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Pseudo-Instantaneous Action-at-a-Distance with Weberian Potentials

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Abstract: In this work, the conceptions of the electrodynamics of action-at-a-distance and of electromagnetic fields are reviewed and confronted, with the aim of clarifying whether it is possible to propose a Weberian theory of action-at-a-distance, which together with a renewed mechanics (both respecting the existence of a limiting velocity *c*) could constitute a framework capable of yielding predictions equivalent to those of the conventional Maxwell-Lorentz and SRT framework. In this work, it is shown that, considering basic assumptions about the interaction process, understanding it as an action exchange process, it is possible to derive the expression of the electrodynamic Phipps' potential energy between two interacting charges. It is also shown that, with the same assumptions and the additional hypothesis that the interaction is supported by a mediating agent (aether or field) which is perturbed by the transmission of action, this perturbation obeys the wave equation (one of the main drawbacks of Weberian electrodynamics). Another aim of the work is to confront the relational electrodynamic formulations (supported by Galilean transformations that assume a universal time) with the Maxwell-Lorentz formulation of electromagnetism (linked to Lorentz transformations that entail local times), in order to elucidate whether the space-time to which the SRT leads us is a mere mathematical artifice or an inescapable reality.

Keywords: weber's electrodynamics; electromagnetism; special relativity; fundamental physics

1. Introduction

Two Rival Programmes

In the period of almost 24 centuries from the time of Aristotle to the present day, only two possible paradigms have been proposed for the interaction of matter: that of so-called *action-at-a-distance* and that of *contact action* mediated through "aethers" or "fields".

Until 1686, with the publication of Newton's *Principia* [1], all previous pictures of the world, following in the wake of the Aristotelian tradition, postulated that the space between objects was completely filled by aethers or eddies. Matter influenced these undetectable fluids and, in turn, was carried along by them. Until the 17th century, this portentous philosophy, however, was never able to make predictions testable by experiment. Nevertheless, its convincing concepts were to be the seeds of all later field theories.

The first mathematical law capable of making testable predictions was a law of action-at-a-distance (AAAD), known as Newton's *law of universal gravitation* [1]. According to it, all bodies with mass attract each other with a force whose intensity is directly proportional to the product of their masses, and inversely proportional to the distance separating them. There is, of course, no mention of the medium (aether) that could sustain this attraction. Almost a century later, at the end of the 18th century, Michell [2] and Coulomb [3] proposed and validated two new force laws (of attraction and repulsion) between magnetic poles and isolated charges, respectively, which were also directly proportional to the product of their sources (poles or charges) and inversely proportional to the square of the separation distance.

Nineteenth-century electrodynamics was dominated, on the one hand, by the theories of action-at-a-distance (or pure particle theories), developed among others by Ampère [4], Gauss [5], Weber

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[6], Riemann [7], Clausius [8] and, on the other hand, by pure field theories, developed mainly by Faraday [9] and Maxwell [10]. The essential feature of AAAD electrodynamic theories is that the fundamental concepts are charges and currents, while the fields are auxiliary, secondary and therefore dispensable elements (a sort of "accounting artifices"). For pure field theories the situation is just the opposite: the fundamental concept is that of the field, and charges and currents are auxiliary, secondary and dispensable entities.

At the end of the 19th century, field theories would eventually prevail. But at the beginning of the 20th century, they had to be revised due to the discovery of the first elementary particles. This forced revision, which added particle concepts to the ontology of pure field theories, gave rise to modern particles-field theories, which can be understood as a fusion between theories of action-at-a-distance and pure field theories.

Despite these changes, there remained supporters of the pure schools in the 20th century. The most important advocate of pure field theories was possibly Einstein, with his search for the unified field equations. Advocates of pure AAAD included Frenkel [11], Schwarzschild [12], Tetrode [13] and Fokker [14], Ritz [15] and, somewhat later, Wheeler and Feynman [16].

Strictly speaking, the first AAAD laws (Newton's, Michell's and Coulomb's) should be called *instantaneous* action-at-a-distance laws. However, Weber's law, that of his master Gauss, those subsequently proposed by Riemann and Clausius, and later Ritz, even if they only include magnitudes referring to the current instant (which would allow us to qualify them as *instantaneous* laws), it is also true that they incorporate a certain limiting velocity at which the action would be transmitted (obviously, without alluding to any underlying mechanism, as befits an AAAD or pure particle law), so they could constitute a different category, which we will call here *pseudo-instantaneous* action-at-a-distance.

