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Article

Electric Vehicle Routing Problem with States of Charging Stations

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Abstract: This paper proposes an electric vehicle routing problem with considering states of charging stations and suggests solution strategies. Charging of electric vehicles is a main issue in the field of electric vehicle routing problem. There are many studies to find locations of charging stations, recharging functions for batteries of vehicles, and so on. However, states of charging stations significantly affect the routes of electric vehicles, which is not much explored. The states may include open or close of charging stations, occupied or empty of charging slots, and so on. This paper investigates how the states of charging stations are estimated and routing strategies are determined. We formulate a mixed integer programming model and suggest how to solve the problem with exact method. Numerical examples provide the optimal routing strategies of electric vehicles in the changing environments of the states of charging stations.

Keywords: electric vehicle routing problem; states of charging stations; location routing problem; mathematical model; optimization

1. Introduction

The fast growth of air pollution and global climate change has driven to consider sustainability in various sectors including energy, manufacturing, logistics, and so on. A country has been forced to reduce the use of fossil fuel as the main cause of global warming. Transportation has accounted for more than 23% of green house gas emission causing air pollution (Xiao et al. 2021 [1]). Accordingly, transportation has been changed to encourage the use of electric vehicles rather than internal combustion vehicles. The study of the vehicle routing problem has been extended to the study of the electric vehicle routing problem. The electric vehicle routing problem has become an important issue in the field of sustainability.

Many studies have conducted for the electric vehicle routing problem. Comparing to conventional vehicle routing problem, the electric vehicle routing problem has additional issues while both problems aim to find the optimal route out of alternatives. The issues of the electric vehicle routing problem include finding optimal location of charging stations, estimating the recharging functions of the battery and energy consumptions, and so on. Most studies are related with charging the battery of the electric vehicle. Without charging issue, the problem is the same as the conventional vehicle routing problem. Location problem is to find the optimal locations of charging stations minimizing the total travel costs. With the vehicle routing, the problem becomes the location routing problem. For the charging function, the problem is to find the change or pattern of charging level by time. The function is assumed as a linear or nonlinear function. Since the charging time is long, there are some studies of partial charging instead of full charging. Research questions are where the charging stations are in place and how to charge the battery.

On the other hand, there are lack of studies for the states of the charging stations. When an electric vehicle arrives at a charging station and all charging slots are occupied, the electric vehicle may have to wait for a long time. If the charging station is closed, the electric vehicle should go to another charging station. The state of charging station can be defined as the availability of the charging station. If the charging station is not available at the moment of the arrival of the vehicle,

the situation significantly affects its route. Thus, this paper considers the states of charging stations for the electric vehicle routing problem. Each charging station has its own state. The states for charging stations may have distributions or patterns. Once the patterns of the states are found, one can find the optimal location routes of electric vehicles.

One of concern of electric vehicle routing problem is that the charging time is too long rather than the fossil fuel. Due to the long charging time, vehicles occupy the charging slots for a long time so that there may be lack of available slots. If the stations are crowded, the availability does matter. For example, assuming a vehicle arrives at a charging station and all charging slots are occupied. If the earliest available slot will be in thirty minutes, the vehicle has to wait at least thirty minutes. The states of charging station are not deterministic, but random or stochastic. Each station has different uncertainty such as open/close, occupied/empty, normal/out of order, and so on. These uncertainties present the availability of the charging station. Assuming multiple deliveries, each vehicle travels twice in a day, needs to recharge one time either in the first time period or the second time period. If a vehicle recharges the battery in the first time period, the vehicle can travel without recharge in the second time period. On the other hand, the vehicle wants to recharge in the second time period, the vehicle can travel without recharging in the first time period. In this case, the vehicle has to decide the time period to recharge to minimize the total travel costs in a day. Depending on the states of charging stations, we can decide when to recharge the battery for each vehicle. Thus, the patterns of states of charging stations can be used as input parameters in the electric vehicle routing problem.

Contributions of this paper are as follows: This paper suggests an electric vehicle routing problem with states of charging stations and an optimization model. Depending on the states of charging stations, the optimal routes may be different. We also propose the strategy to achieve the optimal routing plans for vehicles. Vehicles have to decide when and where to go at the next time period to minimize the total travel costs. Lastly, this paper provides some numerical experiments to show the viability of the proposed solution approach.

In the following sections, this paper presents the literature about this issue in Section 2. Section 3 describes the problem and a mathematical model for electric vehicle routing problem. Section 4 proposes solution methods for the electric vehicle routing problem with states of charging stations. Numerical examples and results are presented in Section 5. Finally, this paper summarizes the paper and discusses future work.

2. Literature Review

Electric vehicle routing problem has become an interesting issue in a variety of different fields such as transportation, energy, sustainability, and so on. Researchers have focused on the charging related issues or solution approaches. This section presents the related works that related with the electric vehicle routing problem rather than the conventional vehicle routing problem.

