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Dynamic Event-Triggered Control for Delayed Nonlinear Markov Jump Systems under Randomly Occurring DoS Attack and Packet Loss

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Abstract: This paper aims to address the exponential stability and stabilization problems for a class of delayed nonlinear Markov jump systems under randomly occurring Denial-of-Service (DoS) attack and packet loss. Firstly, the stochastic characteristics of DoS attack and packet loss are depicted by the attack success rate and packet loss rate. Secondly, a Period Observation Window (POW) method and a hybrid-input strategy are proposed to compensate for the impact of DoS attack and packet loss on the system. Thirdly, A Dynamic Event-triggered Mechanism (DETM) is introduced to save more network resources and ensure the security and reliability of systems. Then, by constructing a general common Lyapunov functional and combining with the DETM and other inequality analysis techniques, the less conservative stability and stabilization criteria for the underlying systems are derived. In the end, the effectiveness of our result is verified through a numerical example.

Keywords: delayed nonlinear Markov jump systems; attack success rate; Packet loss rate; Dynamic Event-triggered Mechanism; exponential stability and stabilization

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1. Introduction

In recent decades, Markov Jump Systems (MJSs) have received widespread attention [1,2] due to their powerful ability to depict the mutation phenomenon that the parameter and structure of real systems often encounter. It is worth mentioning that some undesirable dynamic behaviors, such as cyber attack [3,4], packet loss [5,6], time delay [7,8], non-linearity [9,10], often appear in the real systems, due to the openness of communication network, limited bandwidth, external disturbance and signal propagation. As well known, the stability is a prerequisite to ensure the normal operation of the system. However, such undesirable dynamic behaviors often lead oscillation, chaos, and even instability. Therefore, it is very interesting to study the stability and stabilization problems of the Delayed Nonlinear MJSs (DNMJSs) under cyber attack and packet loss.

Generally, there are two kinds of common cyber attack, named Denial-of-Service (DoS) attack and deception attack. Compared with deception attack, DoS attack is often launched by occupying communication resources to prevent the normal operation of the network and poses strong aggressiveness and ease of implementation, so it has became the most threatening form of cyber attack. Recently, many fruit results on the stability and stabilization problems of DNMJSs under DoS attack or packet loss have been achieved [11,12]. From the perspective of characterization methods, Bernoulli process [11–13] and Markov processes [14,15] are often used to model the stochastic properties of the DoS attack and packet loss. From the viewpoint of compensation strategies, the hold-input strategy and zero-input strategy are often adopted to deal with the impacts of DoS attack and packet loss on the systems. Name a few, the hold-input strategy is used to achieve faster and smoother stability of the systems in [16] and [17]. The stochastic behaviors of the packet loss is modeled by a Bernoulli process, and a zero-input strategy is adopted to compensate for the impacts of packet loss in [12]. However, DoS attack and packet loss are rarely mentioned together, and few papers consider attack success

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rate and packet loss rate, which will limit its applicability in practical applications. Therefore, it is necessary to consider the impacts of attack success rate and packet loss rate on system performance at the same time, which prompts this paper.

On the other hand, with the increasing pressure of network communication, especially in the case of limited network resources, how to improve the resource utilization rate has become the focus of many scholars. To this end, the traditional time-triggered mechanism, such as sampled-data control or impulsive control, are widely used in recent years [18]. However, they cannot determine the triggered time on demand, which results in a waste of communication resources to a certain extent. For event-triggered mechanism, the signals are transmitted only when the system state meets the preset triggered condition, which can effectively overcome the aforementioned obstacle. It is worth noting that according to the type of triggered parameters, event-triggered mechanisms are often divided into Static Event-triggered Mechanism (SETM) [19,20] and Dynamic Event-triggered Mechanism (DETM) [21,22]. Compared with the SETM, DETM contains a non-negative internal dynamic variable in the event-triggered condition relied on the system's state and error state, which make DETM has greater advantages in reducing communication costs. Thus, how to use DETM to study the security control problem of Delayed MJSs under DoS attack and packet loss shall be an interesting topic.

Based on the points discussed above, this paper will further study the dynamic event-triggered security control problem for a class of DNMJSs under randomly occurring DoS attack and packet loss. The main contributions of this paper are summarized as follows:

1. Two independent Bernoulli processes are introduced to describe the stochastic characteristics of attack success rate and packet loss rate during the action-period and sleeping-period, respectively.
2. Considering the physical properties of randomly occurring DoS attack and packet loss, the POW method and hybrid-input strategy are proposed, which are very useful to depict the evolution law of DoS attack and packet loss.
3. By constructing a general common Lyapunov functional, combining with DETM and other inequality analysis techniques, the less conservative security stability criteria are obtained.

Notations: Throughout this paper, $\mathbb{R}$, $\mathbb{Z}_+$ represent the set of real numbers and the set of positive integer numbers, respectively. $\mathbb{R}^n$, $\mathbb{R}^{m\times n}$ and $\mathbb{S}_+^{n\times n}$ stand for a $n-$dimensional Euclidean space, the set of $m \times n$ real matrices and the set of symmetric positive definite matrices, respectively. The symbol $*$ denotes the symmetric entry in the symmetric matrix. $sym\{G\} = G + G^T$. $\langle x, y\rangle$ represents the inner product of vectors $x, y \in \mathbb{R}^n$.

2. Problem Formulation and Preliminary

2.1. System Description

Consider the following DNMJSs:

$$\dot{x}(t) = A(\bar{\lambda}_t)x(t) + A_q(\bar{\lambda}_t)x(t-q(t)) + f(t, x(t)) + B(\bar{\lambda}_t)u(t), \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $q(t)$ denotes the time-varying delay that satisfies $0 \le q(t) \le q, 0 \le \dot{q}(t) \le \mu < 1$, $x(s) = \varphi(s), \forall s \in [-q, 0]$ is the initial condition, $u(t) \in \mathbb{R}^m$ is the control input, $A(\bar{\lambda}_t) \in \mathbb{R}^{n\times n}$, $A_q(\bar{\lambda}_t) \in \mathbb{R}^{n\times n}$, $B(\bar{\lambda}_t) \in \mathbb{R}^{n\times m}$ are known constant matrices, $f(t, x(t))$ is the external disturbance that satisfies Assumption 1, $\{\bar{\lambda}_t, t \ge 0\}$ is a Markov jump process taken values in the finite set $\Gamma = \{1, 2, \cdots, S\}$, and its transition rate matrix $\Pi_\lambda = \{\lambda_{ij}\}$ satisfies:

$$\Pr\{\bar{\lambda}_{t+\Delta} = j|\bar{\lambda}_t = i\} = \begin{cases} \lambda_{ij}\Delta + o(\Delta), & i \ne j \\ 1 + \lambda_{ij}\Delta + o(\Delta), & i = j \end{cases} \tag{2}$$

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where $\Delta > 0$, $\lim_{\Delta \to 0} o(\Delta)/\Delta = 0$. $\lambda_{ij} \geq 0 (i, j \in \Gamma, i \neq j)$ represents the transition rate from the mode i at time t to the mode j at time $t + \Delta$, and $\lambda_{ii} = -\sum_{j=1, j\neq i}^{S} \lambda_{ij}$. For the convenience, denote $A(\bar{\lambda}_t) = A_i, A_q(\bar{\lambda}_t) = A_{qi}, B(\bar{\lambda}_t) = B_i$, when $\bar{\lambda}_t = i$.

Assumption 1 ([23]). *For $\hat{x}_1, \hat{x}_2 \in \mathbb{R}^n$, $f(t, x)$ satisfies one-sided Lipschitz:*

$$\langle f(t, \hat{x}_1) - f(t, \hat{x}_2), \hat{x}_1 - \hat{x}_2 \rangle \leq \rho_0 \|\hat{x}_1 - \hat{x}_2\|^2,$$

where $\rho_0 \in \mathbb{R}$ is one-sided Lipschitz constant.

