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Fuzzy Multi-Item Newsvendor Problem: An Application to Inventory Management

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Abstract: This paper proposes a novel formulation of the fuzzy newsvendor problem for inventory management applications. This new formulation allows the use of any profit function. A new credibility estimation is proposed to explore the neighborhood around the most impactful demand scenarios. Further, a simulation procedure was designed for the different demand scenarios, which allows the comparison of the proposed approach with probabilistic demand curves. The fuzzy newsvendor problem is solved using a modified genetic algorithm (GA), where an initialization mechanism with null values is proposed. The new formulation of the fuzzy newsvendor problem together with the modified GA have shown to improve the average profit up to 55% in problems with low-budget scenarios.

Keywords: newsvendor problem; fuzzy newsvendor problem; inventory management; genetic algorithms; credibility estimation

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1. Introduction

Every day, a newsvendor needs to buy journals based on an uncertain demand. Assuming that each journal has a fixed cost and selling price, if she/he asks for too many journals and the demand is not enough, there is a reduction in the profit. On the other hand, if the demand is higher than the number of journals ordered, potential sales do not occur, resulting in "lost profits" [1]. This dilemma of buying more or less newspapers, which is know as the *newsvendor problem*, can be used to model inventory management problems.

Several solutions can be found to solve several inventory management problems, [1–3]. When multi–items are considered, one deals with the Multi-Item Newsvendor Problem (MINP). In this problem it is important to consider the number of constraints and their type (cost, service level, etc.), the decision-making policies (as e.g. optimize expected profit, service level, etc.). Often, solutions are found using risk–averse techniques. Further, usually MINP use probability density functions to model the uncertain demand [4–7].

However, the demanded probabilistic density functions are difficult to derive in real scenarios, especially for innovative and disruptive products, where there is no sufficient data to accurately predict the demand probability distribution. It is possible to mitigate these limitations by including additional information from human expertise using e.g. fuzzy systems [8].

Fuzzy logic is a suitable tool to incorporate uncertain demands with a proven effectiveness in solving MINP [8–10]. A fuzzy environment can use few data points to describe uncertainty through meaningful membership functions. Furthermore, fuzzy logic offers an ideal environment to describe the vagueness of human thinking through mathematical operations, defining linguistic terms such as "the demand of a product is around 2000" [11].

The first fuzzy solution for an inventory management problem dates back to 1995 [12]. A year later, Petrov proposed the first fuzzy solution for newsvendor problems [8]. Analytical analyses in a fuzzy environment [8,13–16] are useful to specific cases, where it is possible to study a limited number of items in a well isolated economic environment. Problems arise when the number of items and their relations increase, leading to highly nonlinear problems, making analytical approaches hard to

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implement. Most of the recent fuzzy [17–19] and non-fuzzy [20–24] solutions focus on solving highly complex single–item problems, lacking the generalization to multi–item problems.

Fuzzy MINP problems are usually solved recurring to metaheuristic algorithms [9,10]. Inspired by real-world phenomena, metaheuristics use computational power to find solutions when the classical methods cannot, due to time and complexity. However, metaheuristics do not always guarantee that the solutions found are optimal. However, they can provide, at least, good results for highly complex optimization problems [25,26].

Shao proposed a genetic algorithm [27,28], to solve the newsvendor problem with a fuzzy environment [9]. This paper extended the fuzzy objective functions proposed in [8], with the adoption of credibility theory concepts [29,30]. In [9], Shao used the concepts of possibility, necessity and credibility of a fuzzy event, as well as the excepted value of a fuzzy variable [31] to derive objective functions for different decision-making policies.

In 2011, Taleizadeh [10] studied a variety of metaheuristic algorithms to solve a fuzzy single-period newsvendor problem and also proved the suitability of genetic algorithms for this problem.

This paper extends the formulation of the existing fuzzy newsvendor problem from single-item to multi-item problems, allowing its application to inventory problems. The proposed formulation is flexible, as it allows the use of any profit function. Further, this paper proposes the extension of the genetic algorithm in [10] to solve the fuzzy multi-item newsvendor problem, enhancing the work of [9] in both the generation and evaluation of solutions.