Various problems for the electric vehicle routing problem have been considered. Chakraborty et al. (2021, [2]) have dealt with the electric vehicle routing problem to find optimal charging schedule and routes. Schneider et al. (2015, [3]) have introduced time windows constraints in the electric vehicle routing problem. Keskin et al. (2019, [4]) have considered the electric vehicle routing problem with time windows and explored the queues at the charging stations. Hiermann et al. (2016, [5]) have also explored the electric vehicle routing problem with different vehicle size. Charging the vehicle has been explored. Montoya et al. (2017, [6]) have assumed a nonlinear function for the charging rate and suggested proper charging behavior. Froger et al. (2019, [7]) have used a nonlinear function and proposed arc based and path based tracking of time approach rather than node based model to avoid replicating the charging station nodes. If vehicles get fully charged during the travel, the total traveling time will be significantly increased. Instead of full charging, partial charging has been proposed in many studies. Bac & Erdem (2021, [8]) have proposed the electric vehicle routing problem with multiple depots and heterogeneous fleets. Basso et al. (2021, [9]) have also considered partial charging. In their research, the problem has included the uncertainties such as road condition or energy consumption. Hybrid electric vehicle has been considered. In their research, Kasani et al. (2021, [10]) have investigated electric vehicle and hybrid electric vehicle routing and scheduling

problem in private and public sectors. Ma et al. (2021, [11]) have suggested the shared autonomous electric vehicle routing problem with battery swapping and proposed a speed optimization model on the travel arc. Pelletier et al. (2019, [12]) have explored the uncertainty of energy consumption during the delivery tour. Al-dal'ain & Celebi (2021, [13]) have investigated the problem of vehicle compositions of electric and conventional vehicles to minimize the total operational costs in urban area. Brady & O'Mahony (2016, [14]) have tried to find the patterns of the travel of vehicle and charging behavior. They have used dataset of vehicles traveling to resolve the uncertainty.

For the modeling of the electric vehicle routing problem, the studies have adopted optimization models. Mixed integer programming has been mostly used to formulate the problems as many studies have adopted the mixed integer programming model for the vehicle routing problem. The objective function is similar to that of conventional vehicle routing problem. Schneider et al. (2014, [15]) have formulated their model to minimize the total travel distance. Hiermann et al. (2016, [5]) have tried to minimize the total distance and vehicle costs. Ma et al. (2021, [11]) have formulated the objective function to minimize the total travel distance, travel time, and energy consumption. Froger et al. (2019, [7]) and Montoya et al. (2017, [6]) have constructed their objective function to minimize the total driving and charging time. Bac & Erdem (2021, [8]) have tried to minimize the total travel time, time windows deviations, unscheduled jobs, and overtime. Al-dal'ain & Celebi (2021, [13]) have constructed the objective function to minimize the total costs of vehicles: purchasing and salvage value, operational cost, maintenance cost, CO₂ emission cost, and fuel cost. Variants of the mixed integer programming has been used to the model. Pelletier et al. (2019, [12]) have formulated a robust mixed integer programming model. Chakraborty et al. (2021, [2]) have proposed a multiobjective mixed integer programming model for minimizing energy consumption and travel time. Nolz et al. (2022, [16]) have developed their model with mixed integer quadratic programming model as a nonlinear programming model. Data analytical model has also been used for the problem. Basso et al. (2021, [9]) have used the Bayesian regression for the estimation of energy consumption. Brady & O'Mahony (2016, [14]) have suggested the model with Monte Carlo simulation and copula function. Alizadeh et al. (2014, [17]) have used a network model with a single shortest path problem on an extended transportation graph with virtual nodes. Lu & Wang (2019, [18]) have developed a dynamic programming model for the capacitated electric vehicle routing problem. Gan et al. (2013, [19]) have adopted an optimal control problem to find the optimal schedule of electric vehicles.

For the solution method, A variety of different approaches have been used to find the optimal solutions. Like the conventional vehicle routing problem, various heuristic methods have been used. Schneider et al. (2014, [15]) have adopted a variable neighborhood search with tabu search method. In their approach, tabu search is plugged in the variable neighborhood search algorithm. Bac & Erdem (2021, [8]) have used variable neighborhood search algorithm and variable neighborhood descent algorithm. The variable neighborhood search was used to handle the dynamic problem and the variable neighborhood descent algorithm was used to cover deterministic problem. Adaptive large neighborhood search algorithm has also been used to solve the problem. Nolz et al. (2022, [16]) have proposed a template based adaptive large neighborhood search algorithm with a two phase solution scheme. Ma et al. (2021, [11]) have used an adaptive large neighborhood search method with a speed optimization algorithm as a subroutine. Hiermann et al. (2016, [5]) have solve the problem using branch and price combined with adaptive large neighborhood search algorithm. Kessler & Bogenberger (2019, [20]) have estimated the energy consumption model for electric vehicles with statistical analysis. Montoya et al. (2017, [6]) have adopted a hybrid meta heuristic combining iterated local search and heuristic concentration. Basso et al. (2021, [9]) have suggested a two stages approach: first one is to find the paths between all nodes to be visited, the second is to select the best order of the tour to minimize energy consumption. Exact algorithm has also been used. Pelletier et al. (2019, [12]) have used an exact method with reformulation and two phase heuristic based on large neighborhood search for large instances. Keskin et al. (2019, [4]) have solved the problem using the exact algorithm for small instances and the adaptive large neighborhood search for large instances. Al-dal'ain & Celebi (2021, [13]) have also used the exact algorithm with software GAMS. Chakraborty

et al. (2021, [2]) have proposed a multiobjective heuristic algorithm that is a graph based centralized scheduling strategy.

