amount of plastic waste is increasing in number and volume whereas the volume of metals being used is decreasing (Debnath 2019).

Industrially, the resource recovery of e-waste is limited to mechanical recycling and in some cases, recovered metal is transported to sister companies or third-party smelters (Khaliq et al. 2014; Debnath et al. 2018). Here, we need to recall an important feature of EWM, unlike most other waste management protocols. The management of e-waste is globally handled mostly by informal sectors which further compounds the problem as appropriate technologies are often not used, even when available, either due to lack of resources or intention. This is even more so in developing nations (Ghosh et al. 2016). The informal sectors often employ inefficient and rudimentary technologies to extract copper, gold etc. from e-waste (Debnath 2020). This is one of the primary reasons for the non-circularity of materials recovered from the e-waste. This results in an inefficient supply chain which is not a closed loop incumbency, thus affecting business cycles in the longer run. The erratic and uncontrolled use of technology and disposal schemes attribute a strong stochastic element to the e-waste business. Typical examples of this relate to heterogeneous material, hazardous waste disposal, energy efficiency, secondary emissions, recovery efficiency, supply uncertainty, etc. The sustainability of the e-waste business depends upon the sustainability and efficiency of the supply chain network. In other words, the supply chain network and its flexibility play a pivotal role in determining the profitability profile of an e-waste recycling plant. The following subsections outline published literature on the e-waste supply chain network, its sustainability and related issues.

1.1. E-waste Supply Chain Network and Supply Chain Sustainability

1.1.1. General literature on E-waste SCN and Application of MCDM techniques

The SCN of e-waste is a technically complex routine that is even more interesting due to the versatility of the supply chain dynamics (Debnath 2019). There is a broad selection of literature available on e-waste SCN. Hazra et al. (2011) analyze e-waste supply chain issues in India, which serves as a template for the e-waste menace faced by developing countries. Streicher-Porte et al. (2005) have utilized the concept of the supply chain to study material flow analysis. Though the study enabled the material flow paths of recycled materials from e-waste, it failed to ensure a generalized supply chain structure. Sharma et al. (2008) developed an Analytic Hierarchy Process (AHP) multi-criteria decision-making methodology for optimizing supply chain delivery network which accounts for both qualitative and quantitative factors.

Ciocoiu et al. (2011) presented the AHP) as a potential decision-making tool for the implementation of e-waste management systems considering political, economic, social, technical and environmental issues. Lin et al. (2011) used AHP to analyze different criteria of notebook PC supply chain management (SCM) and proposed a Sensitivity Model for effective deployment of SCM

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strategies. Yang et al. (2012) show the two-stage deployment of Quality Function Deployment (QFD) a Japanese Quality Management tool, for constructing the strategic adjustment model in a WEEE management system. Chen et al. (2012) have investigated the inventory management problem for a double-ended fluctuation in the e-waste recycling supply chain. Rakheja et al. (2013) analyze the risk issues in supply chain management using AHP. Ghosh et al. (2014) have highlighted issues and challenges in the e-waste supply chain in India using the QFD tool. The study identified formal e-waste collection, technology availability and storage of the e-waste as the most challenging issue for e-waste supply chain stakeholders.

1.1.2. Issues and Challenges in E-waste SCN

The issue of a sustainable e-waste supply chain has been recently dealt with by Barletta et al. (2015), focusing on the prerequisites of a sustainable e-waste recycling plant. The outcome of this study primarily addressed requirements from the perspective of production and environmental engineers although some of that could qualify as operational management issues as well. Debnath et al. (2015) presented a comprehensive and comparative review of the Waste Electrical and Electronic Equipment (WEEE) management system in India, the UK, and Switzerland. They focus on the issues and challenges of the supply chain network and the legislation.

The first pioneering work in the context of e-waste supply chain issues and challenges was reported by Ghosh et al. (2016). Their study was limited to BRICS nations, yet the issues raised by them were surprisingly generic and not limited to BRICS requirements alone. They projected compliance with the Basel Convention and the transboundary movement of e-waste as the primary issue. Additionally, issues with informal collection of e-waste, crude processing of e-waste, inefficiency of formal collection networks, extended producer responsibility, etc were highlighted as major issues. Cruz-Sotelo et al. (2017) mapped the e-waste supply chain in Mexico and presented a legal framework for sustainable e-waste management in Mexico. Baidya et al. (2019) identified the drawbacks of the e-waste supply chain in India using Analytic Hierarchical Process (AHP) and a sustainable framework was proposed.

1.1.3. Application of Mathematical Modelling of E-waste SCN

From the mathematical modeling perspective, Isernia et al. (2019) analyzed the efficiency of the reverse supply chain of e-waste in Italy within a circular economy framework. They relied on the probability transition matrix method to evaluate the collection efficiency of the collection centers with the threshold targets defined by the European Union. Debnath (2019) mapped the high-level supply chain network of e-waste in India and described the path of material flow and the path of e-waste in detail. This work also identified the prevalent practice of repairing and refurbishing electronic items before the final decision of disposal. Doan et al. (2019) presented a detailed review of the e-waste supply chain and provided future perspectives. Polat et al. (2019) employed fuzzy mathematics to model an e-waste supply chain network considering sales prices, product weights, costs and product demands as the fuzzy parameters which reflect the uncertainties in the real-life model much more efficiently. The model developed by Wang et al. (2019) is structured on a three-echelon game theoretic supply chain model that estimated the optimal pricing decision and government subsidies with stakeholders, considering e-waste remanufacturing utilization rate as a key parameter. Lara et al. (2019) developed a reverse logistics metamodel that they validated against the e-waste management sector in Colombia with Extended Producer Responsibility (EPR) and Shared Responsibility (SR) as two policy components. Ghalehkhondabi et al. (2020) used game theory to model a hazardous waste supply chain considering the disposal facility and the contractor as the players' underpricing and environmental uncertainty. Ghalehkhondabi and Ardjmand (2020) used a game theory approach to model an e-waste supply chain with three players: the government, a recycling center, and a collection center and analyzed different scenarios of material recovery, sustainability, and supply chain profits. Baidya et al. (2020) have used a combined AHP-QFD method to prioritize different issues and challenges prevailing along the e-waste supply chain. They validated their findings through two case studies from two developing countries – India and China. The study compared the

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supply chain networks of India and China and discussed the sustainability aspects qualitatively. Debnath (2020) later developed a generic global e-waste supply chain network that led to an in-depth discussion on the supply side; demand side and internal operations side of the SCN.

1.2. Research Gap and Objectives

The e-waste market is stochastic and susceptible to market volatility. The uncertainties along the Supply Chain Network (SCN) are different from an SME and need further consideration. The stochastic fluctuations reflect uncertainties along the production and distribution process. It is important to identify the most vulnerable nodes to maximize the profit. The supply ambiguity of electronic waste can certainly affect the profitability trend and it needs immediate attention. An e-waste recycling plant has more economic constraints to abide by environmental regulations, an aspect that contributes towards environmental uncertainty. However, this does not rule out the effects of other uncertainties – economic and social uncertainty.

Supply chain literature is replete with examples of applications of Machine & Deep Learning (ML), including Artificial Intelligence (AI) (Zhu et al. 2019; Baryannis et al. 2019). Mixed integer nonlinear programming (Sadeghi et al. 2019); fuzzy mathematics (Salehi et al. 2020); robust optimization (Gholizadeh et al. 2020); intelligent algorithms (Jemmali et al. 2019); scenario programming (Habib et al. 2020); stochastic programming (Nur et al. 2020); multivariate multi-layered AI (Rajesh 2020) have all been used to analyze stochastically driven supply chains. While even e-waste prognosis under an ML/ AI route is not an entirely virgin territory, we need to understand that structurally an e-waste supply chain network is a reverse flow network. Such networks are inherently complex and heterogeneous (Suyabatmaz et al. 2014) and require special treatment and specific models (Suyabatmaz et al. 2014; Lara et al. 2019). Typical examples include but are not limited to the implementation of game theory (Ghalehkhondabi et al. 2020); system dynamics modelling approach (Guo et al. 2018); development of meta-models (Lara et al. 2019); fuzzy mathematics (Polat et al. 2019); combined MCDM techniques (Baidya et al. 2020) etc. Since the reverse supply chain network for electronic waste is distinctively different from any other type of generic supply chain network or that of an SME, there is a clear research gap that can integrate the time evolution of uncertainties evolving from three separate pillars of sustainability i.e., environmental, economic and social amalgamated in a unified framework.

The eventual choice of adapting any of the above lies in the hands of the supply chain manager or the decision maker. Our model integrates all possible cost components based on the 'Utilization-to-cradle' regime. One can add or remove components in the cost function and rework the problem. If it is redundant, then the dimension of the resulting matrix in equations 10 and 13 will be reduced. The choice is ultimately with the concerned recycler, as, after all, they know their SCN best. This inherent self-sufficiency is the beauty aka the novelty-cum-flexibility of the model.