Assumption 2 ([23]). *For $\hat{x}_1, \hat{x}_2 \in \mathbb{R}^n$, $f(t, x)$ is quadratic inter-bounded, if*

$$\|f(t, \hat{x}_1) - f(t, \hat{x}_2)\|^2 \leq \beta_0 \|f(t, \hat{x}_1) - f(t, \hat{x}_2)\|^2 + \alpha_0 \langle \hat{x}_1 - \hat{x}_2, f(t, \hat{x}_1) - f(t, \hat{x}_2) \rangle$$

holds, where $\beta_0, \alpha_0 \in \mathbb{R}$ are known constants.

Definition 1 ([9]). *The system (1) is exponentially mean-square stable, if there are constants $a > 0$ and $c > 0$, such that*

$$E\left\{\|x(t)\|^2\right\} \leq ae^{-ct} \sup_{-q \leq s \leq 0} E\left\{\|\varphi(s)\|^2\right\}.$$

Definition 2 ([9]). *For the Lyapunov functional $V(x(t), \bar{\lambda}_t)$, its infinitesimal operator is defined as follows:*

$$LV(x(t), \bar{\lambda}_t) = \lim_{\Delta \to 0^+} \frac{1}{\Delta} \left[E\left\{ V(x(t+\Delta), \bar{\lambda}_{t+\Delta}) \middle| \ x(t), \bar{\lambda}_t \right\} - V(x(t), \bar{\lambda}_t) \right]. \quad (3)$$

Lemma 1 ([10]). *For any vectors $\zeta_1(t), \zeta_2(t)$, $\sigma_1(t), \sigma_2(t) \in \mathbb{R}$ satisfying $\sigma_1(t) + \sigma_2(t) = 1$, and matrices $Z \in \mathbb{R}^{n \times n}$, $\aleph_1, \aleph_2 \in \mathbb{S}_+^{n \times n}$, the following inequality holds*

$$\frac{1}{\sigma_1(t)} \zeta_1^T(t) \aleph_1 \zeta_1(t) + \frac{1}{\sigma_2(t)} \zeta_2^T(t) \aleph_2 \zeta_2(t) \geq \begin{bmatrix} \zeta_1(t) \\ \zeta_2(t) \end{bmatrix}^T \begin{bmatrix} \aleph_1 & Z \\ * & \aleph_2 \end{bmatrix} \begin{bmatrix} \zeta_1(t) \\ \zeta_2(t) \end{bmatrix} \quad (4)$$

subject to $\begin{bmatrix} \aleph_1 & Z \\ * & \aleph_2 \end{bmatrix} > 0.$

2.2. DoS Attack and Packet Loss

The openness and complexity of the communication network often leads to DoS attacks and packet loss, which can reduce or even destroy the performance of the system. Thus, this paper shall consider the DoS attack and packet loss in the communication network between the zero-order holder (ZOH) and the actuator (as shown in Figure 1), and the DoS attack and packet loss have the following characteristics: (1) the DoS attack and packet loss will not occur in a same time interval. (2) the DoS attack and packet loss will occur randomly.

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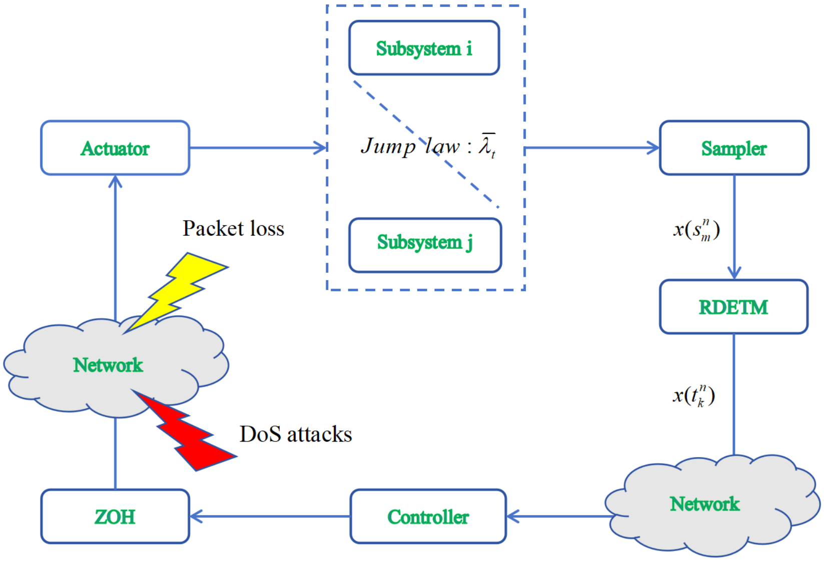

Figure 1. The framework of system (1) under DoS attack and packet loss

To depict such kind of DoS attack and packet loss more intuitively, a Periodic Observation Window (POW) method is proposed to model the first characteristic of DoS attack and packet loss. Specifically, the nth POW is designed as $[n\ell, (n+1)\ell)$, which can be divided into $\left[n\ell, n\ell + \ell_{off}\right)$ and $\left[n\ell + \ell_{off}, (n+1)\ell\right)$, where $n \in \{0, \mathbb{Z}_+\}$, ℓ is an observation period. $\left[n\ell, n\ell + \ell_{off}\right)$ is the sleeping-period of DoS attack, $\left[n\ell + \ell_{off}, (n+1)\ell\right)$ is the action-period of DoS attack, and DoS attack and packet loss will occur in the action-period and sleeping-period, respectively. It is worth noting that during the sleeping-period, the control signal cannot be transmitted to the actuator if the packet loss occur, and during the action-period, the control signal cannot be transmitted to the actuator if the attack succeeds.

Furthermore, two random variables $\varsigma_s(t)$ and $\varsigma_a(t)$ are introduced to model the second characteristic of DoS attack and packet loss, which are independent for each other and obey the Bernoulli distribution, i.e.,

$$\varsigma_s(t) = \begin{cases} 0, & \text{Packet losses} \\ 1, & \text{Packet does not loss} \end{cases} \quad \forall t \in \left[n\ell, n\ell + \ell_{off}\right),$$

$$\varsigma_a(t) = \begin{cases} 0, & \text{Attack succeeds} \\ 1, & \text{Attack does not succeed} \end{cases} \quad \forall t \in \left[n\ell + \ell_{off}, (n+1)\ell\right).$$

From the property of Bernoulli distribution, it is easy to see that $\Pr\{\varsigma_s(t) = 1\} = \varsigma_s$, $\Pr\{\varsigma_s(t) = 0\} = 1 - \varsigma_s$, $\Pr\{\varsigma_a(t) = 1\} = \varsigma_a$, $\Pr\{\varsigma_a(t) = 0\} = 1 - \varsigma_a$, where $\varsigma_s, \varsigma_a \in (0, 1)$ represent the expectation of random variables.

Remark 1. *From the view of defense, the POW method can provide an effective way for defenders to monitor the cyber attack, and also provide a feasible strategy for defenders to compensate for the adverse impacts of Dos attack and packet loss. Furthermore, compare with the existing literature, the characteristics of DoS attack and packet loss considered in this paper is more in line with the actual situation.*

2.3. Dynamic Event-Triggered Mechanism

In order to further reduce the burden of network transmission, a DETM shall be introduced in this section. To this end, it is assumed that the system state is sampled with a fixed sampling period h,

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the mth sampling instant is denoted as s_m^n and satisfies $s_{m+1}^n - s_m^n = h$. Furthermore, the kth triggering instant is denoted as t_k^n and satisfies the following DETM:

$$t_{k+1}^n = t_k^n + \min_{j^n \in \mathbb{Z}_+} \{ j^n h \mid F_{etc}(t_k^n + j^n h) \geq \delta \phi(t_k^n + j^n h) \}, \tag{5}$$

where $F_{etc}(t_k^n + j^n h) = e^T(t_k^n + j^n h) W e(t_k^n + j^n h) - \sigma x^T(t_k^n) W x(t_k^n)$, $\sigma \in (0,1)$ is a triggering threshold, W is a weighting matrix to be determined, $e(t_k^n + j^n h) = x(t_k^n) - x(t_k^n + j^n h)$ stands for the error state between the current sampling state and the latest triggered state. $\phi(t)$ is a dynamic variable that satisfies the following dynamic rule:

$$\dot{\phi}(t) = -2v_1 \phi(t) - \delta \phi(s_m^n) + x^T(s_m^n) \Xi x(s_m^n), \quad t \in [s_m^n, s_{m+1}^n) \tag{6}$$

where $v_1 > 0$ and $\delta > 0$ are the given constants, $\Xi > 0$ is a weighting matrix to be determined. The initial condition is $\phi(0) = \phi_0 \geq 0$.

