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Mueller Matrix Polarizing Power

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Abstract: The transformation of the states of polarization of electromagnetic waves that interact with polarimetrically linear media can be represented by means of the associated Mueller matrix. A global measure of the ability of a linear medium to modify the states of polarization of incident waves, due to any combination of enpolarizing, depolarizing and retarding properties, is introduced as a distance of the Mueller matrix to the identity matrix. This new descriptor, called the polarizing power, is applicable to any Mueller matrix and can be expressed as a function of the degree of polarimetric purity and the trace of the Mueller matrix. The graphical representation of the feasible values of the polarizing power provides a general view of its main peculiarities and features. The values of the polarizing power for several typical devices are analyzed.

Keywords: Mueller matrices; polarimetry; polarizing power

1. Introduction

There are certain well-known descriptors that can be derived from a Mueller matrix **M** and provide information on specific features related to enpolarizing (diattenuation and polarizance [1,2]), depolarizing and retarding properties [3]. Nevertheless, form the point of view of the analysis and exploitation of the information held by **M**, it still lacks the definition of a parameter giving an overall measure of the ability of the medium to transform the states of polarization of incident electromagnetic waves regardless of the origin and nature of the features involved (enpolarization, depolarization or retardation), which usually appear in a combined manner. Such an ability can be characterized through the distance from **M** to the 4×4 identity matrix **I** (which in turn represents a completely neutral polarimetric effect).

Thus, the aim of this work is to introduce a properly defined measure of the polarizing power associated with **M**, which in no way replaces other known descriptors. The contents of this communication are organized as follows. Section 2 summarizes the theoretical concepts and notations necessary to describe the original results presented. Section 3 is devoted to the introduction of the concept of polarizing power associated with a given Mueller matrix, together with some related analyses and graphical representation. Section 4 deals with the invariance properties of the polarizing power with respect to certain polarimetric transformations. The results are illustrated in Section 5 through the inspection of the values taken by the polarizing power for typical polarization devices like diattenuators, retarders and intrinsic depolarizers.

2. Theoretical Background

Hereinafter we will use the term "light" for the sake of brevity, nevertheless it can be understood as the more general "electromagnetic wave".

The transformation of polarized light by the action of a linear medium (under fixed interaction conditions) can always be represented mathematically as $\mathbf{s}' = \mathbf{M}\mathbf{s}$ where $\mathbf{s}$ and $\mathbf{s}'$ are the Stokes vectors that represent the states of polarization of the incident and emerging light beams, respectively, while **M** is the Mueller matrix associated with this kind of interaction and that can always be expressed as [3–6]

$$\mathbf{M} = m_{00} \begin{pmatrix} 1 & \mathbf{D}^T \\ \mathbf{P} & \mathbf{m} \end{pmatrix},$$
$$\mathbf{m} = \begin{pmatrix} k_1 & r_3 & r_2 \\ q_3 & k_2 & r_1 \\ q_2 & q_1 & k_3 \end{pmatrix}, \tag{1}$$
$$\mathbf{k} \equiv \frac{1}{\sqrt{3}} \begin{pmatrix} k_1 \\ k_2 \\ k_3 \end{pmatrix}, \mathbf{D} \equiv \begin{pmatrix} D_1 \\ D_2 \\ D_3 \end{pmatrix}, \mathbf{P} \equiv \begin{pmatrix} P_1 \\ P_2 \\ P_3 \end{pmatrix}, \mathbf{r} \equiv \begin{pmatrix} r_1 \\ r_2 \\ r_3 \end{pmatrix}, \mathbf{q} \equiv \begin{pmatrix} q_1 \\ q_2 \\ q_3 \end{pmatrix},$$

where m_{00} is the mean intensity coefficient (MIC), i.e., the ratio between the intensity of the emerging light and the intensity of incident unpolarized light; **D** and **P** are the diattenuation and polarizance vectors, with absolute values *D* (diattenuation) and *P* (polarizance), and vectors **k**, **r**, **q**, with respective absolute values *k*, *r*, and *q*, are constitutive of the normalized 3×3 submatrix **m** associated with **M**, which provides the complementary information on retardance and depolarization properties.