Related works listed above show that the electric vehicle problem has been explored in variety of ways. Finding optimal locations of charging stations have been studied with the vehicle routing problem. The issues of charging have also been suggested. They are charging functions, charging behaviors, partial charging, and so on. Most studies have formulated the problem with mixed integer programming. Various methods have been proposed to solve the electric vehicle problem. However, although there have been many studies for the electric vehicle routing problems, there are lack of research for the state of charging stations while their states can affect the optimal routes. Thus, this paper can be a trigger to tackle this issue.

3. Electric Vehicle Routing Problem

3.1. Problem Statement

Vehicle routing problem is to find the optimal routes for vehicles that have to travel customer sites geographically dispersed. Vehicle routing problem is a problem combined two problems: traveling salesman problem and bin packing problem. Traveling sales man problem is known as NP-hard problem. Accordingly, the vehicle routing problem is also NP-hard problem. Electric vehicle routing problem has additional issues of charging the battery.

Figure 1 presents the electric vehicle routing problem. There are charging stations that vehicles can visit during their delivery tours. The problem has to consider the charging the battery for the electric vehicle while the conventional vehicle routing problem does not need to consider. In general, there are multiple charging stations so that the vehicle can choose one of them. Selected charging stations can affect the optimal routes for vehicles and total travel costs or distances. Thus, the vehicle has to choose the charging station to minimize the total travel costs.

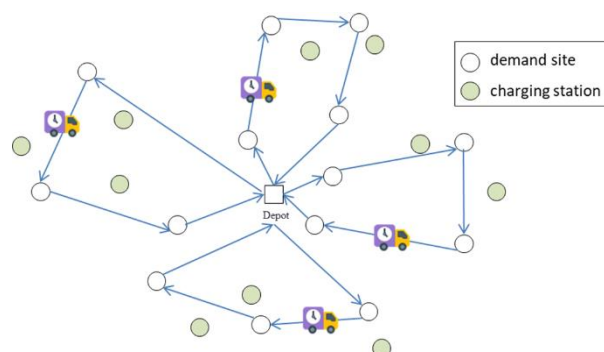


Figure 1. Electric vehicle routing problem.

Assuming many customer sites to deliver, each vehicle has to do the delivery tour twice in a day. We also assume that each vehicle should charge in either the first route or the second one. Thus, each vehicle has to decide when it charges.

Figure 2 denotes the vehicles servicing twice in a day. At the beginning, the customer sites are determined for delivery in the day and they are divided into two groups: the first time period group to visit and the second one. If a vehicle decides to charge in the first time period, the vehicle needs no charging in the second time period. On the other hand, if a vehicle does not charge in the first time period, the vehicle has to charge in the second time period. The problem is to find the optimal routes for two time periods and also when the vehicles charge to minimize the total travel cost for a whole day.

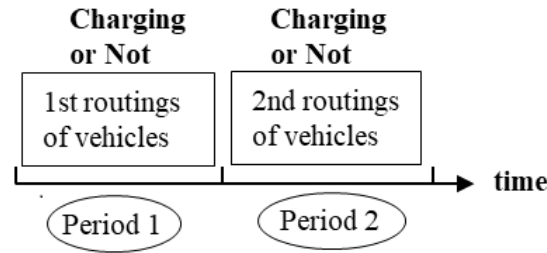


Figure 2. Twice delivery tours in a day.

3.2. Mathematical Model

We consider an electric vehicle routing problem with time windows. There are one depot, multiple vehicles, and multiple customer sites. The model for the problem is as follows.

Sets

$V = \{0, 1, 2, \dots, n, n+1, \dots, n+p\}$: (0: depot node, $1 \sim n+p$: nodes)

$F = \{n+1, \dots, n+p\}$: Charging stations

F' : Dummy charging stations

$N = V \setminus \{0\} \cup F$: Customer sites

$A = \{(i, j) | i, j \in V, i \neq j\}$: Arcs

$M = \{1, \dots, m\}$: Vehicles

$V^+(i) = \{j \in V | (i, j) \in A\}$: Successor nodes of node i

$V^-(i) = \{j \in V | (j, i) \in A\}$: Predecessor nodes of node i

Parameters

$[a_i, b_i]$: Time window for node i

s_i : Service time for node i

q_i : Demand for node i

w_i : Difficulty of using the charging station i

$d_{i,j}$: Travel time from node i to node j

τ : Charging time per unit energy

δ : Energy consumption per unit distance

K : Capacity of a vehicle

Q : Capacity of energy(electricity) for a vehicle

L : Large constant number

Variables

$x_{i,j}^m$: Binary variable, 1 if arc $(i, j) \in A$ belongs to the optimal routes by vehicle m , 0 otherwise

t_i^m : Arrival time of vehicle m at node i (beginning of service)

y_i^m : Energy level of vehicle m at node i

z_i : Binary variable, 1 if charging station i is used, 0 otherwise

(EVRP) $\text{Min } \sum_{m \in M} \sum_{(i,j) \in A} d_{i,j} x_{i,j}^m + \sum_{i \in F'} w_i z_i$ (1)

s. t. $\sum_{m \in M} \sum_{j \in V} x_{i,j}^m = 1, \forall i \in N$, (2)

