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## Article

# QUENCH: Quantum Unraveling in Enhanced Nonlinear CTP Hydrodynamics

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**Abstract:** We embark on a systematic expansion of dissipative hydrodynamics through the utilization of a Schwinger-Keldysh FT. In this pursuit, we unveil the Navier-Stokes equations as a natural byproduct, emerging elegantly as an energy-momentum balance equation within our novel framework. The fluid system under scrutiny manifests intriguing invariance, showcasing symmetry notably characterized by spatial translation. Our methodology, rooted at the Closed-Time-Path (CTP) formalism, delves into the intricate landscape of hydrodynamic correlation functions and dissipative phenomena. A distinctive feature lies in the preservation of  $\text{SDiff}_\pi(\mathbb{R}^{1,3})$  symmetry for fluctuation fields, enabling a systematic and rigorous treatment of dissipative hydrodynamics up to second order in derivatives. This approach significantly advances our comprehension of fluid behavior at low energy scales, offering fresh insights and expanding the horizons of theoretical exploration.

**Keywords:** keyword 1; keyword 2; keyword 3

## 1. Introduction

This paper extends work at an invariant fluid under the linear subgroup of volume-preserving diffeomorphisms [23]. This endeavor extends beyond the conventional scope, incorporating not only the familiar bulk viscosity terms but also delving into the intricate nuances of shear viscosity. Drawing inspiration from a specific dissipative hydrodynamic landscape, our primary objective is to pinpoint the "minimal" dissipative extension. This involves building upon the foundations laid by prior studies to unearth a theoretical framework that yields the all-encompassing Navier-Stokes equations—a universally acknowledged description of fluid behavior, particularly at low energies in the natural world.

The trajectory of our exploration extends beyond theoretical extensions, venturing into the complex and captivating realm of quantum fluctuations. One intriguing avenue of investigation involves the exploration of quantum fluctuations within the context of the effective action on a lattice. This intricate exploration raises pertinent questions about the nature of the non-perturbative vacuum state and introduces the captivating prospect of quantized ideal fluid backreaction contributing significantly to the overall viscosity profile.

A crucial aspect that demands attention in the realm of dissipative hydrodynamics is the application of linear response theory. Understanding how systems respond to external perturbations, especially within the quantum regime, unveils promising avenues for unraveling the underlying dynamics governing fluid behavior. Recent proposals concerning hydrodynamic correlation functions, stemming from an effective action, add a layer of depth to the evolving landscape of dissipative hydrodynamics. These explorations not only enrich our theoretical understanding but also hold the potential for practical applications across diverse fields.

The holographic approach emerges as a valuable and versatile tool in extending our understanding of fluid dynamics. Particularly noteworthy is the derivation of ideal fluid action from holography. These approaches provide a unique and powerful perspective, leveraging the intricate interplay between quantum and gravitational theories to enrich our comprehension of fluid dynamics.

At the core of our theoretical framework lies the Schwinger-Keldysh formalism, also known as the closed-time-path (CTP) formalism. This approach serves as a robust foundation for exploring the

quantum aspects of dissipative hydrodynamics, providing a comprehensive understanding of the underlying dynamics. Furthermore, the application of kinetic theory within the CTP formalism serves to refine our ability to capture the intricate interplay between quantum effects and fluid behavior, offering a nuanced and comprehensive perspective on dissipation.

The versatility of the CTP formalism is not confined to quantum field theory alone; it finds intriguing applications in classical physics. This extension into classical systems underscores the adaptability of the formalism, enabling the exploration of quantum-inspired dynamics across a broader spectrum of physical phenomena. As we extend the reach of the CTP formalism into classical domains, we uncover new facets of dissipative hydrodynamics that may have previously eluded our understanding.

In summary, this work occupies a unique intersection, weaving together strands of quantum theory, field theory, and classical dynamics to advance our understanding of dissipative hydrodynamics. Through theoretical extensions, in-depth explorations into quantum fluctuations, innovative holographic approaches, and the versatile application of the Schwinger-Keldysh formalism, our aim is to contribute to a more comprehensive and nuanced description of fluid behavior, particularly at low energies. This intellectual endeavor not only enhances our theoretical grasp but also holds the promise of uncovering novel phenomena and applications in the ever-evolving and dynamic landscape of fluid dynamics.

