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Article

The Effective Potential of Reduced Models in (2+1)D

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Abstract: The description of the electron-electron interactions in two-dimensional materials has a dimensional mismatch, where electrons live in (2+1)D while photons propagate in (3+1)D. In order to define an action in (2+1)D, one may perform a dimensional reduction of quantum electrodynamics in (3+1)D (QED4) into Pseudo quantum electrodynamics (PQED). The main difference between this model and QED4 is the presence of a pseudo-differential operator in the Maxwell term. However, besides the Coulomb repulsion, electrons in a material are subjected to several microscopic interactions, which are inherent in a many-body system. These are expected to reduce the range of the Coulomb potential, leading to a short-range interaction. Here, we consider the coupling to a scalar field in PQED for explaining such mechanism, which resembles the spontanoues symmetry breaking (SSB) in Abelian gauge theories. In order to do so, we consider two cases: (i) By coupling the gauge field to a Higgs field in scalar quantum electrodynamics in (3+1)D and, thereafter, performing the dimensional reduction and; (ii) By coupling a Higgs field to the gauge field in PQED and subsequently calculating its effective potential. In case (i), we obtain a model describing electrons interacting through the Yukawa potential and, in case (ii), we show that SSB does not occur at one-loop approximation. The relevance of the model for describing electronic interactions in two-dimensional materials is also adressed.

Keywords: Pseudo quantum electrodynamics; Coleman-Weinberg Potential; Spontaneous Symmetry Breaking.

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1. Introduction

The experimental realization of two-dimensional materials [1] in condensed matter physics has attracted the interest of the community in high energy physics due to the emergence of Dirac cones and the possibility of observing some interacting effects [2], such as the Fermi velocity renormalization [3,4] and mass renormalization [5,6]. When considering the effects of electronic interactions in these materials, it is useful to consider a dimensional reduction of QED4, namely, PQED, which provides the physical Coulomb potential among static particles in the spatial plane [7]. Within this approach, several results, including strong and weak interactions, have been obtained and compared to experimental data, see Refs. [5,8–12], just to cite a few. Furthermore, the model has also been coined as reduced quantum electrodynamics in Refs. [13,14]. Although the Coulomb potential has been successful in describing these effects, we also expect that this electron-electron interaction become screened due to the microscopic interactions within the two-dimensional material [15]. These interactions should include impurities and phonons, which would make the model even more complicated and, for the best of our knowledge, a complete solution is yet to be known [9]. On the other hand, because the long-range Coulomb interaction is related to a massless photon, hence, a simplest method to generate a short-range potential is to obtain a massive bosonic field, which is straightforwardly derived from SSB. Indeed, when considering the Higgs mechanism in QED4, one would obtain a Yukawa interaction in (3+1)D, yielding a short-range interaction, as expected [16].

The Higgs mechanism for QED in (2+1)D (QED3) would provide the simplest solution for obtaining a mass term for the gauge field and, therefore, a short-range interaction. However, as discussed before, this mechanism must be considered in PQED rather than in QED3. In PQED, the power of the gauge-field propagator is $\propto (p^2)^{-1/2}$ while in QED it is p^{-2}, where p is the external momentum. Hence, in order to obtain a Yukawa potential in PQED, one must obtain a scalar propagator

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given by $1/\sqrt{p^2+m^2}$. This, on the other hand, is obtained from the dimensional reduction of QED4 in the presence of a mass term, as it has been shown in Ref. [17].

Here, we consider the coupling of the gauge field in PQED with a scalar field. This is meant to effectively describe the effects of the many-body system and, essentially, to generate a mass-like term for PQED and to reduce the range of the Coulomb interaction. Firstly, we review the dimensional reduction which generates reduced models in both (2+1)D and (1+1)D. In order to do so, we apply the dimensional reduction in the two-point Green functions of the classical fields, through the equation of motion. Thereafter, we calculate the effective potential for a reduced scalar model in (2+1)D and discuss SSB. This quantity shows a behavior similar to the effective potential in a self-interacting scalar theory in (3+1)D. However, the reduced model does not allow SSB due to its asymptotic limit, leading to a stable ground state in a symmetric phase. Thereafter, we consider effective models for describing the electronic interactions for Dirac-like quasiparticles in (2+1)D in terms of a gauge field interacting with a scalar field. The realization of SSB, however, may occur either in (3+1)D or (2+1)D, providing two different cases for investigation. In case (i), we conclude that the Yukawa interaction is obtained whenever SSB occurs in (3+1)D and derive an effective action for describing such interaction in (2+1)D. In case (ii), which we call as Abelian Higgs PQED (HPQED) in comparison to scalar QED3 [18], we show that the quantum correction does not provide a SSB, hence, the system remains in its symmetric phase. These results are obtained in the one-loop approximation, using the so-called background field method for calculating the effective potential [16].

This paper is organized as follows. In Sec. II, we review the concept of reduced models, using the classical equation of motion. In Sec. III, we calculate the effective potential for a reduced version of the Klein-Gordon theory in (2+1)D. In Secs. IV and V, we consider the effects of considering both SSB and the dimensional reduction in QED4 plus a Higgs field. In Appendixes A–C, we show some details of the calculations.

2. The Reduced Models

In this section, we derive a reduced model that describes the Yukawa interaction in (2+1) dimensions at classical level. In order to do so, we start with the Yukawa action in (3+1) dimensions, whose *Euclidean* action is given by

$$\mathcal{L}_{4\text{D}} = \frac{\partial_\mu \varphi \partial^\mu \varphi}{2} + \frac{m^2\varphi^2}{2} + g\varphi\bar{\psi}\psi + \mathcal{L}_M[\psi], \tag{1}$$

where g is a dimensionless coupling constant, φ is a real and massive Klein-Gordon field, ψ is the Dirac field, and $\mathcal{L}_M[\psi]$ is the matter action. This model shall be relevant because it will work as a simple example for calculating the dimensional reduction in theories with SSB.