$\sum_{m \in M} \sum_{j \in V} x_{i,j}^m \leq 1, \forall i \in F'$, (3)

$\sum_{j \in V^+(0)} x_{0,j}^m = 1, \forall m \in M$, (4)

$\sum_{i \in V^-(h)} x_{j,i}^m - \sum_{i \in V^+(h)} x_{i,j}^m = 0, \forall j \in N, m \in M$, (5)

$\sum_{i \in V^-(0)} x_{i,0}^m = 1, \forall m \in M$, (6)

$\sum_{i \in N} q_i \sum_{j \in V} x_{i,j}^m \leq K, \forall m \in M$, (7)

$t_i^m + (s_i + d_{i,j})x_{i,j}^m - t_j^m \leq (1 - x_{i,j}^m)L, \forall (i, j) \in A, i, j \in N, m \in M$, (8)

$t_i^m + d_{i,j}x_{i,j}^m + \tau(Q - y_i^m) - t_j^m \leq (L + \tau Q)(1 - x_{i,j}^m), \forall i \in F', j \in N, m \in M$, (9)

$(\delta d_{i,j})x_{i,j}^m - Q(1 - x_{i,j}^m) \leq y_i^m - y_j^m, \forall i, j \in N, m \in M$, (10)

$y_i^m - y_j^m \leq (\delta d_{i,j})x_{i,j}^m + Q(1 - x_{i,j}^m), \forall i, j \in N, m \in M$, (11)

$\sum_{i \in N, j \in F'} x_{i,j}^m = 1, \forall m \in M$, (12)

$$x_{i,j}^m \leq z_i, \forall i \in F', j \in N, m \in M, \quad (13)$$

$$x_{i,j}^m \leq z_j, \forall i \in N, j \in F', m \in M, \quad (14)$$

$$a_i \leq t_i^m \leq b_i, \forall i \in N, m \in M, \quad (15)$$

$$x_{i,j}^m, y_i^m, z_i \in \{0, 1\}, \forall i, j \in V, m \in M \quad (16)$$

The objective function (1) aims to minimize total travel distances and difficulty of visiting charging stations. The second term plays a role as the availability. If a charging station is easy to access, the value of w_i will have small value. That is the state of each charging station. Constraint (2) ensures that each customer site is served once by one vehicle. Constraint (3) presents that each charging station can be used at most once. Constraints (4) and (6) enforces that each vehicle has to leave and return to the depot. Constraint (5) presents that once a vehicle enters a node, the vehicle has to leave the node except the depot. Capacity of each vehicle is in constraint (7). Constraints (8) and (15) are time windows constraints for customer nodes. Constraint (9) denotes the constraint for time of charging. Constraints (10) and (11) present energy consumption through the arc. Constraint (12) forces each vehicle has to visit a charging station once. Constraints (13) and (14) ensure a charging station is open only if a vehicle travels the charging station. Constraint (16) is for binary variables.

If w_i has large value, the charging station i will have low possibility to be chosen by a vehicle. Thus, depending on the value, we can control the availability of the charging station. The model assumes that the capacity of the battery is limited, so each vehicle has to visit a charging station during a delivery tour. In the problem of two time periods, we solve the problem of electric vehicle routing with the above EVRP model. Depending on the locations of customer sites and available charging stations, decisions will be made if the vehicles visit the charging stations in the first time period or not with the solutions from the model. The solution of the model provides the optimal strategy for the electric vehicle routing problem.

4. Solution Method

This section presents how to solve the electric vehicle routing problem with states of charging stations. For a conventional vehicle routing problem, we have to determine the optimal routes for vehicles. When it comes to an electric vehicle routing problem, the locations of charging stations are determined additionally. Considering the states of charging stations, the states will affect the optimal solution. In this paper, we consider two time periods problem. The vehicles have to determine how to travel for time periods. This seems to have two electric vehicle routing problems. The optimal route for each time period may have the visit of charging stations or may not. These decisions are to be made for the problem in this paper.

In Figure 3, the problem is solved with two phases. At first, patterns of states for charging stations are estimated to present the availability of charging station. Each charging station has its own pattern of state of occupation. Collecting the data from charging stations, the patterns are determined by analyzing the data. Once the pattern is found, the pattern can be seen as the availability of charging station. In the second phase, we solve the electric vehicle routing problem of the model proposed in the previous section. The patterns become the parameters in this routing problem. The solution presents the total travel costs with states of charging stations. The solutions for time periods can be used to find the optimal strategy.

