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Quantum Dynamics under Gravitational Potential; Insights from Gravitational-Redshift

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Abstract: This short paper briefly explores the interplay between quantum mechanics and gravity, drawing from the principle of gravitational redshift and bringing it into quantum mechanical frameworks. The exploration looks through quantum mechanical relations and equations such as Planck-Einstein relation, De-Broglie's relation and the relativistic wave equations, and propagators, accounting for the presence of gravitational fields through gravitational redshift of its momentum.

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Introduction

The gravitational potential is a scalar quantity defined as the gravitational potential per unit mass of a body. It defines the strength of the gravitational field.

From Newtonian gravity, the energy of a gravitational field is given as $U = GMm/r$, dividing by mass leads to the field potential $\Phi = GM/r$, alternatively the field potential is written as $\Phi = gr$ for a gravitational potential energy $U = mgr$. The current description of gravity departs from Newtonian ideologies that considers gravity as a force of attraction between massive bodies according to newton's inverse square law, and is now involving the geometry of a 4-dimensional space-time, where gravity is said to be the effect of the curvature of space-time. This theory is the general theory of relativity [14,15,19] developed by Albert-Einstein.

Quantum mechanics [7–9,13] is another popular branch of physics studying the quantization and behavior of particles, and other unique phenomena and properties that these particles possess and experiences.

The dynamics of these particles and quantum fields are studied using frameworks such as relativistic quantum mechanics, canonical quantization, and path integral. Mathematically, Relativistic quantum mechanics uses wave-equations such as the Klein-Gordon's equation for spin-0 scalar fields, and Dirac's equation for spin-1/2 spinor fields.

While these frameworks and equations have proven to be useful tools in the study of particles and quantum fields, fitting in the concept of gravity or general relativity into these frameworks still remains an ongoing area of research referred to as quantum gravity [6].

There are several attempts at developing theories of quantum gravity. One such attempt is loop-quantum gravity [4,5] which is a canonical theory of quantum gravity that takes the space-time continuum to be a complex geometric structure known as spin-networks. A similar attempt in developing a quantum theory of gravity is the Causal Dynamical triangulation [20,21] which discretizes space-time in a lattice structure. CDT employs a path integral style formulation but replaces the integrals with discrete sum which is an approximation to integrals hence leading to the lattice structure. Another prominent attempt is strings theory [3] though this is a much wider theory that defines quantum fields and particles as vibrational modes of a string, where a particular vibrational mode exists that leads to the existence of graviton, a hypothetical particle expected to be the carrier of the gravitational force.

While the theories mentioned above are fully within quantum domains, another theory proposed as a semi-classical approximation to quantum gravity is QFT in Curved Space-time [1,2]. This theory defines quantum field theories in curved background space-time, unlike the usual flat

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background. It is considered semi-classical because we're only treating quantum field quantum mechanically while we treat space-time classically.

Gravitational redshift [16–18] is a theory proposed within the domains of general relativity, it predicts a shift in the wavelength or frequency of light as it goes further away from the source of the gravitational field. This shift would then be noticed as the change in color of the light. According to general relativity this shift in wavelength is attributed to gravitational time dilation effects experienced by the light as it moves further away from the gravitational field.

The quantum and relativistic nature of light as well as its experience of redshift motivates this paper, discussing how the quantum states of light can be influenced by gravitational redshifts.

Gravitational Redshift

Gravitational redshift as stated in the introduction is one of the various phenomena predicted by general relativity, sometimes referred to as Einstein shift. It is the increase in wavelength or decrease in frequency of light or electromagnetic waves as they travel further away from the gravitational field, this is attributed to the fact that the wave loses energy as it travels further away from the gravitational well. The opposite effect referred to as blue shift involves an energy gain as the wave travels into the gravitational well, leading to increase in the frequency and decrease in the wavelength. Before proceeding to the main ideas of this brief paper, we review some of the mathematical expressions of gravitational redshift which will be instrumental in communicating the subject of this article.

The gravitational red-shift is expressed as

$$z_f = \frac{\Delta f}{f_0} = -\frac{\Delta \phi}{c^2}, \quad z_\lambda = \frac{\Delta \lambda}{\lambda_0} = \frac{\Delta \phi}{c^2}$$

z_f *is the frequency shift, and* z_λ *is the shift in the wavelength* From which we can state the following

$$f = f_0\left(1 - \frac{\Delta\phi}{c^2}\right) \qquad \lambda = \lambda_0\left(1 + \frac{\Delta\phi}{c^2}\right)$$

Also the gravitational time dilation according to general relativity is given by $\tau = \frac{t}{\sqrt{1-\frac{\Delta\phi}{c^2}}}$.

These expressions marks the starting point of the subject to be explored in this paper.

Planck-Einstein Relation

The Planck-Einstein relation is one of the foundational equations of quantum mechanics, relating the energy of a photon to its frequency. The relation implies a direct proportionality between the energy of the photon and its frequency with its proportionality constant given to be the Planck's constant.