The paper is organized as follows. Section 2 describes classical and fuzzy multi-item newsvendor problems. The optimization artchtiecture proposed in this paper is described in Section 3. In this section, the optimization algorithm is described (which uses a genetic algorithm), and a novel method to estimate the credibility is proposed. Further, novel problem-specific genetic mechanisms are also proposed. The benchmark case studies are described in Section 4. A general simulation procedure, which is necessary for addressing both classical and fuzzy multi-item newsvendor problems is proposed in Section 5. Section 6 presents the obtained results, and the conclusions and future work are presented in Section 7.

2. Multi-Item Newsvendor Problem

This section introduces a novel fuzzy formulation for the multi-item newsvendor problem. Section 2.1 presents the classical approach with probabilistic demand curves. Section 2.2 explains the fundamentals of the fuzzy multi-item newsvendor problem.

2.1. Classical Multi-Item Newsvendor Problem

The classical formulation suggested in [32] uses a modified form of the original model proposed in 1964 [33]. This form minimizes the expected cost function, being this minimization equivalent to maximize an“expected profit” function [32]. Also, the original model used the salvage value of the leftover items instead of the environmental disposal cost. The model of the classical multi-item newsvendor problem is as follows:

$$\min \quad E = \sum_{i=1}^{N} \left[c_i x_i + h_i \int_0^{x_i} (x_i - d_i) f_i(d_i) d(d_i) + v_i \int_{x_i}^{\infty} (d_i - x_i) f_i(d_i) d(d_i) \right], \tag{1}$$

subject to

$$\sum_{i=1}^{N} c_i x_i \leq B_G \tag{2}$$

The list of variables used in the classical multi-item newsvendor problem, and their respective description, which will be used throughout this paper are the following:

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n	Total number of items
v_i	Cost of revenue loss per unit of item i
h_i	Cost incurred per item i for leftover
c_i	Cost per unit of item i
x_i	Ordering quantity of item i
d_i	Demand of item i
$f_i(d_i)$	Demand probability density function of item i
E	Expected cost function value
B_G	Available budget
R_0	Profit target
G	Total number of generations

2.2. *Fuzzy Multi-Item Newsvendor Problem*

This section describes the fuzzy multi-item newsvendor problem. Section 2.2.1 starts by presenting the definitions of possibility, necessity, credibility and the expected value of a fuzzy demand. Then, Section 2.2.2 explains how membership grades can be estimated. The estimation of possibility, necessity, credibility is explained in Section 2.2.3. Finally, the estimation of the expected profit is described in Section 2.2.4.

2.2.1. Definitions

The possibility, necessity and credibility of fuzzy event presented in [29,30] are used in this paper. The concepts of possibility, necessity and credibility of a fuzzy event $(\xi \geq r)$ are defined as:

$$\text{Pos}\{\xi \geq r\} = \sup\ \mu\{u\}\,,\ u \geq r \tag{3}$$

$$\text{Nec}\{\xi \geq r\} = 1 - \sup\ \mu\{u\}\,,\ u \leq r \tag{4}$$

$$\text{Cr}\{\xi \geq r\} = \frac{1}{2}[Pos\{\xi \geq r\} + Nec\{\xi \geq r\}] \tag{5}$$

where ξ is the fuzzy demand with the membership function $\mu\{u\}$, r is the profit value and u is the generic demand. Considering (3), (4) and (5), the expected value of a fuzzy demand ξ is given by:

$$E[\xi] = \int_0^{\infty} \text{Cr}\{\xi \geq r\}\, dr - \int_{-\infty}^{0} \text{Cr}\{\xi \leq r\}\, dr. \tag{6}$$

In [9,30] the concepts in (3), (4) and (5) are used to define objective functions that describe multiple decision-making policies, particularly the maximization of the expected profit, which will be explained in Section 2.2.4.

2.2.2. Membership grade estimation

In an MINP, the demand contain the proposed quantities for each item. Since each item has its unique demand membership function, it is fundamental to find a way of estimating the grade of a demand. This is the purpose of a conjunctive operator.