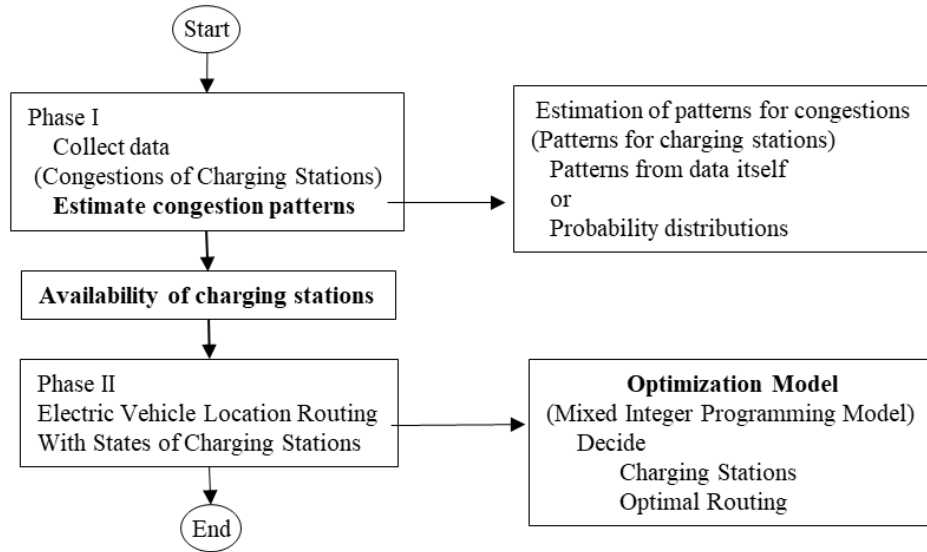


Figure 3. Solution procedure.

Algorithm: solution strategy

Initialize:

Collect dataset E and estimate the patterns P of dataset E

Find states $\Omega(s, t)$ of charging stations

$T = \{1, 2, \dots, n\}$: time horizon for all days

$r_{i,j}$: route in period i with j action where $j = \{0, 1\}$;

where 1 (visit charging station), 0 (no visit charging station)

$D_{i,j}$: demand for period j in day i

Repeat: until $k = n$

$k \rightarrow 1$

For day k

Solve (EVRP) with $D_{k,1}$: z_{k1} (objective value)

Solve (EVRP) with $D_{k,2}$: z_{k2} (objective value)

Solve (EVRP) with $D_{k,1}$ and $\Omega(s, t)$: z_{k3} (objective value)

Solve (EVRP) with $D_{k,2}$ and $\Omega(s, t)$: z_{k4} (objective value)

If $z_{k1} + z_{k4} > z_{k2} + z_{k3}$, $\pi^*(k) = \{r_{1,1}, r_{2,0}\}$

If $z_{k1} + z_{k4} < z_{k2} + z_{k3}$, $\pi^*(k) = \{r_{1,0}, r_{2,1}\}$

$k \rightarrow k + 1$

end

The above algorithm describes how to find the optimal strategy for the problem considering the states of charging stations in two time periods problem. From the solution, each vehicle can visit the charging station only once to minimize the total travel costs.

For the estimation of patterns of states of charging stations, we can derive the probability distribution of the states from data. The state of a charging station is defined as the time to wait when a vehicle arrives at the charging station. Data from a charging station may not be waiting time, so we can derive the waiting time from dataset. Common data at a station is the number of arrivals of vehicles at certain time. With these data, we can derive the distributions of waiting times at charging stations.

Arrivals to a system is usually assumed as following a Poisson distribution in queueing system. Let λ be the parameter of Poisson distribution and denotes the number of events in a given interval. The probability density function is defined as follows.

$$P(X = x) = \frac{\lambda^x e^{-\lambda}}{x!}, x = 0, 1, 2, \dots \quad (17)$$

The above function (17) presents the probability that the number of events is x in a unit time interval. From the probability distribution of arrivals, we can derive the distribution of time intervals between arrivals. The probability distribution of the time interval can be defined as an exponential distribution as follows.

$$f(x) = \lambda e^{-\lambda x}, x \geq 0 \quad (18)$$

(18) denotes the probability that the interarrival time between events is x . These two probability distributions are used to estimate the states of charging stations. The waiting time of a vehicle is calculated by summing of the charging time within the interarrival time. For instance, we assume a simple example for estimating the states from the data.

Table 1 shows the number of vehicles arriving in time intervals. The data can be counted in each charging station. With the data, we can derive the probability distribution of arrivals, which is assumed to follow a Poisson distribution.

Table 1. Arrivals of vehicles in time intervals.

Time interval	Arrivals of vehicles	Time interval	Arrivals of vehicles
11:00-11:10	1	12:20-12:30	1
11:10-11:20	0	12:30-12:40	2
11:20-11:30	0	12:40-12:50	1
11:30-11:40	0	12:50-13:00	0
11:40-11:50	1	13:00-13:10	4
11:50-12:00	2	13:10-13:20	0
12:00-12:10	3	13:20-13:30	2
12:10-12:20	4		

Table 2 presents the counts of arrivals from Table 1. For example, zero arrivals is four times in Table 1 and two vehicle arrivals is three times. Third column is the value of product of two numbers. With the two sums, we can calculate λ of the distribution as follows.

Table 2. Counts of arrivals.

Number of arrivals (A)	Observed counts (B)	A x B
0	4	0
1	4	4
2	3	6
3	1	3
4	2	8
Sum	14	21

$$\lambda = \frac{A \times B}{\text{Sum of observed counts}} = \frac{21}{14} = 1.5 \quad (19)$$

(19) presents the calculation of λ of the distribution of arrivals of vehicles at the charging station. The value of λ denotes that the average number of vehicles in unit time period is 1.5. With this λ , we can derive the interarrival time distribution with exponential distribution. Thus, the average time of interarrival is calculated as $\frac{1}{\lambda} = 0.67$. Adding the charging time from the interarrival can be estimated the waiting time of vehicle.