Supply Chain Networks can be addressed from the perspective of Life Cycle Assessment (LCA) and the range can vary based on the approach taken, such as cradle-to-cradle (Neiro et al. 2016), cradle-to-gate (Westfall et al. 2016), gate-to-gate (Colley et al. 2020) etc. Figure 1 illustrates different boundaries of the supply chain based on LCA concepts. In this study, we consider a generalized version of an e-waste SCN, starting from the consumers and ending with the 3rd party recyclers handling the recovered materials from e-waste, translating to a "utilization-to-cradle" model. Under the current investigation, we have extended a recent method developed by Chattopadhyay et al. (2020) for the optimization of a supply chain cost kernel with uncertainty components affecting it. The underlying technique extrapolates the knowledge base from the physics of mechanics, economics of operations, and the mathematics of stochastic processes to analyze a time-dynamically evolving 'free energy' model. The model used in this present work structurally follows a similar mathematical description as in of Chattopadhyay et al. (2020) but intrinsically differs in the definition of the basic cost function. Three modes of uncertainty, respectively from the three pillars of sustainability, have been used as inputs in our model. The individual (time-evolving) contributions from the three 'supply lines' are then ranked first through an independent AHP, followed by a cross-verification and benchmarking through Principal Component Analysis (PCA). Impacts of unconstrained and

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constrained environments are then separately analyzed as case studies within this structure, leading to a clear identification of the operational windows for a green SCN.

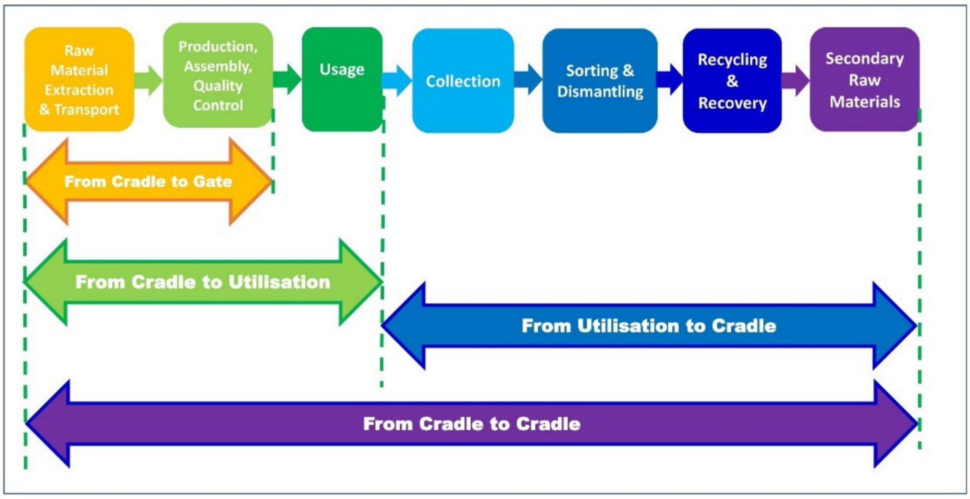

Figure 1. Boundaries of Supply Chain based on LCA.

The optimization of a cost kernel is a central feature of many models reviewed in the previous section. Thus, a proper specification of such a cost kernel is quite central to the modeling effort. That cost function specification has two parts, a theoretical justification, and an empirical counterpart. In this study, we draw upon the business economic theories of duality in multi-product translog production and its dual translog cost function to justify a quadratic cost function. The empirical analysis of production and cost functions received a big boost with the pioneering contributions by Diewert and McFadden (Diewert (1971), and McFadden (1978) establishing the duality between them. The actual specification must reflect the intricate production structure with the underlying separability of production and distribution activities into a hierarchical network. To capture that hierarchical pattern and to derive its specification empirically, we employ the Analytic Hierarchy Process (AHP). The target here is to rank the affecting variables in order of their contribution (through PCA) so that the reweighed cost matrix depicts functional real income against expenditure.

2. Materials and Methods

This study suggests a new e-waste management model outlined in Chattopadhyay et al. (2020). The current investigation exclusively develops a cost-function-based model specifically for the e-waste supply chain network, complementing three uncertainty modules that affect the supply chain sustainability, each weighted by its weight.

The uniqueness of the study is that we have used two popular Machine Learning algorithms i.e., AHP and PCA and integrated them to develop a new hybrid AHP-PCA method for ranking the uncertainty variables. The hybrid AHP-PCA method uses the best of both methods. The algorithm is detailed later in section 3.2.3. The developed model is then converted into a dynamic-constrained problem with the introduction of Hamiltonian and Lagrange Multipliers. The set of equations evolved out of this exercise are simultaneously solved for both constrained and unconstrained cases using MATLAB (bvp4c) with boundary conditions imitating different situations of an e-waste recycling plant. Real-life data obtained from an anonymous e-waste recycling plant is used for validation purposes. The overall methodology has been outlined in the running flowchart depicted in Figure 2.

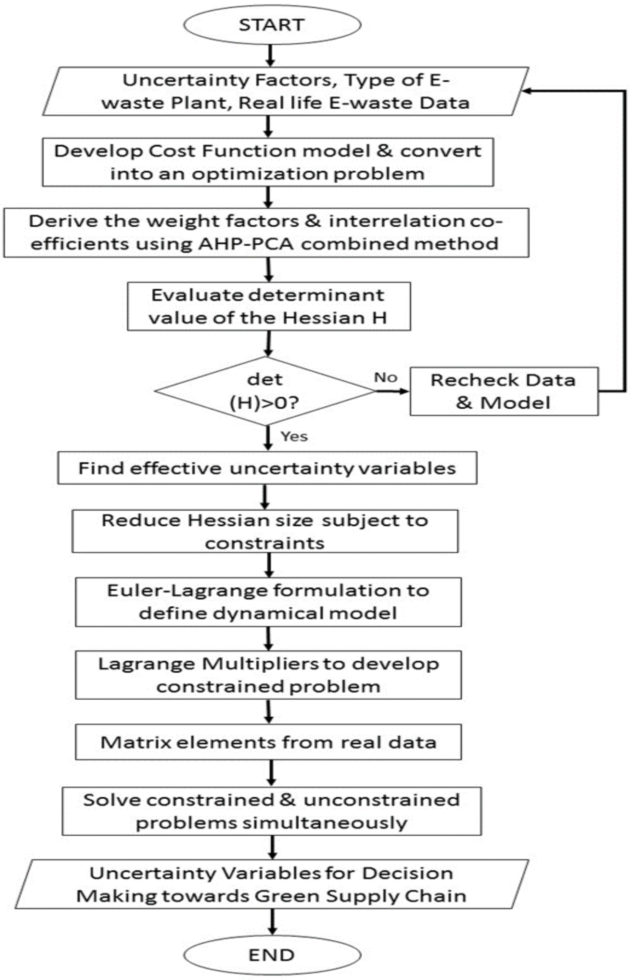

Figure 2. Working Flowchart of the problem-to-solution approach.

3. Mathematics and Modelling

3.1. Modelling Approach

3.1.1. Model Assumptions

The assumptions considered in the model are stated below –

i) Numbers are recorded daily, totaling 300 working days and 10 working hours daily.
ii) The model is developed considering a Material Recovery from E-waste (MREW) (Debnath et al. 2018) facility performing both mechanical recycling and material recovery (wet processes).
iii) The cost of recycled products remains constant over time.
iv) Unit costs remain constant.
v) Legislative costs and costs towards disposal of hazardous materials in a TSDF remain constant annually.
vi) The interdependency of the dependent variables has been assumed to be quadratic order accuracy.

3.1.2. Model Descriptions

The model uses three stochastic 'forces' of volatility as inputs, each of which pertains to the e-waste SCN derived from the three pillars of sustainability (Chattopadyay et al. 2020). We assume, as suggested by the duality theory of production and cost (McFadden (1978)), a quadratic cost-function-based model, where minimization of the cost function kernel defines the time dynamics of the flow

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(Eq. 1). The structure resembles that of Eulerian mechanics (Goldstein 1964) where the cost function plays the role of a 'free energy' potential whose optimized dynamics leads to the paradigmatic Euler-Lagrange model.