Remark 2. *It is easy to see that $\{t_k^n\} \subseteq \{s_m^n\}$, which implies the triggering interval $t_{k+1}^n - t_k^n \geq h > 0$, thus the DETM can avoid the Zeno behavior naturally. Furthermore, as reported in [23], for the given constants $\phi_0 \geq 0$, $v_1 > 0$, $h > 0$ and a weighting matrix $\Xi > 0$, there is always a constant δ satisfying*

$$0 < \delta \leq -2v_1 + \frac{2v_1}{1 - e^{-2v_1 h}}, \tag{7}$$

such that the dynamic variable $\phi(t)$ satisfies $\phi(t) \geq 0$ for $t \in [0, \infty)$. In addition, the DETM designed in this paper relies on the current sampling states of system, and the dynamic variable can be adjusted dynamically with the sampling instants, which result that the data transmission rate can be reduced to a large extent.

2.4. Control Input Strategy

In order to compensate for the adverse influence on the systems from DoS attack and packet loss, this paper shall adopt the hybrid-input strategy, i.e., the zero-input strategy is adopted when the DoS attack and packet loss occur, otherwise the hold-input strategy is adopted. Then, combining with the DETM and the physical characteristics of DoS attack and packet loss, the control input can be designed as

$$u(t) = \begin{cases} \varsigma_s(t) K_i x(t_k^n), & t \in \left[t_k^n, t_{k+1}^n\right) \cap \left[n\ell, n\ell + \ell_{off}\right), \\ \varsigma_a(t) K_i x(t_k^n), & t \in \left[t_k^n, t_{k+1}^n\right) \cap \left[n\ell + \ell_{off}, (n+1)\ell\right), \end{cases} \tag{8}$$

where $K_i \in \mathbb{R}^{m \times n}$ is the controller gain matrix to be determined, $k \in \{1, \cdots, k_s^n, k_s^n + 1, \cdots, k_a^n\}$, where $k_s^n \triangleq \max\{k \in \mathbb{Z}_+ \mid t_k^n < n\ell + \ell_{off}\}$, $k_a^n \triangleq \max\{k \in \mathbb{Z}_+ \mid t_k^n < (n+1)\ell\}$.

Remark 3. *References [16] and [17] only adopt the hold-input strategy when the system is subjected to DoS attack and packet loss. Compared to this case, the hybrid-input strategy adopted in this paper can greatly combat the influence on system from randomly occurring DoS attack and packet loss.*

2.5. Model Transformation

In this section, the input delay method and interval decomposition approach shall be used to describe the control input under the randomly occurring DoS attack and packet loss. Firstly, the relationship between the triggering instants t_k^n and the sleeping-period and action-period of POW should be discussed as follows:

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A) During the sleep-period $\left[n\ell, n\ell+\ell_{off}\right)$:

$$\begin{cases} \Pi^n_{\iota^n_{1,1}} = \left[n\ell + \iota^n_{1,1}h, n\ell + (\iota^n_{1,1}+1)h\right), \iota^n_{1,1} = 0,1,2,\cdots,(\check{\iota}^n_{1,1}-1), \\ \Pi^n_{\iota^n_{1,k}} = \left[t^n_k + \iota^n_{1,k}h, t^n_k + \left(\iota^n_{1,k}+1\right)h\right), \iota^n_{1,k} = 0,1,2,\cdots,(\check{\iota}^n_{1,k}-1), \\ \Pi^n_{\iota^n_1} = \left[t^n_{k^n_s} + \iota^n_1 h, t^n_{k^n_s} + (\iota^n_1+1)h\right), \iota^n_1 = 0,1,2,\cdots,(\check{\iota}^n_1-1), \end{cases} \tag{9}$$

where $\check{\iota}^n_{1,1} \triangleq \max\{\iota^n_{1,1} \in \{0,\mathbb{Z}_+\} \mid n\ell + \iota^n_{1,1}h \le t^n_1\}$, $\check{\iota}^n_{1,k} \triangleq \max\{\iota^n_{1,k} \in \{0,\mathbb{Z}_+\} \mid t^n_k + \iota^n_{1,k}h \le t^n_{k+1}\}$ and $\check{\iota}^n_1 \triangleq \max\{\iota^n_1 \in \{0,\mathbb{Z}_+\} \mid t^n_{k^n_s} + \iota^n_1 h \le n\ell + \ell_{off}\}$. Noted that if $t^n_1 = n\ell$ then $\Pi^n_{\iota^n_{1,1}} = \{t^n_1\}$, otherwise $t^n_1 = n\ell + \iota^n_{1,1}h$. Similarly, $t^n_{k+1} = t^n_k + \check{\iota}^n_{1,k}h$, $n\ell + \ell_{off} = t^n_{k^n_s} + \check{\iota}^n_1 h$, and $k = 1,2,\cdots,(k^n_s - 1)$.

B) During the action-period $\left[n\ell+\ell_{off}, (n+1)\ell\right)$:

$$\begin{cases} \Pi^n_{\iota^n_{2,1}} = \left[n\ell + \ell_{off} + \iota^n_{2,1}h, n\ell + \ell_{off} + (\iota^n_{2,1}+1)h\right), \iota^n_{2,1} = 0,1,2,\cdots,(\check{\iota}^n_{2,1}-1), \\ \Pi^n_{\iota^n_{2,k}} = \left[t^n_k + \iota^n_{2,k}h, t^n_k + \left(\iota^n_{2,k}+1\right)h\right), \iota^n_{2,k} = 0,1,2,\cdots,(\check{\iota}^n_{2,k}-1), \\ \Pi^n_{\iota^n_2} = \left[t^n_{k^n_a} + \iota^n_2 h, t^n_{k^n_a} + (\iota^n_2+1)h\right), \iota^n_2 = 0,1,2,\cdots,(\check{\iota}^n_2-1), \end{cases} \tag{10}$$

where $\check{\iota}^n_{2,1} \triangleq \max\{\iota^n_{2,1} \in \{0,\mathbb{Z}_+\} \mid n\ell + \iota^n_{2,1}h \le t^n_{k^n_s+1}\}$, $\check{\iota}^n_{2,k} \triangleq \max\{\iota^n_{2,k} \in \{0,\mathbb{Z}_+\} \mid t^n_k + \iota^n_{2,k}h \le t^n_{k+1}\}$ and $\check{\iota}^n_2 \triangleq \max\{\iota^n_2 \in \{0,\mathbb{Z}_+\} \mid t^n_{k^n_a} + \iota^n_2 h \le (n+1)\ell\}$. Noted that if $t^n_{k^n_s+1} = n\ell + \ell_{off}$, then $\Pi^n_{\iota^n_{2,1}} = \{t^n_{k^n_s+1}\}$, otherwise $t^n_{k^n_s+1} = n\ell + \ell_{off} + \check{\iota}^n_{2,1}h$. Similarly, $t^n_{k+1} = t^n_k + \check{\iota}^n_{2,k}h$, $(n+1)\ell = t^n_{k^n_a} + \check{\iota}^n_2 h$, and $k = k^n_s+1, k^n_s+2,\cdots,(k^n_a-1)$.