Pros and Cons of Both Programmes

There is a major difference between AAAD theories and field theories. The former predict interactions and forces between directly observable physical objects and can therefore be tested for falsifiability (a requirement of the scientific method). Field theories, on the other hand, predict the value of certain quantities at points in empty space. The only way to test whether these predictions are accurate is to place a detector at each point in space, but then one would only be testing a mathematical theory that is indistinguishable from an AAAD theory. So, deciding which paradigm is best is not straightforward.

Proponents of AAAD argue for its greater simplicity, inherited from greater ontological sparseness. Many other researchers reject the AAAD simply because they prefer to explain the interaction between charges as due to a field, which may itself be constituted by virtual particles that are exchanged; each charge emits these particles that would propagate in space at a finite speed and affect the second charge when it reaches it (which is still a kind of action-at-a-distance, even at small distances). Beyond an aesthetic or intuitive preference, according to Fritch [17], proponents of field theories also find compelling metaphysical arguments for rejecting AAAD, namely the breakdown of locality and the non-conservation of energy.

If the action (in whatever way) were transported at speed c, there must necessarily be a delay between cause and effect, since it would take a finite time for the action to travel from one point in the distribution of charge or current (the point constituting the cause) to any other point in space, where the effect is measured. That is, the action could not be instantaneous in any way. However, according to Pietsch [18], it would be possible to reconcile *locality* with AAAD by simply accepting that the intensity of the interaction decreases rapidly with the distance of separation between the interacting bodies, i.e., without the need for direct contact between the interacting bodies. To describe this idea, Pietsch used the pun *locality of action-at-a-distance*. A similar idea is advocated in the recent work by Li [19].

Another thorny issue that stifles AAAD, also linked to locality, is the conservation of energy. However, according to Piestch [18] again, conservation of energy can simply be reconciled with AAAD, if energy is reinterpreted in a "relational" way. This is usual for potential energy (this is the case for Newton's law of gravitation, Coulomb's, Weber's, and also Phipps', for example), but it is not usual for kinetic energy, which is usually defined with reference to a particular frame of reference. If kinetic energy is also defined relationally, then energy would be a function of the relative distances and velocities of all bodies belonging to a closed system.

In the analysis of the dilemma between the two programmes, one cannot help but come up against the same doubt that worried Ritz. [15] and, years later, Wheeler and Feynman [16]: if, as is well known, Maxwell's equations are symmetric with respect to time, that is, they are compatible with advanced and retarded waves, why do we only observe retarded waves? For Ritz this fact could only mean that something was wrong with Maxwell's equations or that they were incomplete, so it would be preferable to dispense with them and take the retarded Liénard-Wiechert potentials as fundamental. This point gave rise to an interesting debate between Ritz and Einstein [20], the latter being reluctant to exclude the advanced solutions. (However, according to Frisch and Piescht [21], there seems to be good evidence that Einstein came to agree with Ritz that the elementary radiation processes in classical electrodynamics are not symmetric and are fully retarded).

Years later, to get around this same dilemma, Wheeler-Feynman proposed the AAAD theory known as the "absorber theory" [16], which, in addition to taking time symmetry into account, explained why only the retarded solution survived.

Can the Field and AAAD Paradigms be Equivalent?

Faraday put a lot of effort into trying to prove the superiority of his field theory to the then dominant AAAD electrodynamics, advocated by continental physicists such as Ampère or Weber. However, at the end of the 19th century, many of the physics heavyweights of the time, including Maxwell and Thomson, after studying the matter in depth, concluded that pure particle theories and pure field theories were mathematically equivalent. How could this be possible? In fact, any AAAD electrodynamics can be assimilated by a field theory by simply adding the requirement that there must be an ideal emitter and an ideal absorber. Thus, any electrodynamic action will be emitted and absorbed by charged particles, so that the fields can be calculated entirely from the charge distribution, which makes them auxiliary quantities. The way this has usually been implemented is by using retarded potentials (which take into account the transmission time between emitter and receiver) instead of instantaneous ones. This is the strategy followed, for example, to obtain the so-called *retarded Liénard-Wiechert potentials* [22], which are a complete representation of electromagnetic interactions, compatible with SRT, and in which the fields can be considered as auxiliary quantities. (If, however, the fields are still considered as fundamental, this is due to a matter of pure convention).