The equation of motion for φ is promptly obtained from Eq. (1) and reads

$$(-\Box + m^2)\varphi(x) = J(x), \tag{2}$$

where $\Box$ is the d'Alembertian operator and $J(x) \equiv -g\bar{\psi}\psi(x)$ works as an external source for the scalar field. The solutions of Eq. (2) are obtained by inverting the differential operator, i.e,

$$\varphi(x) = \int d^4y\, G_{4D}(x-y)J(y), \tag{3}$$

where

$$G_{4D}(x-y) = \int \frac{d^4k_E}{(2\pi)^4}\frac{e^{ik(x-y)}}{k_E^2+m^2} \tag{4}$$

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is the Fourier transform of the scalar-field propagator. Equation (4), within the static-limit, provides the Yukawa potential. Hence, after using $k_E^2 = k_0^2 + k_1^2 + k_2^2 + k_3^2$ with $k_0 \to 0$ [16], we find

$$V(r) = \int \frac{d^3k_E}{(2\pi)^3} \frac{e^{ikr}}{k_E^2 + m^2} = \frac{e^{-|m|r}}{4\pi r}, \tag{5}$$

where the inverse of m is the interaction length of the model and $r^2 = (x_1 - y_1)^2 + (x_2 - y_2)^2 + (x_3 - y_3)^2$, as expected.

2.1. From (3+1)D to (2+1)D

For calculating a reduced model, we consider that the dynamics of the matter field is constrained to the (2+1)-dimensional space-time. This is obtained by assuming that $J(y) = J(y_0, y_1, y_2)\delta(y_3) \equiv J_{3D}(y)\delta(y_3)$. Thereafter, we perform a dimensional reduction of the two-point Green function of the scalar field. Hence, we obtain the scalar-field propagator at $x_3 = 0$, i.e, $\varphi(x) \to \varphi_{3D}(x) \equiv \phi(x_0, x_1, x_2) = \varphi(x_0, x_1, x_2, x_3 = 0)$. Furthermore, we also adopt the notation, where the x in $\phi(x)$ always do represent the coordinates in the spacetime where the field is defined, hence, for $\phi(x)$, we have $x = (x_0, x_1, x_2)$. On the other hand, for $\varphi(x)$, it follows that $x = (x_0, x_1, x_2, x_3)$. The same holds for loop integrals and propagators in momentum space. After using these conditions in Eq. (3), we find

$$\varphi_{3D}(x) = \int d^3y\, G_{3D}(x-y) J_{3D}(y), \tag{6}$$

where

$$G_{3D}(x-y) = \int \frac{d^3k_E}{(2\pi)^3} \frac{e^{ik(x-y)}}{2\sqrt{k_E^2 + m^2}} \tag{7}$$

is the Fourier transform of the scalar-field propagator in (2+1) dimensions. This also procudes the Yukawa potential in Eq. (5). Indeed, after using $k_E^2 = k_0^2 + k_1^2 + k_2^2$ with $k_0 \to 0$ in Eq. (7), we find

$$V(r) = \int \frac{d^2k_E}{(2\pi)^2} \frac{e^{ikr}}{2\sqrt{k_E^2 + m^2}} = \frac{e^{-|m|r}}{4\pi r}, \tag{8}$$

where $r^2 = (x_1 - y_1)^2 + (x_2 - y_2)^2$.

The main idea of a reduced model is to obtain a theory that yields the reduced propagator in Eq. (7). This may be obtained from

$$\mathcal{L}_{3D} = \frac{\partial^\mu \phi K_E[\Box] \partial_\mu \phi}{2} + g\phi\bar{\psi}\psi + \mathcal{L}_M[\psi] \tag{9}$$

with

$$K_E[\Box] \equiv \frac{2\sqrt{-\Box + m^2}}{-\Box} = \int \frac{d^3k_E}{(2\pi)^3} e^{ikx} \frac{2\sqrt{k_E^2 + m^2}}{k_E^2}. \tag{10}$$

This model describes the dynamical effects of the Yukawa interaction in the plane and some of its quantum effects have been discussed in Ref. [17] when coupled to the Dirac field. Obviously, the Coulomb interaction is promptly obtained by using $m \to 0$ within these calculations. The case of dimensional reduction to (1+1)D is discussed in App. A.

The derivation of the reduced models is more convenient to be performed in the Euclidean spacetime [7]. However, for calculating the Coleman-Weinberg potential, we rather consider the action in the Minkowski space for the sake of comparison with well known results in the literature.

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3. The One-Loop Effective Potential of a Reduced-Scalar Model in (2+1)D

In this section, we discuss the Coleman-Weinberg effective potential of a general scalar action in a D-dimensional spacetime with a Pseudo-diferential operator. When considering an interaction term $U[\phi]$, our toy model action reads

$$\mathcal{L}_D = \frac{\partial^\mu \phi \mathcal{K}[\Box]\partial_\mu \phi}{2} + U[\phi], \tag{11}$$

where the kernel $\mathcal{K}[\Box]$ is an arbitrary pseudo-differential operator in the Minkowski spacetime. The calculations using Eq. (11) shall be useful for our purposes in the next sections, where we discuss the SSB for two-dimensional materials in the light of the PQED formalism.

The partition function of the model is defined as usual and, therefore, reads

$$\mathcal{Z} = \int D\varphi \exp\left\{ i \int d^D x \mathcal{L}_D[\phi] \right\}. \tag{12}$$

The main idea of the one-loop expansion is to expand the action in Eq. (12), using Eq. (11), up to order of $\hbar$, see App. B for more details. Thereafter, we replace the quantum field by its vacuum expectation value (VEV), i.e, $\phi(x) \to \langle \phi(x) \rangle = \rho$, which is acceptable for describing the ground state of the model through its effective potential $V_{\rm eff}(\rho)$. Therefore, we have

$$\mathcal{Z} = e^{-iV_{\rm eff}(\rho)\Omega}, \tag{13}$$

where $\Omega = \int d^3x$ is a constant factor, namely, the spacetime volume. Having these steps in mind and, after some algebra, one finds

$$V_{\rm eff}(\rho) = -U(\rho) - \frac{i}{2}\int \frac{d^D k}{(2\pi)^D} \ln[k^2 \mathcal{K}(k^2) + U''(\rho)], \tag{14}$$

where $U''(\rho)$ means the second derivative of $U[\rho]$. Note that the first term in the rhs of Eq. (14) is the classical potential and the second term is the quantum correction (in order of $\hbar$). Eq. (14) is how far we may go for an arbitrary kernel $\mathcal{K}$.

