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Discrete-Time Adaptive Control for Uncertain Scalar Multiagent Systems with Coupled Dynamics: A Lyapunov-Based Approach

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Abstract: Discrete-time architectures offer a distinct advantage over their continuous counterparts, as they can be seamlessly implemented on embedded hardware without the necessity for discretization processes. Yet, because of the difficulty of ensuring Lyapunov difference expressions, their designs, which are based on quadratic Lyapunov-based frameworks, are highly complex. As a result, various existing continuous-time results using adaptive control methods to deal with system uncertainties and coupled dynamics in agents of a multiagent system cannot be directly applied to the discrete-time context. Furthermore, compared to their continuous-time equivalent, discrete-time information exchange based on periodic time intervals is more practical in the control of multiagent systems. In this paper, we first introduce a discrete-time adaptive control architecture designed for uncertain scalar multiagent systems without coupled dynamics as a preliminary result. We then introduce another discrete-time adaptive control approach for uncertain multiagent systems in the presence of coupled dynamics. Our approach incorporates observer dynamics to manage unmeasurable coupled dynamics, along with a user-assigned Laplacian matrix to induce cooperative behaviors among multiple agents. Our solution includes Lyapunov analysis with logarithmic and quadratic Lyapunov functions for guaranteeing asymptotic stability with both controllers. To demonstrate the effectiveness of the proposed control architectures, we provide an illustrative example.

Keywords: discrete-time; adaptive control; coupled dynamics

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1. Introduction

1.1. Literature Review

The exploration of multiagent systems has seen a growing interest due to their effective and adaptable solutions for tackling intricate real-world tasks. Over the past decade, they have left a significant mark on a diverse range of fields, including scientific, civilian, and military applications such as environmental monitoring, exploration, in/on-space assembly, maintenance and manufacturing, traffic management, and payload and passenger transportation. A key attribute of multiagent systems is their capacity to collaboratively execute missions by operating in specified formations. In the existing literature, the primary focus of general research lies in the development of distributed control algorithms that enable operations with local interactions [1–4]. The presence of uncertainties, such as unknown coefficients from modeling, disturbances, and unknown friction effects, along with coupled dynamics in rigid systems with flexible components or slung load dynamics, can negatively impact the performance and stability of sole agents and/or the overall multiagent system. As a result, systems with coupled dynamics and uncertainty become critical for guaranteeing the entire system's stability [5–8]. To address the challenge posed by uncertainty, effective solutions are presented in continuous-time adaptive and robust control architectures as discussed in [9–15]. Subsequently, in [16–18], the focus shifts to the examination of continuous-time adaptive architectures tailored for managing uncertain multiagent systems characterized by coupled dynamics, particularly in a leader-follower framework. The continuous adaptive control formulations introduced in [16–18] are specifically designed to address various challenges in uncertain multiagent systems with coupled dynamics. Basically, the designs in [16,17] aim to achieve boundedness of the tracking error when dealing with uncertain multiagent systems in the presence of coupled dynamics only and coupled and actuator dynamics together, respectively. The design in [18] aims to achieve asymptotic convergence of the tracking error when dealing with uncertain multiagent systems in the presence of coupled dynamics. Furthermore, the results in [16–18] for a leader-follower setting with a classical command tracking approach (i.e., where all agents converge to the position specified for the leader agent(s) only).

The majority of networked multiagent control systems currently in use are limited in their capacity for creating cooperative behaviors, such that they use the classical command tracking approach. Recognizing the significance of diversifying agent positions in military and civilian applications, one effective strategy involves assigning user-defined positions to each agent, thereby facilitating the creation of continuous-time formations. This can be achieved by manipulating matrices associated with graph theory, employing user-assigned nullspace, and introducing a novel representation for the Laplacian matrix in undirected and connected graphs [19–24]. Specifically, in [19,20], a novel Laplacian matrix is introduced by modifying the degree matrix, while in [21], another novel Laplacian matrix is introduced by modifying the degree matrix to create complex behaviors, but former ones' formulation necessitates precise knowledge of neighboring agent states and none of the controllers in [19–21] can create a robust result when faced with unknown terms. Addressing the robustness, the authors of [22,23] proposed a distributed controller featuring a user-defined Laplacian matrix that is used in [21], offering increased flexibility to agents in the presence of coupled dynamics and actuator dynamics, respectively. Note that all of the controllers are designed in [19–24] are continuous-time algorithms.

Discretizing continuous-time algorithms to apply in embedded code may cause stability margins to be lost [25]. Furthermore, compared to their continuous-time equivalent, discrete-time information exchange based on periodic time intervals is practically more practicable in the control of multiagent systems. Yet, because of the complexity of the resulting Lyapunov difference expressions, discrete-time control designs, which are based on Lyapunov-based frameworks, are highly complex. This issue arises because the controlled physical system's Lyapunov stability cannot be guaranteed by the Lyapunov difference expressions since they cannot be made negative-definite [26–30]. To ensure asymptotic stability for sole systems, the authors in [29–36] solve this issue by logarithmic Lyapunov functions in the Lyapunov analysis. In the context of the multiagent systems, the authors of [4] cover optimal discrete-time cooperative control in multiagent systems, the authors of [37] design adaptive fault-tolerant tracking control for discrete-time multiagent systems via reinforcement learning algorithm, the authors of [38] propose cooperative adaptive optimal output regulation of nonlinear discrete-time multi-agent systems, and the authors of [39] study discrete-time control of multiagent systems with a misbehaving agent.

Note that none of the above results are considered a discrete-time setting for an uncertain multiagent system with coupled dynamics while having the capability of assigning different positions for each agent to achieve complex tasks. To this end, in this paper, discrete-time adaptive control algorithms are designed for an uncertain multiagent system with and without unmeasurable coupled dynamics that ensures asymptotic stability using a Lyapunov candidate composed of logarithmic and quadratic functions and adopts user-assigned Laplacian matrix nullspace yielding flexibility in positioning. Finally, preliminary conference versions of this paper are considered as [40], where this paper goes beyond the conference version by providing detailed proofs of all the results and detailed simulation studies with related discussions.

1.2. Organization

The structure of this paper is as follows. In Section 2, for completeness, the stability analysis of the discrete-time controller for the uncertain multiagent system in the absence of coupled dynamics

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is presented. In Section 3, the stability analysis of the first proposed discrete-time controller for the uncertain dynamical system with coupled dynamics is presented, which guarantees asymptotic convergence of the tracking error. Section 4 validates the theoretical contributions with an illustrative numerical example. In Section 5, concluding remarks are given.

1.3. Notation and Mathematical Preliminaries

A general notation and a graph theoretical notation are used in this paper. We refer to Table 1 for the general notation used in this paper, and the graph theoretical notation is given below.

Table 1. Notation

$\mathbb{N}$	set of non-negative integers
$\mathbb{R}$	set of real numbers
$\mathbb{R}_+$	set of positive real numbers
$\mathbb{R}^n$	set of $n \times 1$ real column vectors
$\mathbb{R}^{n\times m}$	set of $n \times m$ real matrices
$\mathbb{P}^n$	set of $n \times n$ positive definite real matrices
$\triangleq$	equality by definition
$(\cdot)^{\mathrm{T}}$	transpose of a matrix
$(\cdot)^{-1}$	inverse of a matrix
$\mathrm{tr}(\cdot)$	Trace operator
$\ln(\cdot)$	Natural logarithm
$\|\cdot\|_2$	Euclidean norm
$\lambda(A)$	eigenvalues of the real matrix $A \in \mathbb{R}^{n\times n}$
$\overline{\lambda}(A)$	maximum eigenvalue of the real matrix $A \in \mathbb{R}^{n\times n}$
$\underline{\lambda}(A)$	minimum eigenvalue of the real matrix $A \in \mathbb{R}^{n\times n}$
I_n	$n \times n$ identity matrix
0_n	$n \times n$ zero matrix
$\mathrm{diag}(\cdot)$	diagonalized vector

Consider an undirected connected graph $\mathcal{G}$ is defined by set of nodes (i.e., $\mathcal{V}_{\mathcal{G}} = \{1, \ldots, n\}$) and set of edges (i.e., $\mathcal{E}_{\mathcal{G}} \subset \mathcal{V}_{\mathcal{G}} \times \mathcal{V}_{\mathcal{G}}$). A graph $\mathcal{G}$ is then considered as a connected graph with a path between any pair of distinct nodes, where a path $i_0 i_1 \ldots i_L$ is a finite sequence of nodes (i.e., $i_{k-1} \sim i_k$, $k = 1, \ldots, L$), and when the nodes i and j are neighbors (i.e., $(i, j) \in \mathcal{E}_{\mathcal{G}}$), $i \sim j$ denotes the neighboring relation. In addition, the degree matrix is denoted by $\mathcal{D}(\mathcal{G}) \triangleq \mathrm{diag}(d) \in \mathbb{R}^{n\times n}$ with $d = [d_1, \ldots, d_n]^{\mathrm{T}}$ with a degree of a node d_i being equal to the number of its neighbors, the adjacency matrix is denoted by $\mathcal{A}(\mathcal{G}) \in \mathbb{R}^{n\times n}$ with $[\mathcal{A}(\mathcal{G})]_{ij} \triangleq 1$ if $(i, j) \in \mathcal{E}_{\mathcal{G}}$ and $[\mathcal{A}(\mathcal{G})]_{ij} \triangleq 0$ otherwise, and the Laplacian matrix of a graph $\mathcal{G}$ denoted by $\mathcal{L}(\mathcal{G}) \triangleq \mathcal{D}(\mathcal{G}) - \mathcal{A}(\mathcal{G})$, [41,42].