Let one assumes that $u_k = (u_{1k}, u_{2k}, \ldots, u_{nk})$ is a demand vector of n items. The estimated membership grade of the demand vector is given by:

$$\begin{aligned}\mu(u_k) = \mu(u_{1k}) \cap \mu(u_{2k}) \cap \ldots \cap \mu(u_{nk}) = \\ \min(\mu(u_{1k}), \mu(u_{2k}), \ldots \mu(u_{nk}))\end{aligned} \tag{7}$$

$$\begin{aligned}\mu(u_k) = \mu_1(u_{1k}) \cap \mu_2(u_{2k}) \cap \ldots \cap \mu_m(u_{nk}) = \\ \min(\mu_1(u_{1k}), \mu_2(u_{2k}), \ldots \mu_m(u_{nk}))\end{aligned} \tag{8}$$

where $\mu(u_k)$ is the estimated membership grade, and $\mu(u_{nk})$ is the membership grade associated with each ordering quantity.

2.2.3. Possibility, necessity and credibility estimation

The possibility and necessity estimations of multi-item solutions from the demand vectors u_k are computed, respectively, in the following way:

$$\widetilde{\text{Pos}}\{R(x,\xi) \geq R_0\} = \max_{1 \leq k \leq U} \{\mu(u_k) | R(x,u_k) \geq R_0\} \tag{9}$$

$$\widetilde{\text{Nec}}\{R(x,\xi) \geq R_0\} = 1 - \max_{1 \leq k \leq U} \{\mu(u_k) | R(x,u_k) \leq R_0\} \tag{10}$$

where $R(x,u_k)$ is the profit function, U is the total number of random demand vectors and R_0 is a profit target.

The estimation of the credibility is based on the previous estimations of possibility and necessity as follows:

$$\widetilde{\text{Cr}}\{R(x,\xi) \geq R_0\} = \frac{1}{2}[\widetilde{\text{Pos}}\{R(x,\xi) \geq R_0\} + \widetilde{\text{Nec}}\{R(x,\xi) \geq R_0\}] \tag{11}$$

2.2.4. Expected profit estimation

To estimate the expected profit of a solution x, with $x = (x_1, x_2, \ldots, x_n)$, it is possible to use credibility estimations for a high enough number of profit targets. To focus resources on plausible values, theses profit targets should be extracted from the interval defined by:

$$\left[-\sum_{i=1}^{N} c_i x_i, \quad \sum_{i=1}^{N} (v_i - c_i) x_i \right] \tag{12}$$

On the one hand, the interval lower limit corresponds to the scenario where no sales are made. On the other hand, the upper limit corresponds to the scenario where all purchased items are sold.

Assuming that the total number of profit targets is given by S_{cr}, the profit targets are equally distributed and the set of profit targets is defined by $r = (r_1, r_2, \ldots, r_{S_{cr}})$, where $r_1 < r_2 < \ldots < r_{S_{cr}}$. Equations (13), (14) and (15) describe the steps to estimate the expected profit $E$of a solution x:

$$E_1 = -\sum \widetilde{\text{Cr}}\{R(x,u_k) \leq r_j\}, \quad if\ r_j < 0 \tag{13}$$

$$E_2 = E_1 + \sum \widetilde{\text{Cr}}\{R(x,u_k) \geq r_j\}, \quad if\ r_j \geq 0 \tag{14}$$

$$E = E_2 \times \frac{(r_{S_{cr}} - r_1)}{S_{cr}} + \max(0, r_1) + \min(0, r_{S_{cr}}) \tag{15}$$

The number of credibility samples S_{cr} is a crucial variable to this estimation. This variable must be studied to obtain the best possible trade-off between computational time and accuracy.

3. Proposed optimization architecture

The formulation of the fuzzy newsvendor problem is extended in this paper from single-item to multi-item problems, allowing its application to inventory problems. This is accomplished with an optimization architecture that combines a modified genetic algorithm and the expected profit estimation introduced in Section 2.2.4. Along with the common mechanisms of a genetic algorithm, crossover, mutation and selection [27], two novel components are added: a credibility estimation procedure, which is introduced in Section 3.1 and novel problem-specific genetic mechanisms, which are described in Section 3.2.