Mathematically, Mueller matrices are fully characterized by the so-called ensemble criterion [3], which involves two sets of inequalities, namely the passivity, i.e. $m_{00}(1+Q) \leq 1$, with $Q \equiv \max(D,P)$ [7,8], together with the four covariance conditions constituted by the nonnegativity of the eigenvalues of the Hermitian coherency matrix **C** associated with **M**, whose explicit expression in terms of the elements m_{ij} $(i, j = 0,1,2,3)$ of **M**, is [9]

$$\mathbf{C}(\mathbf{M}) = \frac{1}{4} \begin{pmatrix} \begin{matrix} m_{00}+m_{11} \\ +m_{22}+m_{33} \end{matrix} & \begin{matrix} m_{01}+m_{10} \\ -i(m_{23}-m_{32}) \end{matrix} & \begin{matrix} m_{02}+m_{20} \\ +i(m_{13}-m_{31}) \end{matrix} & \begin{matrix} m_{03}+m_{30} \\ -i(m_{12}-m_{21}) \end{matrix} \\ \begin{matrix} m_{01}+m_{10} \\ +i(m_{23}-m_{32}) \end{matrix} & \begin{matrix} m_{00}+m_{11} \\ -m_{22}-m_{33} \end{matrix} & \begin{matrix} m_{12}+m_{21} \\ +i(m_{03}-m_{30}) \end{matrix} & \begin{matrix} m_{13}+m_{31} \\ -i(m_{02}-m_{20}) \end{matrix} \\ \begin{matrix} m_{02}+m_{20} \\ -i(m_{13}-m_{31}) \end{matrix} & \begin{matrix} m_{12}+m_{21} \\ -i(m_{03}-m_{30}) \end{matrix} & \begin{matrix} m_{00}-m_{11} \\ +m_{22}-m_{33} \end{matrix} & \begin{matrix} m_{23}+m_{32} \\ +i(m_{01}-m_{10}) \end{matrix} \\ \begin{matrix} m_{03}+m_{30} \\ +i(m_{12}-m_{21}) \end{matrix} & \begin{matrix} m_{13}+m_{31} \\ +i(m_{02}-m_{20}) \end{matrix} & \begin{matrix} m_{23}+m_{32} \\ -i(m_{01}-m_{10}) \end{matrix} & \begin{matrix} m_{00}-m_{11} \\ -m_{22}+m_{33} \end{matrix} \end{pmatrix}. \tag{2}$$

A measure of the ability of **M** to preserve the degree of polarization (DOP) of totally polarized incident light, is given by the *degree of polarimetric purity* of **M** (also called *depolarization index*) [9], P_Δ, which can be expressed as

$$P_\Delta = \sqrt{\frac{2P_P^2}{3} + P_S^2}, \tag{3}$$

where P_P is the so-called degree of polarizance [11], or *enpolarizance* (a measure of the ability of **M** to increase the degree of polarization of light in either forward or reverse incidence),

$$P_P \equiv \sqrt{\frac{D^2+P^2}{2}}, \tag{4}$$

and P_S is the polarimetric dimension index (also called the degree of spherical purity), defined as [3,12,13]

$$P_S \equiv \sqrt{\frac{3k^2+r^2+q^2}{3}}. \tag{5}$$

Nondepolarizing (or pure) media (i.e., media that do not decrease de degree of polarization of totally polarized incident light) exhibit, uniquely, the maximal degree of polarimetric purity, $P_\Delta = 1$, while $P_\Delta = 0$ is characteristic of perfect depolarizers, with associated Mueller matrix $\mathbf{M}_{\Delta 0} = m_{00}\,\mathrm{diag}(1,0,0,0)$. The maximal value of P_S, $P_S = 1$, entails $P_\Delta = 1$ with $P_P = 0$ (pure and nonenpolarizing media), which corresponds uniquely to retarders (regardless of the value of m_{00}, i.e., regardless of whether they are transparent or exhibit certain amount of isotropic attenuation); the minimal polarimetric dimension index, $P_S = 0$, corresponds to Mueller matrices with $\mathbf{m} = \mathbf{0}$. Maximal enpolarizance, $P_P = 1$, implies $P_\Delta = 1$ and corresponds to perfect polarizers, while the minimal, $P_P = 0$, is exhibited by nonenpolarizing interactions (either pure or depolarizing) [12,14].