Finally, for the definition of the modified version of the Laplacian matrix that allows to assign different positions to each node, let $\omega = [\omega_1, \ldots, \omega_n]^{\mathrm{T}} \in \mathbb{R}^n$ being a vector with entries $\omega_i \in \mathbb{R}_+, i = 1, \ldots, n$, where ω represents the user-assigned nullspace [21]. Next, consider the modified degree matrix given by $\overline{\mathcal{D}}(\mathcal{G}, \omega) \triangleq \mathrm{diag}(\mathcal{A}(\mathcal{G})\omega)(\mathrm{diag}(\omega))^{-1} = \mathrm{diag}(\bar{d}) \in \mathbb{R}^{n\times n}$ with $\bar{d} = [\bar{d}_1, \ldots, \bar{d}_n]^{\mathrm{T}} \in \mathbb{R}^n$. Here, $\mathcal{A}(\mathcal{G})$ is the standard adjacency matrix. Then the modified Laplacian matrix of a graph $\mathcal{G}$ can be represented as $\mathcal{L}(\mathcal{G}, \omega) \triangleq \overline{\mathcal{D}}(\mathcal{G}, \omega) - \mathcal{A}(\mathcal{G})$.

Lemma 1. *In the context of a leader-follower setting, one can define* $\mathcal{K} = \mathrm{diag}\left([\kappa_1, \ldots, \kappa_n]\right) \in \mathbb{R}^{n\times n}$ *with* $\kappa_i \in \{0, 1\}$ *for all* $i = 1, \ldots, n$*, where at least one* κ_i *being equal to* 1 *(for a leader agent* $\kappa_i = 1$*, otherwise it is* 0*). That further yields a modified Laplacian matrix of the leader-following setting that allows assigning different positions that is* $\mathcal{F}(\mathcal{G}, \omega) \triangleq \mathcal{L}(\mathcal{G}, \omega) + \mathcal{K}$.

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2. Preliminary Results: Adaptation For Agent-Based Uncertainty

In this section, a discrete-time adaptive controller is designed that allows one to assign different positions for each agent in the presence of agent-based uncertainties only. To this end, consider the uncertain multiagent system in the absence of the coupled dynamics consisting of n agents given by

$$x_i(k+1) = x_i(k) + \Delta_i\sigma_i(x_i(k)) + u_i(k), \quad x_i(0) = x_{i0}, \quad i = 1, \cdots, n, \quad k \in \mathbb{N}. \tag{1}$$

Here, $x_i(k) \in \mathbb{R}$ represents the agent state, $u_i(k) \in \mathbb{R}$ represents the control input of agent i, $\Delta_i \in \mathbb{R}$ represents an unknown weight uncertainty, and $\sigma_i(x_i(k)) \in \mathbb{R}$ represents a basis function of agent i, where its bound can be represented as $\|\sigma(x_i(k))\|_2 \leq l_{ci} + l_i\|x_i(k)\|_2$, with $l_{ci} \in \mathbb{R}_+$ and $l_i \in \mathbb{R}_+$ as standard in the literature [35].

The control objective of this section is ensuring the states of the agents to track states of the reference model without getting affected by the presence of the uncertainties and able to assign nullspaces to the overall system for creating complex behaviors. Thus, consider the reference model to track given by

$$x_{r_i}(k+1) = x_{r_i}(k) - \epsilon\sum_{i\sim j}\big(\frac{\omega_j}{\omega_i}x_{r_i}(k) - x_{r_j}(k)\big) - \epsilon\kappa_i\big(x_{r_i}(k) - c(k)\big), \quad x_{r_i}(0) = x_{r_{i0}}, \quad k \in \mathbb{N}, \tag{2}$$

where $x_{r_i}(k) \in \mathbb{R}$ and $x_{r_j}(k) \in \mathbb{R}$ are the ideal reference state of agent i and agent j respectively, and $c(k) \in \mathbb{R}$ is a bounded command available only to leader agent(s) and $c(k) = v(k)\omega_i$ with a bounded $v(k) \in \mathbb{R}_+$. In (2), $\epsilon < \frac{1}{\max(\bar{d}_i)+1}$, where modified degree value, $\bar{d}_i$, ensures that the eigenvalues of $I - \epsilon\mathcal{F}(\mathcal{G}, \omega)$ remains within the unit circle.

To overcome the given tracking objective of this section, we propose the below discrete-time adaptive controller

$$u_i(k) = -\epsilon\sum_{i\sim j}\big(\frac{\omega_j}{\omega_i}x_i(k) - x_j(k)\big) - \epsilon\kappa_i\big(x_i(k) - c(k)\big) - \hat{\Delta}_i(k)\sigma_i(x_i(k)), \tag{3}$$

where $\hat{\Delta}_i(k) \in \mathbb{R}$ stands for an estimate of unknown weight uncertainty Δ_i (details below).

Then, using the proposed adaptive control law given by (3) in the uncertain multiagent system given by (1) yields

$$x_i(k+1) = x_i(k) - \epsilon\sum_{i\sim j}\big(\frac{\omega_j}{\omega_i}x_i(k) - x_j(k)\big) - \epsilon\kappa_i\big(x_i(k) - c(k)\big) - \tilde{\Delta}_i(k)\sigma_i(x_i(k)), \quad k \in \mathbb{N}. \tag{4}$$

Here, $\tilde{\Delta}_i(k) \triangleq \hat{\Delta}_i(k) - \Delta_i \in \mathbb{R}$ is the weight estimation error.

Next, the tracking error can be defined as $e_i(k) \triangleq x_i(k) - x_{r_i}(k) \in \mathbb{R}$ an error between the agent state and its reference model, where its dynamics can be rewritten as

$$e_i(k+1) = e_i(k) - \epsilon\sum_{i\sim j}\big(\frac{\omega_j}{\omega_i}e_i(k) - e_j(k)\big) - \epsilon\kappa_i e_i(k) - \tilde{\Delta}_i(k)\sigma_i(x_i(k)), \quad k \in \mathbb{N}. \tag{5}$$

Then, the error dynamics can be written in a combined form for an overall multiagent system as

$$e(k+1) = (I - \epsilon\mathcal{F})e(k) - \tilde{\Delta}^{\mathrm{T}}(k)\sigma(x(k)). \tag{6}$$

Here, $e(k) = [e_1(k), \cdots, e_n(k)]^{\mathrm{T}} \in \mathbb{R}^n$, $\sigma(x(k)) = [\sigma_1(x_1(k)), \cdots, \sigma_n(x_n(k))]^{\mathrm{T}} \in \mathbb{R}^n$, and $\tilde{\Delta}(k) = \mathrm{diag}([\tilde{\Delta}_1(k), \cdots, \tilde{\Delta}_n(k)]) \in \mathbb{R}^n$. In (6), $(I - \epsilon\mathcal{F})$ is Schur, it follows from converse Lyapunov theory [43] that there exists a unique $P \in \mathbb{P}^n$ satisfying the discrete-time Lyapunov equation given by

$$0_n = (I - \epsilon\mathcal{F})^T P(I - \epsilon\mathcal{F}) + R - P \tag{7}$$

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with $R \in \mathbb{P}^n$.

Next, the combined adaptive control inputs with a weight update law can be written as

$$u(k) = -\epsilon \mathcal{F} x(k) + \epsilon \mathcal{K} c(k) - \hat{\Delta}^{\mathrm{T}}(k)\sigma(x(k)) \quad (8)$$

$$\hat{\Delta}(k+1) = \hat{\Delta}(k) + \Big[\frac{\gamma}{1+\mu e^{\mathrm{T}}(k)Pe(k)}\big(e(k+1) - (I-\epsilon\mathcal{F})e(k)\big)\sigma^{\mathrm{T}}(x(k))\Big]^{\mathrm{T}},$$
$$\hat{\Delta}(0) \triangleq \hat{\Delta}_0, \quad k \in \mathbb{N}. \quad (9)$$

Here, $u(k) = [u_1(k), \cdots, u_n(k)]^{\mathrm{T}} \in \mathbb{R}^n$, $x(k) = [x_1(k), \cdots, x_n(k)]^{\mathrm{T}} \in \mathbb{R}^n$, $\hat{\Delta}(k) = \mathrm{diag}([\hat{\Delta}_1(k), \cdots, \hat{\Delta}_n(k)]) \in \mathbb{R}^n$, $\mu \in \mathbb{R}_+$ is a design variable, and $\gamma \in \mathbb{R}_+$ is a learning rate. Then, using (9), weight estimation error dynamics that will be used in the stability analysis of next theorem can be obtained as

$$\tilde{\Delta}(k+1) = \tilde{\Delta}(k) + \Big[\frac{\gamma}{1+\mu e^{\mathrm{T}}(k)Pe(k)}\big(e(k+1) - (I-\epsilon\mathcal{F})e(k)\big)\sigma^{\mathrm{T}}(x(k))\Big]^{\mathrm{T}},$$
$$\tilde{\Delta}(0) \triangleq \hat{\Delta}_0 - \Delta = \tilde{\Delta}_0, \quad k \in \mathbb{N}. \quad (10)$$

Theorem 1. *Consider the uncertain multiagent system given by (1) and the agent reference model given by (2), then the discrete-time adaptive control architecture given by (8) along with the weight update law given by (9) guarantees the Lyapunov stability of the closed-loop system given by (6) and (10) (i.e., boundedness of the $\big(e(k), \tilde{\Delta}(k)\big)$. Moreover, one can conclude the asymptotic tracking error convergence that is*

$$\lim_{k\to\infty} e(k) = 0. \quad (11)$$

Proof. To show the Lyapunov stability of the closed-loop system given by (6) and (10) (i.e., boundedness of the $\big(e(k), \tilde{\Delta}(k)\big)$, one can consider the Lyapunov function candidate composed of logarithmic and quadratic functions given by

$$V(e, \tilde{\Delta}) \triangleq \underbrace{\iota^{-1}\ln\left(1+\mu e^{\mathrm{T}}Pe\right)}_{V_1} + \underbrace{\alpha^{-1}\mathrm{tr}(\tilde{\Delta}^{\mathrm{T}}\tilde{\Delta})}_{V_2}, \quad (12)$$

where $\iota \in \mathbb{R}_+$, and $\alpha \in \mathbb{R}_+$. Note that $V(0,0) = 0$ and $V(e,\tilde{\Delta}) > 0$ for all $(e,\tilde{\Delta}) \neq (0,0)$.

