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Supersymmetry and Particle-like Quantum Black Holes

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Abstract: In this paper, we consider the formation of quantum black holes (QBHs) through a genuine quantum process. We treat particles at the Planck energy as QBHs. The Hawking temperature and entropy of QBHs are estimated. We also show that a QBH has a discrete energy spectrum. By analogy with supersymmetric theory, the mass of the superpartner of a QBH is estimated.

1. Introduction

General relativity (GR) can be considered one of the most successful physical theories. But GR is not free of problems and it presents limitations at Planckian energy scales. There are reasons to believe that a convincing theory of quantum gravity will take over from classical GR and provide the ultraviolet completion. On the other hand, one believes that the ultraviolet divergences of quantum field theory might be removed in a theory in which gravity is quantized, even though renormalization displays a very impressive agreement between high precision measurements in accelerators and the predictions of quantum field theory in presence of radiative corrections. By noticing that the ultraviolet divergences in quantum field theory arise from the lightcone singularities of two-point functions and quantum fluctuations of the space-time metric may smear out the singularities, Pauli suggested that the ultraviolet divergences might be removed in a quantum gravity theory. However, despite the enormous progress in our understanding of quantum gravity in recent years, physics at the Planck scale remains extremely puzzling.

Hawking's description of radiative black holes (BHs) offered the first clue related to quantum gravity [1,2]. Classical BHs solutions display a space-time singularity, a Hawking temperature proportional to the surface gravity and an entropy proportional to the area of horizon. On the other hand, based on the "Heisenberg microscope" ideal experiment that an electron position is determined by observing a scattered photon, the incident photon needs to have higher energy to locate the electron more accurately. When the energy of this photon is above some threshold energy, so huge energy compacted into a tiny region will produce a BH, the so-called hoop conjecture introduced by K. Thorne [3]. In other words, when the Compton wave length and Schwarzschild radius are comparable, classical BHs become quantum black holes (QBHs), which are sensitive to the short distance behavior of gravity. The geometrical descriptions of QBHs are inadequate. Since the Planck scale is the regime where gravity merges with quantum mechanics, QBHs have become an important subject of research [4–6]. A better understanding of these may shed more light on quantum gravity.

A QBH can be produced via a variety of mechanisms. It is well known that the gravitational collapse of vacuum energy fluctuations at the early universe can give rise primordial black holes formation [7,8]. An alternative process for Planckian black hole creation is through an hadronic collision [9,10]. In some studies, the production of QBHs through hadronic collisions at Planckian energy is described in terms of Bose-Einstein graviton condensates [11,12]. Nevertheless, experimental evidence of this process at Large Hadron Collider (LHC) is still lacking. In a new scenario, a QBH model is constructed by considering the evolution of a quantum particle when its energy approaches the Planck scale from below [13]. When the energy of particles is above some threshold energy that we pushed to the Planck mass, a kind of "phase transition" occurs and these particles become QBHs. The horizon radius of QBHs has lost its classical geometrical meaning and acquires the role of wavelength. As a consequence, the event horizon of QBHs becomes uncertain and quantum fluctuating. Here, we treat particles at Planckian energy as another quantum particle, QBHs, but with an uncertain

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event horizon. Our intention was only to test the feasibility of this approach in the simplest possible framework since little was known about quantum gravity. Event horizon fluctuations imply that BH masses should be quantized [14]. By following a quantization scheme introduced by Spallucci [6,13], we obtain a discrete mass spectrum of QBHs. Then, we analyzed the superpartners of QBHs.

This paper is organized as follow. In Sec.2 we described a toy model of QBHs. Sec.3 is dedicated to the quantization of QBHs. In Sec.4 we estimated the mass of superpartners of QBHs. Finally, in Sec.5 we summarize the main results obtained. For convenience, we set c=1.

2. A toy model of QBHs

In fact, the choice of the Planck energy as the lower bound for black hole production depends on the models. In our model, the threshold energy is actually dependent on the spin s and charge q of particles as we will see later on. We now define $r_g = f(s,q)GM$ where $f(s,q) \sim 1$ as the gravitational length scale to produce the horizon. For $\lambda_C > r_g$ with $\lambda_C = \hbar/m$ being the Compton wavelength, the object is an ordinary quantum particle obeying known quantum mechanical rules; for $\lambda_C < r_g$, its energy is above the threshold value and the "phase transition" occurs, a quantum particle develops an event horizon shielding it from the outside world and becomes a QBH.

Even if a black hole is much smaller than a proton, we can still treat it as a classical object as long as their size is large compared to the Planck length. According to the no-hair theorem, no other properties besides mass, electric charge and angular momentum should emerge from beneath the event horizon. Thus, we assume that particles at Planckian energy and corresponding QBHs are indistinguishable and we will use QBHs carrying spin and charge to describe the behaviors.