Regarding measures of the anisotropies involved in **M**, a set of anisotropy coefficients was defined in Ref. [15], leading to an overall parameter called the degree of anisotropy, P_α, which is obtained as a square average of linear and circular anisotropies due to enpolarizing and retarding properties, and can be expressed as follows (except for nondepolarizing diagonal Mueller matrices, for which P_α is undetermined) [15]

$$P_\alpha = \frac{D^2 + P^2 + r^2 + q^2 + 2\mathbf{D}^T\mathbf{P} - 2\mathbf{r}^T\mathbf{q}}{3 - 3k^2 + 2\mathbf{D}^T\mathbf{P} - 2\mathbf{r}^T\mathbf{q}} \,. \tag{6}$$

The values of P_α are limited by $0 \le P_\alpha \le P_\Delta$.

As for depolarizing properties of **M**, complete quantitative information is given by the corresponding indices of polarimetric purity (IPP), defined as follows in terms of the (nonnegative) eigenvalues $(\lambda_1, \lambda_2, \lambda_3, \lambda_4)$ of $\mathbf{C}(\mathbf{M})$, labeled in non-increasing order $(\lambda_1 \ge \lambda_2 \ge \lambda_3 \ge \lambda_4)$ [2,16]

$$P_1 = \frac{1}{\operatorname{tr}\mathbf{C}}(\lambda_1 - \lambda_2),\; P_2 = \frac{1}{\operatorname{tr}\mathbf{C}}(\lambda_1 + \lambda_2 - 2\lambda_3),\; P_3 = \frac{1}{\operatorname{tr}\mathbf{C}}(\lambda_1 + \lambda_2 + \lambda_3 - 3\lambda_4) \,. \tag{7}$$

Leaving aside systems exhibiting magneto-optic effects, the Mueller matrix that represents the same linear interaction as **M**, but with the incident and emergent directions of the light probe swapped, is given by [17,18]

$$\mathbf{M}^r = \operatorname{diag}(1,1,-1,1)\,\mathbf{M}^{\mathrm{T}}\operatorname{diag}(1,1,-1,1), \,. \tag{8}$$

consequently, $D(\mathbf{M}^r) = P(\mathbf{M})$ and $P(\mathbf{M}^r) = D(\mathbf{M})$, showing that D and P share a common nature related to the ability of the medium to enpolarize unpolarized light incoming in either forward or reverse directions [2,11].

3. Polarizing Power

Since a fully neutral polarimetric effect is characterized by $\mathbf{M} = \mathbf{I}$, **I** being the 4×4 identity matrix, any enpolarizing, depolarizing and retarding effect implies that **M** takes a form different from **I**. Thus, the overall ability of **M** to change the incident state of polarization, can be characterized by the polarizing power defined as the normalized distance between matrices **M** an **I**,

$$P_\Omega \equiv \frac{\|\mathbf{M} - \mathbf{I}\|_2}{g}, \tag{9}$$

where $\|\;\|_2$ represents the Frobenius norm and g is an integer parameter that will be determined by imposing the limit $P_\Omega \le 1$.

By considering the partitioned form of **M** in Eq. (1) together with Eqs (3) and (5), and taking into account the relation $\operatorname{tr}\mathbf{m} = \sqrt{3}(k_1 + k_2 + k_3)$, tr representing the trace, we get

$$P_\Omega^2 = \frac{3\left(P_\Delta^2 + 1\right) - 2\operatorname{tr}\mathbf{m}}{g^2} \,. \tag{10}$$