Then, first, taking the Lyapunov difference of $V_1(\cdot)$ and using the error dynamics given by (6) yields

$$\begin{aligned}\Delta V_1 &\triangleq V_1\big(e(k+1)\big) - V_1\big(e(k)\big)\\ &= \iota^{-1}\ln\left(1+\mu e^{\mathrm{T}}(k+1)Pe(k+1)\right) - \iota^{-1}\ln\left(1+\mu e^{\mathrm{T}}(k)Pe(k)\right)\\ &= \iota^{-1}\ln\Big(1+\mu\big[(I-\epsilon\mathcal{F})e(k) - \tilde{\Delta}^{\mathrm{T}}(k)\sigma(x(k))\big]^{\mathrm{T}}P\big[(I-\epsilon\mathcal{F})e(k) - \tilde{\Delta}^{\mathrm{T}}(k)\sigma(x(k))\big]\Big)\\ &\quad -\iota^{-1}\ln\Big(1+\mu e^{\mathrm{T}}(k)Pe(k)\Big).\end{aligned} \quad (13)$$

Note that using natural logarithm property $\ln a - \ln b = \ln(a/b)$ and by adding and subtracting "$\frac{\mu e^{\mathrm{T}}(k)Pe(k)}{1+\mu e^{\mathrm{T}}(k)Pe(k)}$" to the above equality ΔV_1 can be rewritten as

$$\Delta V_1 = \iota^{-1}\ln\Bigg(1+\mu\Big(\frac{\big[(I-\epsilon\mathcal{F})e(k) - \tilde{\Delta}^{\mathrm{T}}(k)\sigma(x(k))\big]^{\mathrm{T}}P\big[(I-\epsilon\mathcal{F})e(k) - \tilde{\Delta}^{\mathrm{T}}(k)\sigma(x(k))\big]}{1+\mu e^{\mathrm{T}}(k)Pe(k)} - \frac{e^{\mathrm{T}}(k)Pe(k)}{1+\mu e^{\mathrm{T}}(k)Pe(k)}\Big)\Bigg). \quad (14)$$

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Then, using the discrete-time Lyapunov equation given by (7) and another natural logarithmic operator property given by $\ln(1+a) \leq a$ when $a \geq -1$ [44], an upper bound for (14) can be obtained as

$$\Delta V_1 \leq \frac{\iota^{-1}\mu}{1+\mu e^{\mathrm{T}}(k)Pe(k)}\Big(-e^{\mathrm{T}}(k)Re(k)+\sigma^{\mathrm{T}}(x(k))\tilde{\Delta}(k)P\tilde{\Delta}^{\mathrm{T}}(k)\sigma(x(k)) -2\sigma^{\mathrm{T}}(x(k))\tilde{\Delta}(k)P(I-\epsilon\mathcal{F})e(k)\Big). \tag{15}$$

Second, taking the Lyapunov difference of $V_2(\cdot)$ along with the weight uncertainty error dynamics given by (10) and using (6) in the dynamics yields

$$\begin{aligned}\Delta V_2 &\triangleq V_2\big(\tilde{\Delta}(k+1)\big)-V_2\big(\tilde{\Delta}(k)\big)\\ &= \alpha^{-1}\mathrm{tr}\Big(\tilde{\Delta}^{\mathrm{T}}(k+1)\tilde{\Delta}(k+1)\Big)-\alpha^{-1}\mathrm{tr}\Big(\tilde{\Delta}^{\mathrm{T}}(k)\tilde{\Delta}(k)\Big)\\ &= \alpha^{-1}\mathrm{tr}\Bigg(\Big(\tilde{\Delta}^{\mathrm{T}}(k)+\frac{\gamma}{1+\mu e^{\mathrm{T}}(k)Pe(k)}(-\tilde{\Delta}^{\mathrm{T}}(k)\sigma(x(k))\sigma^{\mathrm{T}}(x(k)))\Big)\\ &\quad\cdot\Big(\tilde{\Delta}(k)+\frac{\gamma}{1+\mu e^{\mathrm{T}}(k)Pe(k)}(-\sigma(x(k))\sigma^{\mathrm{T}}(x(k))\tilde{\Delta}(k))\Big)\Bigg)-\alpha^{-1}\mathrm{tr}\Big(\tilde{\Delta}^{\mathrm{T}}(k)\tilde{\Delta}(k)\Big).\end{aligned} \tag{16}$$

Using the trace operator property $a^{\mathrm{T}}b = \mathrm{tr}(ba^{\mathrm{T}})$ in (16) and simplifying (16) yields

$$\Delta V_2 = \frac{\alpha^{-1}\gamma}{1+\mu e^{\mathrm{T}}(k)Pe(k)}\sigma^{\mathrm{T}}(x(k))\tilde{\Delta}(k)\Big(-2+\frac{\gamma\sigma^{\mathrm{T}}(x(k))\sigma(x(k))}{1+\mu e^{\mathrm{T}}(k)Pe(k)}\Big)\tilde{\Delta}^{\mathrm{T}}(k)\sigma(x(k)), \tag{17}$$

Note that the bound for $\left\|\frac{\sigma^{\mathrm{T}}(x(k))\sigma(x(k))}{1+\mu e^{\mathrm{T}}(k)Pe(k)}\right\|_2$ with $\|\sigma(x(k))\|_2 \leq l_c + l\|x(k)\|_2$, $l_c \in \mathbb{R}_+$, and $l \in \mathbb{R}_+$ yields $\left\|\frac{\sigma^{\mathrm{T}}(x(k))\sigma(x(k))}{1+\mu e^{\mathrm{T}}(k)Pe(k)}\right\|_2 \leq \eta$ (see Appendix A for details), where $\eta \in \mathbb{R}_+$. Then setting $\gamma = \rho_\gamma\eta^{-1}$, $\rho_\gamma \in (0,1)$, and $\alpha^{-1}\gamma = \iota^{-1}\mu\rho_1$ with ρ_1 being a free variable that will be designed later, an upper bound for (17) can be written as

$$\Delta V_2 \leq \frac{\iota^{-1}\mu}{1+\mu e^{\mathrm{T}}(k)Pe(k)}\Big(\sigma^{\mathrm{T}}(x(k))\tilde{\Delta}(k)(-2\rho_1+\rho_\gamma\rho_1)\tilde{\Delta}^{\mathrm{T}}(k)\sigma(x(k))\Big). \tag{18}$$

Next, using (15) and (18) to compute $\Delta V(\cdot) \triangleq \Delta V_1(\cdot)+\Delta V_2(\cdot)$, and defining the augmented errors as $\tilde{q}^{\mathrm{T}}(k) = \Big[e^{\mathrm{T}}(k), \quad \sigma^{\mathrm{T}}(x(k))\tilde{\Delta}(k)\Big]$, the Lyapunov difference equation can be written as

$$\begin{aligned}\Delta V_1+\Delta V_2 &\leq \frac{\iota^{-1}\mu}{1+\mu e^{\mathrm{T}}(k)Pe(k)}\Big(-e^{\mathrm{T}}(k)Re(k)+\sigma^{\mathrm{T}}(x(k))\tilde{\Delta}(k)[P-2\rho_1+\rho_\gamma\rho_1]\tilde{\Delta}^{\mathrm{T}}(k)\sigma(x(k))\\ &\qquad -2\sigma^{\mathrm{T}}(x(k))\tilde{\Delta}(k)P(I-\epsilon\mathcal{F})e(k)\Big)\\ &= \frac{\iota^{-1}\mu}{1+\mu e^{\mathrm{T}}(k)Pe(k)}\Big(e^{\mathrm{T}}(k)[-R+\beta(I-\epsilon\mathcal{F})^{T}P(I-\epsilon\mathcal{F})]e(k)\\ &\qquad +\sigma^{\mathrm{T}}(x(k))\tilde{\Delta}(k)[P+\frac{1}{\beta}P-2\rho_1+\rho_\gamma\rho_1]\tilde{\Delta}^{\mathrm{T}}(k)\sigma(x(k))\Big)\\ &\quad -\frac{\iota^{-1}\mu}{1+\mu e^{\mathrm{T}}(k)Pe(k)}\tilde{q}^{\mathrm{T}}(k)\underbrace{\begin{bmatrix}\beta(I-\epsilon\mathcal{F})^{\mathrm{T}}P(I-\epsilon\mathcal{F}) & (P(I-\epsilon\mathcal{F}))^{\mathrm{T}}\\ P(I-\epsilon\mathcal{F}) & \frac{1}{\beta}P\end{bmatrix}}_{M\geq 0}\tilde{q}(k).\end{aligned} \tag{19}$$

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Note that M is a positive semi-definite matrix, and with a small value of $\beta \in \mathbb{R}_+$, one can satisfy $-\overline{R} = -R + \beta(I - \epsilon\mathcal{F})^T P(I - \epsilon\mathcal{F}) < 0$. Then, taking an upper bound of (19) yields

$$\Delta V \leq \frac{\iota^{-1}\mu}{1+\mu e^{\mathrm{T}}(k)Pe(k)}\Big(-e^{\mathrm{T}}(k)\overline{R}e(k) -\sigma^{\mathrm{T}}(x(k))\tilde{\Delta}(k)\underbrace{[\rho_1(2-\rho_\gamma)-(1+\frac{1}{\beta})P]}_{N>0}\tilde{\Delta}^{\mathrm{T}}(k)\sigma(x(k))\Big). \tag{20}$$

Note that N is positive definite (see Appendix B for details); hence, an upper bound for (20) can be written as

$$\Delta V \leq \frac{l_0^{-1}\mu}{1+\mu e^{\mathrm{T}}(k)Pe(k)}\Big(-e^{\mathrm{T}}(k)\overline{R}e(k)\Big), \tag{21}$$

which proves the boundedness of the $\big(e(k), \tilde{\Delta}(k)\big)$. It then follows from [Theorem 13.10, [43]] that $\lim_{k\to\infty}(e(k)) = 0$. $\square$

3. Adaptation For Both Agent-Based Uncertainty and Coupled Dynamics

In this section, a discrete-time adaptive controller is designed that allows one to assign different positions for each agent in the presence of agent-based uncertainties and coupled dynamics. Specifically, consider a multiagent system consisting of n agents given by

$$x_i(k+1) = x_i(k) + \Delta_i\sigma_i(x_i(k)) + p_{ui}(k) + u_i(k), \quad x_i(0) = x_{i0}, \tag{22}$$
$$\xi_i(k+1) = f_{ui}\xi_i(k) + g_{ui}x_i(k), \tag{23}$$
$$p_{ui}(k) = h_{ui}\xi_i(k) \quad i = 1,\cdots,n \quad k \in \mathbb{N}. \tag{24}$$

Here, $p_{ui}(k) \in \mathbb{R}$ is the output of the coupled dynamics, $\xi_i(k) \in \mathbb{R}$ is the state of the coupled dynamics, and $f_{ui} \in \mathbb{R}$, $g_{ui} \in \mathbb{R}$, and $h_{ui} \in \mathbb{R}$ are variables related to coupled dynamics, where $f_{ui} \in (-1,1)$ that is standard consideration in the literature [16].