However, the current particle field theory, which results from the fusion of a pure field theory and some notions borrowed from AAAD theories, cannot be mathematically equivalent to a pure field theory or a pure AAAD theory, simply because additional fundamental concepts are involved. SRT further complicates the task. Nevertheless, one should not rule out the possibility of finding an AAAD theory, with certain aspects taken from field theories, which is mathematically equivalent to the conventional Maxwell-Lorentz theory and SRT, and this, without renouncing the Galilean framework.

This new AAAD theory should lead to the fact that it nonetheless transmits 'as if' it propagates wave-like through even a vacuum. To be consistent with the AAAD paradigm, this should be achieved without assuming additional mechanisms about what supports the propagation (which would be more in line with the field paradigm).

The Electrodynamics of Weber and Phipps

From the AAAD approach, Weber extended Coulomb's law in a genuinely relational way, i.e., by formulating a new expression for the potential energy and the interaction force of charge pairs as dependent on the separation distance and the time derivatives of this separation distance. Naturally, to define such a separation distance unambiguously requires absolute simultaneity. For this reason, the approach has had few followers since the advent of Einstein's SRT.

In 1871, Weber proposed the following expression for the electrodynamic potential energy [23]:

$$U_W = \frac{q_1 q_2}{4\pi\varepsilon_0}\frac{1}{r}\left(1-\frac{\dot{r}^2}{2c^2}\right) \tag{1}$$

In this expression, U_W is the potential energy generated by q_1 and q_2, ε_0 is the vacuum's permittivity, c is the velocity of light in the vacuum, r is the distance separating the two charges, and the dot stands for the time derivation, $\dot{r} \equiv dr/dt$, so $\dot{r}$ represents the radial relative velocity. Notice that Equation (1) is fully relational; that is, it is dependent only on the two interacting charges, on the distance between them (r) and on their radial relative velocity ($\dot{r}$).

The Weber potential energy, Eq. (1), can be understood as a generalisation of the Coulomb potential energy, with which it coincides when there is no relative motion between the charges. Therefore, all the results of electrostatics, as well as Gauss' law (the first of Maxwell's equations) can be deduced from Weber's electrodynamics. Similarly, Faraday's law of induction can also be derived from Weber's electrodynamics. In addition, and this was Weber's primary aim, it was also possible to deduce Ampére's law, which gave it an even more fundamental character than this law. These deductions can be found in the treatises by Whittaker [24] and O'Rahilly [25], as well as in the monograph of Assis [26].

Despite these important achievements, Weber was unable to demonstrate the propagation of electrodynamic waves through a vacuum. (Ironically, years before Maxwell published his conclusions that light was an electromagnetic wave, Weber and Kirchhoff, working independently, had shown, on the basis of Weber's law, that electrical signals propagate wave-like at the speed of light through a conducting circuit of negligible resistance).

In order to make it compatible with electromagnetic radiation, Weber's theory was converted into a field theory by Wesley [27,28], introducing a retarded (or advanced) time, so that rapidly varying effects, including radiation, could then be predicted. Also, Kühn [29] derived an inhomogeneous wave equation from Maxwell's equations that was compatible with Weber's electrodynamics under certain conditions (frame of reference at rest, charge moving with uniform velocity, and velocity-dependent value).

Helmholtz strongly criticised Weber's proposal, among other reasons because it admitted the possibility of charges behaving as if they had negative mass, which had never been observed. To overcome this drawback, Phipps [30] proposed a modified potential energy given by:

$$U_P = \frac{q_1 q_2}{4\pi\varepsilon_0}\frac{1}{r}\left(1-\frac{\dot{r}^2}{c^2}\right)^{\frac{1}{2}} \tag{2}$$

It is obvious that both Equation (1) and Equation (2) implicitly contemplate a radial relative velocity that we can call critical. In Weber's expression, this critical velocity takes the value of $\sqrt{2}c$, whereas in Phipps' expression, it takes the value of c. From a strictly mathematical point of view, in Weber's expression the value of $\sqrt{2}c$ needs not be a limiting value, so that higher values are conceivable (although the sign of the energy would change from attractive to repulsive or vice versa). In Phipps' expression, on the other hand, the value of c is a full-fledged limiting value, because higher values would imply non-real (complex) quantities.

It is easy to show that Weber's expression can be seen as an approximate version of Phipps' expression, valid only for situations where $\dot{r}/c \ll 1$ (low velocities). It is sufficient to take Maclaurin's development of Phipps' expression, retaining only the terms up to second order in $\dot{r}/c$; the expression thus obtained coincides exactly with the one proposed by Weber.