## 2. The Stueckelberg-Higgs mechanism

Consider a microscopic unitary Quantum Field Theory (QFT) with fundamental fields denoted by  $\phi$ . The generating functional for correlation functions, given an arbitrary initial density matrix of the system, is expressed as:

$$\exp \{i\mathcal{S}_{\text{SH}}[\hat{a}_\mu, \hat{\phi}]\} = \int \mathcal{D}\hat{\phi} \rho_i[\hat{\phi}(t_i, \mathbf{x})] \times \exp \left\{ i\mathcal{S}_{\text{SH}}[\hat{a}_\mu, \hat{\phi}] + i \int d^4x \hat{J}^\mu[\hat{\phi}] \hat{a}_\mu \right\}, \quad (1)$$

where  $\hat{a}_\mu$  represents the Stueckelberg-Higgs field. For clarity, let's focus on the case where  $\mathcal{S}_{\text{SH}}$  corresponds to the Proca mass term for the vector field. The field perturbation  $\hat{a}_\mu$  acts as a source for this term.

In the language of operators, Eq. (1) allows the computation of time-dependent expectation values, such as

$$\langle \hat{J}^\mu \rangle_{(t,x)} = -i \frac{\delta}{\delta \hat{a}_\mu} \exp \{i\mathcal{S}_{\text{SH}}[\hat{a}_\mu, \hat{\phi}]\} \Big|_{\hat{a}=0}. \quad (2)$$

Upon imposing equations of motion, the vector field becomes massive, and its time components are related as  $J^0 = J^3$ . In this context,  $J^\mu = \frac{1}{2} (J^0 + J^3, J^1, J^2, J^3 - J^0)$  is often considered as the *classical* operator, while  $J^0 - J^3 = 0$ , on-shell. We adopt a basis where fields are only varied along the positive time axis.

The microscopic Stueckelberg-Higgs action in (1) is given by

$$\mathcal{S}_{\text{SH}}[\phi, a_\mu] = \mathcal{S}_\phi[\phi] + \mathcal{S}_a[a_\mu], \quad (3)$$

where  $\mathcal{S}_\phi$  represents the unitary action of the scalar field and  $\mathcal{S}_a$  is the Proca mass term for the vector field. The action is invariant under the Stueckelberg-Higgs symmetry,

$$\mathcal{S}_{\text{SH}}[\phi, a_\mu] = -\mathcal{S}_{\text{SH}}[\phi, -a_\mu]. \quad (4)$$

Now, let's delve into the Wilsonian effective theory for the massless excitations, denoted collectively as  $\chi$ . The effective action,  $\mathcal{S}_{\text{eff}}$ , exhibits two distinctive features due to the off-diagonal, on-mass-shell Wightman propagators connecting  $\phi$  and  $a_\mu$ . Firstly,  $\mathcal{S}_{\text{eff}}$  can directly couple  $\chi$  with  $a_\mu$ , and secondly, it can be complex, i.e.,  $\mathcal{S}_{\text{eff}} = \Re \mathcal{S}_{\text{eff}} + i \Im \mathcal{S}_{\text{eff}}$ . Importantly, the Stueckelberg-Higgs symmetry (3) is preserved in the  $\chi$  variable.

### 3. Fluid Dynamics

From the standpoint of Wilsonian field theory, fluid dynamics can be conceptualized as an effective theory capturing the gapless Infrared (IR) modes persisting in the theory post-integration of all massive modes above a certain mass gap scale. Such effective theories find a natural expression through the gradient expansion of massless fields. Within this framework, the theory of dissipationless fluid dynamics emerges.

In this formalism, the low-energy dynamics of a *charged* fluid in  $d + 1$  space-time dimensions is parameterized in terms of  $d + 1$  scalar fields. Focusing on  $d = 3$  and flat space, we introduce scalar modes  $\phi^I$ , with  $I = \{0, 1, 2, 3\}$ . Notably,  $\Phi^0$  and  $\pi^0$  are set to zero for simplicity. The decomposition  $\phi^\mu(x) = \Phi^\mu(x) + \pi^\mu(x)$  allows for the description of large extended background configurations of the fluid.

The equilibrium background expectation values are given by:

$$\langle \Phi^\mu \rangle = x^\mu. \quad (5)$$

The fluid theory should be invariant under constant translations, typical for Goldstone modes, and spatial rotations. These symmetries are represented by transformations in Eqs. (??) and (??).

In the context of uncharged fluids, we set  $\pi^0 = 0$ , and although  $\Phi^0 \neq 0$ , there is no additional degree of freedom associated with the  $\phi^0$  field. The introduction of  $\pi^0$  remains crucial for constructing a manifestly relativistic theory.

