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Posted Date: 30 November 2023

doi: 10.20944/preprints202311.1978.v1

Keywords: entropy; black holes; oscillating Universe

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Entropy of Black Holes and an Oscillating Universe

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Abstract: The model of an oscillating universe proposed by A. Friedmann in 1922 dominated cosmology for a long time. R. Tolman put forward a thermodynamic argument against the cyclic cosmological model, believing that entropy should accumulate from cycle to cycle. J. Bekenstein and S. Hawking discovered a tremendous amount of entropy associated with black holes. This discovery led to the conclusion that the majority of the Universe's entropy is contained in black holes. N. Poplawski considers a Universe that oscillates within a black hole. This paper analyzes the relationship between the entropy of black holes and the oscillating Universe inside a large black hole. It is demonstrated that the entropy of a black hole can only be defined for an external observer. For an observer inside the black hole, its entropy is zero. This opens up the possibility of constructing a cyclic cosmology in which entropy and the size of the visible part of the Universe change strictly periodically. This does not violate the second law of thermodynamics because the continuous accumulation of entropy from cycle to cycle pertains to the surface of the big black hole, which is invisible to the internal observer.

Keywords: entropy; black holes; oscillating Universe

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1. Introduction

The first model of a pulsating universe within the framework of the general theory of relativity was proposed by A. Friedmann [1]. He estimated the time of the universe's pulsation to be 10 billion years. R. Tolman [2] raised objections to the cyclic universe model based on the second law of thermodynamics. He expressed the opinion that entropy should accumulate in each cycle, making the cyclic universe model impossible. In the classical perspective, the primary source of entropy in the Universe is attributed to baryons, photons, and neutrinos. As indicated by estimates (refer to Table 1), the entropy associated with photons and neutrinos significantly surpasses the entropy of the baryonic matter in our cosmos by eight orders of magnitude. During the 1930s, when Richard Tolman addressed the matter of entropy increase in cyclic Universe models, there was limited knowledge regarding the thermodynamics of black holes. The prevailing belief at that time was that black holes were objects that did not emit radiation. Hawking [3] showed that black holes have temperature and can evaporate, although this effect is negligible for stellar-mass black holes, let alone supermassive ones. Bekenstein [4] and Hawking [5] established that black holes possess enormous entropy proportional to the square of their mass. Black hole entropy, directly linked to their surface area, is closely connected to the concepts of black hole temperature and the phenomenon of Hawking radiation. The necessity to incorporate black hole entropy into thermodynamics stems from the second law of thermodynamics, which stipulates that the entropy of any closed system cannot decrease. However, when a black hole engulfs matter, it effectively conceals the entropy of that matter from an external observer. To adhere to the principles of the second law of thermodynamics, the black hole must essentially "absorb" the entropy of the ingested material. Consequently, in the realm of thermodynamics, black holes of stellar mass started to be perceived as exceedingly cold and sluggishly radiating entities characterized by substantial entropy. Calculations show [6–9] that black holes contain virtually all the entropy of the Universe – see Table 1. Therefore, discussions of entropy issues in cosmology cannot be held without considering black holes.

 doi:10.20944/preprints202311.1978.v1

Table 1. Entropy of the main components of the Universe [1].

Components	S_1[k]	S_2[k]
Stars	$9.5 * 10^{80}$	$3.5 * 10^{78}$
Interstellar medium and intergalactic medium	$7.1 * 10^{81}$	$2.7 * 10^{80}$
Relic neutrinos	$5.2 * 10^{89}$	$1.9 * 10^{88}$
Photons	$5.4 * 10^{89}$	$2.0 * 10^{88}$
Black holes (2.5-5 $M_{\odot}$)	$5.9 * 10^{97}$	$2.2 * 10^{96}$
Supermassive black holes	$3.1 * 10^{104}$	$1.2 * 10^{103}$
Cosmic Event Horizon	-	$2.6 * 10^{122}$

[1] The data is reproduced with the permission of the authors (Egan and Lineweaver [9]) and the American Astronomical Society. The two types of entropy mentioned in Table 1 correspond to different calculation models [9].