The proposed optimization architecture finds the solution with the highest expected profit, according to the fundamentals previously introduced in Section 2. Algorithm 1 details the proposed

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genetic algorithm to maximize the expected profit. This algorithm needs to estimate the credibility $\widetilde{Cr}$, which is explained in detail in Section 3.1 and Algorithm 2.

Algorithm 1 Proposed genetic algorithm.

```
Require: G; i ← 0
while i < G do
    for each individual x of generation i do
        Compute profit interval using (12)
        Compute all profit targets r_j by equally sample profit interval
        Compute the credibility C̃r{R(x, ξ) ≥ r_j} for all j using Algorithm 2
        Compute expected profit E[R(x, ξ)] using (13), (14) and (15)
    Compute next population using the problem specific mechanisms Section 3.2
    i ← i + 1
Select the individual x with the highest E[R(x, ξ)]
```

3.1. Proposed credibility estimation

This section describes the proposed credibility estimation, which is presented in Algorithm 2. The main objective of this approach is estimate the credibility of a solution x generating a profit higher than a profit target R_0. It repeats the estimation K times until it finds it. Estimations for different profit targets R_0 are further used to estimate the expected profit of the solution x, as it was previously described in Section 2.2.4.

The demand vector is randomly generated, and considered quantities that have a membership grade higher than a α_{cut}, which are defined by 10% quantiles, and as so can have the following values:

$$\alpha_{cut} \in T(\alpha_{cut}) = [1.0^{-5},\ 0.1,\ 0.2,\ 0.3,\ 0.4,\ 0.5,\ 0.6,\ 0.7,\ 0.8,\ 0.9] \tag{16}$$

Further, the α_{cut} is always equal or greater than the minimum value between the highest membership grades found for both possibility and necessity. The α_{cut} progressively increases the minimal membership grade of the random demand vectors generated.

This is useful because the possibility estimation requires to find the demand vector with the highest membership grade that generates a profit higher than the profit target R_0, as defined in (9). And, the necessity estimation requires to find the demand vector with the highest membership grade that generates a profit lower than the profit target R_0, as defined in (10).

Note also that sometimes, due to the random generation, for low credibility solutions, the necessity estimation can be higher than the possibility estimation. Those results are impossible due to the nature of the problem, and therefore are automatically rejected by the algorithm.

3.2. Novel problem-specific genetic mechanisms

This section describes problem-specific mechanisms, which are implemented in the architecture proposed in Algorithm 1. These methods enhance the set of solutions by 1) discarding materials in the initial population, 2) scaling chromosomes according to the available budget, and 3) introduce a problem-specific context to the crossover operator. This section proposed thus the chromosomes initialization, the solution resizing and the chromosome normalization, which are described in the next three sections.

3.2.1. Initialization with zero quantities

The initialization with zero quantities initializes chromosomes with a single non-zero ordering quantity x_i. After selecting a random item i for the non-zero ordering quantity, the chromosome is resized as explained in Section 3.2.2 to scale x_i to reach the available budget. If i is a profitable item, the chromosome is selected because the expected profit is higher than chromosomes containing less profitable items. This mechanism aims to select (and further combine) only the most profitable items.

Algorithm 2 Credibility estimation.