The uncertainty in the system arises from different segments of production and distribution functions, starting from uncertainty in the input markets and ending up with the final demand for the main outputs of the firm. We can specify a multiproduct, multi-input production relation as follows:

$$g(q_1,q_2,\ldots.q_s)=f(r_1,r_2,\ldots.r_t;\ l_1,l_2,\ \ldots l_p;\ k_1,k_2,\ldots k_u;\ e), \qquad (1)$$

where $q_1,q_2,\ldots.q_n$ are n different outputs produced by the firm including waste products, $r_1,r_2,\ldots.r_t$ are the different raw materials used in production, $l_1,l_2, \ldots l_p$ are p types of human inputs, $k_1,k_2,\ldots k_u$ are u types of capital goods used in production. Finally, e is the energy input. This relationship is linear in parameters and nonlinear in variables where the left and right sides of Eq. (1) are linear in parameters and are chosen to maximize the canonical correlation (Vinod (1968). Using Shephard's duality, Diewert made a pioneering contribution in establishing a duality relationship between production and cost functions, a translog function being the most prominent one (McFadden (1978). It is a quadratic approximation of the function expressing the logarithm of outputs as logarithms of all the inputs, keeping one of the outputs, such as the main output of the firm as the reference or numeraire commodity. Pulley and Braunstein (1992) specify a quadratic function in logarithms for a multiproduct cost function. Expression in logarithms is a typical choice by economists as it happens to generalize a very popular production function in economics that is just linear in logarithms. The quadratic cost function can be decomposed into three components environmental, social, and economic. At each level of decomposition, it could incorporate an uncertain element. The following quadratic cost function can thus be treated as drawn from the duality relations between production and cost underlying the production of the SMEs we are examining.

3.1.3. Nomenclature

We employ the following nomenclature:

Cost components

F = Cost Function

$C_{Environment}$ = Cost function component for Environmental Uncertainty

C_{Social} = Cost function component for Social Uncertainty

$C_{Economic}$ = Cost function component for Economic Uncertainty

Variables and Parameters

V_{CO2} = Volume of CO_2 generated

E_C = Energy Consumption in the processes involved

W_P = Water used due to the processes involved

W_w = Wastewater generated in the process

N_1 = Number of labourers

N_3 = Number of awareness activities, e.g., adaptation to information, invisible e-waste, repair substituting new

N_4 = No. of recycled products sold

N_5 = No. of operations involved

N_7 = No. of Logistics involved

N_8 = No. of waste materials being send to Treatment, Storage and Disposal Facility (TSDF)

N_9 = No. of Taxes to be paid

f_1 = Unit cost for CO_2 recovery

f_2 = Unit cost of energy used

f_3 = Unit cost for water used

f_4 = Unit cost of wastewater treatment

f_5 = Salary of one labor

f_6 = Average cost of awareness activity

f_7 = Unit revenue earned from product sold

f_8 = Unit cost of each operation
f_{10} = Unit cost of logistics
f_{11} = Unit cost for disposal in TSDF
f_{12} = Unit cost of Taxes
Weight factors
ϵ_i's = Weigh factor for the four cost functions
A_i's = Weigh factor for the main parameters
a_i's, a_{ij}'s & a_i's = Interdependency values for V_{CO2}
b_i's, b_{ij}'s & b_i's = Interdependency values for E_C
c_i's = Interdependency values for W_P
d_i's, d_{ij}'s & d_i's = Interdependency values for W_w
α_i's, α_{ij}'s & α_i's = Interdependency values for N_3
β_i's, β_{ij}'s & β_i's = Interdependency values for N_4
γ = Interdependency value for N_7

$$F = C_{Environmental} + C_{Social} + C_{Economic}, \text{ where} \quad (2)$$

$$C_{Environment} = \sum V_{CO_2} f_1 + \sum E_c\, f_2 + \zeta(\sum W_P\, f_3 + \sum W_w\, f_4) \quad (3)$$

$$C_{Social} = \sum N_1 f_5 + \sum N_3\, f_6 \quad (4)$$

$$C_{Economic} = \sum N_4 f_7 - \sum N_5\, f_8 - \sum N_7\, f_{10} - \sum N_8\, f_{11} - \sum N_9\, f_{12} \quad (5)$$

This model has been developed for an e-waste supply chain network. The topic relates to key environmental concerns. The three basic cost functions are a collection of variables from Environmental, Economic and Social aspects. Eq. 3 collectively represents the cost function of the variables that affect the environment directly i.e., due to the operations, there is environmental impact. In Eq. 5, the cost function of the collective economic variables is defined. For example, No. of Recycled Product (N_4), here the variable is basically "product", the adjective "recycled" is added because the product is a yield from the recycling activities. Just like No. of Products was there in our earlier model. Again, No. of waste materials being sent to the Treatment, Storage and Disposal Facility (TSDF) (N_8), is an economic indicator because the landfilling in TSDF is chargeable. This cost has been considered in the economic part. Similarly, the social cost function includes two components, i.e., the workers part and the awareness generation part. The overall e-waste awareness is low in India which is a big problem. Companies whose data have been used in this study conduct regular awareness activities in schools, colleges and social media, which is the social aspect in the case of e-waste management.

The three function modules outlined in Equations (3) and (4) are derived from the three pillars of sustainability, namely environmental uncertainty (Eq. 3); social uncertainty (Eq. 4) and economic uncertainty (Eq. 5). Each uncertainty function module consists of a linear combination of two or more variables affecting the e-waste supply chain sustainability, categorized as environmental, social or economic uncertainty respectively. These variables have been categorically chosen to study and understand each major and minor perturbation along the e-waste SCN. These variables cover a wide range of aspects as they unify the e-waste pollution impacts as a single component module in the utility function. Also, socioeconomic factors are addressed within the same framework, all technically constrained by the need to maximize green supply chain deliverables.

Combined with the weight factors (represented by the $\epsilon_{i's}$ & A_i's) derived from AHP and the combined AHP-PCA method (as detailed later), the cost function takes the form below –

$$F = \epsilon_1\left(\sum_{i=1}^{N} A_1 V_{CO_2} f_1 + \sum_{i=1}^{N} A_2 E_c f_2 + \sum_{i=1}^{N} \zeta A_3 W_P f_3 + \sum_{i=1}^{N} \zeta A_4 W_w f_4\right) + \epsilon_2\left(\sum_{i=1}^{N} A_5 N_1 f_5 + \sum_{i=1}^{N} A_6 N_3 f_6\right) + \epsilon_3\left(\sum_{i=1}^{N} A_7 N_4 f_7 - \sum_{i=1}^{N} A_8 N_5 f_8 - \sum_{i=1}^{N} A_9 N_7 f_{10} - \sum_{i=1}^{N} A_{10} N_8 f_{11} - \sum_{i=1}^{N} A_{11} N_9 f_{12}\right) \quad (6)$$

Here ζ is a variable whose value is zero when the e-waste plant performs mechanical recycling only, whereas when the e-waste plant is a MREW facility, the value is fixed at unity.

The interdependencies of the variables are expressed as a linear combination of the dependent variables with quadratic accuracy (Nelder 1977). Equations 7a – 7g represent the mathematical expressions for the interdependency of the variables.

$$V_{CO_2} = V_{CO_2}(N_5, N_7) = a_1 N_5 + a_2 N_7 + a_{12} N_5 N_7 + a_1' N_5^2 + a_2' N_7^2 \quad (7a)$$

$$E_C = E_C(N_4, N_5) = b_1N_4 + b_2N_5 + b_{12}N_4N_5 + b'_1N_4^2 + b'_2N_5^2 \quad (7b)$$
$$W_P = W_P(N_5) = W_P^o + c_1N_5 + c_2N_5^2 \quad (7c)$$
$$W_W = W_W(N_5, W_P) = d_1N_5 + d_2W_P + d_{12}N_5W_P + d'_1N_5^2 + d'_2W_P^2 \quad (7d)$$
$$N_3 = N_3(N_4, N_9) = \alpha_1N_4 + \alpha_2N_9 + \alpha_{12}N_4N_9 + \alpha'_1N_4^2 + \alpha'_2N_9^2 \quad (7e)$$
$$N_4 = N_4(E_C, N_5) = \beta_1E_C + \beta_2N_5 + \beta_{12}E_CN_9 + \beta'_1E_C^2 + \beta'_2N_5^2 \quad (7f)$$
$$N_7 = N_7(V_{CO_2}) = \gamma V_{CO_2} \quad (7g)$$

3.2. Uncertainty Analysis

The entire premise of an e-waste portfolio is based on stochastic uncertainty modules that do not allow for absolute prediction of future values. This necessitates appropriate probabilistic approaches to first rank the key contributors and then analyze their interdependence. This is done using a combination of the Analytic Hierarchy Process (AHP) and the Principal Component Analysis (PCA).