Despite the thermodynamic objections raised by Tolman, the model of a pulsating universe remained popular in the mid-20th century (see, for example, Dicke et al. [10]) but receded in the 1980s due to the quantum one-shot model of the Universe. In the 21st century, cyclic cosmological models have once again become actively discussed. N. Poplawski [11,12] is developing the theory of an universe pulsating within a black hole. The idea of such an universe was proposed earlier [13,14]. However, Poplawski's model does not include multiple internal black holes. R. Penrose, the author of another cyclic universe model, emphasized the cosmological significance of black holes [6,7,15]. After the discovery of gravitational waves generated by the merger of stellar-mass black holes (Abbott et al. [16]), hypotheses emerged suggesting that all dark matter consists of black holes (Kashlinsky [17]; Bird et al. [18]; Clesse and Garcia-Bellido [19]). Given that supermassive black holes are found at the center of every galaxy [20,21], one can assert the exceptional importance of black holes for cosmological models. Note the article by Di Valentino et al. [22] about non-zero curvature of space, supporting the idea of a closed universe or a universe inside a black hole. The topic of black hole entropy in cosmology is discussed in many works (see, for example, [23–25]). Roupas [26] discusses the entropy of universes inside black holes. Penrose [7], Ijjas and Steinhardt [27], and Frampton [28,29] explore the topic of entropy in cyclic universe models.

Our article is dedicated to discussing the Tolman entropy problem in a simple cosmological model describing a pulsating universe within a black hole. It is evident that such a model must encompass numerous black holes and account for their dynamics of formation and accumulation. On the other hand, the entropy problem can be examined quite independently of the details of cyclic cosmology, such as the mechanism of the Big Bang or the dynamics of relic radiation (CMB), the contribution of which to the cosmic entropy is negligible compared to the entropy of black holes. As a first step in addressing the Tolman entropy problem, we will consider the simplest scenario of a universe pulsating within a black hole.

2. Materials and Methods

Let's consider the simplest model of a universe inside a huge black hole with a size on the order of a trillion light-years, which we'll call MegaHole (Figure 1). Suppose the universe consists of internal black holes, constituting the majority of its mass, and baryonic matter in the form of galaxies. The compression of such a system (Figure 1a) would lead to a Big Crunch (Figure 1b), during which some of the black holes will inevitably merge into the largest black hole, and the cores of heavy elements will transform back into individual protons and neutrons through photodissociation [10]. For such destruction of nuclei of chemical elements, a relic radiation temperature of 30 billion degrees is required, implying the universe's compression to a size of ~10 light-years. After reaching maximum compression, the Universe experiences a rebound and begins to expand (Figure 1c). The mechanism of this expansion is not crucial for our consideration – it could be an inflationary field; repulsion based on non-Einsteinian theory [11,12], or expansion due to variable gravitational mass [30].