In [31] and [32], studying the high-speed regime with Weberian electrodynamics, it has already been noted that in order to find the known expressions of potential energies and forces of a charge interacting with a whole system of charges (or charged object), i.e., when the *superposition principle* needs to be applied, it is necessary to use a corrected (effective) potential or force, whose explanation is not yet clear, but which could have to do with a kind of synchronisation of the actions coming from charges located at great distances.

The expression of the effective Phipps potential is obtained by the expression:

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$$U^* \equiv U - \dot{r} \cdot \frac{\partial U}{\partial \dot{r}} \tag{3}$$

from which it follows:

$$U_P^* = \frac{q_1 q_2}{4\pi\varepsilon_0} \frac{1}{r} \left(1 - \frac{\dot{r}^2}{c^2}\right)^{-\frac{1}{2}} \tag{4}$$

Further work is required to elucidate and understand why this correction is necessary and what exactly it represents.

Purpose

In the present work we will attempt to show that the Phipps potential energy can be considered a retarded potential energy (albeit expressed in terms of the current time) which is obtained as the geometric mean of the retarded and advanced potential energies. We will also show that, from this very fact, the wave behaviour of the transmission of electrodynamic action (whatever the mechanism underlying it) follows. Finally, we will discuss the possibility that the new framework formed by the Phipps potential energy and a new relational kinetic energy, respectful of the existence of a limiting velocity *c*, could be mathematically equivalent to the conventional Maxwell-Lorentz and SRT framework, without the need for Lorentz transformations and their epistemological implications about space-time, such as the relativity of simultaneity.

2. Electrodynamics of Pseudo-Instantaneous Action-at-a-Distance

The Electrodynamic Potential Energy

Let us assume that when two charges interact, they are exchanging action between them, and that this action is transmitted at a finite speed *c* (equal to the speed of light in vacuum). This means that, if the two charges were to move away from or towards each other at a radial velocity equal to *c*, they would cease to be 'connected', that is, they would cease to interact. We will also assume that the exchange of action involves back and forth transmission; the action parts from one charge, it travels to the other and it returns to the starting charge. We are not interested in speculating anything about the nature of the particular mechanism that makes the transmission of action possible; whether or not there is some kind of mediating agent (either "aether" or "field"), simply because it is our intention to avoid such a mediator appearing explicitly in our mathematical formulation. We will base the following deduction on the above premises and on basic concepts that do not contradict the Galilean transformations. We will adopt a line of thought similar to that followed by Wesley in [33].

The physical quantity known as *action* has *energy* × *time* dimensions. Thus, the rate of change of such an action must have *energy* dimensions. Suppose that what we usually call *electrodynamic potential energy* between two charges q_1 and q_2 measures precisely the rate of change of the action they exchange.

If we call *S* the action exchanged/transmitted between two charges q_1 and q_2 that are separated by a distance *r*, and at relative rest, we will assume that the rate of change of that action will coincide with the well-known *electrostatic Coulomb potential energy*, i.e.,

$$\frac{dS}{dt} \equiv \dot{S} \equiv U_C = \frac{q_1 q_2}{4\pi\varepsilon_0 r} \tag{5}$$

where ε_0, is the vacuum dielectric permittivity constant.

Since we are admitting that the velocity with which the action is transmitted is not infinite, this situation will change if the two charges are in relative motion. Eq. (5) will still represent a reasonable description when the relative velocity between the two charges ($dr/dt \equiv \dot{r}$) is much smaller than the speed with which the action is transmitted, *c*; that is, whenever $\dot{r} \ll c$. Otherwise, in order to know the action exchanged at instant *t*, we must consider the value of the action at an earlier instant (retarded), t_-, and at a later instant (advanced), t_+, and establish some kind of average, once the relevant calculations have been made.

Assuming that the action must make a round trip between q_1 and q_2, the instants and can be interpreted as follows. The action departs from q_1 at instant t_-, reaches q_2 at instant t, and returns to q_1 by reaching it at instant t_+. Or, vice versa. Otherwise, the description would be neither symmetric nor complete. In addition, the roles of q_1 and q_2 must be interchangeable.

The retarded and advanced instants can be defined as:

$$t_- = t - r/c \tag{6}$$

$$t_+ = t + r/c \tag{7}$$

where r represents the relative distance between the charges at instant t.