Based on the interval decomposition in (9) and (10), for $\forall t \in [n\ell,(n+1)\ell)$, it follows from the input delay method that

$$\gamma^n_k(t) = \begin{cases} t - t^{n-1}_{k^{n-1}_a} - \hat{\iota}^{n-1}_2 h, t \in \hat{\Pi}^n_1, \hat{\iota}^{n-1}_2 = \check{\iota}^{n-1}_2+1,\cdots,\check{\iota}^{n-1}_2+\check{\iota}^n_{1,1}-1, \\ t - t^n_1 - \hat{\iota}^n_{1,k}h, t \in \hat{\Pi}^n_2, \hat{\iota}^n_{1,k} = 0,1,\cdots,\check{\iota}^n_{1,k}-1, \\ \qquad k = 1,2,\cdots,(k^n_s-1), \\ t - t^n_{k^n_s} - \hat{\iota}^n_1 h, t \in \hat{\Pi}^n_3, \hat{\iota}^n_1 = 0,1,2,\cdots,\check{\iota}^n_1, \check{\iota}^n_1+1,\cdots,\check{\iota}^n_1+\check{\iota}^n_{2,1}, \\ t - t^n_{k^n_s+1} - \hat{\iota}^n_{2,k}h, t \in \hat{\Pi}^n_4, \hat{\iota}^n_{2,k} = 0,1,2,\cdots,\check{\iota}^n_{2,k}-1, \\ \qquad k = k^n_s+1, k^n_s+2,\cdots,(k^n_a-1), \\ t - t^n_{k^n_a} - \hat{\iota}^n_2 h, t \in \hat{\Pi}^n_5, \hat{\iota}^n_2 = 0,1,2,\cdots,\check{\iota}^n_2-1, \end{cases} \tag{11}$$

where $\hat{\Pi}^n_1 = \Pi^{n-1}_{\check{\iota}^{n-1}_2-1} \cup \left\{\cup^{\check{\iota}^n_{1,1}-1}_{\iota^n_{1,1}=0} \Pi^n_{\iota^n_{1,1}}\right\}$, $\hat{\Pi}^n_2 = \cup^{\check{\iota}^n_{1,k}-1}_{\iota^n_{1,k}=0} \Pi^n_{\iota^n_{1,k}}$, $\hat{\Pi}^n_3 = \left\{\cup^{\check{\iota}^n_1-1}_{\iota^n_1=0} \Pi^n_{\iota^n_1}\right\} \cup \left\{\cup^{\check{\iota}^n_{2,1}-1}_{\iota^n_{2,1}=0} \Pi^n_{\iota^n_{2,1}}\right\}$, $\hat{\Pi}^n_4 = \cup^{\check{\iota}^n_{2,k}-1}_{\iota^n_{2,k}=0} \Pi^n_{\iota^n_{2,k}}$, $\hat{\Pi}^n_5 = \cup^{\check{\iota}^n_2-1}_{\iota^n_2=0} \Pi^n_{\iota^n_2}$. It is easy to find that $\gamma^n_k(t)$ is a piece-wise continuous function, which satisfies $0 \le \gamma^n_k(t) < \gamma = h$ and

$$e^n_k(t) = x(t^n_k) - x(t - \gamma^n_k(t)), \ \forall t \in \hat{\Pi}^n = \cup^5_{\iota^n=1} \hat{\Pi}^n_{\iota^n}. \tag{12}$$

Combining with (8) and (12), the DNMJSs (1) can be rewritten as the following switched systems:

$$\begin{cases} \dot{x}(t) = A_i x(t) + A_{qi} x(t-q(t)) + f(t,x) + u_s(t), \\ u_s(t) = \varsigma_s(t) B_i K_i (x(t-\gamma_k^n(t)) + e_k^n(t)), t \in \left[D_n^{on}, D_n^{off}\right), \\ \dot{x}(t) = A_i x(t) + A_{qi} x(t-q(t)) + f(t,x) + u_a(t), \\ u_a(t) = \varsigma_a(t) B_i K_i (x(t-\gamma_k^n(t)) + e_k^n(t)), t \in \left[D_n^{off}, D_{n+1}^{on}\right), \end{cases} \tag{13}$$

where $\left[D_n^{off}, D_n^{on}\right) = \hat{\Gamma}^n \cap \left[n\ell, n\ell + \ell_{off}\right)$, $\left[D_n^{on}, D_{n+1}^{off}\right) = \hat{\Gamma}^n \cap \left[n\ell + \ell_{off}, (n+1)\ell\right)$.

3. Main Results

Before presenting the main results, the following vectors need to be given.

$$\begin{aligned} \varpi(t) =& col\{x(t), x(t-q(t)), x(t-q), x(t-\gamma_k^n(t)), x(t-\gamma), f(t,x), e_k^n(t)\}, \\ \Gamma_{1i} =& A_i \bar{e}_1 + A_{qi} \bar{e}_2 + \varsigma_s B_i K_i (\bar{e}_4 + \bar{e}_7) + \bar{e}_6, \\ \Gamma_{2i} =& A_i \bar{e}_1 + A_{qi} \bar{e}_2 + \varsigma_a B_i K_i (\bar{e}_4 + \bar{e}_7) + \bar{e}_6, \\ \bar{\Gamma}_{1i} =& A_i \Upsilon_i \bar{e}_1 + A_{qi} \Upsilon_i \bar{e}_2 + \varsigma_s B_i Y_i (\bar{e}_4 + \bar{e}_7) + \bar{e}_6, \\ \bar{\Gamma}_{2i} =& A_i \Upsilon_i \bar{e}_1 + A_{qi} \Upsilon_i \bar{e}_2 + \varsigma_a B_i Y_i (\bar{e}_4 + \bar{e}_7) + \bar{e}_6, \\ \Pi_1 =& \left[\bar{e}_1^T - \bar{e}_4^T, \bar{e}_4^T - \bar{e}_5^T\right]^T, \Pi_2 = \left[\bar{e}_1^T - \bar{e}_2^T, \bar{e}_2^T - \bar{e}_3^T\right]^T, \\ \bar{e}_b =& \left[0_{n\times(b-1)n} \; I_n \; 0_{n\times(7-b)n}\right], \quad b = 1, 2, \cdots, 7. \end{aligned}$$

In this section, the exponential stability and stabilization criteria for the system (13) under the randomly occurring DoS attack and packet loss are established in terms of LMIs.

Theorem 1. *For given positive scalars $\ell, \ell_{off}, q, \sigma, h, \delta, v_1, v_2, \varepsilon_1, \varepsilon_2, \mu$, scalars $\alpha_0, \beta_0, \rho_0$, $v_1 + v_2 > 0$ satisfying (7) and*

$$-v_2\ell + (v_1 + v_2)\ell_{off} > 0, \tag{14}$$

if there exist matrices $W_1, W_2, W_3, J_1, J_2, W, \Xi, P_i \in \mathbb{S}_+^{n\times n}, K_i \in \mathbb{R}^{m\times n}$, and matrices $M_1, M_2 \in \mathbb{R}^{n\times n}$, such that the following LMIs hold:

$$\begin{bmatrix} J_1 & M_1 \\ * & J_1 \end{bmatrix} \geq 0, \tag{15}$$

$$\begin{bmatrix} J_2 & M_2 \\ * & J_2 \end{bmatrix} \geq 0, \tag{16}$$

$$\Phi_{1i} < 0, \tag{17}$$

$$\Phi_{2i} < 0, \tag{18}$$

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where

$$
\begin{aligned}
\Phi_{1i} =& sym\left\{\bar{e}_1^T P_i \Gamma_{1i}\right\} + \bar{e}_1^T\left(2v_2 P_i + \sum_{j=1}^S \lambda_{ij} P_j\right)\bar{e}_1 + \Phi_0, \\
\Phi_{2i} =& sym\left\{\bar{e}_1^T P_i \Gamma_{2i}\right\} + \bar{e}_1^T\left(-2v_2 P_i + \sum_{j=1}^S \lambda_{ij} P_j\right)\bar{e}_1 + \Phi_0, \\
\Phi_0 =& sym\left\{(\varepsilon_1\rho_0 + \varepsilon_2\beta_0)\bar{e}_1^T\bar{e}_1 + (\varepsilon_2\alpha_0 - \varepsilon_1)\bar{e}_1^T\bar{e}_6 - \varepsilon_2\bar{e}_6^T\bar{e}_6\right\} \\
&+ \bar{e}_1^T(W_1 + W_2 + W_3)\bar{e}_1 - e^{-2v_1\gamma}\bar{e}_5^T W_1\bar{e}_5 - e^{-2v_1 q}(1-\mu)\bar{e}_2^T W_2\bar{e}_2 \\
&- e^{-2v_1 q}\bar{e}_3^T W_3\bar{e}_3 + \Gamma_{2i}^T\left(\gamma^2 J_1 + q^2 J_2\right)\Gamma_{2i} + \bar{e}_4^T\Xi\bar{e}_4 + \sigma(\bar{e}_4 + \bar{e}_7)^T W(\bar{e}_4 + \bar{e}_7) \\
&- \bar{e}_7^T W\bar{e}_7 - e^{-2v_1\gamma}\Pi_1^T\begin{bmatrix} J_1 & M_1 \\ * & J_1 \end{bmatrix}\Pi_1 - e^{-2v_1 q}\Pi_2^T\begin{bmatrix} J_2 & M_2 \\ * & J_2 \end{bmatrix}\Pi_2.
\end{aligned}
$$

then the system (13) is exponentially mean-square stable.