The objective here is to achieve asymptotic convergence of the tracking error in the presence of not only agent-based uncertainties but also coupled dynamics. To this end, an observer dynamics is used to estimate the state of the unmeasurable coupled dynamics. Specifically, the adaptive controller is now designed as

$$u_i(k) = -\epsilon\sum_{i\sim j}\Big(\frac{\omega_j}{\omega_i}x_i(k) - x_j(k)\Big) - \epsilon\kappa_i\big(x_i(k) - c(k)\big) - \hat{\Delta}_i(k)\sigma_i(x_i(k)) - h_{ui}\hat{\xi}_i(k), \tag{25}$$

where $\hat{\xi}_i(k) \in \mathbb{R}$ stands for an estimate of the coupled dynamics of agent i with the estimation dynamics given by

$$\hat{\xi}_i(k+1) = f_{ui}\hat{\xi}_i(k) + g_{ui}x_i(k), \quad \hat{\xi}(0) = \hat{\xi}_0, \quad k \in \mathbb{N}, \tag{26}$$
$$\hat{p}_{ui}(k) = h_{ui}\hat{\xi}_i(k). \tag{27}$$

Here, $\hat{p}_{ui}(k) \in \mathbb{R}$ is the estimated output of the coupled dynamics.

Then, using the proposed adaptive control law given by (25) in the uncertain multiagent system with coupled dynamics given by (22) yields

$$x_i(k+1) = x_i(k) - \epsilon\sum_{i\sim j}\Big(\frac{\omega_j}{\omega_i}x_i(k) - x_j(k)\Big) - \epsilon\kappa_i\big(x_i(k) - c(k)\big) - \tilde{\Delta}_i(k)\sigma_i(x_i(k)) - h_{ui}\tilde{\xi}(k). \tag{28}$$

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Here, $\tilde{\Delta}_i(k) \triangleq \hat{\Delta}_i(k) - \Delta_i \in \mathbb{R}$ is the weight estimation error, and $\tilde{\xi}_i(k) \triangleq \hat{\xi}_i(k) - \xi(k) \in \mathbb{R}$ coupled dynamics estimation error.

Next, the tracking error dynamics can be rewritten as

$$e_i(k+1) = e_i(k) - \epsilon \sum_{i\sim j}(\frac{\omega_j}{\omega_i} e_i(k) - e_j(k)) - \epsilon\kappa_i e_i(k) - \tilde{\Delta}_i(k)\sigma_i(x_i(k)) - h_{ui}\tilde{\xi}(k), \quad k \in \mathbb{N}. \quad (29)$$

Then, the error dynamics can be written in a combined form for an overall multiagent system as

$$e(k+1) = (I - \epsilon\mathcal{F})e(k) - \tilde{\Delta}^{\mathrm{T}}(k)\sigma(x(k)) - H\tilde{\xi}(k). \quad (30)$$

Here, $\tilde{\xi}(k) = [\tilde{\xi}_1(k), \cdots, \tilde{\xi}_n(k)]^{\mathrm{T}} \triangleq \hat{\xi}(k) - \xi(k) \in \mathbb{R}^n$ is the coupled dynamics combined observer error with $\xi(k) = [\xi_1(k), \cdots, \xi_n(k)]^{\mathrm{T}} \in \mathbb{R}^n$ and $\hat{\xi}(k) = [\hat{\xi}_1(k), \cdots, \hat{\xi}_n(k)]^{\mathrm{T}} \in \mathbb{R}^n$. In addition, in (30), $H = \mathrm{diag}([h_{u1}, \cdots, h_{un}]) \in \mathbb{R}^{n\times n}$.

Next, the combined adaptive control input can be written as

$$u(k) = -\epsilon\mathcal{F}x(k) + \epsilon\mathcal{K}c(k) - \hat{\Delta}^{\mathrm{T}}(k)\sigma(x(k)) - H\hat{\xi}(k) \quad (31)$$

with the same augmented weight update law given in (9) and the below augmented observer dynamics

$$\hat{\xi}(k+1) = F\hat{\xi}(k) + Gx(k), \quad \hat{\xi}(0) = \hat{\xi}_0, \quad k \in \mathbb{N}, \quad (32)$$

$$\hat{p}_u(k) = H\hat{\xi}(k), \quad (33)$$

where $F = \mathrm{diag}([f_{u1}, \cdots, f_{un}]) \in \mathbb{R}^{n\times n}$, $G = \mathrm{diag}([g_{u1}, \cdots, g_{un}]) \in \mathbb{R}^{n\times n}$, and $\hat{p}_u(k) = [\hat{p}_{u1}(k), \cdots, \hat{p}_{un}(k)]^{\mathrm{T}} \in \mathbb{R}^n$. In (32), F is Schur, it follows from converse Lyapunov theory [43] that there exists a unique $S \in \mathbb{P}^n$ satisfying the discrete-time Lyapunov equation given by

$$0_n = F^{\mathrm{T}}SF + R_F - S \quad (34)$$

with $R_F \in \mathbb{P}^n$.

Finally, weight estimation error dynamics that will be used in the stability analysis of next theorem satisfies the dynamics given by

$$\tilde{\xi}(k+1) = F\tilde{\xi}(k), \quad \tilde{\xi}(0) = \tilde{\xi}_0, \quad k \in \mathbb{N}. \quad (35)$$

Theorem 2. *Consider the uncertain agent system given by (22) subject to the unmeasurable coupled dynamics given by (23) and (24), and the agent reference model given by (2), then the discrete-time adaptive control architecture given by (31) along with the weight update law given by (9) and the observer dynamics given by (26) and (27) guarantees the Lyapunov stability of the closed-loop system given by (30), (10), and (35) (i.e., boundedness of the $(e(k), \tilde{\Delta}(k), \tilde{\xi}(k))$. Moreover, one can conclude the asymptotic tracking error convergence that is*

$$\lim_{k\to\infty} e(k) = 0. \quad (36)$$

Proof. To show the Lyapunov stability of the closed-loop system given by (30), (10), and (35) (i.e., boundedness of the $(e(k), \tilde{\Delta}(k), \tilde{\xi}(k))$, one can consider the Lyapunov function candidate composed of logarithmic and quadratic functions given by

$$V(e, \tilde{\Delta}, \tilde{\xi}) \triangleq \underbrace{\iota^{-1}\ln\left(1 + \mu e^{\mathrm{T}}Pe\right)}_{V_1} + \underbrace{\alpha^{-1}\mathrm{tr}(\tilde{\Delta}^{\mathrm{T}}\tilde{\Delta})}_{V_2} + \underbrace{\nu^{-1}\tilde{\xi}^{\mathrm{T}}S\tilde{\xi}}_{V_3}, \quad (37)$$

where $\nu \in \mathbb{R}_+$. Note that $V(0,0,0) = 0$ and $V(e, \tilde{\Delta}, \tilde{\xi}) > 0$ for all $(e, \tilde{\Delta}, \tilde{\xi}) \neq (0,0,0)$.

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Then, first, taking the Lyapunov difference of $V_1(\cdot)$ and using the error dynamics given by (30) yields

$$\begin{aligned}\Delta V_1 &\triangleq V_1(e(k+1)) - V_1(e(k)) \\ &= \iota^{-1}\ln\left(1+\mu e^{\mathrm{T}}(k+1)Pe(k+1)\right) - \iota^{-1}\ln\left(1+\mu e^{\mathrm{T}}(k)Pe(k)\right) \\ &= \iota^{-1}\ln\Big(1+\mu\big[(I-\epsilon\mathcal{F})e(k) - \tilde{\Delta}^{\mathrm{T}}(k)\sigma(x(k)) - H\tilde{\xi}(k)\big]^{\mathrm{T}} \\ &\qquad \cdot P\big[(I-\epsilon\mathcal{F})e(k) - \tilde{\Delta}^{\mathrm{T}}(k)\sigma(x(k)) - H\tilde{\xi}(k)\big]\Big) - \iota^{-1}\ln\Big(1+\mu e^{\mathrm{T}}(k)Pe(k)\Big). \end{aligned} \tag{38}$$

Note that using natural logarithm property $\ln a - \ln b = \ln(a/b)$ and by adding and subtracting "$\frac{\mu e^{\mathrm{T}}(k)Pe(k)}{1+\mu e^{\mathrm{T}}(k)Pe(k)}$" to the above equality ΔV_1 can be rewritten as

$$\begin{aligned}\Delta V_1 &= \iota^{-1}\ln\Bigg(1+\mu\bigg(\big[(I-\epsilon\mathcal{F})e(k) - \tilde{\Delta}^{\mathrm{T}}(k)\sigma(x(k)) - H\tilde{\xi}(k)\big]^{\mathrm{T}} \\ &\qquad \cdot\frac{P\big[(I-\epsilon\mathcal{F})e(k) - \tilde{\Delta}^{\mathrm{T}}(k)\sigma(x(k)) - H\tilde{\xi}(k)\big]}{1+\mu e^{\mathrm{T}}(k)Pe(k)} - \frac{e^{\mathrm{T}}(k)Pe(k)}{1+\mu e^{\mathrm{T}}(k)Pe(k)}\bigg)\Bigg). \end{aligned} \tag{39}$$

using another natural logarithmic operator property $\ln(1+a) \le a$ when $a \ge -1$ [44], an upper bound for (39) can be obtained as

$$\begin{aligned}\Delta V_1 &\le \frac{\iota^{-1}\mu}{1+\mu e^{\mathrm{T}}(k)Pe(k)}\Big(-e^{\mathrm{T}}(k)Re(k) + \sigma^{\mathrm{T}}(x(k))\tilde{\Delta}(k)P\tilde{\Delta}^{\mathrm{T}}(k)\sigma(x(k)) + \tilde{\xi}^{\mathrm{T}}(k)H^{\mathrm{T}}PH\tilde{\xi}(k) \\ &\quad -2\sigma^{\mathrm{T}}(x(k))\tilde{\Delta}(k)P(I-\epsilon\mathcal{F})e(k) - 2\tilde{\xi}^{\mathrm{T}}(k)H^{\mathrm{T}}P(I-\epsilon\mathcal{F})e(k) + 2\sigma^{\mathrm{T}}(x(k))\tilde{\Delta}(k)PH\tilde{\xi}(k)\Big), \end{aligned} \tag{40}$$

where "$-R \triangleq (I-\epsilon\mathcal{F})^T P(I-\epsilon\mathcal{F}) - P \in \mathbb{R}_-^{n\times n}$" given by (7) is used.