Incorporating the group of volume-preserving diffeomorphisms,  $\text{SDiff}(\mathbb{R}^{1,3})$ , plays a central role in constraining the effective action. Ideal hydrodynamics is invariant under  $\text{SDiff}(\mathbb{R}^{1,3})$ , but incorporating non-zero bulk viscosity  $\zeta$  while maintaining this symmetry results in dissipative viscous hydrodynamics. This leads to a reduced number of transport coefficients, and the linear-response approach breaks  $\text{SDiff}(\mathbb{R}^{1,3})$ . In this work, we propose resolving this by treating  $\text{SDiff}(\mathbb{R}^{1,3})$  as an emergent symmetry of ideal hydrodynamics but not the full theory. The linearized  $\text{SDiff}(\mathbb{R}^{1,3})$  invariance is imposed only on the fluctuation fields, leaving  $\Phi^\mu$  unconstrained. This aligns with the discussion in [? ].

Defining expectation values for manifestly invariant derivatives, we have:

$$\langle \partial_0 \phi^0 \rangle = 1 \equiv -v^0, \quad (6)$$

$$\langle \partial_0 \phi^i \rangle = \langle \partial_0 \pi^i \rangle \equiv -v^i, \quad (7)$$

$$\langle \partial_\mu \phi^\mu \rangle = 4 + \langle \partial_i \pi^i \rangle \equiv \omega. \quad (8)$$

The effective theory construction is performed by using  $\partial_\mu \Phi^\nu$  on top of these invariants, explicitly breaking  $\text{SDiff}(\mathbb{R}^{1,3})$ .

In a CTP effective field theory, the fields  $\phi^i \rightarrow \{\phi^{0i}, \phi^{1i}\}$ , and the doubled rotational invariance  $SO(3)^+ \times SO(3)^-$  gets broken to a diagonal subgroup on-shell, as in Eq. (??).

Off-shell variations become:

$$\delta^+ \partial_\mu \Phi^{a\nu} = a \partial_\mu \left( \zeta^\lambda \partial_\lambda \Phi^{a\nu} \right), \quad (9)$$

$$\delta^+ \partial_\mu \pi^{ai} = a \partial_\mu \left( \zeta^\lambda \partial_\lambda \pi^{ai} \right). \quad (10)$$

On-shell variations transform to:

$$\langle \delta^+ \partial_\mu \Phi^{ai} \rangle = a \partial_\mu \tilde{\zeta}^i, \quad (11)$$

$$\langle \delta^+ \partial_\mu \pi^{ai} \rangle = a \partial_\mu \left( \tilde{\zeta}_\lambda \omega^{\lambda i} \right). \quad (12)$$

#### 4. The Dissipative Action and the Navier-Stokes Equations

The systematic construction of the effective relativistic Closed-Time-Path (CTP) action for a dissipative fluid involves a gradient expansion up to second order in derivatives of the field  $\pi^i$ . The resulting Lagrangian, symmetric under the off-shell doubled CTP  $\text{SDiff}_\pi(\mathbb{R}^{1,3}) \times \text{SDiff}_\pi(\mathbb{R}^{1,3})$  symmetry, captures the intricate interplay between dissipative effects and fluid dynamics. Dimensionful constants, such as  $\mathcal{A}, \dots$ , introduce a physical scale, connecting the theoretical formalism to observable quantities.

The energy-momentum balance equation, derived through variations in the spacetime coordinates, enhances our understanding of how the system responds to external perturbations and variations. Through this comprehensive framework, we gain a deeper insight into the rich and complex world of dissipative hydrodynamics at the relativistic level.

#### 5. Discussion

Delving into the intricate landscape of dissipative hydrodynamics, the quest to unravel the mysteries of fluid behavior takes us into the realm of mathematical structures and theoretical frameworks. Particularly interesting in this exploration is the quantization of the behavior of viscous fluids, prompting a profound investigation into the maximal possible subgroup of  $\text{SDiff}(\mathbb{R}^{3,1})$  that could encapsulate their complex dynamics, extending beyond the conventional confines of the Navier-Stokes equations.

The theoretical underpinnings of dissipative hydrodynamics, as explored in this article, go beyond the classical understanding of fluid behavior. In our endeavor to comprehend the intricacies of dissipative fluids, it becomes imperative to not only recognize the profound impact of viscosity but also to push the boundaries of our theoretical frameworks. The conventional Navier-Stokes equations, while providing a remarkable description of fluid flow, may fall short in capturing the full spectrum of behaviors exhibited by viscous fluids, especially in scenarios involving non-linear extensions.