As expected, the lower limit for P_Ω^2 is zero and corresponds, uniquely, to $\mathbf{M} = \mathbf{I}$, while the upper limit takes the value $8/g^2$, which corresponds necessarily to either of the following three diagonal half-wave retarders

$$\begin{aligned} \mathbf{M}_{Rd1} &= \operatorname{diag}(1,1,-1,-1), \\ \mathbf{M}_{Rd2} &= \operatorname{diag}(1,-1,1,-1), \,. \\ \mathbf{M}_{Rd3} &= \operatorname{diag}(1,-1,-1,1), \end{aligned} \tag{11}$$

all of them exhibiting retardance π and respective eigenvalues linear horizontal/vertical, linear $\pm\pi/4$ and circular right/left handed. Thus, the choice $g = \sqrt{8}$ is what matches the normalization criterion $P_\Omega \le 1$ and therefore $0 \le P_\Omega \le 1$, so that we define the polarizing power of **M** as

$$P_\Omega = \sqrt{\frac{3\left(P_\Delta^2 + 1\right) - 2\operatorname{tr}\mathbf{m}}{8}} \,. \tag{12}$$

Since the physical meanings of the degree of polarimetric purity and the polarizing power are well established, the above definition provides an interpretation of the quantity $\operatorname{tr}\mathbf{m}$ in terms of P_Δ and P_Ω

$$\mathrm{tr}\,\mathbf{m} = \frac{3}{2}\left(P_\Delta^2 + 1\right) - 4P_\Omega^2 . \tag{13}$$

Note that $-1 \le \mathrm{tr}\,\mathbf{m} \le 3$, where the minimal achievable value $\mathrm{tr}\,\mathbf{m} = -1$ corresponds necessarily to one of the diagonal half-wave retarders in Eq. (11) $(P_\Delta = 1,\ P_\Omega = 1)$, while the maximum $\mathrm{tr}\,\mathbf{m} = 3$ is reached, uniquely, when $\mathbf{M} = \mathbf{I}$ $(P_\Delta = 1,\ P_\Omega = 0)$.

The feasible region for pairs of values $(\mathrm{tr}\,\mathbf{m}, P_\Omega^2)$ is shown in Figure 1. Achievable values are limited by the triangle with vertices AEF. Edge *c* (AE) corresponds, uniquely, to nondepolarizing Mueller matrices, where point A corresponds uniquely to $P_\Omega = 0$ $(\mathbf{M} = \mathbf{I})$, while, as seen above, point E is reached exclusively by diagonal half-wave retarders. All physical situations for depolarizing Mueller matrices have associated points in the triangle AEFA, excluding the above described edge AE. The values of the quantities $(\mathrm{tr}\,\mathbf{m}, P_\Omega^2, P_\Delta^2)$ for each of the points indicated in the figure are the following $\mathrm{A}(3,0,1)$, $\mathrm{B}(2,1/4,1)$, $\mathrm{C}(1,1/2,1)$, $\mathrm{D}(0,3/4,1)$, $\mathrm{E}(-1,1,1)$, $\mathrm{F}(0,3/8,0)$, $\mathrm{G}(1,1/4,1/3)$, $\mathrm{H}(2,1/8,2/3)$.

Along edge *a* (EF), values of P_Δ decrease from 1 to 0 as $\mathrm{tr}\,\mathbf{m}$ increases from -1 (point E) to 0 (point F). Along edge *b* (FA) values of P_Δ increase from 0 to 1 as $\mathrm{tr}\,\mathbf{m}$ increases from 0 (point F) to 3 (point A).