Second, taking the Lyapunov difference of $V_2(\cdot)$ along with the weight uncertainty error dynamics given by (10) and using (30) in the dynamics yields

$$\begin{aligned}\Delta V_2 &\triangleq V_2(\tilde{\Delta}(k+1)) - V_2(\tilde{\Delta}(k)) \\ &= \alpha^{-1}\mathrm{tr}\Big(\tilde{\Delta}^{\mathrm{T}}(k+1)\tilde{\Delta}(k+1)\Big) - \alpha^{-1}\mathrm{tr}\Big(\tilde{\Delta}^{\mathrm{T}}(k)\tilde{\Delta}(k)\Big) \\ &= \alpha^{-1}\mathrm{tr}\Bigg(\Big(\tilde{\Delta}^{\mathrm{T}}(k) + \frac{\gamma}{1+\mu e^{\mathrm{T}}(k)Pe(k)}(-\tilde{\Delta}^{\mathrm{T}}(k)\sigma(x(k))\sigma^{\mathrm{T}}(x(k)) - H\tilde{\xi}(k)\sigma^{\mathrm{T}}(x(k)))\Big) \\ &\qquad \cdot\Big(\tilde{\Delta}(k) + \frac{\gamma}{1+\mu e^{\mathrm{T}}(k)Pe(k)}(-\sigma(x(k))\sigma^{\mathrm{T}}(x(k))\tilde{\Delta}(k) - \sigma(x(k))\tilde{\xi}^{\mathrm{T}}(k)H^{\mathrm{T}})\Big)\Bigg) \\ &\qquad -\alpha^{-1}\mathrm{tr}\Big(\tilde{\Delta}^{\mathrm{T}}(k)\tilde{\Delta}(k)\Big). \end{aligned} \tag{41}$$

Then using the trace operator property $a^{\mathrm{T}}b = \mathrm{tr}(ba^{\mathrm{T}})$ and simplifying (41) yields

$$\begin{aligned}\Delta V_2 &= \frac{\alpha^{-1}\gamma}{1+\mu e^{\mathrm{T}}(k)Pe(k)}\sigma^{\mathrm{T}}(x(k))\tilde{\Delta}(k)\Big(-2+\frac{\gamma\sigma^{\mathrm{T}}(x(k))\sigma(x(k))}{1+\mu e^{\mathrm{T}}(k)Pe(k)}\Big)\tilde{\Delta}^{\mathrm{T}}(k)\sigma(x(k)) \\ &\quad +\frac{\alpha^{-1}\gamma^2\sigma^{\mathrm{T}}(x(k))\sigma(x(k))}{(1+\mu e^{\mathrm{T}}(k)Pe(k))^2}\tilde{\xi}^{\mathrm{T}}(k)H^{\mathrm{T}}H\tilde{\xi}(k) \\ &\quad +\frac{\alpha^{-1}\gamma}{1+\mu e^{\mathrm{T}}(k)Pe(k)}\tilde{\xi}^{\mathrm{T}}(k)H^{\mathrm{T}}\Big(-2+2\frac{\gamma\sigma^{\mathrm{T}}(x(k))\sigma(x(k))}{1+\mu e^{\mathrm{T}}(k)Pe(k)}\Big)\tilde{\Delta}^{\mathrm{T}}(k)\sigma(x(k)). \end{aligned} \tag{42}$$

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Note that using $\left\| \frac{\sigma^{\mathrm{T}}(x(k))\sigma(x(k))}{1+\mu e^{\mathrm{T}}(k)Pe(k)} \right\|_2 \leq \eta$, and setting $\gamma = \rho_\gamma \eta^{-1}$ and $\alpha^{-1} = \iota^{-1}\mu\rho_1\gamma^{-1}$ an upper bound for (42) can be written as

$$\begin{aligned} \Delta V_2 \leq\ & \frac{\iota^{-1}\mu}{1+\mu e^{\mathrm{T}}(k)Pe(k)} \Big(\sigma^{\mathrm{T}}(x(k))\tilde{\Delta}(k)\big(-2\rho_1 + \rho_\gamma\rho_1\big)\tilde{\Delta}^{\mathrm{T}}(k)\sigma(x(k)) + \rho_\gamma \tilde{\xi}^{\mathrm{T}}(k)H^{\mathrm{T}}H\tilde{\xi}(k) \Big) \\ & + \frac{\iota^{-1}\mu}{1+\mu e^{\mathrm{T}}(k)Pe(k)} \tilde{\xi}^{\mathrm{T}}(k)H^{\mathrm{T}}\Big(-2\rho_1 + 2\gamma\eta\rho_1\Big)\tilde{\Delta}^{\mathrm{T}}(k)\sigma(x(k)). \end{aligned} \tag{43}$$

Third, taking the Lyapunov difference of $V_3(\cdot)$ and using the error in observer dynamics (35) yields

$$\begin{aligned} \Delta V_3 &\triangleq V_3\big(\tilde{\xi}(k+1)\big) - V_3\big(\tilde{\xi}(k)\big) \\ &= \nu^{-1}\tilde{\xi}^{\mathrm{T}}(k+1)S\tilde{\xi}(k+1) - \nu^{-1}\tilde{\xi}^{\mathrm{T}}(k)S\tilde{\xi}(k) \\ &= \nu^{-1}\tilde{\xi}^{\mathrm{T}}(k)F^{\mathrm{T}}SF\tilde{\xi}(k) - \nu^{-1}\tilde{\xi}^{\mathrm{T}}(k)S\tilde{\xi}(k) \\ &= -\nu^{-1}\tilde{\xi}^{\mathrm{T}}(k)(S - F^{\mathrm{T}}SF)\tilde{\xi}(k) \\ &= -\nu^{-1}\tilde{\xi}^{\mathrm{T}}(k)R_F\tilde{\xi}(k). \end{aligned} \tag{44}$$

Setting $R_F = H^{\mathrm{T}}R_H H$ with $R_H \in \mathbb{P}^n$, and $\nu = \iota\mu^{-1}\rho_2^{-1}$ with ρ_2 being a free variable that will be designed later, an upper bound for (44) can be written as

$$\Delta V_3 \leq -\frac{\iota^{-1}\mu}{1+\mu e^{\mathrm{T}}(k)Pe(k)} \tilde{\xi}^{\mathrm{T}}(k)H^{\mathrm{T}}\rho_2 R_H H\tilde{\xi}(k). \tag{45}$$

Then, using (40), (43), and (45) to compute $\Delta V(\cdot) \triangleq \Delta V_1(\cdot) + \Delta V_2(\cdot) + \Delta V_3(\cdot)$, and defining $\tilde{q}_1^{\mathrm{T}}(k) = \Big[e^{\mathrm{T}}(k), \quad \sigma^{\mathrm{T}}(x(k))\tilde{\Delta}(k), \quad \tilde{\xi}^{\mathrm{T}}(k)H^{\mathrm{T}} \Big]$, the combined Lyapunov difference equation can be written as

$$\begin{aligned} & \Delta V_1 + \Delta V_2 + \Delta V_3 \\ & \leq \frac{\iota^{-1}\mu}{1+\mu e^{\mathrm{T}}(k)Pe(k)} \Big(-e^{\mathrm{T}}(k)Re(k) + \sigma^{\mathrm{T}}(x(k))\tilde{\Delta}(k)\big[P - 2\rho_1 + \rho_\gamma\rho_1\big]\tilde{\Delta}^{\mathrm{T}}(k)\sigma(x(k)) \\ & \quad + \tilde{\xi}^{\mathrm{T}}(k)H^{\mathrm{T}}\big[P - R_H\rho_2 + \rho_\gamma\rho_1\big]H\tilde{\xi}(k) - 2\sigma^{\mathrm{T}}(x(k))\tilde{\Delta}(k)P(I - \epsilon\mathcal{F})e(k)\Big) \\ & \quad - 2\tilde{\xi}^{\mathrm{T}}(k)H^{\mathrm{T}}P(I - \epsilon\mathcal{F})e(k) + 2\sigma^{\mathrm{T}}(x(k))\tilde{\Delta}(k)\big[P - \rho_1\big]H\tilde{\xi}(k) + \sigma^{\mathrm{T}}(x(k))\tilde{\Delta}(k)\big[2\rho_1\gamma\eta\big]H\tilde{\xi}(k)\Big) \\ & = \frac{\iota^{-1}\mu}{1+\mu e^{\mathrm{T}}(k)Pe(k)} \Big(e^{\mathrm{T}}(k)[-R + \beta(I - \epsilon\mathcal{F})^T P(I - \epsilon\mathcal{F})]e(k) \\ & \quad + \sigma^{\mathrm{T}}(x(k))\tilde{\Delta}(k)\big[P + \frac{1}{\beta}P - 2\rho_1 + \rho_\gamma\rho_1\big]\tilde{\Delta}^{\mathrm{T}}(k)\sigma(x(k)) \\ & \quad + \tilde{\xi}^{\mathrm{T}}(k)H^{\mathrm{T}}\big[2P - R_H\rho_2 + \rho_\gamma\rho_1\big]H\tilde{\xi}(k) + \sigma^{\mathrm{T}}(x(k))\tilde{\Delta}(k)\big[2\rho_1\gamma\eta\big]H\tilde{\xi}(k)\Big) \\ & \quad - \frac{\iota^{-1}\mu}{1+\mu e^{\mathrm{T}}(k)Pe(k)} \tilde{q}_1^{\mathrm{T}}(k) \underbrace{\begin{bmatrix} \beta(I - \epsilon\mathcal{F})^T P(I - \epsilon\mathcal{F}) & (P(I - \epsilon\mathcal{F}))^{\mathrm{T}} & (P(I - \epsilon\mathcal{F}))^{\mathrm{T}} \\ P(I - \epsilon\mathcal{F}) & \frac{1}{\beta}P & (-P + \rho_1)^{\mathrm{T}} \\ P(I - \epsilon\mathcal{F}) & -P + \rho_1 & P \end{bmatrix}}_{\bar{F} \geq 0} \tilde{q}_1(k). \end{aligned} \tag{46}$$