This is the equation $E = hf_0$ *where* f_0 *is the natural or actual frequency of light*

Upon a gravitational red-shift, the frequency becomes

$$f = f_0\left(1 - \frac{\Delta\phi}{c^2}\right)$$

Expressing Planck's relation in terms of its observed/shifted frequency $E = hf$, the expression yields

$$E = h\left[f_0\left(1 - \frac{\Delta\phi}{c^2}\right)\right]$$

Which we alternatively write as $E = h\left(f_0 - \frac{\Delta\phi}{c^2} f_0\right)$ now implying an energy shift.

By doing this the relation accounts for the effect of gravity on the energy of the photon.

However we understand that the Planck-Einstein relation is not limited to photons only, and that it applies to any particle as long as they exhibits wave properties. And if particles such as

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electrons have wave-properties it can be theorized that they may also be capable of experiencing shifts in their energy, frequency and wavelength when climbing out of a gravitational well.

If this be the case, then the modified Planck's relation given above may apply to all matter waves.

De-Broglie's Relation

The De-Broglie's relation developed by Louis De-Broglie is based on the wave-nature of matter, the relation describes the natural/actual wavelength of matter waves as

$$\lambda_0 = \frac{h}{p}$$

Taking momentum to be the subject of the relation leaves us with $p = \frac{h}{\lambda_0}$ considering gravitational red shift of the natural frequency to the observed frequency, gives the momentum shift

$$p = \frac{h}{\lambda_0(1 + \frac{\Delta\phi}{c^2})}$$

When a matter wave departs from a source of spacetime curvature, both the Planck Einstein relation and the de-broglie's relation would include the effects of gravitational redshifts.

Relativistic Wave Equations

While gravitational redshift may influence energy and momentum of matter waves. This is not directly evident in the mass energy equivalence $E = Mc^2$, as the speed of light "c" is a constant value, and its mass remains unchanged in the presence of gravity, which neglects the effect of gravitational redshift. But when the energy of a matter-wave under gravitational redshift is defined based on the full relativistic energy equation, where the gravitational redshift is accounted for through the momentum, the influence of gravitationally redshift on the overall energy is revealed.

This is mathematically expressed below

$$E = \sqrt{p^2c^2 + m^2c^4} \quad where \ \ p = \frac{h}{\lambda_0(1 + \frac{\Delta\phi}{c^2})}$$

Quantization as will be seen below involves promotion of the momentum and energy variables in the relativistic energy equation to momentum and energy operators acting on fields, gives rise to the relativistic wave equation [11,12]

$$\hat{E}^2\psi = \hat{p}^2\psi c^2 + m^2\psi c^4 \ , \qquad \hat{E}^2 = i\hbar^2\frac{\partial^2}{\partial t^2} \ , \qquad \hat{p}^2 = i\hbar^2\nabla^2$$

taking the respective substitutions

$$i\hbar^2\frac{\partial^2}{\partial t^2}\psi = i\hbar^2c^2\nabla^2\psi + m^2c^4\psi$$

Dividing through by "c" leaves us with the klein-gordon's equation

$$\frac{i\hbar^2}{c^2}\frac{\partial^2}{\partial t^2}\psi = i\hbar^2\nabla^2\psi + m^2c^2\psi$$

On the left hand side of the resulting equation is the energy operator acting on scalar field ψ and on the right hand side of the equation we have the momentum operator multiplied by the speed of light also acting on the same scalar field and we also have the additional term m^2c^2.

Another relativistic equation put forward by Paul-Dirac describes the dynamics of spinor field, taking the spins of the particle into consideration. Although initially developed as a theory for studying the dynamics of electrons, Dirac's equation also applies to fermions with ½ spins. Dirac's equation is typically given by

$$(i\hbar\gamma^\mu\partial_\mu - mc)\psi = 0$$

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The spin of the particle is accounted for through the gamma-matrices satisfying the relation $\{\gamma^{\mu}, \gamma^{\nu}\} = 2\eta^{\mu\nu} I$

And this draws a major distinction between Dirac's equation and klein-gordon's equation, where ψ is a spinor field.

Feynman Propagators

With the klein-gordon's equation already given, a propagator accounting for the gravitational redshift would be necessary, such propagator may be given by

$$G(x-y) = \int \frac{d^4k}{(2\pi)^4} \frac{i}{k^2 - m^2 + i\epsilon} \quad , \qquad k = \frac{h}{\lambda_0(1 + \frac{\Delta\phi}{c^2})}$$

For the above propagator, the momentum "k" is said to be influenced by gravitational redshift. It should be noted that given propagator does not directly account for spins of particle.

To describe the spin-1/2 fields corresponding to Dirac's equations the gamma matrices is then introduced, such that the propagator becomes

$$G(x-y) = \int \frac{d^4k}{(2\pi)^4} \frac{i}{\gamma^{\mu}k^2 - m^2 + i\epsilon} \quad , \qquad k = \frac{h}{\lambda_0(1 + \frac{\Delta\phi}{c^2})}$$

In this case the gamma matrices should satisfy the relation in curved spacetime

$$\{\gamma^{\mu}, \gamma^{\nu}\} = 2g^{\mu\nu} I$$

The propagator for a particle departing from a source of spacetime curvature are then considered integrals over gravitationally red-shifted momenta.