The Kerr-Newman metric is a vacuum solution to the classical Einstein field equations, describing a charged BH of mass M rotating with angular momentum J. The line element in Boyer-Lindquist coordinates (t, r, θ, φ) is given by

$$\begin{aligned} ds^2 = &-\left(1 - \frac{2GMr - GQ^2}{\rho^2}\right) dt^2 + \frac{\rho^2}{\Delta} dr^2 + \rho^2\, d\theta^2 \\ &+ \left(r^2 + a^2 + \frac{(2GMr - GQ^2)a^2}{\rho^2}\sin^2\theta\right)\sin^2\theta d\varphi^2 - \frac{2(2GMr - GQ^2)a}{\rho^2}\sin^2\theta d\varphi dt, \end{aligned} \tag{1}$$

where

$$\begin{aligned} \rho^2 &= r^2 + a^2\cos^2\theta \\ \Delta &= r^2 - 2GMr + a^2 + GQ^2 \end{aligned} \tag{2}$$

with $a = J/M$ being the angular momentum per unit mass. We identify the two roots of the equation $\Delta = 0$ with the radii of the inner (Cauchy) and outer (Killing) horizon, i.e.

$$r_\pm = GM \pm \sqrt{G^2M^2 - a^2 - GQ^2} = GM\left(1 \pm \sqrt{1 - \left(\frac{a}{GM}\right)^2 - \frac{Q^2}{GM^2}}\right). \tag{3}$$

In the case of QBHs, we will ignore the orbital angular momentum (transverse to the helicity axis), and consider the QBHs spin parallel to the helicity axis. From (3) we construct the effective geometry of QBHs as

$$r_\pm = Gm\left(1 \pm \sqrt{1 - \left(\frac{s\hbar/m}{Gm}\right)^2 - \frac{q^2\hbar}{Gm^2}}\right), \tag{4}$$

where m, s and q represent the mass, spin and electric charge of a QBH, respectively. Quantum spin in is quite different from classical angular momentum. However, based on the linear Regge trajectory [15], we can suppose that $s = \alpha' m^2$ with α' being the Regge slope. Note that for a spin-1 QBH, $s\hbar/m$ is the Compton wavelength. For simplicity, we only consider the charge of QBHs under U(1). However,

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nothing prevents that QBHs carry a color charge under $SU(3)_c$. The procedure outline here can be used to include QBHs with a color charge in the future, if desired. From (4), we deduce that

$$\left(\frac{sM_P^2}{m^2}\right)^2 + \left(\frac{qM_P}{m}\right)^2 \leq 1. \tag{5}$$

A properly chosen value of $f(s,q)$ can allow QBHs to have spin states (0, 1/2, 1, 3/2, 2) provided that the electric charge is not too large. However, the event horizon of the QBHs vanishes due to high spin and the nakedness of the singularity offends the cosmic censorship conjecture.

We notice that (4) and (5) as generalized formulas will allow us to describe a black hole in thermodynamical terms. The first interesting thermodynamical quantity is the Hawking temperature, which is given by

$$T_H = \frac{\hbar\kappa_+}{2\pi k_B} = \frac{\hbar}{2\pi k_B}\frac{\sqrt{G^2m^2 - (s\hbar/m)^2 - q^2\hbar G}}{2G^2m^2 - q^2\hbar G + 2Gm\sqrt{G^2m^2 - (s\hbar/m)^2 - q^2\hbar G}}, \tag{6}$$

where κ_+ is the surface gravity. In the large mass limit $m \gg M_P$, the QBH transitioned to classical black holes described by GR. Consider the case where s (or q) $\neq 0$ and for the lightest QBH, the so-called extremal QBH, we have

$$\left(\frac{sM_P^2}{m^2}\right)^2 + \left(\frac{qM_P}{m}\right)^2 = 1. \tag{7}$$

The "phase transition" occurs at the critical mass:

$$m_{ext} = \frac{1}{\sqrt{2}}\left(q^2M_P^2 + \sqrt{q^4M_P^4 + 4s^2M_P^4}\right)^{1/2} \tag{8}$$

In this case, the temperature T_H drops to zero and it means that at the Planck scale, the thermodynamical description is no longer valid. For a BH, the area of the horizon is

$$A_H = 4\pi\left(r_+^2 + \frac{s^2\hbar^2}{m^2}\right). \tag{9}$$

Then, the entropy has the form

$$S = \frac{k_B A_H}{4G\hbar} = \frac{\pi k_B q^2}{2} + \frac{\pi k_B\sqrt{q^4 + 4s^2}}{2} + \frac{2\pi k_B s^2}{q^2 + \sqrt{q^4 + 4s^2}}. \tag{10}$$

It is worth signaling that in this extremal limit, the temperature goes to zero, the thermodynamical description is no more an adequate one but we can use this limiting entropy as the definition of the zero temperature entropy.