If we were bent on preserving the Galilean framework, we should assume that the action starts from q_1 at speed c measured with respect to this emitting charge. Then Eq. (6) could be considered a very good approximation as long as the relative radial acceleration is negligible. On the other hand, Eq. (7) could only be considered a reasonable approximation when $\dot{r} \ll c$. But, if we further admit the *secondary emission hypothesis* of Ritz [15] according to which, the charge q_2 returns the received action towards q_1 with speed c with respect to q_1 (the original emitting charge) and not with respect to q_2, then Eqs. (6) and (7) would both be very good approximations, provided that, as before, the radial acceleration could be neglected. (A detailed discussion of the scope and controversial aspects of the Ritz's emission hypothesis, of its repeated refutations many of which have subsequently been revised and invalidated, can be found in [20]).

Now, since we want our final description to be expressed only in terms of t, the current instant, and not of any other past or future instant, we will define the net rate of change of action at instant t, by some average value of the rates of change of the retarded action (i.e., evaluated at t_-, $S_- = S(t_-)$) and of the advanced action (i.e., evaluated at t_+, $S_+ = S(t_+)$).

Applying the chain rule for the derivation, we obtain that the rate of change of the retarded action may be expressed as:

$$\frac{dS_-}{dt} \approx \dot{S} \cdot \frac{dt_-}{dt} = \dot{S} \cdot \left(1 - \frac{\dot{r}}{c}\right) \tag{8}$$

and, similarly, the rate of change of the advanced action will be expressed as:

$$\frac{dS_+}{dt} \approx \dot{S} \cdot \frac{dt_+}{dt} = \dot{S} \cdot \left(1 + \frac{\dot{r}}{c}\right) \tag{9}$$

In view of Eqs. (8) and (9), it is obvious that we must discard the arithmetic mean as a way of calculating the net rate of change, since the terms in $\dot{r}/c$ would cancel out. A perfectly legitimate alternative is the *geometric mean*, according to which:

$$\left\langle \frac{dS}{dt} \right\rangle_{net} = \operatorname{sgn}(\dot{S}) \cdot \left[\frac{dS_-}{dt} \cdot \frac{dS_+}{dt} \right]^{\frac{1}{2}} \approx \dot{S} \cdot \left(1 - \frac{\dot{r}^2}{c^2}\right)^{\frac{1}{2}} \tag{10}$$

One advantage of this approach is that, since the final result shares the minus sign of the rate of change in the retarded action, we would thus be justifying why we only observe retarded and not advanced actions: the issue that already troubled Ritz and later Wheeler and Feynman.

Another advantage is that by considering both the rate of change of the advanced and the retarded action, we maintain the time symmetry exhibited by Maxwell's equations. Thus, Eq. (10) turns out to be insensitive to the direction of time (an aspect it shares with SRT [34]). This is so because the right-hand side of Eq. (10) can be written as $\dot{S} \cdot \left[(1 - \dot{r}/c) \cdot (1 + \dot{r}/c)\right]^{\frac{1}{2}}$ and, then, reversing the direction in which time flows, also involves reversing the +/– signs preceding the radial velocities

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(because the time reversal converts approaching velocities into receding velocities and vice versa), so that, on the whole, the final result remains the same.

It should be noted that the geometric mean has been repeatedly suggested as a method to obtain relativistic expressions from classical expressions referring to the transmitter or receiver, as in the case of the Doppler effect (e.g., [35]).

According to Eq. (5), the net rate of change of the action, which we will rename *electrodynamic potential energy*, *U*, will be expressed as:

$$U = U_C \cdot \left(1 - \frac{\dot{r}^2}{c^2}\right)^{\frac{1}{2}} \tag{11}$$

which is the same expression proposed by Phipps in [30], and recovered in [32], in both cases following very different arguments to those presented here. Note that, despite being formally a retarded potential energy of second order, the expression only contains variables evaluated at the current time; the same is true for the retarded Liénard-Wiechert potentials.

Naturally, if the relative velocity is zero, then Eq. (11) becomes Eq. (5); that is, the Coulomb expression of electrostatics. If it is satisfied that $\dot{r}/c \ll 1$, then, as we said, the expression proposed by Weber, Eq. (1), is recovered.