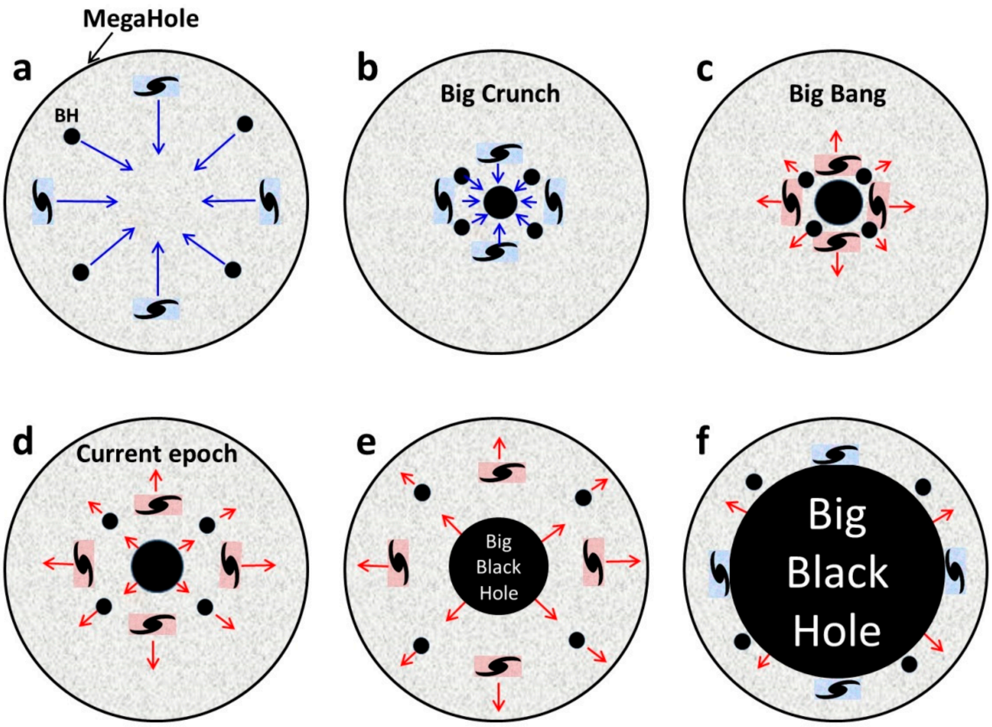

Figure 1. One cycle of the simplest universe model consisting of baryons (galaxies) and black holes (black dots). The universe oscillates within a black hole (MegaHole). The cycle begins with compression (a), which transitions into the Big Crunch (b). The universe's collapse is followed by the Big Bang (c). Stage (d) corresponds to our time. The black hole will continue to grow (e) until it engulfs all galaxies and black holes (f), after which the cycle returns to stage (a).

The accretion growth of black holes is proportional to the square of their mass, m (see, for example, [31]):

$$\frac{dm}{dt} = \frac{27\pi G^2}{c^3}\rho m^2 \tag{1}$$

where ρ is the density of the surrounding medium. The expansion of the universe is accompanied by the accretion growth of the largest black hole, which we will call the Big Black Hole (BBH). Due to its mass, BBH grows much faster than other black holes.

The stage of the universe's development shown in Figure 1d corresponds to our epoch. The Big Black Hole, which presumably began its growth from a size on the order of a light-year, will inevitably grow so rapidly that it will start catching up with the receding galaxies (Figure 1e). There are no mechanisms that would prevent the Big Hole from engulfing all the matter in the universe, thereby equating itself with the MegaHole, effectively replacing it (Figure 1f). Our Galaxy will also be swallowed up; due to weak tidal forces, this absorption will go practically unnoticed, with the only change being the shift from the redshift of galaxies to a blueshift.

The simple model under consideration doesn't involve any serious assumptions except for the assumptions that there should exist a mechanism for the Big Bang and that the minimum diameter of the universe would be on the order of or less than 10 light-years. From a dynamic standpoint, the discussed cycle should repeat infinitely because all accumulated chemical elements are transformed back into hydrogen, which allows for the emergence of new stars and galaxies. All components in such a model must evolve cyclically, moving from birth to destruction and back again. The most complex question lies in the cyclic nature of the black hole population, which is born in each cycle during supernova explosions but is indestructible.

Can the population of continuously growing indestructible objects be cyclic? This question was extensively explored in the work of Gorkavyi and Tyul'bashev [31]. It was demonstrated that the entire population of black holes, even assuming it constitutes all dark matter, easily fits within a sphere with a diameter of about 10 light-years. During compression, a small part of the black hole population transforms into the Big Black Hole. The Big Bang scatters black holes that did not enter the Big Hole throughout the Universe. In this process, supermassive black holes (SMBH) surviving the maximum compression can serve as seeds for the formation of galaxies [21].

The hypothesis that black holes can survive the Big Crunch and enter another cycle of the Universe was first proposed by Carr and Coley [32]. Gorkavyi and Tyul'bashev [31] showed that the cyclic nature (and stationarity over timescales larger than one cycle) of the black hole population is ensured by the expansion of the Big Black Hole to the size of the MegaHole. For all observers living in galaxies and entering the Big Hole at the end of the cosmological cycle, the absorption by the Big Hole means its disappearance from the black hole population.

The discussed model of the universe pulsating within the MegaHole appears realistic from a dynamic perspective. What can thermodynamics tell us about it?

According to Bekenstein [4] and Hawking [5], black holes possess entropy S proportional to the area of their surface or the square of their mass m:

$$S = \frac{4\pi G k}{\hbar c} m^2 \tag{2}$$

where k is the Boltzmann constant. Taking into account numerical values of constants, we obtain the following estimate of the total entropy for the population of black holes with a population size N:

$$S \sim 10^{77} m^2 N \; [k] \tag{3}$$

Gorkavyi and Tyul'bashev [31] obtained the distribution of black holes in the cyclic Universe. In Table 2, using these calculations and formula (3), the entropy of populations of various types of black holes is estimated. The average mass of the black hole, m, and the population size, N, are taken from Model 4 in the paper by Gorkavyi and Tyul'bashev [31].