These estimated waiting times are the parameters for the electric vehicle routing problem as the model (EVRP). The routing problem can be solved with the exact algorithm. We show some numerical examples for the routing problem in the next section.

5. Experimental Results

This section presents simple numerical examples for the electric vehicle routing problem. This paper considers an instance modified from a Solomon benchmark problem.

Figure 4 shows an instance for an electric vehicle routing problem. The instance is randomly modified from a Solomon benchmark problem. There are one depot, multiple customer sites, and multiple charging stations. In the figure, square denotes the depot, circles are for charging stations, and stars present customer sites. All customer sites have to be visited, but some of the charging stations are selected to visit. A vehicle has to visit a charging station once during the deliver tour.

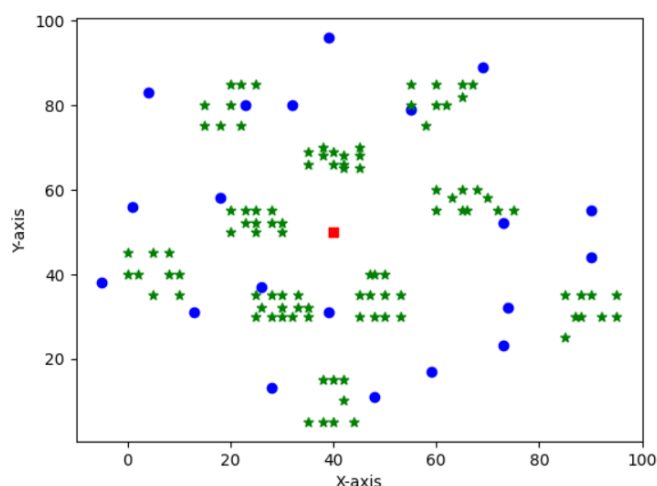
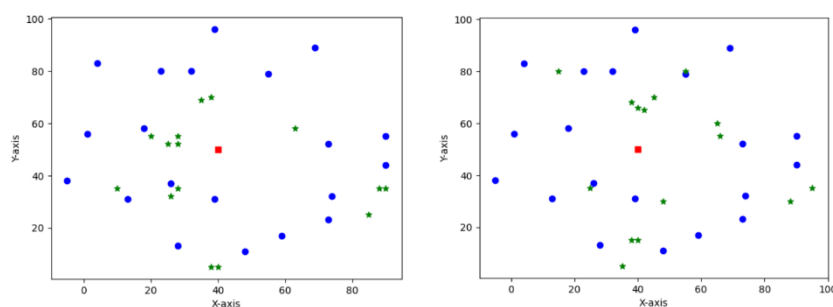


Figure 4. Instance of an electric vehicle routing problem.

Solving a mixed integer programming problem with an exact algorithm is time consuming task. Thus, we have sampled five small instances from the problem of Figure 4 reducing the size with 15 customer sites. Figure 5 shows the sample example 1 from the instance in Figure 4, so the instance has 1 depot, 21 potential charging stations, and 15 customer sites for each time period. Vehicles have to service all customer sites for two time periods and take charging either period 1 or period 2. Potential charging stations are selected to minimize the total travel costs.



(Period 1) (Period 2)

Figure 5. Instance example 1 for period 1 & 2 in a day.

In this paper, we have implemented (EVRP) model with C# with concert technology of CPLEX software. Solving the instance in Figure 5 for period 1 and period 2, we have the following results.

Figure 6 and 7 show the solution of example 1. Three vehicles are used to service the customer sites. In Figure 6, vehicles travel without visiting charging stations. On the other hand, vehicles are visiting charging stations once in Figure 7 and the number with asterisk presents the charging station the vehicle visits. From the results, it can be seen that the optimal routes are changed when the vehicles visit the charging stations. The objective values as total travel costs are also changed. For the optimal strategy, we can calculate $z_{11} + z_{14}$ and $z_{12} + z_{13}$ and compare them.

Period 1	0 -> 11 -> 10 -> 14 -> 2 -> 1 -> 0	Objective value	385
	0 -> 12 -> 13 -> 15 -> 0		
	0 -> 4 -> 3 -> 5 -> 6 -> 7 -> 9 -> 8 -> 0		
Period 2	0 -> 8 -> 9 -> 10 -> 5 -> 6 -> 0	Objective value	485.9
	0 -> 13 -> 11 -> 12 -> 14 -> 7 -> 0		
	0 -> 1 -> 3 -> 15 -> 2 -> 4 -> 0		

Figure 6. Solution of example 1 not visiting charging stations.