3.1. *The case $D = 4$ and $\mathcal{K} = 1$*

This is the most standard case and the effective potential reads [16]

$$V_{\rm eff}^{\rm KG}(\rho, \mu) = -U(\rho) - \frac{(U'')^2}{64\pi^2} \ln\left[\frac{4\pi\mu^2}{-U''}\right], \tag{15}$$

which is derived through the dimensional regularization scheme. Note that μ is a renormalization point and that $V_{\rm eff}^{\rm KG}(\rho)$ is expected to be unchanged by a scale transformation in μ. For a classical potential $U[\rho] = \lambda\rho^4/4!$ with $\lambda < 0$, we conclude that the VEV of the field does not vanish and it is double degenerated. hence, the discrete symmetry $\phi \to -\phi$ is broken by the ground state [16]. Our main goal is to understand what are the effects of considering a pseudo-diferential operator in a scalar action, regarding the mechanism of SSB. Thereafter, we apply these results for PQED with a Higgs field.

3.2. *The case $D = 3$ and $\mathcal{K} = 2/\sqrt{-k^2}$*

This is the case where $\mathcal{K}[\Box] = 2(\Box)^{-1/2}$ in Eq. (11). This corresponds to a reduced Klein-Gordon model. Here, we calculate its effective potential, using both the dimensional regularization and the cutoff regularization. As it shall be clear later, both these methods provide the same logarithmic term for the effective potential.

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3.2.1. The dimensional regularization scheme

Within this regularization scheme, Eq. (14) is written as

$$\begin{aligned} V_{\text{eff}}(\rho,\mu) &= -U(\rho) - \frac{i\mu^{3-D}}{2}\int \frac{d^D k}{(2\pi)^D}\ln[k^2\mathcal{K}[k^2] + U''(\rho)] \\ &\equiv -U(\rho) + V_1(\rho,\mu), \end{aligned} \tag{16}$$

where μ is an energy scale included in order to preseve the units of the integral and V_1 is the quantum correction. In App. C we calculate this correction and obtain

$$\begin{aligned} V_{\text{eff}}(\rho,\mu) &= -U(\rho) + \frac{(1/2)^{D-1}\Gamma[D]2\pi^{D/2}(-U'')^3}{(2\pi)^D 4D\Gamma[D/2]} \\ &\times \left(\frac{\mu}{-U''}\right)^{(3-D)}\Gamma[1-D]. \end{aligned} \tag{17}$$

Expanding Eq. (17) around $D \to 3$, we have

$$\begin{aligned} V_{\text{eff}}(\rho,\mu) &= -U(\rho) - \frac{(U'')^3}{48\pi^2} \\ &\times \left[\frac{1}{2(3-D)} + \frac{3}{4} - \frac{\gamma}{2} + \frac{1}{2}\ln\left(\frac{\mu}{-U''}\right)\right], \end{aligned} \tag{18}$$

where $\gamma \approx 0.58$ is the Euler's constant. Finally, we use the minimal subtraction scheme (where the pole of the Gamma function is neglected as well as some extra constants) in Eq. (18) and find the renormalized effective potential, namely,

$$V_{\text{eff}}(\rho,\mu) = -U(\rho) - \frac{(U'')^3}{96\pi^2}\ln\left(\frac{\mu}{-U''}\right). \tag{19}$$

Note that Eq. (19) resembles the effective potential of the scalar model in (3+1)D, given by Eq. (15). In particular, it has an energy-scale term which depends on μ. This is a surprisingly conclusion, because in an usual scalar theory in (2+1)D, such term does not exist. Obviously, the pseudo-differential operator plays a central role for this result. Next, let us consider the cutoff regularization scheme, which shall be useful when we include a finite lattice regulator, as it occurs for effective models in condensed matter physics [16].

3.2.2. The cutoff regularization scheme

In this case, we also perform the Wick rotation to the Euclidean spacetime, i.e, $d^D k \to i d^D k_E$ and $k^2 \to -k_E^2$. Thereafter, we include an ultraviolet cutoff Λ in the integral, where $0 \leq k_E \leq \Lambda$ in Eq. (14). Furthermore, for solving this integral, we use that

$$\begin{aligned} \int k_E^2 dk_E \ln[2k_E - U''] &= -\frac{1}{12}\left(k_E^2 U'' + k_E U''^2\right) \\ &- \frac{U''^3}{24}\ln[2k_E - U''] \\ &+ \frac{k_E^3}{3}\ln[2k_E - U''] - \frac{k_E^3}{9}. \end{aligned} \tag{20}$$

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Next, using Eq. (20) in Eq. (14) and expanding this for $\Lambda \gg U''$, we find

$$V_{\text{eff}}(\rho,\Lambda) = -U(\rho) - \frac{U''^3}{288\pi^2} - \frac{U''^2\Lambda}{32\pi^2} - \frac{U''\Lambda^2}{16\pi^2} - \frac{(U'')^3}{96\pi^2}\ln\left(\frac{2\Lambda}{-U''}\right). \tag{21}$$

Note that the coefficient of the logarithmic term, i.e, $-(U'')^3/96\pi^2$ is the same as in Eq. (19). This is an relevant feature for calculating the beta functions of the theory in Eq. (11), when considering the continuum limit, i.e, $\Lambda \to \infty$ [16]. Here, because the dimensional reduction is related to two-dimensional materials, we consider the case when $\Lambda \propto 1/a$ is a large but finite constant, where $a \approx 10^{-10}$m is the lattice parameter for a typical crystal. For example, in graphene and other two-dimensional materials, we have $\Lambda \approx 1$eV [1].

3.2.3. The vacuum stability of the reduced-scalar model with a ϕ^4-self-interaction

For a more straightforward application, let us consider a classical potential given by

$$U[\phi] = \frac{M^2\phi^2}{2} + \frac{\lambda\phi^4}{4!}, \tag{22}$$

where M is the bare mass term and λ is the bare coupling constant. The analysis of the vacuum stability of Eq. (21) is similar to the stability given by the effective potential in Eq. (15), where we consider $\lambda < 0$ in order to have a real-valued effective potential. In Figure 1, we plot Eq. (21), using Eq. (22). From this, we may conclude that the ground state of the system remains symmetric at $\rho = 0$, hence, no SSB occurs.