Proof of Theorem 1. Construct the following Lyapunov functional:

$$U(x(t), \bar{\lambda}_t, t) = V(x(t), \bar{\lambda}_t) + \phi(t), \tag{19}$$

where

$$
\begin{aligned}
V(x(t), \bar{\lambda}_t) =& x^T(t)P(\bar{\lambda}_t)x(t) + \int_{t-\gamma}^{t} e^{-2v_1(t-\xi)}x^T(\xi)W_1 x(\xi)d\xi \\
&+ \int_{t-q(t)}^{t} e^{-2v_1(t-\xi)}x^T(\xi)W_2 x(\xi)d\xi \\
&+ \int_{t-q}^{t} e^{-2v_1(t-\xi)}x^T(\xi)W_3 x(\xi)d\xi \\
&+ \gamma\int_{t-\gamma}^{t}\int_{s}^{t} e^{-2v_1(t-\xi)}\dot{x}^T(\xi)J_1\dot{x}(\xi)d\xi ds \\
&+ q\int_{-q}^{0}\int_{t+s}^{t} e^{-2v_1(t-\xi)}\dot{x}^T(\xi)J_2\dot{x}(\xi)d\xi ds.
\end{aligned}
$$

According to Definition 2, it follows:

$$
\begin{aligned}
LV(x(t), \bar{\lambda}_t) \leq& -2v_1 V(x(t), \bar{\lambda}_t) + 2x^T(t)P_i\dot{x}(t) + 2v_1 x^T(t)P_i x(t) \\
&+ x^T(t)\left(\sum_{j=1}^S \lambda_{ij}P_j\right)x(t) + x^T(t)(W_1 + W_2 + W_3)x(t) \\
&- e^{-2v_1\gamma}x^T(t-\gamma)W_1 x(t-\gamma) - e^{-2v_1 q}x^T(t-q)W_3 x(t-q) \\
&- e^{-2v_1 q}(1-\mu)x^T(t-q(t))W_2 x(t-q(t)) \\
&+ \dot{x}^T(t)\left(\gamma^2 J_1\right)\dot{x}(t) - \gamma\int_{t-\gamma}^{t} e^{-2v_1(t-\xi)}\dot{x}^T(\xi)J_1\dot{x}(\xi)d\xi \\
&+ \dot{x}^T(t)\left(q^2 J_2\right)\dot{x}(t) - q\int_{t-q}^{t} e^{-2v_1(t-\xi)}\dot{x}^T(\xi)J_2\dot{x}(\xi)d\xi.
\end{aligned}
\tag{20}
$$

By using the Jensen integral inequality in [10] and Lemma 1, with the help of (15) and (16), the last two integral quadratic terms of (20) can be rewritten as:

$$
\begin{aligned}
&-\gamma\int_{t-\gamma}^{t} e^{-2v_1(t-\xi)}\dot{x}^T(\xi)J_1\dot{x}(\xi)d\xi - q\int_{t-q}^{t} e^{-2v_1(t-\xi)}\dot{x}^T(\xi)J_2\dot{x}(\xi)d\xi \\
\leq& -e^{-2v_1\gamma}\varpi^T(t)\Pi_1^T\begin{bmatrix} J_1 & M_1 \\ * & J_1 \end{bmatrix}\Pi_1\varpi(t) - e^{-2v_1 q}\,\varpi^T(t)\Pi_2^T\begin{bmatrix} J_2 & M_2 \\ * & J_2 \end{bmatrix}\Pi_2\varpi(t).
\end{aligned}
\tag{21}
$$

From Assumption 1, we have

$$\begin{aligned} &2\varepsilon_2\beta_0 x^T(t)x(t) + 2\varepsilon_2\alpha_0 x^T(t)f(t,x) - 2\varepsilon_2 f^T(t,x)f(t,x), \\ &+ 2\varepsilon_1\rho_0 x^T(t)x(t) - \varepsilon_1 x^T(t)f(t,x) - \varepsilon_1 f^T(t,x)x(t) \geq 0, \end{aligned} \tag{22}$$

where the scalars $\varepsilon_1 > 0$ and $\varepsilon_2 > 0$.

Case A: During the sleeping-period $[n\ell, n\ell + \ell_{off})$, one can obtain from (5) and (6) that,

$$\begin{aligned} \dot{\phi}(t) \leq& -2v_1\phi(t) + x^T(t-\gamma_k^n(t))\Xi x(t-\gamma_k^n(t)) - (e_k^n(t))^T W e_k^n(t) \\ &+ \sigma[x(t-\gamma_k^n(t)) + e_k^n(t)]^T W[x(t-\gamma_k^n(t)) + e_k^n(t)]. \end{aligned} \tag{23}$$

Combining with (20) – (23), we have

$$LU(t) \leq -2v_1 U(t) + \varpi^T(t)\{\Phi_{1i}\}\varpi(t). \tag{24}$$

Thus, it follows from (17) and (24) that,

$$LU(t) \leq -2v_1 U(t). \tag{25}$$

Then, for $t \in \left[D_n^{off}, D_n^{on}\right)$, one can obtain from (25) and Dynkin formula [2] that

$$\begin{aligned} EU(t) \leq& e^{-2v_1\left(t-D_n^{off}\right)} EU\left(D_n^{off}\right) \\ \leq& e^{-2v_1\left(t-\ell_{n-1}^{on}\right)+2(v_1+v_2)\left(\ell-\ell_{off}\right)} EU\left(D_{n-1}^{on}\right) \\ \leq& e^{-2v_1\left(t-\ell_{n-1}^{off}\right)+2(v_1+v_2)\left(\ell-\ell_{off}\right)} EU\left(D_{n-1}^{off}\right) \\ &\vdots \\ \leq& e^{-2v_1\left(t-\ell_0^{off}\right)+2n(v_1+v_2)\left(\ell-\ell_{off}\right)} EU\left(D_0^{off}\right), \end{aligned} \tag{26}$$

It follows from $D_0^{off} = 0$ that

$$EU(t) \leq e^{-2v_1 t + 2n(v_1+v_2)\left(\ell-\ell_{off}\right)} U(0), \tag{27}$$

For $t \geq D_n^{off} \geq n\ell$, from (14), we have

$$EU(t) \leq e^{-2\lambda n} U(0), \tag{28}$$

where $\lambda = -v_1\ell + (v_1+v_2)\ell_{off}$. And because of $t \leq D_n^{on} = n\ell + \ell_{off}$, then $n \geq \frac{t-\ell_{off}}{\ell}$. Thus

$$EU(t) \leq e^{2\lambda\frac{\ell_{off}}{\ell}} e^{-\frac{2\lambda}{\ell}t} U(0). \tag{29}$$

Case B: During the action-period $[n\ell + \ell_{off}, (n+1)\ell)$, one can obtain from (5), (6) and $v_1 + v_2 > 0$ that

$$\begin{aligned} \dot{\phi}(t) =& -2v_1\phi(t) - \delta\phi(t-\gamma_k^n(t)) + x^T(t-\gamma_k^n(t))\Xi x(t-\gamma_k^n(t)) \\ \leq& 2v_2\phi(t) + x^T(t-\gamma_k^n(t))\Xi x(t-\gamma_k^n(t)) - (e_k^n(t))^T W e_k^n(t) \\ &+ \sigma[x(t-\gamma_k^n(t)) + e_k^n(t)]^T W[x(t-\gamma_k^n(t)) + e_k^n(t)], \end{aligned} \tag{30}$$