Period 1	0 -> 7 -> 12* -> 9 -> 8 -> 0	Objective value	436.4
	0 -> 4 -> 3 -> 5 -> 6 -> 10* -> 2 -> 1 -> 0		
	0 -> 11 -> 10 -> 14 -> 21* -> 12 -> 13 -> 15 -> 0		
Period 2	0 -> 10 -> 11 -> 12 -> 2* -> 13 -> 14 -> 0	Objective value	489
	0 -> 5 -> 15 -> 4* -> 1 -> 3 -> 2 -> 4 -> 0		
	0 -> 6 -> 14* -> 8 -> 9 -> 7 -> 0		

Figure 7. Solution of example 1 visiting charging stations.

$$z_{11} + z_{14} = 385 + 489 = 874 < z_{12} + z_{13} = 485.9 + 436.4 = 922.3 \quad (20)$$

From the comparison (20), $z_{11} + z_{14}$ is less than $z_{12} + z_{13}$. Accordingly, vehicles are recommended to visit charging stations in period 2 rather than period 1.

Figure 8~11 show the experimental results of example 2~5. The examples are generated by randomly sampling 15 customer sites from the instance in Figure 4. Optimal routes are determined by solving (EVRP) model with the exact method. For the strategy for a day, we calculate $z_{k1} + z_{k4}$ and $z_{k2} + z_{k3}$ for each example.

	No charging stations		Visit charging stations	
	Obj. Value	Optimal Routes	Obj. Value	Optimal Routes
Period 1	448.6	0 -> 11 -> 1 -> 2 -> 9 -> 10 -> 8 -> 0	468.8	0 -> 11 -> 9 -> 10 -> 8 -> 12* -> 6 -> 7 -> 0
		0 -> 15 -> 14 -> 12 -> 13 -> 0		0 -> 1 -> 4* -> 15 -> 14 -> 12 -> 13 -> 0
		0 -> 4 -> 5 -> 6 -> 3 -> 7 -> 0		0 -> 4 -> 5 -> 3 -> 8* -> 2 -> 0
Period 2	516.2	0 -> 12 -> 15 -> 13 -> 14 -> 1 -> 0	525.1	0 -> 9 -> 10 -> 2* -> 11 -> 0
		0 -> 3 -> 8 -> 6 -> 10 -> 5 -> 7 -> 0		0 -> 12 -> 15 -> 13 -> 14 -> 6* -> 2 -> 1 -> 0
		0 -> 9 -> 4 -> 2 -> 11 -> 0		0 -> 3 -> 4 -> 5 -> 15* -> 8 -> 6 -> 7 -> 0

Figure 8. Solution of example 2.

	No charging stations		Visit charging stations	
	Obj. Value	Optimal Routes	Obj. Value	Optimal Routes
Period 1	352.9	0 -> 10 -> 11 -> 13 -> 15 -> 14 -> 0	404.7	0 -> 6 -> 7 -> 8 -> 16* -> 9 -> 0
		0 -> 12 -> 2 -> 3 -> 4 -> 5 -> 1 -> 0		0 -> 10 -> 11 -> 13 -> 14 -> 2* -> 15 -> 0
		0 -> 6 -> 7 -> 9 -> 8 -> 0		0 -> 12 -> 1 -> 6* -> 2 -> 3 -> 4 -> 5 -> 0
Period 2	392.3	0 -> 3 -> 5 -> 7 -> 6 -> 8 -> 0	435.5	0 -> 13 -> 10 -> 11 -> 12 -> 14 -> 4* -> 15 -> 0
		0 -> 13 -> 10 -> 11 -> 12 -> 14 -> 0		0 -> 1 -> 2 -> 10* -> 5 -> 6 -> 0
		0 -> 4 -> 9 -> 15 -> 1 -> 2 -> 0		0 -> 3 -> 4 -> 7 -> 8 -> 16* -> 9 -> 0

Figure 9. Solution of example 3.

	No charging stations		Visit charging stations	
	Obj. Value	Optimal Routes	Obj. Value	Optimal Routes
Period 1	393	0 -> 14 -> 9 -> 8 -> 0	420	0 -> 7 -> 6 -> 5 -> 6* -> 1 -> 3 -> 2 -> 0
		0 -> 10 -> 11 -> 7 -> 6 -> 5 -> 0		0 -> 14 -> 12 -> 13 -> 4* -> 15 -> 4 -> 0
		0 -> 1 -> 15 -> 3 -> 12 -> 13 -> 2 -> 4 -> 0		0 -> 10 -> 11 -> 17* -> 9 -> 8 -> 0
Period 2	442.3	0 -> 4 -> 5 -> 2 -> 3 -> 14 -> 0	467.5	0 -> 4 -> 5 -> 7 -> 14* -> 6 -> 8 -> 0
		0 -> 10 -> 12 -> 11 -> 13 -> 9 -> 7 -> 0		0 -> 15 -> 14 -> 2* -> 10 -> 12 -> 11 -> 13 -> 9 -> 0
		0 -> 6 -> 8 -> 15 -> 1 -> 0		0 -> 1 -> 6* -> 2 -> 3 -> 0

Figure 10. Solution of example 4.