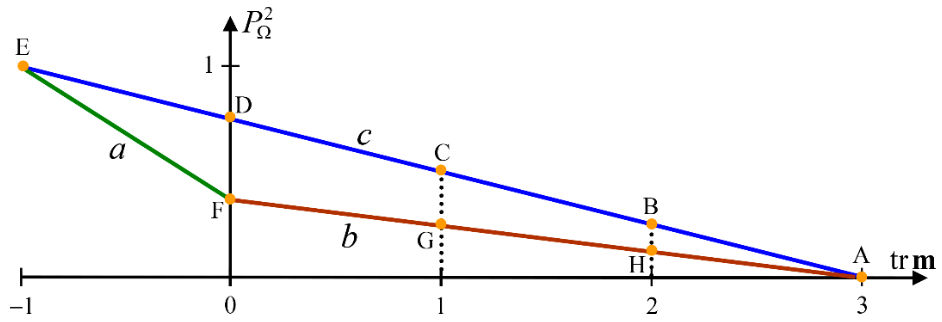

Figure 1. The feasible values for pairs $(\mathrm{tr}\,\mathbf{m}, P_\Omega^2)$ correspond to the triangle limited by lines *a* (EF), *b* (FA) and *c* (AE), which are respectively determined by the functions $P_\Omega^2 = (3 - 5\,\mathrm{tr}\,\mathbf{m})/8$, $P_\Omega^2 = (3 - \mathrm{tr}\,\mathbf{m})/8$ (with $0 \le \mathrm{tr}\,\mathbf{m} \le 3$), and $P_\Omega^2 = (3 - \mathrm{tr}\,\mathbf{m})/4$ (with $0 \le \mathrm{tr}\,\mathbf{m} \le 3$). The coordinates $(\mathrm{tr}\,\mathbf{m}, P_\Omega^2)$ of the points shown in the figure are the following: $\mathrm{A}(3,0)$, $\mathrm{B}(2,1/4)$, $\mathrm{C}(1,1/2)$, $\mathrm{D}(0,3/4)$, $\mathrm{E}(-1,1)$, $\mathrm{F}(0,3/8)$, $\mathrm{G}(1,1/4)$, $\mathrm{H}(2,1/8)$. Except for vertices A ($\mathrm{tr}\,\mathbf{m} = 0$) and E $(\mathrm{tr}\,\mathbf{m} = -1)$, for a fixed value of $\mathrm{tr}\,\mathbf{m}$ infinite values for P_Ω^2 are achievable depending on the corresponding feasible values of P_Δ. In the same way, leaving aside vertices A and E, for a fixed value of P_Ω^2 different values for $\mathrm{tr}\,\mathbf{m}$ are achievable. Points in the line *c* are uniquely covered by nondepolarizing Mueller matrices $(P_\Delta = 1)$. Also, Vertex A corresponds uniquely to a neutral Mueller matrix $(\mathbf{M} = \mathbf{I})$; vertex E is achieved uniquely by either of the diagonal retarders indicated in Eq. (11), and vertex F corresponds, uniquely, to perfect depolarizers $(P_\Delta = 0)$.

4. Invariance

Given a Mueller matrix **M**, transformations $\mathbf{M}_R^T \mathbf{M}\,\mathbf{M}_R$, where $\mathbf{M}_R$ is a proper orthogonal matrix, i.e., $\mathbf{M}_R^T = \mathbf{M}_R^{-1}$ and $\det \mathbf{M}_R = +1$, so that $\mathbf{M}_R$ represents a retarder, are called single retarder transformations [19] and play an important role in polarization theory. The general form of $\mathbf{M}_R$ is

$$\mathbf{M}_R = \begin{pmatrix} 1 & \mathbf{0}^T \\ \mathbf{0} & \mathbf{m}_R \end{pmatrix}, \quad \left[\mathbf{0} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \quad \mathbf{m}_R^T = \mathbf{m}_R^{-1}, \quad \det \mathbf{m}_R = +1 \right], \tag{14}$$

so that it represents a rotation in the mathematical space of the Poincaré sphere. It should be noted that a rotation of angle α about the direction of light propagation (in the Cartesian laboratory coordinate system) are performed through matrices whose general form is [21]

$$\mathbf{M}_G(\alpha) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos 2\alpha & \sin 2\alpha & 0 \\ 0 & -\sin 2\alpha & \cos 2\alpha & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \tag{15}$$

and constitute a subclass of matrices $\mathbf{M}_R$ that, in fact, are mathematically indistinguishable from those of circular retarders [3]. Consequently, the single retarder transformation invariance includes such a kind of rotation invariance.

Let us now observe that both quantities, P_Δ and $\operatorname{tr}\mathbf{m}$, appearing in the definition (12) of the polarizing power are invariant under single retarder transformations, which implies that P_Ω is also invariant under such transformations.