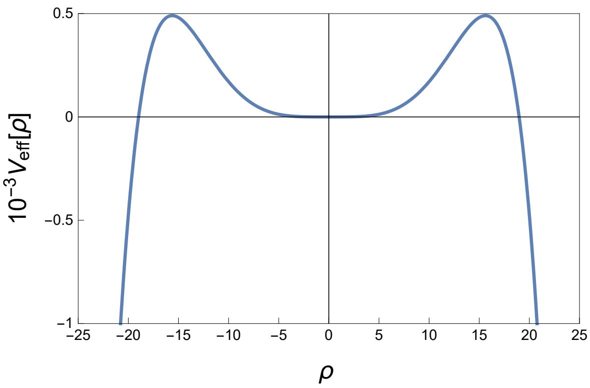

Figure 1. The effective potential of the reduced-scalar model. We plot Eq. (21) with $M = 0$, $\lambda = -0.5$, and $\Lambda = 1.0$. Note that the local minimum in $\rho = 0$ is the only acceptable ground state whether we assume that ρ is always much less than Λ.

Although it is interesting that the reduced model has a similar structure for the effective potential, in comparison to the scalar field in (3+1)D, a few comments have to be addressed. (i) Our example holds for the Z_2 symmetry $\phi \to -\phi$, however, SSB is defined for continuous symmetries, hence, our calculation works only as a first approximation for calculating SSB. Despite this, the generalization for a complex field is simple and the results are clearly the same; (ii) The effective potential of the scalar field in (2+1)D does not have the logarithmic term [18], which is a striking difference in comparison to both the higher dimensional model (see Eq. (15)) and our reduced-scalar model (see Eq. (21); (iii) Because we are assuming a finite Λ, hence, our effective potential is also finite and we do not need to deal with divergencies.

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Next, we shall consider a scalar version of PQED with a complex scalar field. This also provides a more physical situation, allowing us to make predictions regarding the interactions between quasiparticles in two-dimensional materials.

4. From SSB to the Dimensional Reduction

The static approximation of PQED describes a system of electrons interacting through the Coulomb potential. However, due to screening effects within the two-dimensional material, it is expected that the Coulomb potential becomes a short-range interaction [19]. This mechanism, however, must be considered either before or after the dimensional reduction of the Gauge field. This lead us to two important cases, which we shall consider next. First, let us assume that the SSB occurs in (3+1)D and then we perform the dimensional reduction. Thereafter, we consider the opposite case, where the Higgs field is included in PQED.

We start with QED4 and include a Higgs field, hence, our action in the Euclidean spacetime reads

$$\mathcal{L}_{\rm HQED4} = \frac{1}{4}\bar{F}^{\mu\nu}\bar{F}_{\mu\nu} - \frac{1}{2\alpha}(\partial_\mu \bar{A}^\mu)^2 + |D_\mu\varphi|^2 + U[|\varphi|] + \bar{\psi}(i\partial\!\!\!/ - m)\psi + e\bar{\psi}\gamma_\mu\psi\bar{A}^\mu, \tag{23}$$

where $D_\mu \to \partial_\mu + ig\bar{A}_\mu$, with $\mu = 0,1,2,3$, is the covariant derivative and g is the dimensionless charge of the complex scalar field φ. $\bar{A}_\mu$ is the gauge field and $\bar{F}_{\mu\nu} = \partial_\mu\bar{A}_\nu - \partial_\nu\bar{A}_\mu$ is its field strength tensor. Furthermore, α is the gauge fixing parameter. Because of charge conservation, this parameter is not relevant for our purposes, hence, we simply neglect this term. φ is a massive complex scalar field coined Higgs field and $U[|\varphi|]$ represents its self-interacting potential. ψ is the Dirac field, which is meant to describe the quasi-particles in the two-dimensional material [5,9], and m is its bare mass while e is the dimensionless electric charge.

The standard approach for SSB in Eq. (23) is to consider a polar representation of the scalar field, i.e, $\varphi(x) = \xi(x)e^{i\theta(x)}$. Furthermore, we assume that $U[|\varphi|] = \mu^2\xi^2/2 + \lambda\xi^4/4!$, where (μ,λ) are known constants. After some algebra, we conclude that the scalar field $\theta(x)$ may be removed from the model by a gauge transformation, namely, $\bar{A}_\mu \to \bar{A}_\mu - \partial_\mu\theta/g$ [16]. Hence, we find

$$\mathcal{L}_{\rm HQED4} = \frac{1}{4}\bar{F}^{\mu\nu}\bar{F}_{\mu\nu} + \partial_\mu\xi\partial^\mu\xi + U[\xi] + g^2\xi^2\bar{A}_\mu\bar{A}^\mu + \bar{\psi}(i\partial\!\!\!/ - m)\psi + e\bar{\psi}\gamma_\mu\psi\bar{A}^\mu. \tag{24}$$

Finally, we consider that the Higgs field is in the broken phase and take its lowest-order solution, given by $\xi(x) \to \rho_0$. Therefore,

$$\mathcal{L}_{\rm HQED4} = \frac{1}{4}\bar{F}^{\mu\nu}\bar{F}_{\mu\nu} + g^2\rho_0^2\bar{A}_\mu\bar{A}^\mu + \bar{\psi}(i\partial\!\!\!/ - m)\psi + e\bar{\psi}\gamma_\mu\psi\bar{A}^\mu. \tag{25}$$