Require: x; R_0; $R(x, u_k)$; $T(\alpha_{cut})$; K
$\alpha_{cut} \leftarrow T(\alpha_{cut})$
Pos $\leftarrow 0$; Nec $\leftarrow 0$; $n \leftarrow 0$; $stop \leftarrow False$
while $stop = False$ **do**
  Compute random μ_k with $\min(\mu(u_{1k}), \ldots, \mu(u_{nk})) \geq \alpha_{cut}$
  Compute $\mu(u_k)$ using (7)
  ====== Possibility and Necessity Update ======
  if $R(x, u_k) \geq R_0$ and $\mu(u_k) > Pos$ **then**
    Pos $\leftarrow \mu(u_k)$
  else if $R(x, u_k) < R_0$ and $\mu(u_k) > \widetilde{Nec}$ **then**
    $\widetilde{\text{Nec}} \leftarrow \mu(u_k)$
  ========== Threshold Update ==========
  if $\min(\text{Pos}, \widetilde{\text{Nec}}) \geq \alpha_{cut}$ **then**
    $\alpha_{cut} \leftarrow \min(\text{Pos}, \widetilde{\text{Nec}})$ (soft threshold update)
  else if $n \geq K$ and $\alpha_{cut} \geq \max(T(\alpha_{cut}))$ **then**
    Assign $stop \leftarrow True$ (finish execution)
  else if $n \geq K$ **then**
    $\alpha_{cut} \leftarrow \min(T(\alpha_{cut}) > \alpha_{cut})$ (hard threshold update)
    $n \leftarrow 0$
  else
    $n \leftarrow n + 1$
========== Finish Execution ==========
Nec $\leftarrow 1 - \widetilde{\text{Nec}}$
if Pos $>$ Nec **then**
  Compute $\text{Cr}\{R(x, \xi) \geq R_0\}$ using (11)
else
  $\text{Cr}\{R(x, \xi) \geq R_0\} = 0$ (solution rejected)

3.2.2. Solution resizing

The solution resizing mechanism scales the chromosome ordering quantities x_i to use all the available budget. It scales the quantities up or down without changing the relative proportions between them. On the one hand, this mechanism can transform over-budget solutions into feasible solutions by scaling them down. On the other hand, it can scale up under-budget solutions to use all the available budget. To apply this mechanism, the ordering quantities x_i are resized to quantities x_{ri} by multiplying it by a resizing ratio, as follows:

$$x_{ri} = x_i \times \frac{B_G}{\sum_{i=1}^{N} c_i x_i} \tag{17}$$

3.2.3. Chromosome normalization

This normalization makes the crossover independent of the absolute values in x, by normalizing them according to the items with most expected demand value μ_i. In this paper, the most expected demand value comes from the item with the probabilistic density function $f_i(D_i)$, but it could also be the fuzzy value with the highest grade. To apply the chromosome normalization, first the ordering quantities x_i must be normalized by applying the transformation:

$$x_{ni} = \frac{x_i}{\mu_i} \tag{18}$$

where x_i is the ordering quantity of item i, μ_i is the expected demand value of the probabilistic distribution of item i, and x_{ni} is the normalized ordering quantity of item i. After the crossover has been applied, all normalized ordering quantities x_{ni} must be de-normalized by multiplying them by μ_i.

4. Case Studies

The two benchmark case studies used in this paper have been presented in [32,34]. The optimization method presented in these papers is a Generic Iterative Method (GIM) with two different use cases: one with exponential demand distributions, which is described in Section 4.1, and the other with normal demand distributions, described in Section 4.2.

4.1. Case Study 1: Exponential demand distribution

The first case study was proposed in [32], where the item demand is exponentially distributed. An exponential distribution for demand d_i with a mean value μ_i is described in (19) and (20), defining its probability density function and cumulative distribution function, respectively.

$$f_i(d_i, \mu_i) = \begin{cases} 0, & d_i < \mu_i \\ \frac{1}{\mu_i} e^{-\frac{d_i}{\mu_i}}, & d_i \geq \mu_i \end{cases} \tag{19}$$

$$F_i(d_i, \mu_i) = \begin{cases} 0, & d_i < \mu_i \\ 1 - e^{-\frac{d_i}{\mu_i}}, & d_i \geq \mu \end{cases} \tag{20}$$

In [32], the exponential demand distribution considered a problem with six items, as presented in Table 1, and a budget of 3500 currency units (CU).

Table 1. Data for Case study 1: revenue loss per unit, cost for leftover, cost per unit, and mean of demand for the six items.

Item	v_i (CU)	h_i (CU)	c_i (CU)	μ_i
1	7	1	4	200
2	12	2	8	225
3	30	4	20	112.5
4	30	4	10	100
5	40	2	13	75
6	45	5	15	30

The GIM proposed in [32] solved this optimization problem by relaxing the problem constraint, applying the Leibniz Rule and using a Lagrangian optimization with a Lagrangian multiplier. The obtained solution with the ordering quantities per item is presented in Table 2.