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Note that setting $\rho_1 = (1+\frac{1}{\beta})P \in \mathbb{P}^n$ one can show the $\bar{F}$ matrix is positive semi-definite as shown in Appendix C. Then using $-\overline{R} = -R + \beta(I-\epsilon\mathcal{F})^T P(I-\epsilon\mathcal{F}) < 0$, and taking an upper bound of (46) yields

$$\begin{aligned}\Delta V \;\leq\;& \frac{\iota^{-1}\mu}{1+\mu e^{\mathrm{T}}(k)Pe(k)}\Big(-e^{\mathrm{T}}(k)\overline{R}e(k) - \sigma^{\mathrm{T}}(x(k))\tilde{\Delta}(k)\underbrace{\big[(1+\frac{1}{\beta})P-(1+\frac{1}{\beta})\rho_\gamma P\big]}_{\rho_3}\tilde{\Delta}^{\mathrm{T}}(k)\sigma(x(k)) \\ & -\tilde{\xi}^{\mathrm{T}}(k)H^{\mathrm{T}}\underbrace{\big[-2P+R_H\rho_2-\rho_\gamma\rho_1\big]}_{\rho_4}H\tilde{\xi}(k)+\sigma^{\mathrm{T}}(x(k))\tilde{\Delta}(k)\big[2\rho_1\gamma\eta\big]H\tilde{\xi}(k)\Big), \\ \leq\;& \frac{\iota^{-1}\mu}{1+\mu e^{\mathrm{T}}(k)Pe(k)}\Big(-e^{\mathrm{T}}(k)\overline{R}e(k)\Big) - \frac{\iota^{-1}\mu}{1+\mu e^{\mathrm{T}}(k)Pe(k)}\tilde{q}_2^{\mathrm{T}}(k)\underbrace{\begin{bmatrix}\rho_3 & -\rho_1\rho_\gamma \\ -\rho_1\rho_\gamma & \rho_4\end{bmatrix}}_{\rho_5\geq 0}\tilde{q}_2(k),\end{aligned} \tag{47}$$

where $\tilde{q}_2^{\mathrm{T}}(k) = \begin{bmatrix}\sigma^{\mathrm{T}}(x(k))\tilde{\Delta}(k) & \tilde{\xi}^{\mathrm{T}}(k)H^{\mathrm{T}}\end{bmatrix}$. In (47), $\gamma = \rho_\gamma\eta^{-1}$ is used. One can then show positive definiteness of ρ_5 as shown in Appendix D. Finally, one can obtain an upper bound for (47) that is

$$\Delta V \;\leq\; \frac{l_0^{-1}\mu}{1+\mu e^{\mathrm{T}}(k)Pe(k)}\Big(-e^{\mathrm{T}}(k)\overline{R}e(k)\Big), \tag{48}$$

which proves the boundedness of the $\big(e(k), \tilde{\Delta}(k), \tilde{\xi}(k)\big)$. It then follows from [Theorem 13.10, [43]] that $\lim_{k\to\infty}(e(k)) = 0$. □

Remark 1. *The given proposed discrete-time adaptive control architecture can be sequentially executed in embedded code. Table 2 presents one possible sequential operation of this architecture. Note that no knowledge of any signal from step $(k+1)$ is required to execute the architecture.*

Table 2. Sequential Operation

Initial Execution: k=0
1: Using $x_r(0)$, and $c(0)$, calculate $x_r(1)$
2: Using $x(0)$, $c(0)$, $\hat{\Delta}(0)$, and $\hat{\xi}(0)$, calculate $u(0)$
3: Apply $u(0)$ to get $x(1)$ from the physical system
Repetitive Execution: k≥1
4: Using $x(k-1)$ and $x_r(k-1)$, calculate $e(k-1)$
5: Using $x(k)$ and $x_r(k)$, calculate $e(k)$
6: Using $e(k-1)$, $e(k)$, $x(k-1)$, and $\hat{\Delta}(k-1)$, calculate $\hat{\Delta}(k)$
7: Using $\hat{\xi}(k-1)$, and $x(k-1)$, calculate $\hat{\xi}(k)$
8: Using $x_r(k)$, and $c(k)$, calculate $x_r(k+1)$
9: Using $x(k)$, $c(k)$, $\hat{\Delta}(k)$, and $\hat{\xi}(k)$, calculate $u(k)$
10: Apply $u(k)$ to get $x(k+1)$ from the system

4. Illustrative Numerical Examples

In order to illustrate the efficacy of the proposed discrete-time control architecture, consider a group of 5 agents on a line graph with the third agent being a leader. See Figure 1 for the graph formation chosen for the illustrative numerical examples of this paper. For the simulations, we select the uncertain weights as $\Delta = [0.075, 0.05, 0.015, 0.0375, 0.0606]$ and the coupled dynamics matrices as $f_{ui} = [-0.1, -0.05, -0.02, -0.15, -0.2]$, $g_{ui} = [0.6, 0.5, 0.4, 0.3, 0.2]$, and $h_{ui} = [1.3, 1.2, 1, 0.9, 0.7]$.

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Figure 1. Directed graph representation for an undirected line graph with five agents, where the third agent is the leader.

Here, we choose the nullspace $\omega = [1;2;3;2;1]$ and use the command $c(k) = v(k)w_3 = 1.5$ (i.e., $v(k) = 0.5$) such that $\lim_{k\to\infty} x(k) = v(k)\omega$. Thus, the expectation for convergences is that the first agent should converge to 0.5, the second agent should converge to 1, the third (the leader) agent should converge to 1.5, the fourth agent should converge to 1, and the fifth agent should converge to 0.5.

Regarding the update law, we calculated weight update law with $\gamma = 0.43$ and $\mu = 1$, where the known basis functions are selected as $\sigma_1(x_1(k)) = x_1^2(k)$, $\sigma_2(x_2(k)) = \cos x_2(k)$, $\sigma_3(x_3(k)) = x_3(k)$, $\sigma_4(x_4(k)) = \sin x_4(k)$, and $\sigma_5(x_5(k)) = x_5^3(k)$. We set the time step Δt to 0.2, the control parameter ϵ to $1/6$, and Lyapunov equation matrices R to $0.1I$, and R_F to $2I$.

For motivating the necessity of the controller that is proposed in Section 3, we first simulated the uncertain multiagent system in the presence of the unmeasurable coupled dynamics with the proposed controller of Section 2. Figures 2, 3, and 4 show the closed-loop system response with the proposed discrete-time adaptive control architecture given in Section 2. These are tracking response, control input, and unknown weight estimates, respectively. As you see from Figures 2, 3, and 4, convergence to the assigned position cannot be achieved.

Figures 5, 6, 7, and 8 then show the closed-loop system response, control input, unknown weights estimates, and observer state, respectively, for each agent in the presence of the agent-based uncertainty and the coupled dynamics with the proposed discrete-time adaptive control architecture given in Section 3, where the effects of system uncertainties and coupled dynamics are suppressed successfully while achieving the convergence to the assigned position.

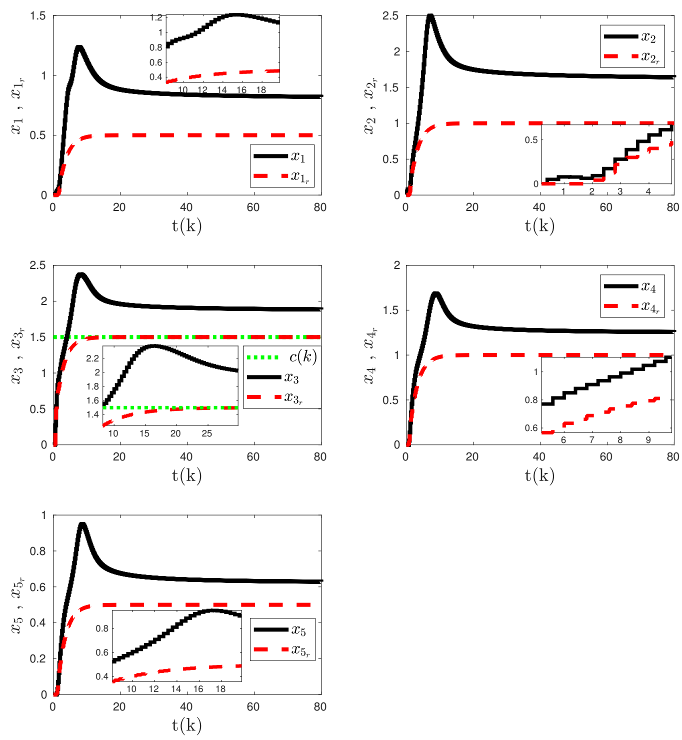

Figure 2. Uncertain multiagent system response in the presence of coupled dynamics with the proposed discrete-time adaptive control method given in Section 2.