Next, we use Eq. (25) for calculating the dimensional reduction. Note that whenever $\rho_0 \neq 0$ we may conclude that the Gauge field has acquired a mass-like term. The dimensional reduction, on the other hand, will follow exactly the same steps as we did in Sec. II. This also may be performed through the generating functional of the current-current correlation functions, as it has been done in Ref. [7] for PQED. Hence, it follows that the reduced version of Eq. (25) is

$$\begin{aligned}\mathcal{L}_{\rm HRQED} &= \frac{1}{4}\bar{F}^{\mu\nu}K_E[\Box]\bar{F}_{\mu\nu} + \bar{\psi}(i\partial\!\!\!/ - m)\psi \\ &+ \frac{1}{2\alpha}\bar{A}^\mu\partial_\mu K_E[\Box]\partial_\nu\bar{A}^\nu + e\bar{\psi}\gamma_\mu\psi\bar{A}^\mu,\end{aligned} \tag{26}$$

which is fully defინied in (2+1)D and

$$K_E[\Box] = \frac{2\sqrt{-\Box + 2g^2\rho_0^2}}{-\Box}. \tag{27}$$

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Despite our notation, the gauge field in Eq. (26) is not the same as in Eq. (23). This model describes the Yukawa interaction between Dirac-like electrons that are confined to move, for example, in a two-dimensional material [17]. Furthermore, because of the SSB, the interaction become short-ranged and given by

$$V(r) = \frac{e^{-\sqrt{2}g\rho_0 r}}{4\pi r}, \tag{28}$$

which is obtained similarly to the calculations in Sec. II. The mechanism we use in this section does not allow us to discuss further about SSB. Indeed, it occurs in the higher dimensional model and we simply performed the dimensional reduction. Nevertheless, we could also consider that the Higgs mechanism is driven in (2+1)D instead of in (3+1)D. In this new case, we should also ask whether the effective potential is minimized in the broken phase or not.

5. From Dimensional Reduction to SSB

In this section, we consider a different path for discussing SSB in PQED. Here, we start with PQED and then include a Higgs field, in the Minkowski spacetime, for calculating the SSB. In this case, we may use some results in Sec. III regarding the loop integrals. Therefore, we have

$$\mathcal{L}_{\rm HPQED} = -\frac{1}{2}F^{\mu\nu}(\Box)^{-1/2}F_{\mu\nu} + |D_\mu\phi|^2 - \frac{1}{2\alpha}\frac{(\partial^\mu A_\mu)^2}{(\Box)^{1/2}} + U[|\phi|], \tag{29}$$

where the subscript HPQED stands for Higgs-PQED, $D_\mu \to \partial_\mu + ieA_\mu$, with $\mu = 0, 1, 2$, is the covariant derivative, and e is the dimensionless electric charge of the complex scalar field ϕ. A_μ is the gauge field of PQED, $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is its field strength tensor, and α is the gauge-fixing parameter. ϕ is a massive complex scalar field, coined Higgs field, and $U[|\phi|]$ represents its symmetry-breaking potential. We neglect the Dirac term, because it does not play any role regarding the SSB and may be included later.

The model in Eq. (29) has been investigated in the perturbation theory in Ref. [19], within the symmetric phase. Here, we shall discuss its broken phase in the light of the effective potential. For the sake of simplicity, we assume the Feynman gauge, where the longitudinal part of the gauge-field propagator vanishes. As matter of fact, the effective potential in scalar QED4 is also gauge-dependent. Nevertheless, the physical predictions, such as the ratio between the masses of both gauge and scalar fields, are expected to independent on α [16]. Using the one-loop approximation, we may expand the action as

$$\begin{aligned}\mathcal{L}_{\rm HPQED} &= A^\mu(\Box)^{1/2}A_\mu - ieA_\mu[\phi^*(\partial^\mu\phi) - (\partial^\mu\phi^*)\phi] \\ &- \phi^*\Box\phi + e^2A_\mu A^\mu|\phi|^2 + U[|\phi|].\end{aligned} \tag{30}$$

Next, we consider the one-loop approximation for calculating the effective potential. Similar to the calculation in Sec. III, we assume that the scalar field may have a nonzero VEV, such that $\langle\phi\rangle = \rho$ and $\langle A_\mu\rangle = 0$. Using these assumptions in Eq. (30), and after neglecting some irrelevant constants, we find

$$\begin{aligned}V_{\rm eff}^{\rm HPQED}(\rho) &= -U(\rho) - i\int\frac{d^3k}{(2\pi)^3}\ln[k^2 + U''] \\ &- \frac{3i}{2}\int\frac{d^3k}{(2\pi)^3}\ln[(-k^2)^{1/2} + e^2\rho^2].\end{aligned} \tag{31}$$

Eq. (31) is the main result of this section and should be compared to the standard scalar-QED in (2+1)D [18]. In order to do so, we must replace $(-k^2)^{1/2} \to (-k)^2$ for describing the Maxwell propagator. In this case, we may conclude that the quantum corrections to the effective potential are finite in the sense that they do not have the logarithmic term, which depends either on μ or Λ, depending on the regularization scheme. Here, nevertheless, the third term in rhs of Eq. (31) provides a

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logarithmic term, such as in the previous case of the reduced-scalar model. This term is clearly related to the pseudo-differential operator.

5.1. The Higgs term

The second term in the rhs of Eq. (31) is the Higgs-field contribution to the effective potential. Within the cutoff regularization, after going to the Euclidean space, it yields

$$\frac{1}{2\pi^2}\int_0^\Lambda k_E^2 dk_E \ln\left[k_E^2 - U''\right]. \tag{32}$$

After solving the integral over k_E, keeping only the ρ-dependent terms, and expanding for $\Lambda \gg U''$, we find

$$-\frac{(U''\Lambda)}{2\pi^2} - \frac{(-U'')^{3/2}}{6\pi}. \tag{33}$$

A Maxwell-like propagator for the gauge field would essentially provide a similar result.

5.2. The PQED term

The third term in the rhs of Eq. (31) is the PQED contribution. Using the same assumptions as before, it yields

$$\frac{3}{4\pi^2}\int_0^\Lambda k_E^2 dk_E \ln\left[k_E + e^2\rho^2\right]. \tag{34}$$

Solving the integral over k_E and expanding for $\Lambda \gg e^2\rho^2$, see Eq. (20), we have

$$\frac{(e^2\rho^2)^3}{12\pi^2} - \frac{3\Lambda e^4\rho^4}{8\pi^2} + \frac{3\Lambda^2 e^2\rho^2}{8\pi^2} + \frac{3(e^2\rho^2)^3}{24\pi^2}\ln\left(\frac{\Lambda^2}{(e^2\rho^2)^2}\right). \tag{35}$$

Note that due to the pseudo-differential operator we have obtained a logarithmic term that resembles the standard effective potential in the scalar QED in (3+1)D [16].