Table 2. Case Study 1: benchmark solution with ordering quantities per item.

Item	1	2	3	4	5	6
x_i	78.41	58.16	30.06	81.74	70.91	25.29

4.2. Case Study 2: Normal demand distribution

The second case study was proposed in [34] , and considers a normal distribution for demand d_i. A demand distribution d_i has the mean value μ_i and the standard deviation σ_i, see (21) and (22), defining its probability density function and cumulative distribution function, respectively.

$$f_i(d_i, \mu_i) == \frac{1}{\sigma_i\sqrt{2\pi}} e^{-\frac{1}{2}(\frac{d_i-\mu_i}{\sigma_i})^2} \tag{21}$$

$$F_i(d_i, \mu_i) == \frac{1}{2}\left[1 + erf\left(\frac{d_i - \mu_i}{\sqrt{2}\sigma_i}\right)\right] \tag{22}$$

where:

$$erf(x) = \frac{1}{\sqrt{\pi}} \int_{-x}^{x} e^{-t^2} dt \tag{23}$$

This case study includes 17 items, as shown in Table 3, and the available budget of 2500 CU.

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Table 3. Data for Case study 2: revenue loss per unit, cost for leftover, cost per unit, mean and standard deviation of demand for the 17 items.

Item	v_i (CU)	h_i (CU)	c_i (CU)	μ_i	σ_i
1	7	1	4	102	51
2	12	2	8	73	18.3
3	30	4	19	123	30.8
4	30	4	17	95	23.8
5	40	2	23	62	15.5
6	45	5	15	129	43
7	16	1	10	69	34.5
8	21	2	10	83	41,5
9	42	3	40	120	30
10	34	5	20	89	22.3
11	20	3	10	115	38.3
12	15	5	7	91	30.3
13	10	3	4	52	17.3
14	20	3	12	76	38
15	47	2	33	66	16.5
16	35	4	21	147	36.8
17	22	1	11	104	34.7

The expected profit-maximizing solution proposed in [34] is presented in Table 4.

Table 4. Case Study 2: solution showing the ordering quantities per item.

Item	x_i	Item	x_i
1	0	**10**	0
2	0	**11**	15.58
3	0	**12**	42.2
4	0	**13**	34.56
5	0	**14**	0
6	106.86	**15**	0
7	0	**16**	0
8	14.02	**17**	15.23
9	0		

5. Simulation procedure

This section presents a general simulation procedure for addressing both classical and fuzzy multi-item newsvendor problems. Studies on the fuzzy multi-item newsvendor problem [9,10,35] have been focusing on evaluating the performance of the solutions solely based on maximizing a fuzzy objective function. This approach raises questions, such as: "Is the objective function a good representation of reality?" or "Will the solution generate the expected results in a real scenario?".

To address these issues, we propose the Algorithm 3, which uses demand vectors u, based on each item's demand D_i, to evaluate solutions, both fuzzy and non-fuzzy. The procedure simulates real scenarios by using a diverse range of demand vectors u, with a greater emphasis on probable vectors while still accounting for less likely ones. By computing the profit for each demand vector u with a given solution x, the average profit and profit standard deviation across all vectors can be determined.

Algorithm 3 Simulation procedure.

Require: n (number of demands vectors to generate)
Derive n demand vectors d_i using the open-source library in Python NumPy, where each vector element i comes from the demand probability density functions $f_i(d_i)$.
For each demand vector d_i, calculate the profit R.
Compute the average profit and the profit standard deviation across all demand vectors d_i.

6. Results

This section presents the results using the proposed optimization architecture proposed in Section 3. The results are compared with previous classical and fuzzy newsvendor problems. The case studies presented in Section4 are used to asses the performance of the proposed intelligent framework. These comparisons intend to understand if a more flexible framework — with the ability to incorporate complex profit functions and human expertise — can obtain better results in some sense than previous classical and fuzzy approaches that lack this kind of flexibility.