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Combining with (20), (21) and (30) ,

$$LU(t) \leq 2v_2 U(t) + \varpi^T(t)\Phi_{2i}\varpi(t). \tag{31}$$

According to (18) and (31), it follows that

$$LU(t) \leq 2v_2 U(t). \tag{32}$$

Then, for $t \in \left[D_n^{on}, D_{n+1}^{off}\right)$, one can obtain from (32) and Dynkin formula [2] that

$$\begin{aligned} EU(t) \leq& e^{2v_2(t-D_n^{on})} EU(D_n^{on}) \\ \leq& e^{2v_2\left(t-D_n^{off}\right)-2(v_1+v_2)\ell_{off}} EU\left(D_n^{off}\right) \\ \leq& e^{2v_2\left(t-\ell_{n-1}^{on}\right)-2(v_1+v_2)\ell_{off}} EU(D_{n-1}^{on}) \\ &\vdots \\ \leq& e^{2v_2\left(t-\ell_0^{on}\right)-2n(v_1+v_2)\ell_{off}} EU(D_0^{on}) \\ \leq& e^{2v_2\left(t-\ell_0^{off}\right)-2(n+1)(v_1+v_2)\ell_{off}} EU\left(D_0^{off}\right), \end{aligned} \tag{33}$$

From $D_0^{off} = 0$, we have

$$EU(t) \leq e^{2v_2 t - 2(n+1)(v_1+v_2)\ell_{off}} U(0), \tag{34}$$

For $t \leq D_{n+1}^{off} = (n+1)\ell$, it can be obtained from (14)

$$EU(t) \leq e^{-2\lambda(n+1)} U(0), \tag{35}$$

And because of $t \leq (n+1)\ell$,

$$EU(t) \leq e^{-\frac{2\lambda}{\ell}t} U(0). \tag{36}$$

Denote $\eta_1 = \min_{i\in\{1,\cdots,S\}}\{\lambda_{\min}(P_i)\}$, $\eta_2 = \min_{i\in\{1,\cdots,S\}}\{\lambda_{\max}(P_i)\}$, $\eta_3 = \eta_2 + \gamma\lambda_{\max}(W_1) + q\lambda_{\max}(W_2) + q\lambda_{\max}(W_3) + \gamma^2\lambda_{\max}(J_1) + q^2\lambda_{\max}(J_2)$. From (29) and (36), there is a scalar $d > 1$ satisfies

$$EU(t) \geq \eta_1 E\left\{\|x(t)\|^2\right\}, \tag{37}$$

$$U(0) \leq d\eta_3 E\left\{\sup_{-\gamma\leq s\leq 0}\|\varphi(s)\|_\gamma^2\right\} + \|\phi(0)\|. \tag{38}$$

Furthermore, for given φ and $\phi(0)$, there always exists a scalar $\eta_4 > 0$ such that $\|\phi(0)\| \leq \eta_4 E\{\sup_{-\gamma\leq s\leq 0}\|\varphi(s)\|_\gamma^2\}$. Thus, combining with (29), (36), (37) and (38), we have

$$E\left\{\|x(t)\|^2\right\} \leq \frac{\vartheta\eta_5}{\eta_1} e^{-\frac{2\lambda}{\ell}t} E\left\{\sup_{-\gamma\leq s\leq 0}\|\varphi(s)\|_\gamma^2\right\}, \tag{39}$$

where $\vartheta = \max\left\{e^{2\lambda\frac{\ell_{off}}{\ell}}, 1\right\}$ and $\eta_5 = d\eta_3 + \eta_4$. Therefore, system (13) is exponentially mean-square stable. The proof is finished. □

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Next, based on Theorem 1, we shall to solve the controller gain matrix K_i and the weighting matrices Ξ_i and W_i in the DETM.

Theorem 2. *For given positive scalars $\ell, \ell_{off}, \gamma, q, \sigma, \delta, v_1, v_2, \varepsilon_1, \varepsilon_2, \mu, \kappa_1, \kappa_2$, scalars $\alpha_0, \beta_0, \rho_0$, and $v_1 + v_2 > 0$, satisfying (14) and (7), if there exist matrices $\bar{W}_{1i}, \bar{W}_{2i}, \bar{W}_{3i}, \bar{J}_{1i}, \bar{J}_{2i}, \bar{\Xi}_i, \bar{W}_i, \Upsilon_i \in \mathbb{S}_+^n$ and matrices $\bar{M}_{1i}, \bar{M}_{2i} \in \mathbb{R}^{n\times n}$, such that the following LMIs hold:*

$$\begin{bmatrix} \bar{J}_{1i} & \bar{M}_{1i} \\ * & \bar{J}_{1i} \end{bmatrix} \geq 0, \tag{40}$$

$$\begin{bmatrix} \bar{J}_{2i} & \bar{M}_{2i} \\ * & \bar{J}_{2i} \end{bmatrix} \geq 0, \tag{41}$$

$$\begin{bmatrix} \bar{\Phi}_{1i} & * & * & * & * \\ \gamma\bar{\Gamma}_{1i} & -(2\kappa_1\Upsilon_i - \kappa_1^2\bar{J}_{1i}) & * & * & * \\ q\bar{\Gamma}_{2i} & 0 & -(2\kappa_2\Upsilon_i - \kappa_2^2\bar{J}_{2i}) & * & * \\ \bar{\Phi}_{14i} & 0 & 0 & \bar{\Phi}_{44} & * \\ \Phi_{15} & 0 & 0 & 0 & \Phi_{55} \end{bmatrix} < 0, \tag{42}$$

$$\begin{bmatrix} \bar{\Phi}_{2i} & * & * & * & * \\ \gamma\bar{\Gamma}_{1i} & -(2\kappa_1\Upsilon_i - \kappa_1^2\bar{J}_{1i}) & * & * & * \\ q\bar{\Gamma}_{2i} & 0 & -(2\kappa_2\Upsilon_i - \kappa_2^2\bar{J}_{2i}) & * & * \\ \bar{\Phi}_{14i} & 0 & 0 & \bar{\Phi}_{44} & * \\ \Phi_{15} & 0 & 0 & 0 & \Phi_{55} \end{bmatrix} < 0, \tag{43}$$

where

$$\begin{aligned}
\bar{\Phi}_{1i} =& sym\Big\{\bar{e}_1^T\bar{\Gamma}_{1i}\Big\} + \bar{e}_1^T(2v_1\Upsilon_i + \lambda_{ii}\Upsilon_i)\bar{e}_1 + \bar{\Phi}_{0i}, \\
\bar{\Phi}_{2i} =& sym\Big\{\bar{e}_1^T\bar{\Gamma}_{2i}\Big\} + \bar{e}_1^T(-2v_2\Upsilon_i + \lambda_{ii}\Upsilon_i)\bar{e}_1 + \bar{\Phi}_{0i}, \\
\bar{\Phi}_{0i} =& sym\Big\{(\varepsilon_2\alpha_0 - \varepsilon_1)\bar{e}_1^T\Upsilon_i\bar{e}_6 - \varepsilon_2\bar{e}_6^T\bar{e}_6\Big\} - e^{-2v_1\gamma}\bar{e}_5^T\bar{W}_{1i}\bar{e}_5 \\
&+ \bar{e}_1^T(\bar{W}_{1i} + \bar{W}_{2i} + \bar{W}_{3i})\bar{e}_1 - e^{-2v_1 q}(1-\mu)\bar{e}_2^T\bar{W}_{2i}\bar{e}_2 \\
&- e^{-2v_1 q}\bar{e}_3^T\bar{W}_{3i}\bar{e}_3 + \bar{e}_4^T\bar{\Xi}_i\bar{e}_4 - \bar{e}_7^T\bar{W}_i\bar{e}_7 + \sigma(\bar{e}_4 + \bar{e}_7)^T\bar{W}_i(\bar{e}_4 + \bar{e}_7) \\
&- e^{-2v_1\gamma}\,\Pi_1^T\begin{bmatrix} \bar{J}_{1i} & \bar{M}_{1i} \\ * & \bar{J}_{1i} \end{bmatrix}\Pi_1 - e^{-2v_1 q}\Pi_2^T\begin{bmatrix} \bar{J}_{2i} & \bar{M}_{2i} \\ * & \bar{J}_{2i} \end{bmatrix}\Pi_2, \\
\bar{\Phi}_{14i} =& \begin{bmatrix} \sqrt{\lambda_{i1}}\Upsilon_i & 0 & 0 & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \sqrt{\lambda_{i(i-1)}}\Upsilon_i & 0 & 0 & 0 & 0 & 0 & 0 \\ \sqrt{\lambda_{i(i+1)}}\Upsilon_i & 0 & 0 & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \sqrt{\lambda_{iS}}\Upsilon_i & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \\
\bar{\Phi}_{44} =& diag\{-\Upsilon_1, \cdots, -\Upsilon_{i-1}, -\Upsilon_{i+1}, \cdots, -\Upsilon_S\}, \\
\Phi_{15} =& \begin{bmatrix} (\varepsilon_1\rho_0 + \varepsilon_2\beta_0) & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \Phi_{55} = -\frac{1}{2}(\varepsilon_1\rho_0 + \varepsilon_2\beta_0).
\end{aligned}$$