The Relational Kinetic Energy

In [32], starting from Coulomb's law and the principles of conservation of energy and mass-energy equivalence, the expression of Phipps' potential energy was derived together with the (fully relational) expression of the kinetic energy of two interacting charges q_1 and q_2. This kinetic energy was given by:

$$T = (\mu c^2 + E) \cdot \left[1 - \left(1 - \frac{\dot{r}^2}{c^2}\right)^{\frac{1}{2}}\right] \tag{12}$$

where *E* is the total energy of the system and μ is the reduced mass of the system, which is calculated from the individual masses of each charge, m_1 and m_2,

$$\mu = \frac{m_1 \cdot m_2}{m_1 + m_2} \tag{13}$$

Eq. (12), although different from the relativistic expression, also converges like the latter to the Newtonian expression of the kinetic energy, in the limit of low energies and low velocities, $\dot{r}/c \ll 1$.

For this isolated system, the principle of conservation of energy imposes that the sum of potential energy (Phipps') and kinetic energy (the new expression) must be constant:

$$\frac{q_1 q_2}{4\pi\varepsilon_0 r}\left(1 - \frac{\dot{r}^2}{c^2}\right)^{\frac{1}{2}} + (\mu c^2 + E) \cdot \left[1 - \left(1 - \frac{\dot{r}^2}{c^2}\right)^{\frac{1}{2}}\right] = E \tag{14}$$

which we can rewrite as

$$\frac{q_1 q_2}{4\pi\varepsilon_0 r}\left(1 - \frac{\dot{r}^2}{c^2}\right)^{\frac{1}{2}} + \mu c^2 \cdot \left[1 - \left(1 - \frac{\dot{r}^2}{c^2}\right)^{\frac{1}{2}}\right] = E \cdot \left(1 - \frac{\dot{r}^2}{c^2}\right)^{\frac{1}{2}} \tag{15}$$

from which, simplifying, we obtain

$$\frac{q_1 q_2}{4\pi\varepsilon_0 r} + \mu c^2 \cdot \left[\left(1 - \frac{\dot{r}^2}{c^2}\right)^{-\frac{1}{2}} - 1\right] = E \tag{16}$$

which is the same expression that could be obtained using Coulomb's law and the relativistic expression for the kinetic energy.

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The above is only an indication that the two approaches can produce formally overlapping results. However, Phipps' electrodynamics (and Weber's) are not without difficulties (as the interpretation of the effective potential, e.g.,).

The Wave Equation

According to the above deduction, whatever mechanism underlies the interaction, it will depend on the value of the outgoing action (retarded action, S_{-}) and the value of the return action (advanced action, S_{+}). Thus, if we were willing to assume that the transmission of the action requires a mediating agent that sustains it and that is altered or perturbed during the process, whatever the perturbation is could be described by a certain function $\Psi = \Psi(t_{-}, t_{+}; r, t)$, where the instants t_{-} and t_{+} are given by Eqs. (6) and (7), and *r* is the position at time *t*. These considerations suffice to deduce that the second partial derivatives of the function Ψ with respect to *r* and *t* satisfy the expression:

$$\frac{\partial^2 \Psi}{\partial r^2} = \frac{1}{c^2}\frac{\partial^2 \Psi}{\partial t^2} \tag{17}$$

which is the well-known "homogeneous one-dimensional wave equation".

To demonstrate this, taking into account Eqs. (6) and (7), we will first calculate two partial derivatives that will be used, repeatedly, later on (for simplicity, it will be introduced the notation $t_{\mp}$, to refer jointly to the retarded and advanced instants):

$$\frac{\partial t_{\mp}}{\partial t} = 1 \tag{18}$$

$$\frac{\partial t_{\mp}}{\partial r} = \mp \frac{1}{c} \tag{19}$$

Next, we calculate the first and second order time partial derivatives:

$$\frac{\partial \Psi}{\partial t} = \frac{\partial \Psi}{\partial t_{-}} \cdot \frac{\partial t_{-}}{\partial t} + \frac{\partial \Psi}{\partial t_{+}} \cdot \frac{\partial t_{+}}{\partial t} = \frac{\partial \Psi}{\partial t_{-}} + \frac{\partial \Psi}{\partial t_{+}} \tag{20}$$

$$\frac{\partial^2 \Psi}{\partial t^2} = \frac{\partial}{\partial t}\left(\frac{\partial \Psi}{\partial t_{-}} + \frac{\partial \Psi}{\partial t_{+}}\right) = \frac{\partial^2 \Psi}{\partial t_{-}^2} \cdot \frac{\partial t_{-}}{\partial t} + \frac{\partial^2 \Psi}{\partial t_{+}^2} \cdot \frac{\partial t_{+}}{\partial t} = \frac{\partial^2 \Psi}{\partial t_{-}^2} + \frac{\partial^2 \Psi}{\partial t_{+}^2} \tag{21}$$