3.2.1. Analytical Hierarchical Process (AHP)

In this paper, two exclusive AHP analyzes have been carried out similar to Chattopadhyay et al. (2020). However, the structural specifications differ as the present focus is on e-waste SCN. The first AHP (Figure 3) model addresses three criteria enumerating the three uncertainties occurring from the three pillars of sustainability; the key (three) variables are linked through eleven criteria nodes. The second is a layered AHP (Figure 4) that utilizes the same criteria as the first but has two layers of alternatives. The layers are created in such a way that the structure not only connects the alternatives with the criteria but also the individual interdependencies of the alternatives. The first layer of alternatives consists of those variables that have dependencies on the variables in the second layer (function of a function, i.e., a functional). The second AHP is executed to find the interdependencies whereas the first AHP is designed to rank the variables. A registered student version of the commercial software package "Super Decision" is used for the AHP calculations (http://www.superdecisions.com/). The determination of compound and square interdependencies follows the methodology of our previous work. The detailed AHP flowchart is outlined below:

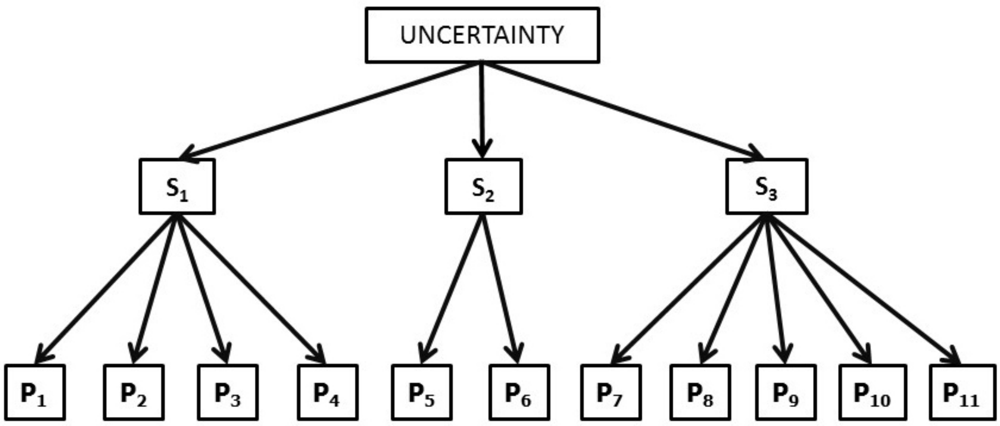

Figure 3. AHP model 1 to determine the general alternative rankings. Environmental uncertainty (S_1), Social uncertainty (S_2), and Economic uncertainty (S_3) are in the criteria layer. Volume of CO_2 generated (P_1), Energy Consumption in the processes involved (P_2), Water used due to the processes involved (P_3), Wastewater generated in the whole process (P_4) are the alternatives connected to S_1; No. of laborers (P_5), No. of awareness activities (P_6) are the alternatives connected to S_2; No. of recycled products sold (P_7), No. of operations involved (P_8), No. of logistics involved (P_9), No. of waste materials being send to Treatment, Storage and Disposal Facility (TSDF) (P_{10}), No. of taxes (P_{11}) are the alternatives connected to S_3.

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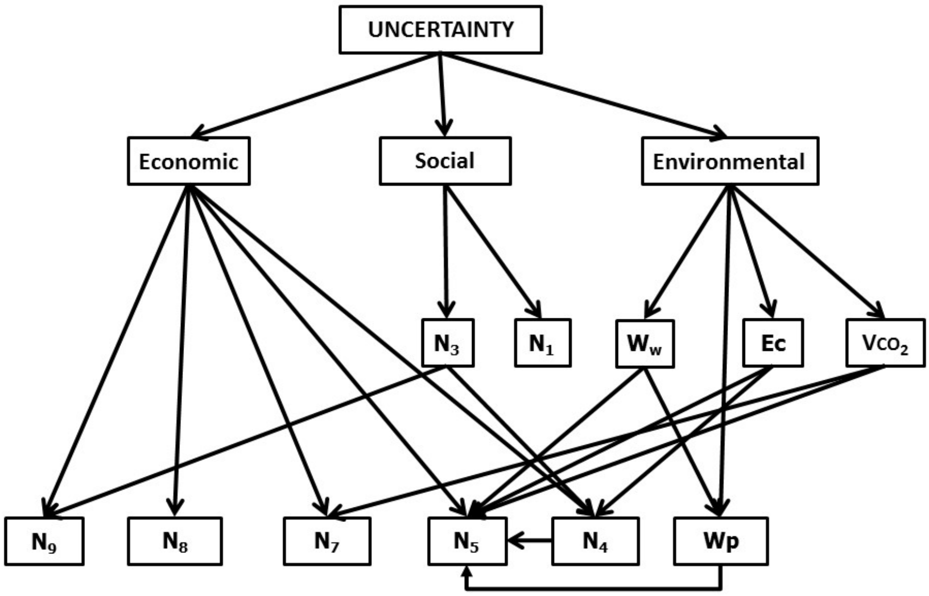

Figure 4. Layered AHP model for determination of interrelationship values.

3.2.2. Multivariate Study - Principal Component Analysis (PCA)

Principal component analysis (PCA) is perhaps the primogenial and one of the well-known multivariate analysis techniques. PCA was first introduced by Pearson (1901) and later developed by Hotelling (1933) independently (Jolliffe 2002). The fundamental idea of PCA is to reduce the dimensionality of a huge data set with interrelated variables, increasing interpretability while retaining maximum information (Jolliffe and Cadima 2016). The methodology involves a transformation of the original dataset to a new set of variables aka the principal components (PCs), which are uncorrelated, and ranked where the top few PCs capture the maximum variation present in all the original variables (Jolliffe 2002).

In the context of supply chains, the PCA has been used for a wide range of multivariate analyzes, e.g., damage and fault detection (Pozo and Vidal 2018), hypothesis testing (Pozo and Vidal 2018), constrained PCA-based method development (Takane 2013), Chemometrics (Kundu et al. 2017); radiative transfer computational advancement (Gray 2017), rankings and preferences (da Costa 2015) etc. PCA is also a popular method among supply chain managers for its versatility (Agrawal and Saxena 2018). In this study, PCA has been used as a hybrid method, combined with the AHP analysis in step 1, for further refining the values obtained from the generic AHP, specifically to quantify the interdependencies of the dependent variables as well as to measure their relative positions in the coefficient matrix. The method is detailed in the following subsection.

3.2.3. Hybrid AHP-PCA method

Under the current investigation, a hybrid AHP-PCA method is developed for ranking and finding the interdependencies of the variables of the utility function, the first such approach known. The uniqueness of this method is that it utilizes the best of both, generic ranking (PCA) and interdependency calibration (AHP). The super-weighted matrix has been used as input for the PCA. By doing this, the results obtained using AHP are being cross-checked and verified. This method thus unifies Multi-Criteria Decision Making (MCDM) with statistical approaches. The working algorithm of this method is given below –

Step 1: Develop the AHP models with 'm' (here m=3) criteria and 'n' alternatives (here n=11)– the linear one for alternative rankings and the layered structure for interdependencies.

Step 2: Input ratings for the pairwise comparison matrix for both regular and layered AHP models to derive the ranking of the alternatives.

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Step 3: Record the AHP outputs and the super-weighted matrices.

Step 4: Reduce the super weighted matrix obtained from the crown structure AHP into 'p x p' matrix (here p=7).

Step 5: Use the 'p x p' matrix obtained in step 4 as input and run PCA.

Step 6: Use the principal components as alternative rankings for the variables.

Step 7: Replace infinitesimally small values with zero in the correlation matrix obtained from PCA. In this case, we reduce 25 entries to zero while the matrix dimension changes to 7x6.

Step 8: Find the norm of the newly developed correlation matrix.

Step 9: Divide each element of the new correlation matrix by the norm values.

Step 10: Map the matrix developed in Step 9 with the matrix in Step 4 and derive the interdependency factors by matching the positions.

Step 11: IF, any required values obtained from PCA are zero then use the equivalent AHP value. Further, normalize it and use the resultant values as weight factors.

For zero entries from the benchmarking table, AHP values are given preference over PCA as with AHP, the rankings are already given using the eigenvalues.

We recall that F is the cost. The lambda values are chosen here through the AHP and AHP-PCA analysis. As was shown in the previous paper (Chattopadhyay (2020), these are proportional to the epsilon values. The final values of lambda and other weight factors are provided in the supplementary material to this paper.

When PCA is reapplied, the less important alternatives i.e., the options with the smallest eigenvalues, were simply converted to zeroes (given no weight) to emphasize the prioritized (data or logic-driven) options. In a realistic scenario involving an e-waste supply chain, we may not be allowed to resort to such oversimplification though. This double screening through AHP→PCA filters ensures that the finally obtained values offer reliable estimates for relative weight and interdependency factors. The compound and square interdependencies are derived using the methodology of Chattopadhyay et al. (2020). The compound interdependencies have been taken as the product of the concerned coefficient, e.g., value of a_{23} = value of a_2 x value of a_3. The squared coefficient has been taken as the square root of the concerned co-efficient. For example, the value of a'_3 = square root of a_3.