5.3. The Effective Potential

Finally, the effective potential, after taking the two terms, reads

$$\begin{aligned} V_{\text{eff}}^{\text{HPQED}}(\rho,\Lambda) = -U(\rho) - \frac{(U''\Lambda)}{2\pi^2} - \frac{(-U'')^{3/2}}{6\pi} + \frac{(e^2\rho^2)^3}{12\pi^2} - \frac{3\Lambda e^4\rho^4}{8\pi^2} \\ + \frac{3\Lambda^2 e^2\rho^2}{8\pi^2} + \frac{3(e^2\rho^2)^3}{24\pi^2}\ln\left(\frac{\Lambda^2}{(e^2\rho^2)^2}\right). \end{aligned} \tag{36}$$

Next, let us discuss the possibility of finding a symmetry-breaking solution in Eq. (36). Let us consider a classical potential, given by Eq. (22) with $M^2 = 0$. In this case, we find a stable ground state at $\rho = 0$ and the effective potential resembles Fig. 1. Otherwise, when $\lambda > 0$, we do not have a real-valued potential.

Although our results indicate that HPQED does not admits a SSB at one-loop approximation, we could also imagine that the scalar field has a nonzero VEV at tree level, i.e, an explicit breaking of symmetry. In this case, the interaction between the electrons is given by a two-dimensional Fourier transform of $1/(2\sqrt{p^2} + 2e^2\rho_0^2)$. Indeed, such result has been considered in Ref. [20] in the light of superconductivity driven by a topological phase transition. Clearly, this is quite different in comparison to the Yukawa potential we have discussed in Sec. IV. Indeed, it is not hard to conclude that the resulting potential behaves as $\ln(r)$ for $r \to 0$ and goes to zero as $r \to \infty$. This only proves the need of taking care between the order of the dimensional reduction and SSB.

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6. Summary

The major relevance of SSB is in describing quantum states of matter, such as ferromagnetism and superconductivity. This is typically made by considering a Landau-Ginzburg theory [21]. For field theories, scalar QED4 is a relativistic generalization of the Landau-Ginzburg theory, where the Dirac field of QED4 is replaced by a scalar field with a self-interacting term [16]. However, when considering a (2+1)D version of scalar QED, one concludes that the effective potential behaves quite different in comparison with the higher-dimensional version of the model. For example, its effective potential trivially obeys the renormalization group equation [18]. Indeed, because the spatial dimension is reduced by one, hence, the ultraviolet divergences are reduced and all of the beta functions vanish.

In order to have a proper description of the electronic interactions between electrons in a condensed-matter system, we must consider the reduced version of QED4, namely, PQED [7]. This dimensional reduction also may be performed to preserve the Yukawa interaction in the plane [17]. Here, we show that the SSB does not occur in PQED, using the one-loop approximation within the background field method. Although we have considered a real scalar field, the generalization for the complex field is straightforward. Thereafter, we consider the Abelian version of scalar PQED in (2+1)D. In this case, we must be careful about the order in which we realize both the SSB and the dimensional reduction. When the SSB is considered in scalar QED4, hence, after the dimensional reduction, we obtain a version of PQED that describes the Yukawa interaction between electrons in (2+1)D. Nevertheless, starting with PQED and coupling it to a scalar field, we realize that SSB is not realized and the system remains in its symmetric phase. Obviously, our approach is not the only path for describing screening effects and it only has the virtue that it follows from a quite simple and well known method, namely, the coupling of the gauge and scalar fields. Indeed, screening effects due to the fermionic loop are also considered in literature of quantum field theory [16] and may also be relevant for two-dimensional materials [10].

It would be interesting to investigate the SSB in the Abelian PQED where both the gauge and scalar fields have a pseudo-differential operator as well as the effect of including a thermal bath. We shall discuss this elsewhere.

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7. From (3+1)D to (1+1)D

In this case, we have to consider $J(y) = J(y_0, y_1)\delta(y_2)\delta(y_3)$ and $\varphi(x) \to \varphi_{2D}(x; a) \equiv \Theta(x_0, x_1; a) = \varphi(x_0, x_1, x_2 = a, x_3 = 0)$. Note that, for the sake of convenience, we have introduced a scale parameter $x_2 = a$. Similarly to the previous case, we obtain

$$\varphi_{2D}(x; a) = \int d^2y\, G_{2D}(x - y; a) J_{2D}(y), \tag{37}$$

where

$$G_{2D}(x - y; a) = \int \frac{d^2k_E}{(2\pi)^2} e^{ik(x-y)} \frac{K_0(\sqrt{k^2 + m^2}a)}{2\pi} \tag{38}$$

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is the Fourier transform of the scalar-field propagator in (1+1) dimensions. K_0 is the modified Bessel function of second kind. Obviously, this also procudes the Yukawa potential in Eq. (5), but the derivation is more subtle.

The static potential may be obtained by using $k_E^2 = k_0^2 + k_1^2$ with $k_0 \to 0$ in Eq. (38), hence,

$$V(x_1;a) = \int_{-\infty}^{+\infty} \frac{dk_1}{(2\pi)} e^{ik_1 x_1} \frac{K_0(\sqrt{k_1^2 + m^2}a)}{2\pi}. \tag{39}$$

Next, we use a parametrization in ξ, given by

$$\frac{K_0(\sqrt{k_1^2 + m^2}a)}{2\pi} = \int_{-\infty}^{+\infty} \frac{d\xi}{2\pi} \frac{e^{i\xi a}}{2\sqrt{\xi^2 + k_1^2 + m^2}}. \tag{40}$$

Using Eq. (40) in Eq. (39) and changing variables to $\rho^2 = k_1^2 + \xi^2$ and $\tan\theta = k_1/\xi$, after some algebra, we find

$$V(x_1;a) = \int_0^{\infty} \frac{\rho d\rho}{4\pi} \frac{J_0(\rho R)}{\sqrt{\rho^2 + m^2}}, \tag{41}$$

where $R^2 = x_1^2 + a^2$. Finally, solving the integral over ρ in Eq. (41) and using $a \to 0$, we have

$$V(x_1) = \frac{e^{-|mx_1|}}{4\pi|x_1|}, \tag{42}$$

which is the Yukawa potential in (1+1) dimensions.