3. Quantization of QBHs

Since the horizon radius of a QBH has lost its classical geometrical meaning and acquires the role of wavelength, one can encode the horizon dynamics into the motion of a relativistic particle trapped in a gravitational potential [6]. In this framework the horizon oscillations are effectively described by the wave equation of this test particle. The horizon wave equation of QBHs is given by

$$\mathcal{H}\psi(r) = E^2\psi(r), \tag{11}$$

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where $\mathcal{H} = \boldsymbol{p}^2 + m^2$ is a relativistic Hamiltonian. The mass m of a QBH is expressed in terms of the horizon radius r as

$$m = \frac{r}{2G}\left(1 + \frac{(s\hbar/m)^2 + q^2\hbar G}{r^2}\right). \tag{12}$$

The mass can be obtained by solving this cubic algebraic equation. For $m \gg M_P$, the horizon freezes into a static configuration, one gets $s\hbar/m \to 0$. For $m \sim M_P$, $s\hbar/m$ is of order Planck length l_p. The corresponding spherically symmetric radial wave equation is

$$\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\psi}{\partial r}\right)\right]\psi(r) + \left[E^2 - m^2 - \frac{l(l+1)}{r^2}\right]\psi(r) = 0. \tag{13}$$

The angular dependent part of the wave function can be expressed in terms of spherical harmonics $Y_l^m(\theta,\phi)$, where l is a non-negative integer. For a large mass, the solution of radial wave equation is

$$\psi_n(r) = N_n\frac{r^{2d}}{(2G)^d}\exp\left(-r^2/4G\right)L_n^{2d+1/2}\left(\frac{r^2}{2G}\right), \tag{14}$$

where $L_n^{2d+1/2}(\frac{r^2}{2G})$ is the generalized Laguerre polynomials, N_n is a normalization factor and

$$4d = \sqrt{q^4 + (2l+1)^2} - 1. \tag{15}$$

A discrete energy spectrum of a QBH is given approximately by

$$2GE_n^2 - q^2\hbar - \lambda_1(E_n)s^2\hbar = \sqrt{q^4 + \lambda_2(E_n)s^4 + (2l+1)^2} + 4n + 2, \tag{16}$$

where $\lambda_1(E_n) \sim \frac{M_P^2}{E_n^2}$ and $\lambda_2(E_n) \sim \frac{M_P^4}{E_n^4}$ are numerical factors that drop to zero when one considers the classical limit $E_n \gg M_P$. For a neutral and spin-0 QBH, we have

$$2GE_n^2 = 4n + 2l + 3. \tag{17}$$

4. Supersymmetry and QBHs

Supersymmetry as a possible extension of the Standard Model stirs together fermions and bosons and carries the implication that each elementary particle has a superpartner. It would be interesting to investigate superpartners of QBHs. In the case of QBHs, we expect that a QBH and its superpartner have equal masses.

Let us set aside supersymmetry for now and just focus on energy. We now generalize it to a more complicated framework that the "partner" of a QBH not only has spin differing by 1/2 unit, but also charge differing by 1 unit. In other words, we consider a transformation involving spin and charge. Finally, we will restrict our considerations to the case that each pair of superpartners shares the same charge. Intuitively, consider two charged complex scalar field mix with a neutral Marojana spinor field. Thus, the mass shift of QBHs deriving from incremental spin cancel out the mass shift deriving from reduced charge and vice versa. We have

$$E_n(s_1, q_1) = E_n(s_2, q_2), \tag{18}$$

where $E_n(s,q)$ denotes the mass of QBHs of spin s and charge q (a QBH and its "partner" are labeled by 1 and 2). Of course, it can be generalized to the case of a QBH carrying a color charge.

In the Minimal Supersymmetric Standard Model (MSSM), with perfectly unbroken supersymmetry, each pair of superpartners shares the same mass and $SU(3)_c \times SU(2)_L \times U(1)$ quantum numbers besides spin. However, the mass of QBHs depends on the spin and charge as can be seen from Eq. (16). The mass of a spin-1/2 QBH will change if this QBH with spin differing by 1/2

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unit becomes a spin-0 QBH. As a consequence, they are unlikely to have the same mass. In order to achieve this, it is necessary to include additional self-energy.

In quantum physics, every particle moves through a “medium” consisting of the quantum fluctuations of all particles present in the theory. However, the above considerations only involve “bare" mass. We may expect that a QBH and its “partner" have the same physical mass. This implies

$$E_n(s_1, q_1) + \Delta E_{\mathrm{em}1} = E_n(s_2, q_2) + \Delta E_{\mathrm{em}2}, \tag{19}$$

where E_{em} is the mass shift due to the electromagnetic field. It is worth mentioning that E_n is the “bare" mass of a QBH.