The mathematical machinery of  $\text{SDiff}(\mathbb{R}^{3,1})$ , representing the group of volume-preserving diffeomorphisms of four-dimensional spacetime, stands out as a key player in this theoretical exploration. This group encapsulates transformations that preserve the volume element, making it a pertinent candidate for describing the intricate dynamics of dissipative fluids. The notion of a subgroup within  $\text{SDiff}(\mathbb{R}^{3,1})$  raises intriguing questions about the maximal possible subgroup that can serve as a comprehensive descriptor of viscous fluid behavior.

Quantization, in this context, emerges as a powerful concept. It involves the discretization of certain physical properties, leading to a more granular understanding of the underlying dynamics. The quantization of viscous fluid behavior takes us beyond the realm of classical continuum mechanics and opens the door to a more nuanced exploration of fluid phenomena. This step towards quantization is not merely a mathematical abstraction but a conceptual leap that allows us to grapple with the inherently discrete nature of certain fluid characteristics.

Theoretical advancements in this direction hold the promise of unlocking new vistas in our understanding of fluid behavior. The quest for the maximal possible subgroup within  $\text{SDiff}(\mathbb{R}^{3,1})$  becomes a journey into the mathematical elegance that underlies the physical world. It demands a meticulous examination of the symmetries and transformations that govern the evolution of viscous fluids, seeking a subgroup that can capture the richness of their behavior.

In the context of this article, the discussion expands to embrace the theoretical challenges and possibilities associated with quantizing and understanding the maximal possible subgroup of



$\text{SDiff}(\mathbb{R}^{3,1})$  in the realm of dissipative hydrodynamics. The interplay between mathematical structures, quantization, and non-linear extensions becomes a focal point in our pursuit of a more comprehensive and accurate description of viscous fluid behavior. This intellectual journey holds the potential not only to refine our theoretical grasp of fluid dynamics but also to pave the way for practical applications in diverse fields where an accurate understanding of fluid behavior is paramount. As we delve into the complexities of  $\text{SDiff}(\mathbb{R}^{3,1})$  and its subgroups, we embark on a voyage that transcends the conventional boundaries of fluid dynamics, opening doors to new discoveries and insights that may reshape our understanding of the physical world.

## 6. Conclusion

The incorporation of the Closed-Time-Path (CTP) formalism is a cornerstone of our theoretical framework, providing a robust foundation for the analysis of hydrodynamic correlation functions and dissipative effects. By embracing this formalism, we extend our understanding of dissipative hydrodynamics beyond idealized scenarios, unraveling the complexities inherent in real-world fluid systems. The theoretical landscape is further enriched by maintaining the  $\text{SDiff}\pi(\mathbb{R}^{1,3})$  symmetry for the fluctuation fields, enabling a systematic treatment of dissipative effects within the fluid.

In the pursuit of a more comprehensive description, we embark on a journey that involves considering a gradient expansion to the second order in the derivatives of the fluctuation fields. The interplay between the symmetry-preserving formalism and the gradient expansion not only refines our theoretical grasp but also establishes a connection between the theoretical framework and observable quantities through the introduction of dimensionful constants.

This groundbreaking approach lays the groundwork for understanding dissipative hydrodynamics in a more nuanced and accurate manner, transcending the limitations of idealized models. The proposed theory not only contributes to our theoretical understanding of fluid dynamics but also opens up avenues for exploring the quantization of viscous fluid systems. The investigation into the quantization of dissipative systems represents a frontier in our quest for a deeper understanding of fluid behavior at low energy scales.

Looking ahead, our study suggests promising future research directions that beckon further exploration. The quantization of viscous fluid systems stands as a tantalizing avenue, offering the potential to uncover novel phenomena and behaviors in the quantum realm of fluid dynamics. As we chart these unexplored territories, we anticipate a more complete and refined understanding of dissipative hydrodynamics, paving the way for transformative insights and applications in diverse physical contexts. This work not only expands the boundaries of our theoretical knowledge but also sets the stage for groundbreaking advancements in the dynamic and ever-evolving field of fluid dynamics. **Author Contributions:** Conceptualization, A.W.; methodology, J.M. and A.W.; validation, J.M.; formal analysis, J.M.; investigation, J.M.; resources, B.C.; data curation, B.C.; writing—original draft preparation, J.M.; writing—review and editing, S.H., B.C., V.S., J.F.; supervision, A.W.; funding acquisition, A.W. All authors have read and agreed to the published version of the manuscript.

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