then, the system (13) is exponentially mean-square stable. Moreover, we get $K_i = \Upsilon_i\Upsilon_i^{-1}$, $\Xi_i = \Upsilon_i^T\Xi\Upsilon_i$ and $W_i = \Upsilon_i^T W\Upsilon_i$.

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Proof of Theorem 2. Denote $\Upsilon_i = P_i^{-1}$, $G_i = \Upsilon_i G \Upsilon_i$, $G \in \{W_1, W_2, W_3, J_1, J_2, \Xi, W, N_1, N_2, M_1, M_2\}$, $Y_i = K_i\Upsilon_i$, $G_u = \{\Upsilon_i, \Upsilon_i, \Upsilon_i, \Upsilon_i, \Upsilon_i, I_n, \Upsilon_i, I_n, I_n, \hat{\Upsilon}_i, I_n\}$ and $\hat{\Upsilon}_i = diag\underbrace{\{I_n, \cdots, I_n\}}_{S-1}$. Pre- and post-multiplying (15) and (16) by G_u, we have (40) and (41). Pre- and post-multiplying (17) and (18) by G_u, respectively, then by using $-J_1^{-1} = -\Upsilon_i J_{1i}{}^{-1}\Upsilon_i \leq -2\kappa_1\Upsilon_i + \kappa_1^2 J_{1i}$, $-J_2^{-1} = -\Upsilon_i J_{2i}{}^{-1}\Upsilon_i \leq -2\kappa_2\Upsilon_i + \kappa_2^2 J_{2i}$ and Schur complement, we can get (42) and (43), respectively. □

4. Numerical Example

Example 1. *Consider the system* (13) *with the following parameters:*

$$A_1 = \begin{bmatrix} -1 & 0 \\ 0.8 & -1.4 \end{bmatrix}, A_{q1} = \begin{bmatrix} 0.85 & 1.75 \\ 0 & -1.6 \end{bmatrix}, B_1 = \begin{bmatrix} 0.7 \\ 0.4 \end{bmatrix},$$
$$A_2 = \begin{bmatrix} 1 & 0.8 \\ 0 & -1.2 \end{bmatrix}, A_{q2} = \begin{bmatrix} -0.49 & 0 \\ -1.45 & -0.19 \end{bmatrix}, B_2 = \begin{bmatrix} 0.4 \\ 0.7 \end{bmatrix},$$

and other parameters are given as $\ell = 2, h = 0.05, v_1 = 0.2, v_2 = 0.5, \sigma = 0.1, \delta = 0.5, \varepsilon_1 = 0.1, \varepsilon_2 = 0.6, \alpha_0 = -0.2, \beta_0 = 0.1, \rho_0 = -0.4, \mu = 0.68, \kappa_1 = 0.17, \kappa_2 = 0.15, q = 0.02$, *the attack success rate and packet loss rate are set as* $\varsigma_s = 0.7, \varsigma_a = 0.3$, *respectively. The mode transition rate is given as* $\Pi_\lambda = \begin{bmatrix} -3 & 3 \\ 5 & -5 \end{bmatrix}$.

Based on the above parameters, by using the Matlab LMI toolbox to solve the LMIs in Theorem 2, we can get: $\ell_{off} = 1.66s$ and

$$K_1 = [-5.8501 \ \ -3.0855], K_2 = [-4.5665 \ \ -2.7979],$$
$$W_1 = \begin{bmatrix} 75.0127 & 16.2939 \\ 16.2939 & 37.2556 \end{bmatrix}, W_2 = \begin{bmatrix} 38.0562 & 14.8402 \\ 14.8402 & 41.0452 \end{bmatrix},$$
$$\Xi_1 = \begin{bmatrix} 5.4950 & -0.0791 \\ -0.0791 & 7.4208 \end{bmatrix}, \Xi_2 = \begin{bmatrix} 3.4657 & 1.5007 \\ 1.5007 & 7.6517 \end{bmatrix}.$$

Furthermore, let the nonlinear function $f(t,x) = -0.2\sin(-1.2x(t))$, the initial condition $x(0) = col\{0.5, -0.5\}, \phi_0 = 20$, combining with the above gain matrices, the simulation results of systems (13) are given in the following figures. As shown in Figure 2, system (13) cannot achieve stability without control. As shown in Figure 3, the system state gradually reaches the stable state under DETM. Figure 4 is the control input of system (13) under DETM. Figure 5 shows the dynamic evolution process of the function $\lg\phi(t)$, which shows a downward trend as a whole. Figure 6 shows the relationship between triggered instants and triggered intervals. In summary, this example demonstrates that DETM (5) can not only stabilize the system under the influence of DoS attack and packet loss, but also alleviate network communication pressure to a certain extent.