3.2.4. Unconstrained Problem

The central mathematical outline follows the schematic in Chattopadhyay et al. (2020), leading to a Euler -Lagrange structure (Goldstein 1964) that depicts the optimized (from the cost function) time evolution of the interacting variables defining the income-outcome cost matrix. The perturbed dynamics close to the linearly stable fixed points can then be represented by the following dynamical system:

$$\delta\left(\frac{d}{dt}\begin{bmatrix} \frac{\partial F}{\partial V_{CO_2}} \\ \frac{\partial F}{\partial E_C} \\ \frac{\partial F}{\partial W_P} \\ \frac{\partial F}{\partial W_W} \\ \frac{\partial F}{\partial N_1} \\ \frac{\partial F}{\partial N_3} \\ \frac{\partial F}{\partial N_4} \\ \frac{\partial F}{\partial N_5} \\ \frac{\partial F}{\partial N_7} \\ \frac{\partial F}{\partial N_8} \\ \frac{\partial F}{\partial N_9} \end{bmatrix}\right) = \begin{bmatrix} m_{11}\; m_{12}\; m_{13}\; m_{14}\; 0\; 0\; m_{17}\; m_{18}\; m_{19}\; 0\; 0 \\ m_{21}\; m_{22}\; m_{23}\; m_{24}\; 0\; m_{26}\; m_{27}\; m_{28}\; m_{29}\; 0\; 0 \\ m_{31}\; m_{32}\; m_{33}\; m_{34}\; 0\; 0\; m_{37}\; m_{38}\; m_{39}\; 0\; 0 \\ m_{41}\; m_{42}\; m_{43}\; m_{44}\; 0\; 0\; m_{47}\; m_{48}\; m_{49}\; 0\; 0 \\ 0\; 0\; 0\; 0\; 0\; 0\; 0\; 0\; 0\; 0\; 0 \\ 0\; m_{62}\; 0\; 0\; 0\; m_{66}\; m_{67}\; 0\; 0\; 0\; 0 \\ m_{71}\; m_{72}\; m_{73}\; m_{74}\; 0\; m_{76}\; m_{77}\; m_{78}\; 0\; 0\; 0 \\ m_{81}\; m_{82}\; m_{83}\; m_{84}\; 0\; 0\; m_{87}\; 0\; m_{89}\; 0\; 0 \\ m_{91}\; m_{92}\; m_{93}\; m_{94}\; 0\; 0\; 0\; m_{88}\; m_{89}\; 0\; 0 \\ 0\; 0\; 0\; 0\; 0\; 0\; 0\; 0\; 0\; 0\; 0 \\ 0\; 0\; 0\; 0\; 0\; 0\; 0\; 0\; 0\; 0\; 0 \end{bmatrix} \begin{bmatrix} \delta V_{CO_2} \\ \delta E_C \\ \delta W_P \\ \delta W_W \\ \delta N_1 \\ \delta N_3 \\ \delta N_4 \\ \delta N_5 \\ \delta N_7 \\ \delta N_8 \\ \delta N_9 \end{bmatrix} \qquad (8)$$

Focusing on the leading dynamic variables (V_{CO_2}, E_C, N_3, N_4) for the specific recycler whose data we seek to compare against, given that the other variables are largely fixed for them, Eq. (8) can be easily simplified

$$\delta\left(\frac{d}{dt}\begin{bmatrix} \frac{\partial F}{\partial V_{CO_2}} \\ \frac{\partial F}{\partial E_C} \\ \frac{\partial F}{\partial N_3} \\ \frac{\partial F}{\partial N_4} \end{bmatrix}\right) = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix} \begin{bmatrix} \delta V_{CO_2} \\ \delta E_C \\ \delta N_3 \\ \delta N_4 \end{bmatrix}. \qquad (9)$$

Note that other variables like W_P, W_W, etc. could also have non-trivial contributions for different recyclers in which case they would have to be considered as well. Also, it is relevant to note that the zero rows ascribed to variables N_1, N_8 and N_9 in Eq. (8) above, respectively relating to the labor capacity (N_1), number of waste materials sent to TSDF (N_8), and tax (N9), remain largely unchanged throughout the operation cycle of the unit, and hence do not contribute to the recycling dynamics. The rearrangement of equation (9) leads to –

$$\frac{d^2}{dt^2}\begin{bmatrix} \rho_1 \delta V_{CO_2} \\ \rho_2 \delta E_C \\ \rho_3 \delta N_3 \\ \rho_4 \delta N_4 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix} \begin{bmatrix} \delta V_{CO_2} \\ \delta E_C \\ \delta N_3 \\ \delta N_4 \end{bmatrix} \qquad (10)$$

The second-ordered time derivative in Eq. (10) mimics an 'underdamped' model (Risken 1996) of mechanics, that from the perspective of a supply chain, represents a 'lightly' constrained SCN where many of the constraints are outliers but not necessarily boundary conditions.

3.2.4. Constrained Problem

The constrained version of the problem is formulated by introducing Lagrange multipliers (Goldstein 1964; Elton et al. 2009). This helps in solving the optimization problem without explicit parameterization in terms of the constraint (Tur et al. 2009). The values of the individual Lagrange multipliers are considered to be proportional to the epsilon values that replicate the corresponding weightage of the individual uncertainties in the cost function. The Lagrangian '$\mathcal{L}$' is defined as –

$$\mathcal{L} = F - \lambda_1\left(V_{CO_2} f_1 - V\right) - \lambda_2(N_1 f_5 + N_3 f_6 - E) - \lambda_3(N_4 f_7 - R) \qquad (11)$$

where λ_i's are the Lagrange multipliers. The realistic system restrictions (constraints) are expressed through the quantities joined with the Lagrange multipliers which we enforce on the system. We impose three constraints on V, E, and R, which we have chosen in consultation with the e-waste recycler, on Eq. (11): 1) V, the cost associated with CO_2 emission control; 2) E, the maximum expenditure budget accorded for wages of the labours and employees and awareness activities, and

3) R, the maximum revenue target. Overall, this amounts to a suitably recalibrated greener supply chain within viable operation lines.

The constrained version of the problem takes the following form –

$$\delta\left(\frac{d}{dt}\begin{bmatrix}\frac{\partial\mathcal{L}}{\partial V_{CO_2}}\\ \frac{\partial\mathcal{L}}{\partial E_C}\\ \frac{\partial\mathcal{L}}{\partial N_3}\\ \frac{\partial\mathcal{L}}{\partial N_4}\end{bmatrix}\right)=\begin{bmatrix}c_{11} & c_{12} & c_{13} & c_{14}\\ c_{21} & c_{22} & c_{23} & c_{24}\\ c_{31} & c_{32} & c_{33} & c_{34}\\ c_{41} & c_{42} & c_{43} & c_{44}\end{bmatrix}\begin{bmatrix}\delta V_{CO_2}\\ \delta E_C\\ \delta N_3\\ \delta N_4\end{bmatrix} \quad (12)$$

Rearrangement of Eq. (12) leads to -

$$\frac{d^2}{dt^2}\begin{bmatrix}\omega_1\delta V_{CO_2}\\ \omega_2\delta E_C\\ \omega_3\delta N_3\\ \omega_4\delta N_4\end{bmatrix}=\begin{bmatrix}c_{11} & c_{12} & c_{13} & c_{14}\\ c_{21} & c_{22} & c_{23} & c_{24}\\ c_{31} & c_{32} & c_{33} & c_{34}\\ c_{41} & c_{42} & c_{43} & c_{44}\end{bmatrix}\begin{bmatrix}\delta V_{CO_2}\\ \delta E_C\\ \delta N_3\\ \delta N_4\end{bmatrix} \quad (13)$$

Equations (10) and (13) are solved using data obtained from an anonymous multi-award winner Indian E-waste recycler company. MATLAB R2019a (bvp4c) was used to solve the system of equations concerning the solutions of the corresponding boundary value problems (Table 1). V_{CO2} values are in tons of carbon dioxide emissions, Energy consumption values are in Gigawatt, the number of awareness activities are plain numbers whereas product sales are in % of sales target. The initial conditions represent the current status, whereas the boundary conditions represent the targets to be achieved.

Table 1. Boundary conditions for evaluation of results.

Sl.	**VCO_2**	**E_c**	**N_3**	**N_4**
	Volume of CO_2 (tons of CO_2)	Energy Consumption (% Energy Consumed in GW)	Number of Awareness Activities (No.)	Product Sales (% of sales target)
		Initial Conditions (IC)		
1	1.2	0.002	2	0.3
		Boundary Conditions (BC)		
1	0.82	0.0015	4	0.6

4. Results & Discussion

The time dynamic behavior of the leading dynamic variables, both for constrained and unconstrained environments, are discussed below. First, we need to have an essence of what the standalone constrained and unconstrained systems represent. In simple parlance, they jointly characterize the dystopian and utopian case scenarios respectively. Both cases are ranked using AHP and hybrid AHP-PCA methods and compared. For a real e-waste recycler, the SCN is stochastic and highly sensitive to minor logistic perturbations, technically represented as SCN strategies. Appropriate Initial and Boundary Conditions represent such strategies in our time-varying model. The precise nature of these initial and boundary conditions are subjective of the SCN where the initial conditions would represent the present state of the system, while the desired state of the system would define the terminal conditions, thus leading to a fixed end point formulation of the problem. It is desirable sometimes to examine how the solution changes with different terminal conditions, and hence it is interesting to solve a variable endpoint problem. Such a solution can identify the tradeoffs between SCN's total cost F and the relaxation of the terminal condition.