Similarly to the previous case in Eq. (9), we may define a full action in (1+1)D that generates the scalar-field propagator in Eq. (38), given by

$$\mathcal{L}_{2D} = \frac{\partial^\mu \Theta L[\Box] \partial_\mu \Theta}{2} + g\Theta\bar{\psi}\psi + \mathcal{L}_M[\psi], \tag{43}$$

where

$$\begin{aligned} L[\Box] &\equiv \frac{2\pi}{(-\Box)K_0(\sqrt{-\Box + m^2}a)} \\ &= \int \frac{d^2k_E}{(2\pi)^2} e^{ikx} \frac{2\pi}{k_E^2 K_0(\sqrt{k_E^2 + m^2}a)}. \end{aligned} \tag{44}$$

This concludes our general example of reduced models. Our main results are easily generalized for gauge theories, such as QED4.

8. The one-loop approximation

In this appendix, we derive Eq. (14). We start with Eq. (12) and perform an expansion for $\phi(x) \approx \phi_{cl}(x)$, given by

$$\begin{aligned} \mathcal{L}_D[\phi] &= \mathcal{L}_D[\phi_{cl}] + \frac{\delta \mathcal{L}_D}{\delta\phi}|_{\phi=\phi_{cl}}(\phi - \phi_{cl}) \\ &+ \frac{1}{2}\frac{\delta^2 \mathcal{L}_D}{\delta\phi\delta\phi}|_{\phi=\phi_{cl}}(\phi - \phi_{cl})^2 + ..., \end{aligned} \tag{45}$$

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where the classical field $\phi_{cl}(x)$ is just a solution of the equation of motion. In particular, from Eq. (11), note that

$$\frac{\delta^2 \mathcal{L}_{\mathrm{D}}}{\delta\phi\delta\phi} = (-\Box)\mathcal{K} + U'' = G_{\phi}^{-1}[-\Box] + U'', \tag{46}$$

where $G_{\phi}^{-1}[-\Box] \equiv (-\Box)\mathcal{K}$ is, essentially, the inverse of the free propagator of the scalar field. Using Eq. (45) and Eq. (46) in Eq. (12), we find

$$\mathcal{Z} = e^{iS_{\mathrm{cl}}[\phi_{\mathrm{cl}}]} \int D\bar{\phi} \exp\left\{ i\frac{1}{2}\bar{\phi}[G_{\phi}^{-1}[-\Box] + U'']\bar{\phi} \right\}, \tag{47}$$

where $\bar{\phi} \to \phi - \phi_{cl}$. Solving the integral over $\bar{\phi}$ in Eq. (47) yields

$$\mathcal{Z} = e^{iS_{\mathrm{cl}}[\phi_{\mathrm{cl}}]} \det\{-i[G_{\phi}^{-1}[-\Box] + U'']\}^{-1/2}. \tag{48}$$

Next, we use that $\det A = \exp\{\mathrm{Tr}\} \ln A$, where Tr is a trace operation over the space of the arbitrary matrix A, i.e, $\mathrm{Tr} \ln A = \int d^D x \langle x| \ln A |x\rangle$. Therefore,

$$\begin{aligned} &- \frac{1}{2}\int d^D x \langle x| \ln[G_{\phi}^{-1}[-\Box] + U'']|x\rangle = \\ &- \frac{\Omega}{2}\int \frac{d^D k}{(2\pi)^D} \ln[G_{\phi}^{-1}(k) + U''], \end{aligned} \tag{49}$$

where $\Omega = \int d^D x$ and an irrelevant $-i$ factor has been eliminated in the first line of Eq. (49). The rhs of Eq. (49) is the one-loop quantum correction, proportional to $\hbar$, and we write this as $\Omega V_1[\phi_{\mathrm{cl}}]$. Hence, using Eq. (49) in Eq. (48), we have

$$\mathcal{Z} = e^{iS_{\mathrm{cl}}[\phi_{\mathrm{cl}}] + \Omega V_1[\phi_{\mathrm{cl}}]}. \tag{50}$$

Finally, having in mind the constant field configuration where $\phi_{\mathrm{cl}}(x) \to \rho$, we find that $S_{\mathrm{cl}}[\phi_{\mathrm{cl}}] \to \Omega U(\rho)$. Therefore, after comparing Eq. (50) and Eq. (49) with Eq. (13), it follows Eq. (14).

9. The D-Dimensional Integral for the Reduced Model

In this appendix, we calculate Eq. (17). Firstly, we must convert the D-dimensional integral to the Euclidean spacetime, which is easily performed by using $k^2 \to -k_E^2$ and $d^D k \to i d^D k_E$. Hence, we find

$$V_1(\rho,\mu) = \frac{\mu^{3-D}}{2}\int \frac{d^D k_E}{(2\pi)^D} \ln[2\sqrt{k_E^2} - U''(\rho)]. \tag{51}$$

Next, we derive Eq. (14) in respect to U'', hence, we find

$$\frac{dV_1}{dU''} = -\frac{\mu^{3-D}}{2}\int \frac{d^D k_E}{(2\pi)^D} \frac{1}{2\sqrt{k_E^2} - U''(\rho)}. \tag{52}$$

For solving the integral in Eq. (52), we use the following identity

$$\int \frac{d^D k_E}{(2\pi)^D} \frac{1}{\sqrt{k_E^2} + A} = \frac{2\pi^{D/2}}{\Gamma[D/2]} \frac{A^{D-1}}{(2\pi)^D} \Gamma[D]\Gamma[1-D], \tag{53}$$

where A is an arbitrary constant. Using Eq. (53) in Eq. (52) and integrating over U'', one finds Eq. (17).

It is interesting to notice that Eq. (53) is not the same as the D-dimensional integral we would find for the usual scalar theory, where the denominator would be like $k_E^2 + A$. Hence, let us briefly derive

Eq. (53). Let us call I_D as the lhs of Eq. (53) and use $d^D k_E = k_E^{D-1} dk_E\, S_D$, where $S_D = 2\pi^{D/2}/\Gamma[D/2]$. Thereafter, we make a variable change to $x \equiv A/(k+A)$ and, after some algebra, we find

$$I_D = \frac{S_D}{(2\pi)^D} A^{D-1} \int_0^1 dx x^{-D}(1-x)^{D-1}. \tag{54}$$

The beta function $B(a,b)$ reads

$$B(a,b) = \frac{\Gamma[a]\Gamma[b]}{\Gamma[a+b]} = \int_0^1 dx x^{a-1}(1-x)^{b-1}, \tag{55}$$

where a and b are real positive constants. After using the identity in Eq. (55) into Eq. (54), we find Eq. (53).