The Holographic Principle claims that the whole dynamics of a QBH is the dynamics of its horizon. Let us now just make the simple assumption that the charge can be treated as being distributed over the surface of the event horizon and one can easily estimate the electromagnetic self-energy E_{em}. However, for $E_n \sim M_P$, the semiclassical theory breaks down because of large fluctuations of the event horizon. We regard $r = r_+$ as the characteristic length and obtain

$$\Delta E_{\mathrm{em}} = \frac{\alpha \hbar q^2}{2\sqrt{r^2 + \frac{s^2\hbar^2}{m^2}}}, \tag{20}$$

where α is the fine structure constant.

The elementary particles of the Standard Model must reside in either a chiral or gauge supermultiplet. As an example, by analogy with two complex scalar fields as the superpartners of the left-handed and right-handed parts of the electron Dirac field, consider a spin-1/2 QBH and its spin-0 superpartner. We now take $q_1 = q_2 = 1$. By using Eq. (20), the equation corresponding to Eq. (19) is

$$m_1 + \frac{\alpha\hbar}{2\sqrt{r_1^2 + \frac{\hbar^2}{4m_1^2}}} = m_2 + \frac{\alpha\hbar}{2r_2}, \tag{21}$$

where

$$m_1 = E_n(s_2 = \frac{1}{2}, q_1 = 1) \tag{22}$$

and

$$m_2 = E_n(s_2 = 0, q_2 = 1). \tag{23}$$

For an extremal black hole, it reduces to

$$m_1 + \frac{\alpha\hbar}{2\sqrt{G^2 m_1^2 + \frac{\hbar^2}{4m_1^2}}} = m_2 + \frac{\alpha\hbar}{2Gm_2}. \tag{24}$$

Of course, a more likely scenario is that a QBH and its superpartner indeed have different physical mass when we treat the spin as energy. The difference in their mass is of order M_P.

5. Conclusions and Discussion

In this paper, we considered a genuine quantum formation of QBHs by considering the evolution of a quantum particle when its energy approaches the Planck scale from below. At the extremely high energy scale, the gravity enters the stage and the particles become QBHs. Thus, we use QBHs carrying spin and charge to describe the particles at the Planck energy. The cosmic censorship conjecture imposes restrictions on the spin and charge of QBHs. Since the horizon radius of a QBH has lost its classical geometrical meaning and acquires the role of wavelength, we follow a quantization scheme [6] and show that QBHs have a discrete mass spectrum. The excited states of QBHs are labeled by quantum numbers s, q, n and l.

Then, by analogy with supersymmetric theory, we study the superpartners of QBHs. Since the mass of QBHs is spin dependent, it would be unreasonable to insist that a superpartner with spin differing by $1/2$ has the same mass. If we insist that the total mass of QBHs is invariant under the transformation, it is necessary to include additional self-energy. Here, we consider a electromagnetic self-energy. Based on the Holographic Principle, the self-energy of a charged QBH has a finite value. A more likely scenario is that a QBH and its superpartner have equal total energy but different physical mass. We may expect that these considerations could be applied to the case of particles since they just reflect the energy conservation. It is fair to assume that an electron with spin differing by $1/2$ unit as a scalar particle has a different mass. Similarly, we can also speculate that the mass of an electron changes when its electric charge shifts from 1 unit to 0, transforming into an electrically neutral particle. Unlike some quantum numbers, the spin and charge are associated with energy. Thus, we can establish energy equivalence equations analogous to Einstein's mass-energy equation $E = m$, introducing the charge-energy equivalence equation as $E = q$ and the spin-energy equivalence equation as $E = \sqrt{s}$. On the other hand, gravity knows about a particle with spin differing by $1/2$ unit. A simple way to understand this is to recall that the vacuum energy of the zero point fluctuations is positive for bosons and negative for their fermionic partners. After the spontaneous soft supersymmetry breaking, gravity knows about the net vacuum energy since gravity couples to energy and momentum. In the MSSM, each pair of superpartners shares the same interactions. Thus, the mass of the *selectron*, the superpartner of the electron, should be of order M_P once we we treat the spin as energy. In this case, we may include radiative corrections to the mass due to gravity and the heavy superpartners can produce larger gravitational self-energy. In the framework of "large extra-dimension quantum gravity" [16], the gravitational (or string) scale is actually very much lower than the Planck scale, perhaps even as low as a few TeV. The compactified extra dimensions affect the strength of gravity and the physical mass of superpartners can be lowered down to the TeV scale. However, this guess needs a further exploration. Further more, if a superpartner is electrically neutral, it interacts only weakly with ordinary matter, and so can provide a good candidate for dark matter that seems to be required by cosmology.

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