This study strategizes how decreasing environmental load with increasing social accountability may still conform to the economic profitability of relevant SMEs. The effect of different ranking methods is explored in the chosen variables both in constrained and unconstrained conditions, separately at intervals of one-year, three-year and five-year timelines. The 1-year results replicate the immediate effect, whereas the 5-year timer represents a long-time effect. The 3-year results provide an understanding of events at intermediate time scales. This intermediate time scale is strategically

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important because this provides a clear numerical grasp of the state the company is in at that point and offers scopes of strategizing for the future. The time dynamic behavior of the 4 key variables e.g., carbon dioxide emission volume (V_{CO2}), energy consumption (E_C), number of awareness activities (N_3) and product sales (N_4) all rely on the hybrid AHP→PCA ranking method outlined in the preceding sections to identify best performance strategies.

4.1. Volume of Carbon Dioxide Emission

The hybrid method performs better than the simple AHP method. The Hybrid method de-emphasizes the variables that do not contribute much to the cost kernel that is minimized. It emphasises the first environmental cost component and hence concentrates on reducing CO_2 emissions, while AHP concentrates on variables, even those that do not contribute much to environmental cost reduction.

A way to check this is to ascribe different (larger) weights to environmental costs in AHP itself and analyze if the AHP solution so obtained corresponds to that of the hybrid problem with lower weight for environmental cost.

We note that AHP is still based on expert ratings, hence, the resulting weightage could have some human bias. On the other hand, the hybrid method refines the whole output of AHP and removes those which are contributing less in the overall scenario. This is the generalized scenario. Now, if we look at this VCO2 case, the hybrid method does better, because the weightage is changed as well and the other least-contributing variables are cut loose. Hence, the overall solution is getting a better shape.

Figures 5a,b compare the time dependence of the volume of CO_2 generated (V_{CO2}) in a constrained environment respectively for 1 and 5 years. In both cases, two ranking methods are used for comparison – AHP (dash-dotted line) and hybrid AHP-PCA method (solid line). It is clear from the figures that the hybrid AHP-PCA method provides a better ranking than any individual scoring methods (AHP or PCA for us) as the solid-line curves capture the immediate effects much better than the dash-dotted lines. For a 1-year timeline, the AHP results suggest that the carbon dioxide emission will reach a minimum within the fifth and the sixth month and rise again before reaching the boundary value whereas the hybrid AHP-PCA results suggest that carbon dioxide emission will smoothly decrease to the targeted value. The AHP curve suggests when to resurrect the strategy change, represented by the point of inflection. Whereas, the AHP-PCA curve suggests that within a year it is quite not possible to resurrect a strategy change, rather it is a period of observation. We find that the hybrid AHP-PCA results are more realistic, as it is practically impossible to run into a record low-emission figure (~79% of the starting value) within 4 months.

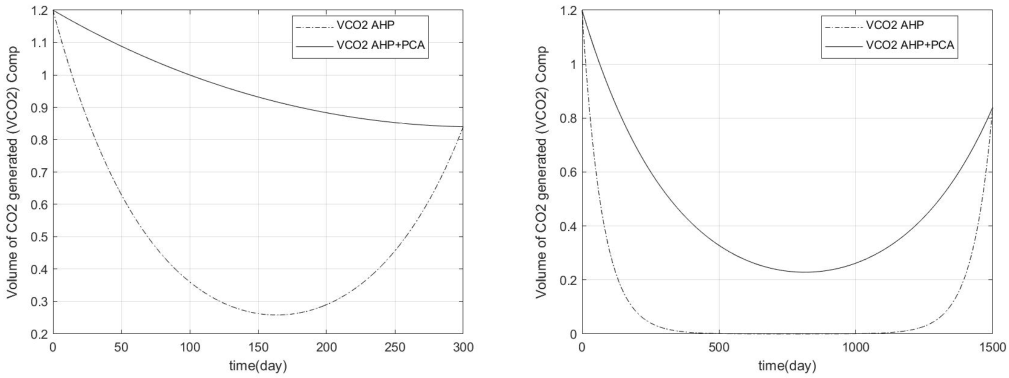

Figure 5. Time dependency of V_{CO2} in a constrained environment, using AHP for ranking (dash-dotted line) and hybrid AHP-PCA for ranking (solid line) obtained from simultaneous solution of Eqs. (9) and (12): (a) 1-year time span; (b) 5-year time span.

In the 5-year timelines, the hybrid AHP-PCA curve identifies the inflection point (the minima) after 2 years and then converges again to its target value. This suggests that in the long term, it is

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important to devise a policy change to lower the emissions, i.e., savings in environmental efforts, combining technological efforts in emission reduction, carbon credits, positive environmental impacts through Corporate Social Responsibility (CSR) and other green activities like greening the supply chain, cleaner production lines and zero waste efforts. Compared to the hybrid AHP-PCA ranked results, the standalone AHP ranked results fail to capture the 'negative-emission' characteristic of the curve as the curve flattens to zero after 1st year till towards the end of the fourth year. Unless there is a very enthusiastic supply chain manager, the AHP results will lead to a happy-face decision of offsetting the carbon footprint in the long run. Hence there may not be a continual improvement unless it is mentioned in the quality (ISO 9000) and environmental (ISO 14000) policy of the recycling company concerned.

Figure 6 depicts the time dependence of the volume of CO_2 generated (V_{CO2}) in a constrained environment, starting at the end of the 3-year year timespan with AHP (dash-dotted line) and hybrid AHP-PCA method (solid line) as the ranking methods. This is an example of hierarchical module training, based on Machine Learning inputs from the hybrid AHP→PCA model. In the unconstrained environment, the solutions are non-convergent indicating an unstable system. The sample result curve for the unconstrained environment is provided in Appendix 1. The curves resemble the ones with 5-year timelines. However, in this case, the hybrid AHP-PCA curve ensures an "operation window" within 400 – 600 days. This indicates that it is possible to achieve further reduction in emissions driven by a policy change. Hence, it is quite an intelligent approach to set targets by the 3-year timeline for carbon dioxide emission and revise/ tailor as required within the next 2 years' timeline. This will allow the supply chain manager to locate operation windows in a less risk-prone period enabling short-term strategy enforcement towards better-fitted solutions. This also opens opportunities to explore unconventional and newly developed approaches for pilot studies which is a good way to strengthen the much-required industry-academia bond.

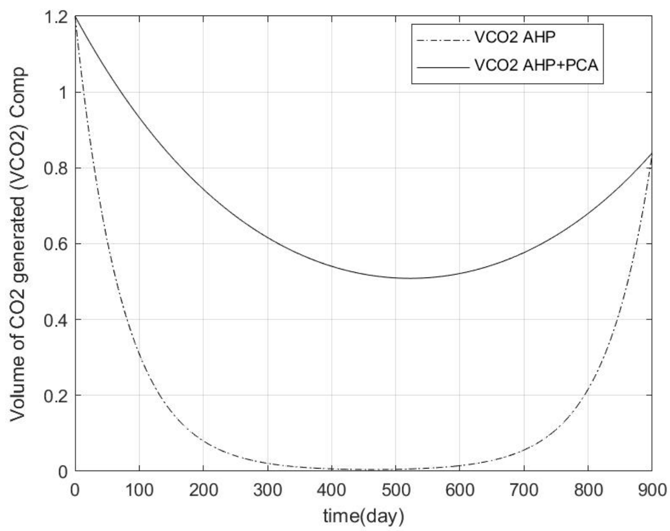

Figure 6. Time dependency of V_{CO2} in a constrained environment, using AHP for ranking (dash-dotted line) and hybrid AHP-PCA for ranking (solid line) obtained from simultaneous solution of Eqs. (9) and (12) over 3 years.

4.2. Energy Consumption

In the 1-year timeline, the results from both AHP and hybrid AHP-PCA ranking methods show a similar trend. Both the curves increase smoothly only to reach the target value. Figure 7a suggests increasing energy consumption as we interpret that the mechanical recycling of e-waste is highly energy intensive. In the 5-year timeline (Figure 7b), the AHP curve has a hyperbola shape. The curve shows a steep fall in the beginning reaching a minimum by the end of the second year, then rising to a higher value (nearly double) by the end of the fifth year. Alternatively, the hybrid AHP-PCA curve

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shows a similar trend to its 1-year scenario, which is an increasing profile. We interpret that, the system is inherently stochastic and energy consumption is a very critical and sensitive parameter which needs attention. In the long run, the AHP curve identifies an "operation window" for devising policy changes. A decrease in energy consumption implies an organization heading towards bankruptcy. However, the variations may be attributed to the supply of e-waste demand uncertainty or even an economically damaging pandemic that shuts off the entire work cycle. As the AHP-PCA curve fails to provide any inflection point, we interpret that the current energy policy in the e-waste organization needs immediate attention. Again, this behavior could be attributed to the overestimation of the AHP-PCA method or the underestimation of AHP.