Feynman Diagrams

When calculating amplitudes through a Feynman diagram the gravitational redshift can also be included, just as with the propagator.

We can give for a generic or assumed diagram that

$$\prod_{n=1}^{N} \left(\int \frac{d^4k}{(2\pi)^4} \frac{i}{k^2 - m^2 - j_{int} + i\epsilon} \right)^n \quad , \qquad k = \frac{h}{\lambda_0(1 + \frac{\Delta\phi}{c^2})}$$

With the interaction term and coupling constant "j_{int}" accounting for vertices on the diagram, it can also be said that as gravitational redshift influences the energy and momentum at certain scales, the interaction strength would also be influenced by the shifting.

Gravitational Blueshift

This is a necessary concept to discuss being the opposite of gravitational redshift, this happens when the wave moves closer to the source of the gravitational potential and not away from it. In this case the matter wave gains energy and increases its frequency while decreasing its wavelength.

Similar mathematical approaches taken for its redshift counterpart is applicable to the concept of blue shift, with just the opposite effect being considered.

Conclusions

Gravitational redshift is a concept that lies in between general relativity and the wave nature of light, revealing how the properties of light waves such as its wavelength, energy and frequency are altered as it climbs out of the gravitational well.

However this wave nature of light can be studied within the domains of the fundamental principle of quantum mechanics, fundamental principle such as the planck-einstein relation and the de-Broglie's relation, where the redshifts of the wavelength and frequencies is considered in the relations. Due to the wave properties of matter it can also be theorized that matter-waves apart from just photons may also be able to experience gravitational red-shifts.

The relativistic energy equation as discussed in the article accounts for gravitational redshift through the effect of shift in momentum. Upon quantization of energy and momentum we arrive at klein-gordon's equation.

The propagators corresponding to the equations are then defined to be integrals over gravitationally red shifted momenta. This implies that gravity may influence the propagation and interaction of particles, by reason of the gravitational shift in momentum both at high energy or low energy scales.

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References

1. Bryce. S. DeWitt, Quantum field theory in curved space-time, Physics Reports, 19(6), 295-357, (1975).
2. Stefan Hollands, & Robert. M. Wald, Quantum fields in curved spacetime, Physics Reports, 574, 1-35, (2015).
3. Thibault Damour & Alexander. M. Polyakov, String theory and gravity, General Relativity and Gravitation, 26, 1171-1176, (1994).
4. Abhay Ashetekar & Eugenio Bianchi, A short review of loop quantum gravity, Reports on Progress in Physics, 84(4), 042001, (2021).
5. Carlo Rovelli, Loop quantum gravity, Living Reviews in relativity, 11, 1-69, (2008).
6. Carlo Rovelli, Quantum gravity, Cambridge University Press, (2004).
7. Roland Omnes, Understanding quantum mechanics, Princeton University press, (1999).
8. Werner Heinsberg, Development of Concepts in the history of quantum theory, the physicist's conception of nature, 264-275, (1973).
9. Louis de-Broglie, Interpretation of quantum mechanics by the double solution theory, Annales de la fondation Louis de Broglie 12(4), 1-23, (1987).
10. Albert Einstein, Elementary derivation of the equivalence of mass and energy, Bulletin of the American mathematical society, 41(4), 223-230, (1935).
11. LL. Foldy, Relativistic wave-equations, Pure and applied physics, 10, 1-46, (1962).
12. Paul Adrien Maurice Dirac, Relativistic wave-equations, Proceedings of the Royal society of London. Series A-mathematical and Physical science, 155(886), 447-459, (1936).
13. Hans Bethe, Quantum theory, Review of modern physics, 71(2), S1, (1999).
14. Michael Paul Hobson et-al, General relativity: An introduction for physicists, Cambridge university press, (2006).
15. Robert. M. Wald, General relativity, University of Chicago Press, (2010).
16. John Earman & Clark Glymor, The gravitational red-shift as a test of general relativity: History and analysis, Studies in History and philosophy of science part-A, 11(3), 175-214, (1980).
17. Randy-Wayne, Rethinking the concept of spacetime in the general theory of relativity: The deflection of starlight and the gravitational red shift, The African review of Physics, 7, (2012).
18. Satya pal Singh et-al, the redshift in relativity, European Journal of Physics Education, 2(2), 24-48, (2011).
19. John Earman & Michael Janssen, Einstein's Explanation of the motion of the Mercury's perihelion, the attraction of gravitation: New studies in the history of general relativity, 129, (1993).
20. Renate Loll, Quantum gravity from Causal dynamical triangulations; a review, Classical and quantum gravity, 37(1), 013002, (2019).
21. Jan Ambjorn et-al, Quantum gravity via Causal dynamical triangulation, Springer handbook of spacetime, 723-741, (2014).

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