	No charging stations		Visit charging stations	
	Obj. Value	Optimal Routes	Obj. Value	Optimal Routes
Period 1	309.6	0 -> 5 -> 4 -> 11 -> 12 -> 9 -> 10 -> 0	388.3	0 -> 13 -> 2 -> 1 -> 4* -> 15 -> 14 -> 0
		0 -> 13 -> 15 -> 14 -> 0		0 -> 3 -> 14* -> 7 -> 8 -> 6 -> 0
		0 -> 3 -> 7 -> 8 -> 6 -> 2 -> 1 -> 0		0 -> 5 -> 4 -> 9 -> 17* -> 11 -> 12 -> 10 -> 0
Period 2	354.2	0 -> 4 -> 5 -> 1 -> 2 -> 3 -> 15 -> 0	436.7	0 -> 4 -> 5 -> 6 -> 12* -> 10 -> 7 -> 9 -> 8 -> 0
		0 -> 10 -> 7 -> 6 -> 9 -> 8 -> 0		0 -> 11 -> 12 -> 2* -> 13 -> 14 -> 0
		0 -> 13 -> 11 -> 12 -> 14 -> 0		0 -> 15 -> 4* -> 1 -> 2 -> 3 -> 0

Figure 11. Solution of example 5.

$z_{21} + z_{24} = 448.6 + 525.1 = 973.7 < z_{22} + z_{23} = 516.2 + 468.8 = 985$ (21)

$z_{31} + z_{34} = 352.9 + 435.5 = 788.4 < z_{32} + z_{33} = 392.3 + 404.7 = 797$ (22)

$z_{41} + z_{44} = 393 + 467.5 = 860.5 < z_{42} + z_{43} = 442.3 + 420 = 862.3$ (23)

$z_{51} + z_{54} = 309.6 + 436.7 = 746.3 > z_{52} + z_{53} = 354.2 + 388.3 = 742.5$ (24)

(21)~(24) present the comparisons of total travel costs for days. In examples 2, 3, and 4, the total travel costs of $z_{k1} + z_{k4}$ are smaller than $z_{k2} + z_{k3}$. Thus, visiting charging stations in period 2 is better than the other way. On the other hand, vehicles are better to visit the charging stations in period 1. In this way, we can make decisions how and when the vehicles take charging during the delivery tours.

Figure 12 presents the solutions as the difficulties of charging at stations changes. The difficulty of charging is the weight in the objective function of model (EVRP). As the difficulty changes, the optimal routes changes. If the change is small, the optimal solution does not change. However, if the change is big enough, the optimal route will be different. The model can find the threshold of the difficulty to change the optimal routes. Thus, with the values, we can control the routing strategy for vehicles.

Difficulty of charging			Optimal routes
w_2	w_6	w_{15}	0 -> 9 -> 10 -> 2* -> 11 -> 0
5	5	5	0 -> 3 -> 4 -> 5 -> 15* -> 8 -> 6 -> 7 -> 0
			0 -> 12 -> 15 -> 12 -> 13 -> 6* -> 2 -> 1 -> 0
w_2	w_6	w_{15}	0 -> 9 -> 10 -> 21* -> 11 -> 0
10	5	5	0 -> 3 -> 4 -> 5 -> 15* -> 8 -> 6 -> 7 -> 0
			0 -> 12 -> 15 -> 12 -> 13 -> 6* -> 2 -> 1 -> 0
w_2	w_6	w_{15}	0 -> 9 -> 10 -> 2* -> 11 -> 0
5	7	5	0 -> 3 -> 4 -> 5 -> 15* -> 8 -> 6 -> 7 -> 0
			0 -> 12 -> 15 -> 12 -> 13 -> 4* -> 2 -> 1 -> 0
w_2	w_6	w_{15}	0 -> 9 -> 10 -> 2* -> 11 -> 0
5	5	8	0 -> 3 -> 8 -> 6 -> 7 -> 16* -> 4 -> 5 -> 0
			0 -> 12 -> 15 -> 12 -> 13 -> 6* -> 2 -> 1 -> 0

Figure 12. Solutions depending on the states of charging stations.

6. Conclusions

This paper suggests the electric vehicle routing problem with considering the states of charging stations. Charging issue is an important issue for the electric vehicle routing problem. Charging functions, energy consumption, and location of charging stations have been explored in many studies. However, the state of a charging station is also important to find the optimal route in the electric vehicle routing problem. The state of a charging station can be seen as the availability of the charging station. If charging is not available when a vehicle arrives, the travel costs will significantly increase. In this paper, we consider the states of charging stations, formulate the model, and suggest the solution approach.

The first step of the solution approach is to collect the data from all charging stations. The arrivals of vehicles to the station are collected as raw data. The data has been analyzed to find the patterns of states of charging stations. Estimating the probability distribution of arrivals and interarrival times can be used to find the pattern of waiting times when a vehicle arrives the charging station. Once the

patterns of states of charging stations are determined, the values are used as input parameters for the model of the electric vehicle routing problem. With the solutions, we can find the optimal routing strategy for two time periods problem.

Experimental results provide how to find the optimal strategy for the electric vehicle routing problem with states of charging stations. In two time periods problem, we are able to determine in which time period the vehicles visit the charging stations. The difficulty of charging can be used to find the threshold the optimal routes changes. Thus, we can control the electric vehicle routing with considering the availability of the charging stations.

Assuming the nonstationary distributions for the states of charging stations have to be explored in the future work. Investigating the uncertainty of charging durations for vehicles is also a challenging issue. Obtaining real world scenarios of states of charging stations and investigating the charging behaviors of vehicles are the future works of our study.

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