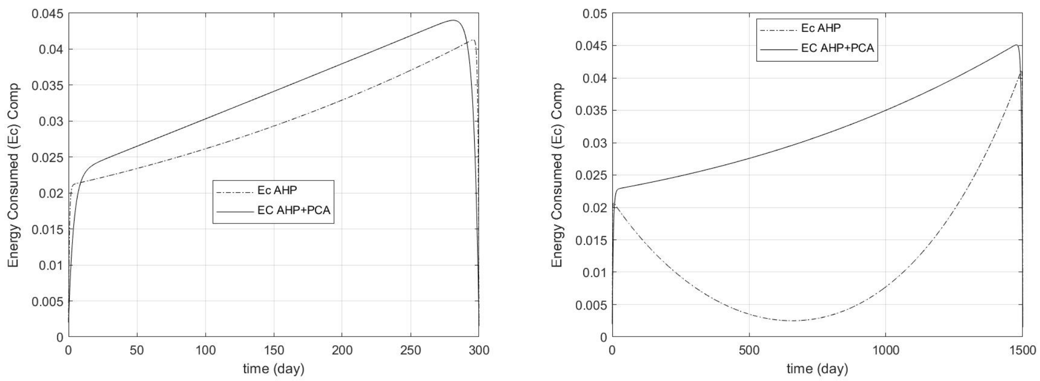

Figure 7. Time dependency of E_c in a constrained environment, using AHP for ranking (dash-dotted line) and hybrid AHP-PCA for ranking (solid line) obtained from simultaneous solution of Eqs. (9) and (12): (a) 1-year time span; (b) 5-year time span.

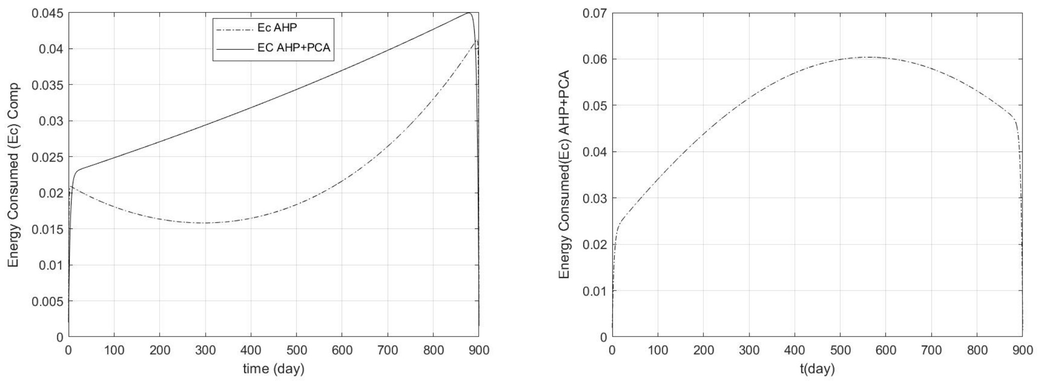

Figure 8. Time dependency of E_c: (a) in a constrained environment, using AHP for ranking (dash-dotted line) and hybrid AHP-PCA for ranking (solid line) and (b) in an unconstrained environment using hybrid AHP-PCA for ranking obtained from simultaneous solution of Eqs. (9) and (12) in a 3-year timespan.

In the constrained environment, the hybrid AHP-PCA ranked curve imitates the 1-year & 5-year AHP ranked curve. This is quite realistic as in reality, the energy consumption of an e-waste recycling plant will increase over time as supply increases, which is an indicator of sustainable business. Alternatively, from a mathematical point of view perhaps AHP-PCA method is overestimating the dynamics of the system. On the other hand, the AHP ranked curve (constrained) carries a similar profile to its 5-year appearance. In this case, an operation window within 200 – 400 days is obtained. Whereas the hybrid AHP-PCA ranked curve in the unconstrained environment exhibits an increasing parabolic profile, which reaches a maximum within 500 – 600 days and then reduces. The sensitivity of this parameter is quite high compared to the other cases; hence such behavior of curves has appeared. However, comparing the results of constrained and unconstrained cases of the hybrid AHP-PCA method, we interpret that there might be a case of over-prediction in the constrained case.

We should also be cautious about the possibility of recurrent overestimation accruing from our hybrid AHP-PCA ranking method, as has been discussed above. Additionally, both methods are seen to contribute towards parameter sensitivity and are differentially adaptive to the ambient response (AHP is more stable than PCA on this). E-waste recycling facilities performing mechanical recycling operations are highly energy intensive and hence energy consumption is ought to be a critical factor for business sustainability.

4.3. Number of Awareness Activities

In developing countries, the awareness level of e-waste disposal is a big issue and needs proper attention (Debnath et al. 2015; Baidya et al. 2020). The level of awareness is proportional to the business of an e-waste recycler. Hence, it is for the best of the recycler's interest, that awareness activities need to be taken seriously and as a CSR activity as well. Such practice is also visible among e-waste recyclers around the globe. Time dependency of several awareness activities (N_3) in a constrained environment is presented in Figures 9a,b, for 1-year and 5-year timespan respectively. The results obtained using standalone AHP as the ranking method are in dash-dot lines whereas the results obtained using the hybrid AHP-PCA method are in solid lines.

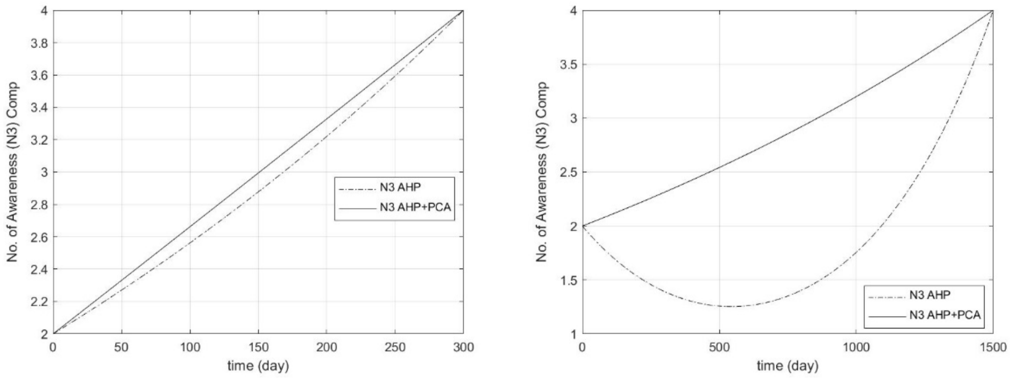

Figure 9. Time dependency of N_3 in a constrained environment, using AHP for ranking (dash-dotted line) and hybrid AHP-PCA for ranking (solid line) obtained from simultaneous solution of Eqs. (9) and (12): (a) 1-year time span; (b) 5-year time span.

As shown in the 1-year scenario, both curves depict a smooth increasing profile. While the solid line is almost straight, the dash-dotted one is slightly bent in between. Certainly, both curves give an outlook that over time, an increase in awareness activities will be helpful. In the long-term scenario (5-year), the dash-dotted curve depicts a minimum around the 500th day and sharply increases to reach the target value. In contrast, the solid line depicts the social responsibility of the recycler with a smoothly increasing curve which helps the social image and eventually increases the business potential of the recycler. The dash-dot curve implies that the e-waste recycler might struggle to sufficiently increase the number of awareness activities in the initial years but after some time they will eventually gear up to increase the number of awareness activities. The minimum is obtained in the second quarter of the second year which means that is the point when further decision needs to be taken for higher social accountability based on company policy and budgets. We interpret that the hybrid AHP-PCA model might be giving the perfect fit as the path shown is more realistic.

In the constrained environment, the solid line exhibits a straight line translating to a realistic scenario. Compared to this, the dash-dotted curve is a repetition of the 5-year scenario and hence needs no further discussion. In the unconstrained environment, the AHP ranking method provides a wave-like profile however the negative minimum is unphysical (as shown in the insets of Figure 10b). On the other hand, the hybrid AHP-PCA method of ranking creates a semi-parabolic profile

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that offers a maximum value (~ 4.5) at the 600th-day timestamp. This means that in an arbitrage condition, the recycler can keep increasing the awareness activities.