10. The Dimensional Reduction of the QED4 theory coupled with Higgs field

In this appendix, we calculate the dimensional reduction of Eq. (23). Firstly, we expand

$$|D_\mu \varphi|^2 = \partial^\mu \varphi^* \partial_\mu \varphi - ig\bar{A}^\mu[\varphi^*\partial_\mu \varphi - \varphi \partial_\mu \varphi^*] + g^2 \bar{A}_\mu \bar{A}^\mu \varphi^2. \tag{56}$$

Next, we the scalar field is written as $\varphi(x) = \xi(x)e^{i\theta(x)}$, where (ξ, θ) are real-valued functions. Hence, it follows that

$$|D_\mu \varphi|^2 = \partial^\mu \xi \partial_\mu \xi + \xi^2 \partial^\mu \theta \partial_\mu \theta + 2g\xi^2 \bar{A}^\mu \partial^\mu \theta + g^2 \bar{A}_\mu \bar{A}^\mu \xi^2. \tag{57}$$

After using the gauge transformation $\bar{A}^\mu \to \bar{A}^\mu - \partial^\mu \theta/g$, we can eliminate the $\theta(x)$ dependence on the model and obtain

$$|D_\mu \varphi|^2 = \partial^\mu \xi \partial_\mu \xi + g^2 \xi^2 \bar{A}_\mu \bar{A}^\mu. \tag{58}$$

This is essentially given in Eq. (24). Furthermore, we consider that the Higgs field is in the broken phase and take its lowest-order solution, given by $\xi(x) \to \rho_0$ and find Eq. (25).

In order to perform the dimensional reduction, we set the generating functional of the Green functions for A_μ, namely,

$$Z = Z_0 \int D\bar{A}_\mu e^{-\int d^4x \left[\frac{1}{4}\bar{F}^{\mu\nu}\bar{F}_{\mu\nu} - \frac{1}{2\alpha}(\partial_\mu \bar{A}^\mu)^2 + g^2\rho_0^2 \bar{A}_\mu \bar{A}^\mu + e j_\mu \bar{A}^\mu\right]}, \tag{59}$$

The first two terms of the gauge field action are conveniently written as

$$\frac{1}{4}\bar{F}_{\mu\nu}\bar{F}^{\mu\nu} - \frac{1}{2\alpha}(\partial_\mu \bar{A}^\mu)^2 = -\frac{1}{2}\bar{A}^\mu \left[-\delta_{\mu\nu}\Box - \left(1 + \frac{1}{\alpha}\right)\partial_\mu \partial_\nu\right]\bar{A}^\nu. \tag{60}$$

Nevertheless, because the theory is gauge invariant we may neglect the gauge-dependent term, for when it is coupled to a current it vanishes. Next, we use the following identity

$$\int D\varphi_B e^{-\frac{1}{2}\int dx[\varphi_B K \varphi_B] + \int dx \varphi_B J} = \det\{K\}^{-1/2} e^{\frac{1}{2}\int dx[JK^{-1}J]}, \tag{61}$$

which holds for any bosonic field φ_B. Furthermore, K is an arbitrary matrix and J is an external source. Therefore, we have

$$\begin{aligned} Z &= \frac{Z_0}{e} \int D(e\bar{A}_\mu) e^{-\int d^4x \left[\frac{1}{2}(eA^\mu)\frac{[-\Box\delta_{\mu\nu} + 2g^2\rho_0^2\delta_{\mu\nu}]}{e^2}(eA^\nu) + j_\mu(e\bar{A}^\mu)\right]} \\ &= \frac{Z_0}{e} \det\left\{(-\Box + 2g^2\rho_0^2)/e^2\right\}^{-4/2} e^{\int d^4x \left[\frac{e^2}{2} j^\mu \frac{1}{(-\Box + 2g^2\rho_0^2)} j_\mu\right]}. \end{aligned} \tag{62}$$

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Obviously, we may use the fact that the physics is unchanged by modifying the constant Z_0 and obtain

$$Z[j_\mu] = e^{-\int d^4x \left[-\frac{e^2}{2} j^\mu \frac{1}{(-\Box+2g^2\rho_0^2)} j_\mu\right]}, \tag{63}$$

where it is defined that $Z[0] = 1$. Now, we confine the charges to move in a two-dimensional space, using the condition

$$j_\mu = \begin{cases} j_\mu^{3D}(x_0, x_1, x_2)\delta(x_3), \ \mu = 0,1,2 \\ 0, \ \mu = 3 \end{cases} . \tag{64}$$

Therefore,

$$\begin{aligned} \frac{1}{-\Box + 2g^2\rho_0^2}\Big|_{x_3=y_3=0} &= \int \frac{d^4k}{(2\pi)^4} \frac{e^{ik(x-y)}}{k^2 + 2g^2\rho_0^2}\Big|_{x_3=y_3=0} \\ &= \int \frac{d^4k}{(2\pi)^4} \frac{e^{ik_{3D}(x-y)}}{k_3^2 + k_{3D}^2 + 2g^2\rho_0^2} \\ &= \int \frac{d^3k}{(2\pi)^3} e^{ik_{3D}(x-y)} \int_{-\infty}^{+\infty} \frac{dk_3}{2\pi} \frac{1}{k_3^2 + k_{3D}^2 + 2g^2\rho_0^2} \\ &= \int \frac{d^3k}{(2\pi)^3} \frac{e^{ik_{3D}(x-y)}}{2\sqrt{k_{3D}^2 + 2g^2\rho_0^2}} \\ &= \frac{1}{2} \frac{1}{(-\Box_{3D} + 2g^2\rho_0^2)^{1/2}}. \end{aligned} \tag{65}$$

Hence, we obtain the generating functional, given by

$$Z = e^{-\int d^3x \left[-\frac{e^2}{2} j^\mu_{3D} \frac{1}{2} \frac{1}{(-\Box_{3D}+2g^2\rho_0^2)^{1/2}} j_\mu^{3D}\right]}. \tag{66}$$

Finally, it is fair to conclude that the effective action associated with this generating functional is given by Eq. (26).

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