On the other hand, the first and second order spatial partial derivatives will be:

$$\frac{\partial \Psi}{\partial r} = \frac{\partial \Psi}{\partial t_{-}} \cdot \frac{\partial t_{-}}{\partial r} + \frac{\partial \Psi}{\partial t_{+}} \cdot \frac{\partial t_{+}}{\partial r} = \frac{1}{c} \cdot \left(-\frac{\partial \Psi}{\partial t_{-}} + \frac{\partial \Psi}{\partial t_{+}}\right) \tag{22}$$

$$\frac{\partial^2 \Psi}{\partial r^2} = \frac{1}{c} \cdot \frac{\partial}{\partial r}\left(-\frac{\partial \Psi}{\partial t_{-}} + \frac{\partial \Psi}{\partial t_{+}}\right) = \frac{1}{c} \cdot \left(-\frac{\partial^2 \Psi}{\partial t_{-}^2} \cdot \frac{\partial t_{-}}{\partial r} + \frac{\partial^2 \Psi}{\partial t_{+}^2} \cdot \frac{\partial t_{+}}{\partial r}\right) = \frac{1}{c^2} \cdot \left(\frac{\partial^2 \Psi}{\partial t_{-}^2} + \frac{\partial^2 \Psi}{\partial t_{+}^2}\right) \tag{23}$$

Comparing the second order partial derivatives, given by Eq. (21) and Eq. (23), it is obvious that Eq. (17) is fulfilled, which is what we wanted to prove. The wave propagation velocity described is clearly *c*, in principle, referring to the 'medium' supporting the disturbance, although other interpretations are also possible.

3. Discussion

Pseudo-Instantaneous Action-at-a-Distance: a Second Opportunity

The derivation of the wave equation from pseudo-instantaneous AAAD assumptions shown in the previous section is an important result. Since the perturbation represented by the function Ψ is the transmitter of the electrodynamic action, the fact that this function satisfies the wave equation means that the interaction propagates at speed *c* through the perturbed 'medium' (the vacuum), with the properties we attribute to waves. Of course, other interpretations are possible, but that would force us to speculate about the nature of the 'medium', and that is not what we want.

One of the main reasons for dismissing (and forgetting) the electrodynamic line of research undertaken by Weber was his apparent inability to predict the propagation of electromagnetic waves in a vacuum, such as light itself. Since Maxwell's theory was able to do so, Maxwell's theory, along with the notion of *field* that underpinned it, was elevated, and eventually prevailed over theories of action-at-a-distance, which included Weber's theory.

The simple ideas shown in this paper, however, indicate that the line of electrodynamic research initiated by Weber was abandoned prematurely, and that the work programme of the AAAD can continue to provide solutions. Fukai, for example, showed in [36] that Weber's law is not only able to explain most of classical electrodynamics, but also to explain the propagation of radiation through a vacuum, for which only certain assumptions about the vacuum, which are common in quantum physics, need to be made. A similar line was followed by Li in [37], where, using Weber electrodynamics, and assuming a neutral but polarisable vacuum, he derived the wave equation, for waves propagating at speed *c*.

Another reason for the abandonment of the AAAD programme was the inconsistency with the relativity of simultaneity arising from SRT, which seemed to undermine the very foundations of this programme. But, is this insurmountable obstacle? Could the frameworks of electromagnetism (Maxwell-Lorentz and SRT) and electrodynamics (Phipps potential and appropriate mechanics) be equivalent?

The Omniscient Narrator's Point of View

Perhaps, the current electromagnetic framework (Maxwell-Lorentz equations and SRT) and the electrodynamic framework (Phipps' potential and renewed mechanics) are just different points of view leading to similar results. Something like telling a story through two different modes: a subjective one, told by any of the characters in the story, and an objective one, told by an omniscient narrator. The subjective version corresponds to the relativistic point of view, which democratises all reference frames by introducing local times and lengths, Lorentz transformations and advocates the locality of field theory. The objective version would correspond to the Weberian action-at-a-distance formulation, fully relational and assuming Galileo transformations and universal time. The two narratives could be equivalent (subject to stylistic considerations), but they would have to respect certain rules in order to do so. For example, in the story told by a character, he can guess what other characters are thinking, but cannot act with certainty unless news of their actions has had time to reach him (i.e., the actions of one object cannot influence another, if this implies that the information should travel at a speed greater than *c*). Similarly, although the omniscient narrator knows all the thoughts of all the characters, the narrative rules impose that he can never reveal the secrets of one character to another, i.e., the omniscient narrator is not just another character, but an impartial observer (so, again, the actions of one object cannot be motivated by those of another, if this implies that the information would have had to travel at a speed greater than *c*). Nor can there be aberrant time jumps in either narrative mode. If these simple rules are respected, in essence, the stories told in both cases will be very similar, although they may seem completely different to the reader.