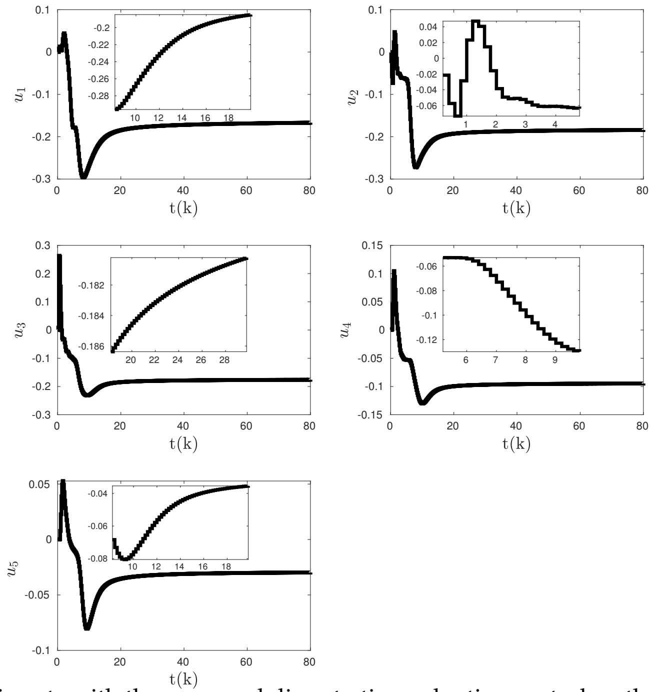

Figure 3. Control inputs with the proposed discrete-time adaptive control method given in Section 2.

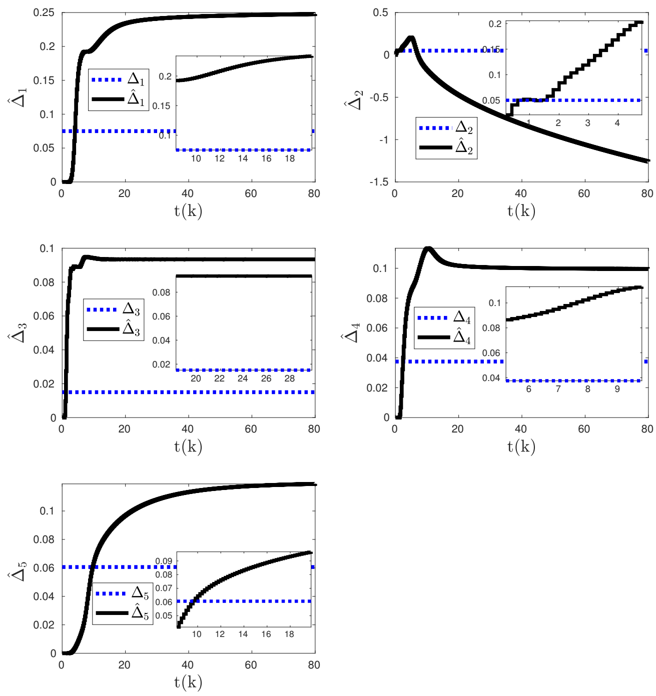

Figure 4. Agent-based uncertainty estimations of the proposed discrete-time adaptive control method given in Section 2.

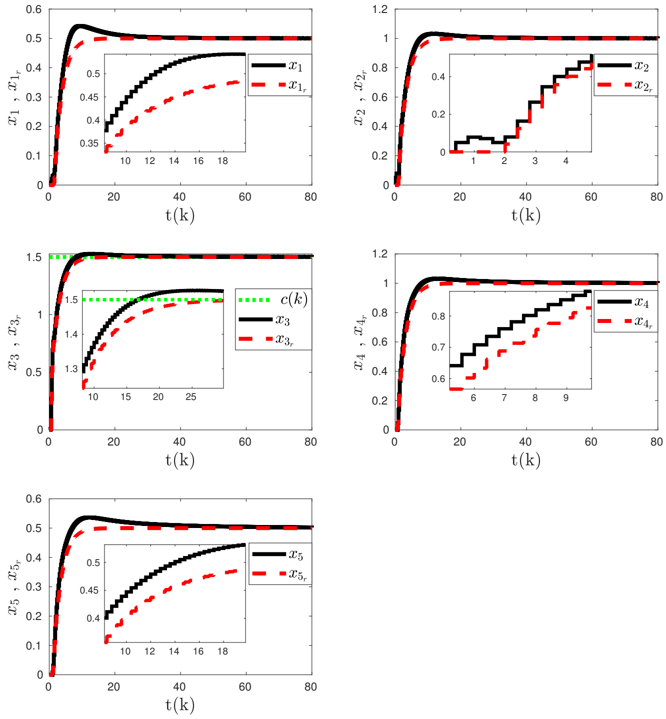

Figure 5. Uncertain multiagent system response in the presence of coupled dynamics with the proposed discrete-time adaptive control method given in Section 3.

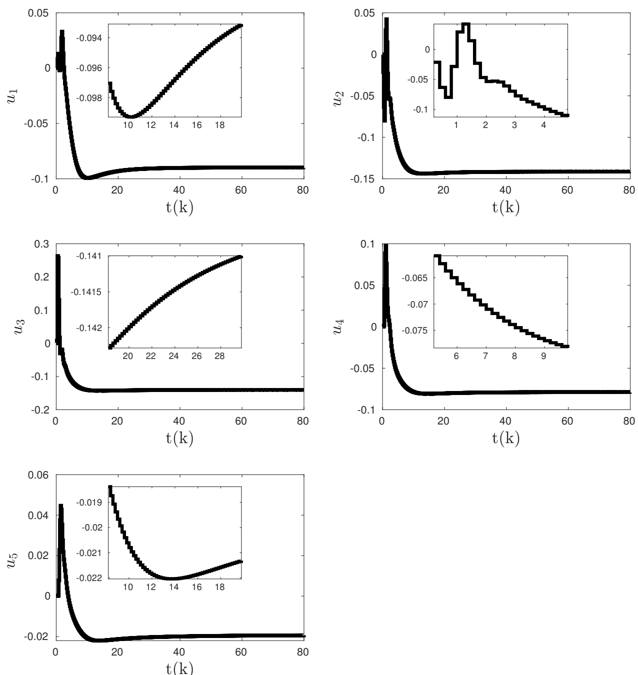

Figure 6. Control inputs with the proposed discrete-time adaptive control method given in Section 3.

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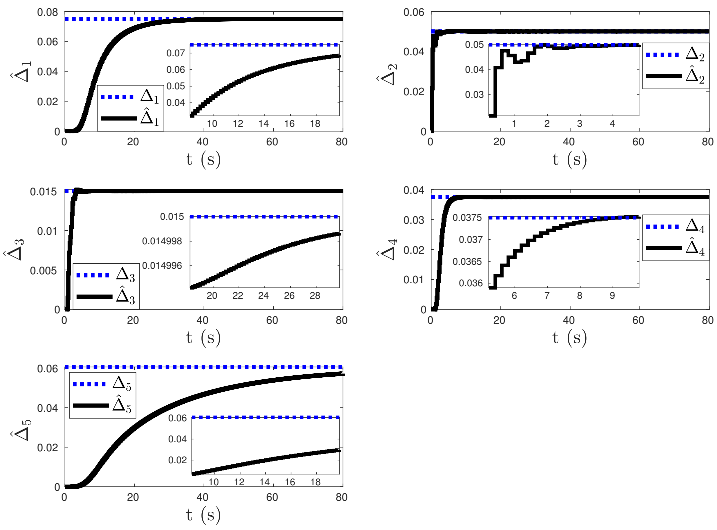

Figure 7. Agent-based uncertainty estimations of the proposed discrete-time adaptive control method given in Section 3.

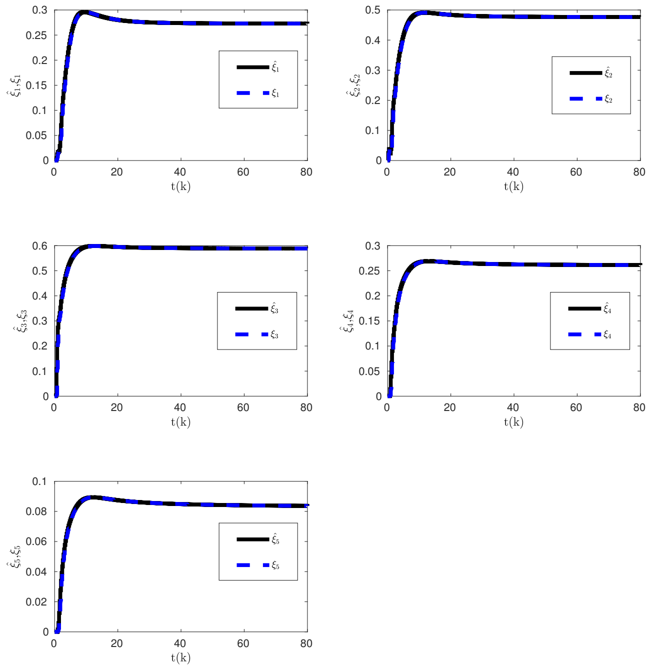

Figure 8. Agent-based coupled dynamics of the proposed discrete-time adaptive control method given in Section 3.

5. Conclusion

This paper addresses the challenges and complexities associated with discrete-time architectures in the context of multiagent systems with uncertain scalar dynamics and coupled interactions. The paper introduces discrete-time adaptive control architecture with observer dynamics for managing unmeasurable coupled dynamics. Additionally, a user-assigned Laplacian matrix is incorporated to induce cooperative behaviors among multiple agents. The proposed control architecture is accompanied by Lyapunov analysis employing logarithmic and quadratic Lyapunov functions to guarantee asymptotic stability. Through an illustrative example, the paper demonstrates the effectiveness of the introduced control architecture, showcasing its ability to address uncertainties and

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coupled dynamics in multiagent systems. These contributions open avenues for further research in the development and application of adaptive control strategies in discrete-time scenarios. The future research direction can include adding actuator dynamics and unknown control degradation to the multiagent system model as well as experimentally validating the theoretical results of this paper with the system with multiple robots.