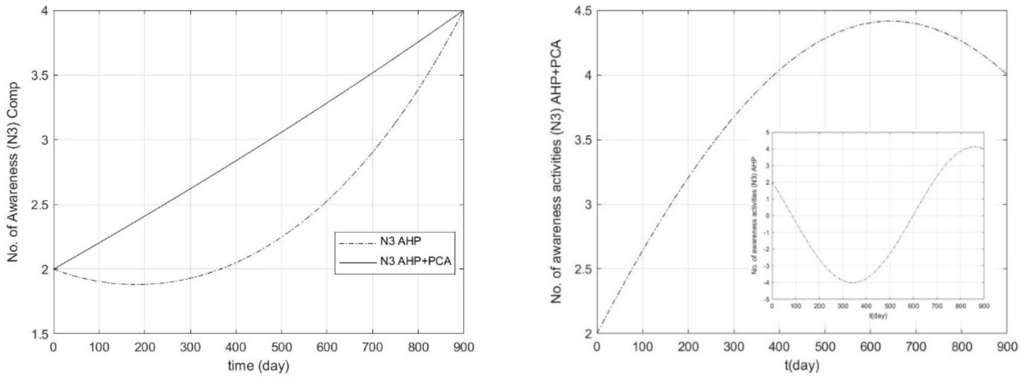

Figure 10. Time dependency of N_3: (a) in a constrained environment, using AHP for ranking (dash-dotted line) and hybrid AHP-PCA for ranking (solid line) and (b) in an unconstrained environment using hybrid AHP-PCA for ranking and AHP for ranking (provided in the insets) obtained from simultaneous solution of Eqs. (9) and (12).

The awareness activity (N_3) relates to the social uncertainty contributing to the overall cost function. Positive growth in social aspects is always a good deal and the hybrid AHP-PCA ranking method can predict the realistic trends in both constrained and unconstrained scenarios. On the other hand, the standalone AHP method underpredicts and fails to describe the dynamic system both in constrained and unconstrained scenarios. Our model shows that in this case, the decision-making on several awareness activities is guided by the budget constraint. Despite the fact, that an e-waste recycling facility can survive on its own or operate on break-even mode in an arbitrage condition of continuous supply of e-waste, it is suggested that regular monitoring and cross-validation of existing policy should be carried out for business sustainability.

4.4. Product Sales

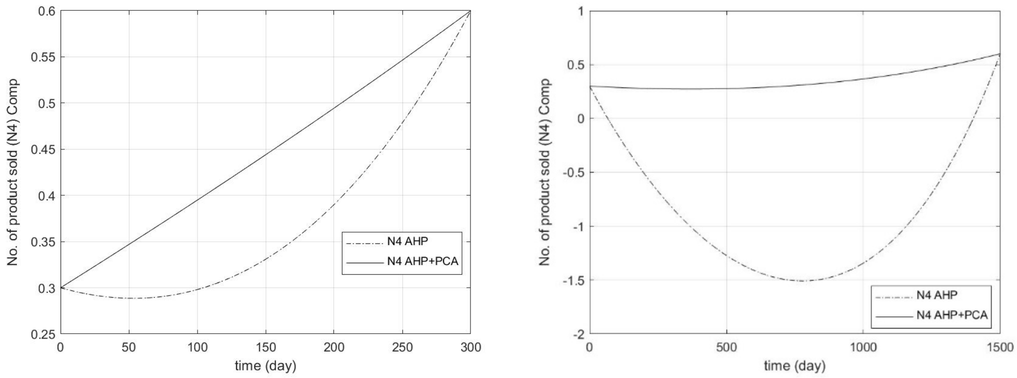

Figure 11. Time dependency of N_4 in a constrained environment, using AHP for ranking (dash-dotted line) and hybrid AHP-PCA for ranking (solid line) obtained from simultaneous solution of Eqs. (9) and (12): (a) 1-year time span; (b) 5-year time span.

In the 1-year timeline, using AHP (dash-dotted line) as the ranking method predicts that immediately after the beginning, the e-waste plant incurs loss as shown by the dipping curve and starts to peak after 3 months to reach a target value. This parabolic curve (dash-dotted line) suggests that in the current scenario, the company may face some issues at the start. On the other hand, using the hybrid AHP-PCA ranking method (solid line) dictates a straight line. Over the 5-year scenario, the dash-dotted line exhibits a parabolic profile with a minimum at the 750th-day timestamp. Clearly,

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AHP underpredicts the dynamics of the system. In contrast, the solid line demonstrates a smooth increasing curve. The market price volatility of recycled products is a major issue; hence it is suggested that both methods should be tested in the interest of a greener supply chain with a cleaner production line leading to sustainable business.

The curves in Figure 12a represent results in the constrained environment using AHP (dash-dotted line) and hybrid AHP-PCA (solid line) for ranking. Both the curves display a similar profile to the 5-yearly results. Hence the interpretation remains unchanged. In the unconstrained scenario, the hybrid AHP-PCA method points to interesting outcomes. The results using the AHP ranking method are provided in the insets of Figure 12b. As seen in the previous cases, here also standalone AHP fails to obtain rational results in an unconstrained scenario (Figure 12b inset). The hybrid AHP-PCA ranked result shows a parabolic curve. This shows the robustness of the hybrid AHP-PCA method as it captures the system dynamics in the unconstrained scenario. The curve profile suggests that even in arbitrage conditions, the company may not have a steady growth profile.

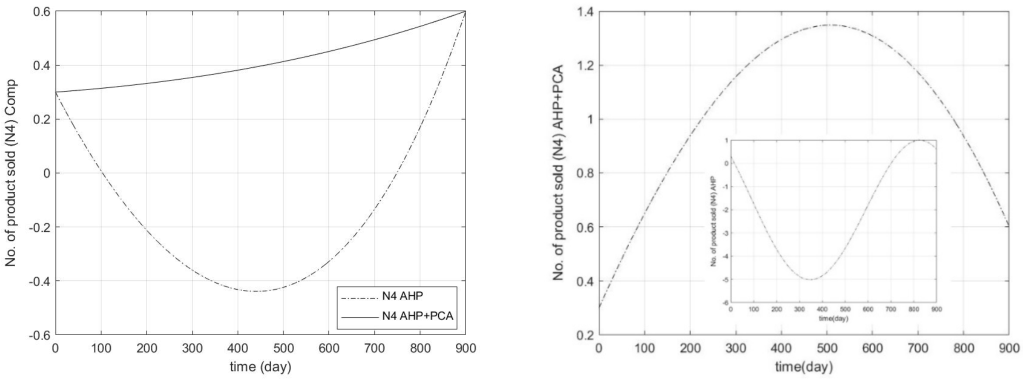

Figure 12. Time dependency of N_4: (a) in a constrained environment, using AHP for ranking (dash-dotted line) and hybrid AHP-PCA for ranking (solid line) and (b) in an unconstrained environment using hybrid AHP-PCA for ranking and AHP for ranking (provided in the insets) obtained from simultaneous solution of Eqs. (9) and (12).

Economic sustainability is the most important of all from the business perspective (Debnath and Ghosh 2019). From that sense, a product sold (N_4) is the most important parameter which needs to be nurtured for maximum profit. A greener supply chain network with a sustainable production line is a utopian case but we can always look forward to reaching as close as possible to the target values. That is exactly what these boundary conditions have helped us to do. The alluded case gives an outlook of comparison of both the methods, but the choice of boundary conditions lies in the hand of the supply chain manager of the respective plant. For business sustainability, it is suggested that regular monitoring of critical parameters and policy changes at certain intervals (identified through analysis) will help in greening the supply chain.

5. Conclusion

Under the current investigation, we consider a generic “Utilization-to-cradle” e-waste supply chain network. Following the working architecture of our previous work, we have developed a cost function model which considers uncertainties arising from three pillars of sustainability i.e., Environmental, Economic and Social. The model is supported by the individual weights and interdependencies of the uncertainty parameters. Two methods are compared to identify the weight factors and interdependencies namely – AHP and hybrid AHP-PCA method. The market dynamics are defined by the Euler-Lagrange equation with two optimization scenarios – unconstrained and constrained. The optimization is executed by solving the boundary value problems using MATLAB 2019b.

We used anonymous Indian E-waste Recyclers data to test the robustness of the SCN model in 1, 3 and 5-year timelines respectively. The boundary conditions replicate the strategy of decreasing

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environmental load while increasing social accountability together with economic profitability. The results show that the hybrid AHP-PCA ranking method is superior to the standalone AHP method of ranking. However, certain cases emerged where the hybrid method overestimated the dynamics of the system. The model was able to identify the operation windows for the supply chain managers for reinvigoration of the policies. Volume of carbon dioxide and energy consumption emerged as the most important parameters while energy consumption was the most sensitive parameter of the system. The model also identifies that for social accountability practice, the decision should be purely guided by the budget constraints to maintain a sustainable business. On the other hand, the product sales which is the economic driver reflected an increasing profile to the current case. The numbers and outputs are likely to change with the change in case and constraints. The study almost unerringly replicates the results which is realistic and provides a guideline for developing a cleaner production line with a sustainable profitability margin. Future studies with separate data sets for the hypothetical MREW facility are underway with more intense machine-learning techniques.

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