The relativistic view, which began with the contributions of Poincaré, Lorentz and Larmor [38], was masterfully culminated by Einstein, and is considered valid by today's scientific community. The Weberian view has not been so lucky; it has not been developed to its ultimate consequences, nor is it considered valid by many researchers (quite the opposite, as it is very often considered erroneous and heretical). Einstein's theory replaced Galileo's transformations by Lorentz transformations, from which a whole set of new results were derived: length contraction, time dilation, relativity of simultaneity, among others. As difficult to accept as they initially seemed, they were eventually accepted based on the conviction that the SRT represented the only plausible explanation. This, however, is a merit that SRT never had. Einstein himself acknowledged [38] that the Lorentz rest aether theory was able to explain all the experimental results that SRT explained, but it relied on an

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unnecessary and superfluous hypothesis: the existence of an undetectable aether. However, the mere existence of an alternative explanation should spur us to continue to search for viable alternatives.

In this sense, Weber's electrodynamics is still of interest, even if the construct is not yet complete. To complete the Weberian vision, not only a modernisation is required to make it compatible with the existence of a limiting speed *c* (which is already achieved by Phipps electrodynamics and the associated new mechanics), but also to find all the rules by which the "omniscient narrator" is governed. Some of these rules may have been in Weberian theory from the beginning: the respect for Galilean transformations and its fully relational approach (Gauss's clear-sighted inheritance). But more must be missing. Perhaps, the effective potential energy, Eqs. (3) and (4), is part of these rules; the way in which action-at-a-distance is regulated so that the results can be the same as those obtained with the relativistic view.

It may be possible to use whatever approach we want (subjective or objective, fields or AAAD), as if it were a matter of style, as long as we get the right equations that have been tested by experiments. However, it should be borne in mind that experiments can only test the suitability of new equations for modelling physical phenomena, not the various ideas used to derive the equations. If our conception alters previous formulae or invents new ones, we can test the suitability of the resulting equations for modelling physical phenomena, but not the concepts used to derive these formulas. This does not imply that all models of classical electromagnetism are the same. Some may be easier to understand, others may be cheaper in terms of the number of assumptions used, or others may be more useful for modelling a specific phenomenon. The only way to be sure is to find the limits or domain of validity of each of the different models.

In this connection, it is worth remembering that *Ampère's law* was meticulously tested experimentally by Ampère and has never been disproved to this day. Maxwell praised it, saying of it in his *Treatise* [10] that it "was the cardinal equation of electrodynamics". However, this law does not appear in today's textbooks, pushed into a corner by *Lorentz's law*, which, however, was not tested experimentally in as many situations as Ampère's law was, and with which it is well known that it does not coincide except when considering the interaction of closed circuits. Is Ampère's law wrong? Weber's law (or any of its modernisations) is perfectly compatible with Ampère's law, but not with Lorentz's law. Thus, when comparing the predictions of the current Maxwell-Lorentz-SRT framework with those of an eventual, definitive and complete Weberian electrodynamics (such as Phipps, perhaps), it should be kept in mind that any discrepancies that arise should not automatically invalidate the new predictions, but perhaps indicate that there are fully accepted models that have been pushed beyond their verified range of validity.

5. Conclusions

In this work it has been proved that it is possible to derive the Phipps electrodynamic potential energy expression between two charges, considering the interaction process as the result of an exchange of action (in a round trip) between the two charges. It has also been proved that, under the same assumptions and adding the additional hypothesis that the interaction is supported by some kind of mediating agent (either aether or field) which is perturbed during the transmission of action, the perturbation of this medium obeys the wave equation, thus overcoming one of the main drawbacks of Weberian electrodynamics. It can be concluded, therefore, that the Weberian electrodynamics line of research was abandoned prematurely, and that the programme of work on action-at-a-distance, suitably renewed, can continue to provide solutions.

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