Appendix A

In this appendix, the upper bound for $\left\| \frac{\sigma^{\mathrm{T}}(x(k))\sigma(x(k))}{1+\mu e^{\mathrm{T}}(k)Pe(k)} \right\|_2 \leq \eta$ is obtained. Given that $\|x(k)\|_2 \leq \|e(k)\|_2 + x_{\mathrm{r}}^*$, $\|\sigma(x(k))\|_2 \leq l_c + l\|x(k)\|_2$, and setting $d^* = l_c + l x_{\mathrm{r}}^*$ then the bound on $\sigma^{\mathrm{T}}(x(k))\sigma(x(k))$ can be written as

$$\begin{aligned}
\|\sigma^{\mathrm{T}}(x(k))\sigma(x(k))\|_2 &\leq \left(l_c + l\|x(k)\|_2\right)^2 \\
&= l_c^2 + 2ll_c(\|e(k)\|_2 + x_{\mathrm{r}}^*) + l^2\|x(k)\|_2^2 \\
&= l_c^2 + 2ll_c(\|e(k)\|_2 + x_{\mathrm{r}}^*) + l^2(\|e(k)\|_2^2 + 2x_{\mathrm{r}}^*\|e(k)\|_2 + x_{\mathrm{r}}^{*2}) \\
&= l_c^2 + 2l(l_c + l x_{\mathrm{r}}^*)\|e(k)\|_2 + 2l(l_c + l x_{\mathrm{r}}^*)x_{\mathrm{r}}^* - l^2 x_{\mathrm{r}}^{*2} + l^2\|e(k)\|_2^2 \\
&= l_c^2 + 2ld^*\|e(k)\|_2 + 2ld^* x_{\mathrm{r}}^* - l^2 x_{\mathrm{r}}^{*2} + l^2\|e(k)\|_2^2 \\
&= l^2\|e(k)\|_2^2 + 2ld^*\|e(k)\|_2 + d^{*2} - 2l^2 x_{\mathrm{r}}^{*2} - 2ll_c x_{\mathrm{r}}^* + 2ld^* x_{\mathrm{r}}^* \\
&= (l\|e(k)\|_2 + d^*)^2.
\end{aligned}$$

Then an upper bound for $\left\| \frac{\sigma^{\mathrm{T}}(x(k))\sigma(x(k))}{1+\mu e^{\mathrm{T}}(k)Pe(k)} \right\|_2$ can be written as

$$\begin{aligned}
\left\| \frac{\sigma^{\mathrm{T}}(x(k))\sigma(x(k))}{1+\mu e^{\mathrm{T}}(k)Pe(k)} \right\|_2 &\leq \left(\frac{l}{\sqrt{\mu\underline{\lambda}(P)}} + d^*\right)^2 \\
&= \frac{l^2}{\mu\underline{\lambda}(P)} + d^{*2} + \frac{2ld^*}{\sqrt{\mu\underline{\lambda}(P)}} \\
&\leq \frac{2l^2}{\mu\underline{\lambda}(P)} + 2d^{*2} = \eta,
\end{aligned}$$

where the Young's inequality "$2xy \leq \frac{x^2}{2} + \frac{y^2}{2}$" is used at the last step.

Appendix B

In this appendix, the below N is proven to be positive definite

$$N \triangleq \rho_1(2 - \rho_\gamma) - (1 + \frac{1}{\beta})P.$$

Note that setting $\rho_1 \triangleq \frac{1+\beta}{\beta}P$ yields

$$N \triangleq (1 + \frac{1}{\beta})P(2 - \rho_\gamma) - (1 + \frac{1}{\beta})P.$$

Recall that $\rho_\gamma \in (0, 1)$; and hence, "$(2 - \rho_\gamma) > 1$". That concludes the positive definiteness of

$$N \triangleq (1 + \frac{1}{\beta})P(1 - \rho_\gamma) > 0.$$

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Appendix C

In this appendix, the below $\bar{F}$ matrix is proven to be positive semi-definite

$$\bar{F} \triangleq \begin{bmatrix} \beta(I-\epsilon\mathcal{F})^T P(I-\epsilon\mathcal{F}) & \big(P(I-\epsilon\mathcal{F})\big)^{\mathrm{T}} & \big(P(I-\epsilon\mathcal{F})\big)^{\mathrm{T}} \\ P(I-\epsilon\mathcal{F}) & \frac{1}{\beta}P & (-P+\rho_1)^{\mathrm{T}} \\ P(I-\epsilon\mathcal{F}) & -P+\rho_1 & P \end{bmatrix} \geq 0.$$

To prove the positive semi-definiteness of the above matrix, one needs to prove that the minors of the matrix are positive semi-definite [45]. Let p_1, p_2,and p_3 be the minors of $\bar{F}$ given by

$$\begin{aligned}
p_1 &= \beta(I-\epsilon\mathcal{F})^T P(I-\epsilon\mathcal{F}) > 0, \\
p_2 &= (I-\epsilon\mathcal{F})^T P(I-\epsilon\mathcal{F})P - (I-\epsilon\mathcal{F})^T P(I-\epsilon\mathcal{F})P = 0, \\
p_3 &= (I-\epsilon\mathcal{F})^T P(I-\epsilon\mathcal{F})\Big(P^{\mathrm{T}}P - \beta P^{\mathrm{T}}P + 2\beta\rho_1 P - \beta\rho_1^2\Big) \\
&\quad -(I-\epsilon\mathcal{F})^T P(I-\epsilon\mathcal{F})\Big(P^{\mathrm{T}}P + P^{\mathrm{T}}P - \rho_1 P\Big) \\
&\quad +(I-\epsilon\mathcal{F})^T P(I-\epsilon\mathcal{F})\Big(-P^{\mathrm{T}}P + \rho_1 P - \frac{1}{\beta}P^{\mathrm{T}}P\Big) \\
&= (I-\epsilon\mathcal{F})^T P(I-\epsilon\mathcal{F})\Big(-\beta P^{\mathrm{T}}P + 2\beta\rho_1 P - \beta\rho_1^2 - 2P^{\mathrm{T}}P + 2\rho_1 P - \frac{1}{\beta}P^{\mathrm{T}}P\Big) \\
&= (I-\epsilon\mathcal{F})^T P(I-\epsilon\mathcal{F})\Big(-\beta P^{\mathrm{T}}P - 2P^{\mathrm{T}}P - \frac{1}{\beta}P^{\mathrm{T}}P\Big) \\
&\quad +(I-\epsilon\mathcal{F})^T P(I-\epsilon\mathcal{F})\Big(\rho_1(2\beta P + 2P) - \beta\rho_1^2\Big) \\
&= (I-\epsilon\mathcal{F})^T P(I-\epsilon\mathcal{F})\Big(\frac{-(\beta+1)^2}{\beta}P^{\mathrm{T}}P\Big) \\
&\quad +(I-\epsilon\mathcal{F})^T P(I-\epsilon\mathcal{F})\Big(2\rho_1 P(\beta+1) - \beta\rho_1^2\Big)
\end{aligned}$$

where p_1 is positive definite since $\beta \in \mathbb{R}_+$ and $P \in \mathbb{P}_+$, and $p_2 = 0$. To prove $p_3 = 0$, one needs to use $\rho_1 \triangleq \frac{1+\beta}{\beta}P$ that is defined in Appendix B. Then one can conclude $p_3 = 0$ as

$$p_3 = (I-\epsilon\mathcal{F})^T P(I-\epsilon\mathcal{F})\Big(\frac{-(1+\beta)^2}{\beta}P^{\mathrm{T}}P\Big) + (I-\epsilon\mathcal{F})^T P(I-\epsilon\mathcal{F})\Big(2\frac{(1+\beta)^2}{\beta}P^{\mathrm{T}}P - \frac{(1+\beta)^2}{\beta}P^{\mathrm{T}}P\Big) = 0$$

This concludes the proof that the matrix $\bar{F}$ is positive semi-definite.

Appendix D

In this appendix, the below ρ_5 matrix is proven to be positive definite

$$\rho_5 = \begin{bmatrix} \rho_3 & -\rho_1\rho_\gamma \\ -\rho_1\rho_\gamma & \rho_4 \end{bmatrix}$$

with $\rho_1 \triangleq \frac{1+\beta}{\beta}P$, $\rho_3 \triangleq \big[(1+\frac{1}{\beta})P - (1+\frac{1}{\beta})\rho_\gamma P\big] = (1-\rho_\gamma)\rho_1$, and $\rho_4 \triangleq \big[-2P + R_H\rho_2 - \rho_\gamma\rho_1\big]$. To prove the positive definiteness of the above matrix, one needs to prove that the minors of the matrix are positive semi-definite [45]. Let p_1 and p_2 be the minors of ρ_5 given by

$$\begin{aligned}
p_1 &= \rho_3 = (1-\rho_\gamma)\rho_1, \\
p_2 &= \rho_3\rho_4 - \rho_\gamma^2\rho_1\rho_1 \\
&= (1-\rho_\gamma)\rho_1(-2P + R_H\rho_2 - \rho_\gamma\rho_1) - \rho_\gamma^2\rho_1\rho_1 \\
&= \rho_1\big(-2P + (1-\rho_\gamma)R_H\rho_2 - \rho_\gamma\rho_1 + 2\rho_\gamma P\big).
\end{aligned}$$

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Here, p_1 is positive definite since $\rho_\gamma \in (0,1)$. Then setting $\rho_2 \triangleq \frac{1}{\underline{\lambda}(R_H)}(1-\rho_\gamma)^{-1}(2P+\rho_\gamma\rho_1-2\rho_\gamma P)$ yields

$$\begin{aligned} p_2 &= \rho_1\big(\frac{R_H}{\underline{\lambda}(R_H)}-R_H\big)(2P+\rho_\gamma\rho_1-2\rho_\gamma P) \\ p_2 &= \rho_1\underbrace{\big(\frac{R_H}{\underline{\lambda}(R_H)}-R_H\big)}_{>0}\underbrace{(2P(1-\rho_\gamma)}_{>0}+\rho_\gamma\rho_1). \end{aligned}$$

This concludes the proof that the matrix ρ_5 is positive definite.

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