6.1. Impact of the novel problem-specific mechanisms

This section evaluates the impact of specific features on the performance of the intelligent framework. Each of the introduced problem-specific generic mechanisms presented in Section 3.2, namely: initialization with zero quantities, solution resizing and chromosome normalization are analysed separately.

First, Table 5 presents the influence of the initialization with zero quantities, introduced in Section 3.2.1, is only advantageous for case study 2. Therefore, it must be always tested for different instances.

Table 5. Results using the initialization with zero quantities

Case study	Init. with zero quantities	Fitness	Simulated profit
1	Yes	4167.9	2860.3
1	**No**	**4940.9**	**2853.0**
2	**Yes**	**3741.9**	**3797.3**
2	No	2229.9	2446.6

The effect of the solution resizing feature, presented in Section 3.2.2, is shown in Table 6. It has a clear positive effect by eliminating unfeasible solutions and improving the profit. Therefore, this feature is always used in genetic algorithm.

Table 6. Results using solution resizing.

Case study	Sol. resizing	Fitness	Unfeasible sol.	Simulated profit
1	**Yes**	**4167.9**	**0**	**2860.3**
1	No	3945.1	250.4	2293.2
2	**Yes**	**3741.9**	**0**	**3797.3**
2	No	2208.8	122.2	2305.6

The effect of normalizing the chromosome described in Section 3.2.3 is presented in Table 7. It can be seen the performance in terms of fitness and simulated profit is very slightly affected. Therefore, it is advantageous to try this mechnism in different instances.

Table 7. Results using chromosome normalization.

Case study	Chromosome normalization	Fitness	Simulated profit
1	Yes	4889.5	2827.4
1	No	4940.9	2853.0
2	Yes	3713.7	3755.7
2	No	3741.9	3797.3

6.2. Main Results

In this section, we present the main results of our study, which compares the performance of three different methods for solving the case studies in Section 4. The first method is the classical approach, introduced in Section 2.1, the second methods is the fuzzy genetic algorithm proposed in [9] and the third the fuzzy optimization architecture proposed in this paper, see Section 3.

Table 8 presents these results. It is clear for both case studies that the proposed approach clearly outperform the other two approaches in terms of profit for both case studies. It is however slightly worse in terms of standard deviation than the other two approaches for case study 2.

Table 8. Simulation results in terms of average profit and standard deviation of the profit.

Case Study	Method	Average Profit	Profit St. Dev.
1	Classical approach	2877.1	2.0
1	Fuzzy GA from [9]	2889.8	1.2
1	Proposed approach	2914.7	1.5
2	Classical approach	3869.4	0.5
2	Fuzzy GA (as in [9])	2833.6	0.2
2	Proposed approach	3878.7	0.8

7. Conclusions

A novel formulation of the fuzzy newsvendor problem for inventory management applications was proposed. The designed framework implemented a fuzzy formulation to solve cases where there is insufficient data to predict the demand distributions and it is necessary to integrate human-expertise knowledge. One of the main contributions of this work is the redesign of the credibility estimation procedure, introducing a dynamic adjustment of a $alpha_{cut}$ threshold to generate meaningful demand vectors, instead of using a purely random vector generation.

The proposed fuzzy optimization genetic algorithm is compared in two benchmark case studies. The proposed approach slightly outperform the classical approach. However, it clearly outperform the previous fuzzy approach. In the most complex case, case study 2, it improves the profit in 55%. The performance increase is the result of introducing a new initialization with null values that proved to be a valuable mechanism in low-budget scenarios, where there is the need for rejecting less profitable items.

The main advantage of this algorithm it is its flexibility. Despite using fixed costs to prove effectiveness against analytical approaches, this solution can work with nonlinear pricing models. To perform this, one only needs to integrate the pricing information when calculating profits in the credibility estimation. This is suggested for future work. Moreover, there is the possibility of changing performance measures. Profit was used to prove the effectiveness against analytical approaches, but the algorithm could prioritize the solutions that most satisfied possible costumer demand, by replacing the profit calculation with a service-level calculation.

Additionally, the proposed algorithm can also be implemented using parallel computing in a cloud environment, which drastically reduces execution time and makes the solution applicable in a real scenario.

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