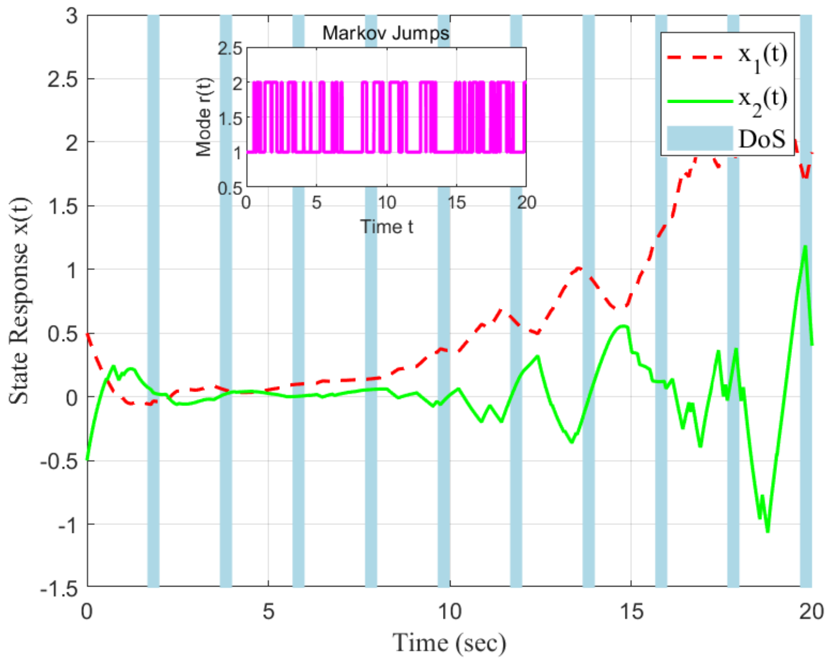

Figure 2. The state response of system (13) without control

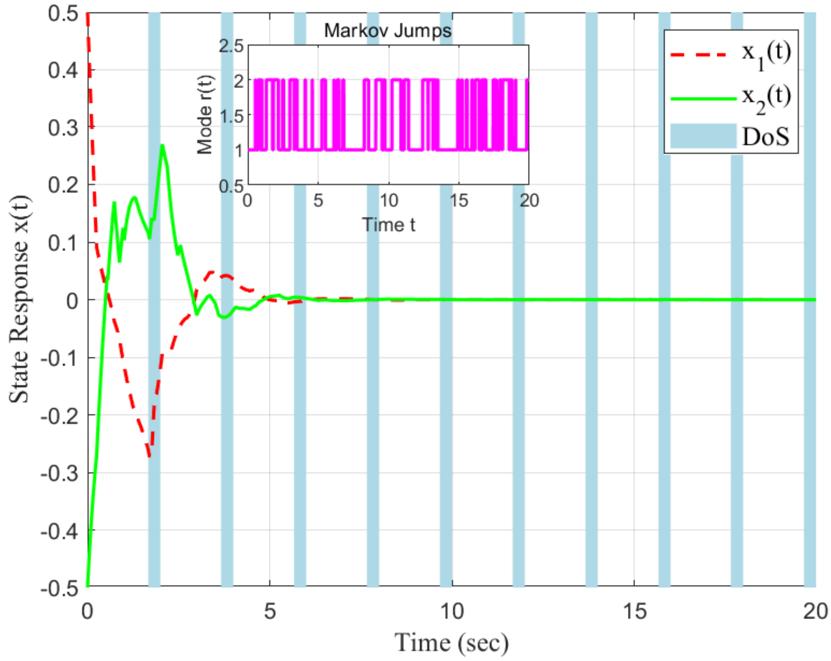

Figure 3. The state response of system (13) under DETM

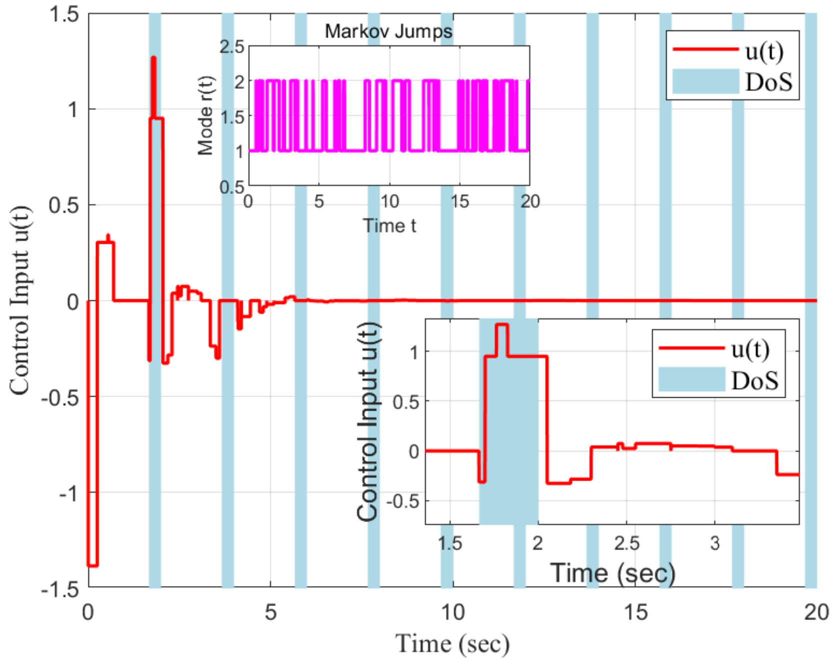

Figure 4. The control input of system (13) under DETM

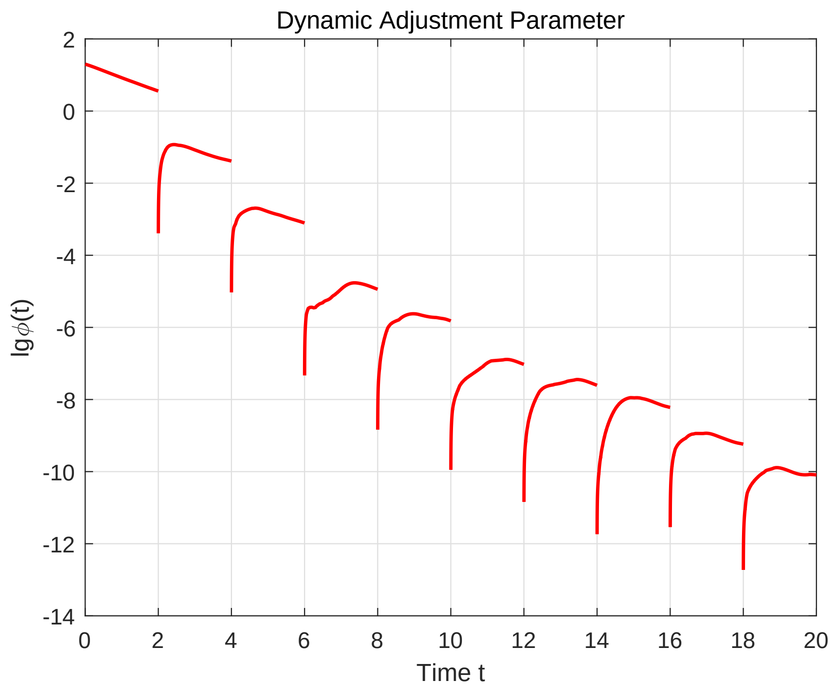

Figure 5. The dynamic evolution of $\lg \phi(t)$

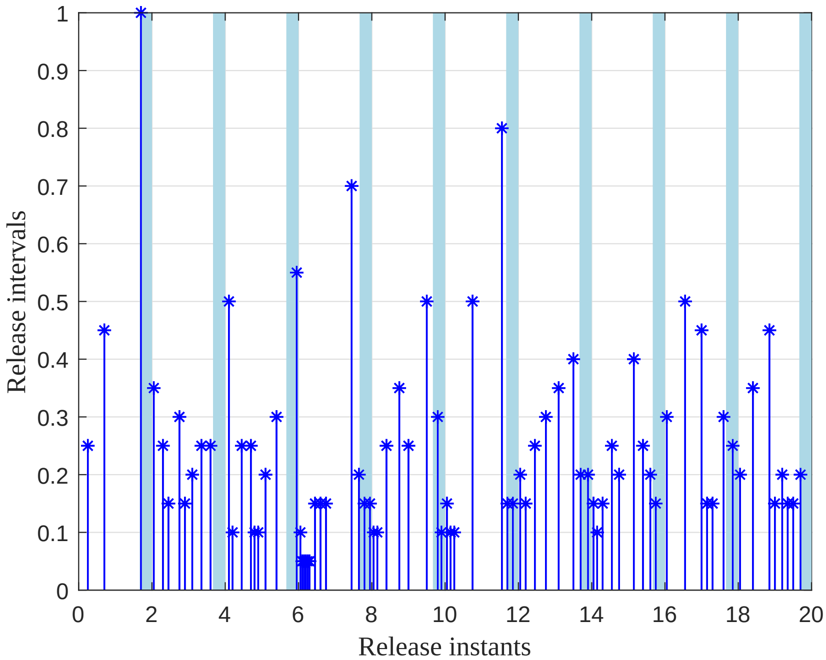

Figure 6. The release instants and release intervals of DETM

5. Conclusions

This paper has studied the exponential stability and stabilization problems of a class of DNMJSs under randomly occurring DoS attack and packet loss. The attack success rate and packet loss rate have been used to describe the stochastic characteristics of DoS attack and packet loss. The POW mehtods and hybrid-input strategy have been proposed to compensate for the impacts of DoS attack and packet loss on the systems. By constructing a general common Lyapunov functional, and combining with the DETM, and other analysis approaches, the less conservative stability criteria are obtained. Finally, a simulation example was used to verify the validity of our results. In the future, the results provided in this paper shall be helpful to investigate other analysis and synthesis of Markov jump systems under cyber attacks and packet loss.

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