Regarding the swapping of the incident and exit directions of light, which corresponds to the replacement of $\mathbf{M}$ (forward Mueller matrix) by $\mathbf{M}^r$ (reverse Mueller matrix), it should be noted that, from its very definition, $P_\Omega(\mathbf{M}^r) = P_\Omega(\mathbf{M})$. Recall that both single retarder transformations and reciprocity invariances also hold for polarimetric quantities like P_Δ, $\operatorname{tr}\mathbf{m}$, m_{00}, P_S, P_P, P_α, P_1, P_2 and P_3.

5. Polarizing Power of Typical Devices

For a more detailed view of the peculiar features of the polarizing power, we next analyze its value for certain kinds devices typically found in polarimetry and polarization theory, like diattenuators, retarders and intrinsic depolarizers (also called diagonal depolarizers).

Diattenuators

Diattenuators constitute a subclass of nondepolarizing systems, characterized by the fact that they produce differential intensity attenuation on their two polarization eigenstates. Diattenuators whose Mueller matrix is symmetric are called normal [22,23] or homogeneous [1].

The Mueller matrix of a normal diattenuator oriented at 0º has the generic form

$$\mathbf{M}_{DL0}(m_{00}, D) = m_{00}\begin{pmatrix} 1 & D & 0 & 0 \\ D & 1 & 0 & 0 \\ 0 & 0 & K & 0 \\ 0 & 0 & 0 & K \end{pmatrix}, \quad \left[K = \sqrt{1-D^2}\right], \tag{16}$$

where m_{00} is the MIC, D is the diattenuation and K is called the counterdiattenuation [20]. In the general case of an elliptical normal diattenuator with arbitrary orientation, its Mueller matrix, $\mathbf{M}_D$, can always be expressed though the single retarder transformation $\mathbf{M}_D = \mathbf{M}_R^T \mathbf{M}_{DL0} \mathbf{M}_R$. Therefore, $P_\Omega(\mathbf{M}_{DL0}) = P_\Omega(\mathbf{M}_D)$, and by applying definition (12) we get the following expression for the polarizing power of normal diattenuators

$$P_\Omega(\mathbf{M}_D) = \sqrt{\frac{1-K}{2}}, \tag{17}$$

whose maximal value $P_\Omega = \sqrt{1/2}$ is achieved by perfect depolarizers, while P_Ω decreases as D decreases down to $P_\Omega = 0 \quad (D = 0)$.

For a given value D of diattenuation, the structure of Mueller matrices of nonnormal diattenuators feature more asymmetry than normal ones and consequently exhibit larger values of P_Ω. For instance, in the case of nonnormal perfect diattenuators with asymmetric Mueller matrices like

$$\begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{18}$$

(together with arbitrary single retarder transformations of them), the polarizing power reach the maximal achievable value among diattenuators $P_\Omega = \sqrt{3/4}$

Retarders

Retarders constitute a subclass of nondepolarizing systems, characterized by the fact that they produce differential phase shift on their two mutually orthogonal polarization eigenstates. The Mueller matrix of a retarder has the general form considered above and, as with normal diattenuators, is normal in the sense that its eigenstates are mutually orthogonal (represented by antipodal points in the Poincaré sphere [20]).

The Mueller matrix of a linear retarder, with retardance Δ, and oriented at 0º has the form

$$\mathbf{M}_{RL0}(\Delta)=\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&\cos\Delta&\sin\Delta\\0&0&-\sin\Delta&\cos\Delta\end{pmatrix},\tag{19}$$

which allows for expressing the Mueller matrix $\mathbf{M}_R$ of a general elliptical retarder through a single retarder transformation $\mathbf{M}_R=\mathbf{M}_{R'}^T\mathbf{M}_{RL0}\mathbf{M}_{R'}$ (not in a unique manner), $\mathbf{M}_{R'}$ being the Mueller matrix of an arbitrary retarder. Thus, the polarizing power of a retarder is given by

$$P_\Omega(\mathbf{M}_R)=\sqrt{\frac{1-\cos\Delta}{2}},\tag{20}$$

in such a manner that $P_\Omega=0$ when $\Delta=0$ $(\mathbf{M}=\mathbf{I})$ and P_Ω increases up to the maximal $P_\Omega=1$, which corresponds to $\Delta=\pi$ (symmetric retarders [3]).

Intrinsic depolarizers

The Mueller matrices associated with depolarizing systems $(P_\Delta<1)$ can have very different forms. Among them, we consider here the so-called intrinsic depolarizers, which have the simple diagonal form $\mathbf{M}_{\Delta I}=m_{00}\,\mathrm{diag}(1,a,b,c)$ [6,24].

The covariance conditions imply the following set of inequalities corresponding to the nonnegativity of the eigenvalues of the associated coherency matrix

$$1+a+b+c\geq 0,\ 1+a-b-c\geq 0,\ 1-a+b-c\geq 0,\ 1-a-b+c\geq 0\,.\tag{21}$$

Thus,

$$P_\Omega(\mathbf{M}_{\Delta I})=\sqrt{\frac{3+a^2+b^2+c^2-2(a+b+c)}{8}},\tag{22}$$

in such a manner that, as expected, the minimum $P_\Omega(\mathbf{M}_{\Delta I})=0$ corresponds to $a=b=c=1$ $(\mathbf{M}_{\Delta I}=\mathbf{I})$, while the maximum $P_\Omega(\mathbf{M}_{\Delta I})=\sqrt{3/8}$ is achieved for perfect depolarizers $(a=b=c=0)$ (recall that the covariance conditions imply $|a|\leq 1,\ |b|\leq 1,\ |c|\leq 1$, besides no more than two diagonal elements of $\mathbf{M}_{\Delta I}$ can be negative).

Particular interesting examples beyond the above considered perfect depolarizer are the following

$$\mathbf{M}_{\Delta a}=m_{00}\begin{pmatrix}1&0&0&0\\0&a&0&0\\0&0&0&0\\0&0&0&0\end{pmatrix},\quad \mathbf{M}_{\Delta ab}=m_{00}\begin{pmatrix}1&0&0&0\\0&a&0&0\\0&0&b&0\\0&0&0&0\end{pmatrix},\tag{23}$$

together with alternative forms obtained by permuting the diagonal elements of their 3×3 submatrices **m**. The respective polarizing powers are

$$\begin{aligned}&P_\Omega(\mathbf{M}_{\Delta a})=\sqrt{\frac{3+a^2-2a}{8}}, && \underset{a=1}{1/2}\leq P_\Omega(\mathbf{M}_{\Delta a})\leq\underset{a=-1}{\sqrt{6/8}},\\ &P_\Omega(\mathbf{M}_{\Delta ab})=\sqrt{\frac{3+a^2+b^2-2(a+b)}{8}}, && \underset{a=b=1}{\sqrt{1/8}}\leq P_\Omega(\mathbf{M}_{\Delta ab})\leq\underset{a=-b,\,|a|=1}{\sqrt{5/8}}\,,\end{aligned}\tag{24}$$

6. Conclusion

The parameter P_Ω introduced as a measure of the overall polarizing power of a medium (or interaction) represented by any kind of Mueller matrix **M**, involves all non-neutral polarimetric effects of **M** on the polarization states of incident electromagnetic waves, including enpolarizance, retardance and depolarization. From the natural definition of P_Ω as the normalized distance from **M** to the identity matrix **I**, the different contributions to the polarizing power are combined in an unbiased and peculiar manner.

The polarizing power space represented in Figure 1, illustrates the main features and physically achievable values of P_Ω. As occurs with other relevant polarimetric quantities, P_Ω is fully determined from **M** and it is invariant under single retarder transformations (including rotations of the Cartesian laboratory coordinate system) and under the replacement of $\mathbf{M}$ (forward Mueller matrix) by $\mathbf{M}^r$ (reverse Mueller matrix). Thus, P_Ω provides deeper insight in the interpretation of the information held by a measured Mueller matrix and also admits its application in imaging Mueller polarimetry to generate new images based on the representation of the point-to-point values of P_Ω.

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