Table 2. The entropy of black holes in the cyclic Universe.

Type of black holes	Massa m (in $M_\odot$)	Number N	Entropy [k]
SBH *	5.28	$4.9 * 10^{22}$	$1.4*10^{101}$
IMBH **	479	$1.4 * 10^{14}$	$3.2*10^{96}$
SMBH ***	$1.15*10^{6}$	$8.2 * 10^{10}$	$1.1*10^{100}$
BBH ****	$3*10^{12}$	1	$9*10^{101}$
MegaHole	$3*10^{24}$	1	$9*10^{125}$

* Stellar-mass Black Hole (<100 $M_\odot$). ** Intermediate-Mass Black Hole (10^2-10^5 $M_\odot$). *** SuperMassive Black Hole (10^5-10^{10} $M_\odot$). **** Big Black Hole (with an initial radius of ~1 light-year).

3. Results

Black holes are the primary component in the entropy balance of the universe (see Table 1). Therefore, the growth of the Universe's entropy is primarily linked to the growth of black holes. However, when looking at the contribution to the entropy of the Universe from various populations of black holes, it becomes evident that the majority of the Universe's entropy is contained within the Big Black Hole, even when it is just forming (see Table 2). The growth of the Big Black Hole to the size of the MegaHole results in a 24-order increase in entropy. The second most significant component of the Universe's entropy is composed of stellar-mass black holes, which make up dark matter. It is clear that in Figure 1a–e, the entropy of the system increases due to the formation and growth of the Big Black Hole. An exception is the transition between stages 1f and 1a, when all objects in the Universe fall into the Big Black Hole. As a result, the BBH disappears from the distribution of black holes [31].

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The entropy of a black hole depends on its temperature, which is only defined for an external observer (since a black hole only emits radiation outward). The temperature of a black hole is introduced under the assumption of a flat metric at infinity and disappears even for a free-falling observer [7], not to mention an observer who has fallen into the black hole. It is logical to propose the following assumption: the temperature of a black hole is not defined for an internal observer, consequently, the entropy of the BBH is not observable for them.

An external observer sees a stationary MegaHole, which possesses a constant entropy $S_{MegaHole}$:

$$S_{ext} = S_{MegaHole} \; and \; \frac{dS_{ext}}{dt} = 0 \tag{4}$$

Thus, for an external observer, there is no violation of the second law of thermodynamics. An internal observer must record cyclic changes in the entropy of the observable part of the universe, but we do not consider periodic decreases in entropy a violation of the second law of thermodynamics. It is useful to introduce the quantity $S_{stars+BHs}$, which represents the entropy of stars and black holes, i.e., all the entropy of the Universe from the perspective of an internal observer, but without the entropy of the Big Black Hole. If we add the entropy of the growing Big Black Hole S_{BBH} to the reduced entropy of the observable part of the Universe $S_{stars+BHs}$, then the sum of these entropies S_{inn} for an internal observer will only increase over time (see Figure 2).

$$\frac{dS_{inn}}{dt} = \frac{dS_{BBH}}{dt} + \frac{dS_{stars+BHs}}{dt} > 0 \tag{5}$$

When a falling observer crosses the boundary of the Big Black Hole, the entropy of this black hole disappears for them. Equation (5) transforms into a simpler formula:

$$\frac{dS_{inn}}{dt} = \frac{dS_{stars+BHs}}{dt} > 0 \tag{6}$$

In fact, entropy growth, from the perspective of an internal observer, never ceased. However, at the moment of crossing the boundary of a black hole, the initial level of entropy of the observable Universe instantly decreased by many orders of magnitude. An observer capable of measuring the entropy of BBH will confirm that the overall entropy of the Universe has not decreased.

Figure 2 qualitatively illustrates the behavior of entropy for different components of the universe and for different observers: S_{ext}, S_{inn}, S_{BBH}, $S_{stars+BHs}$. For clarity, the curves are plotted without adhering to scale.

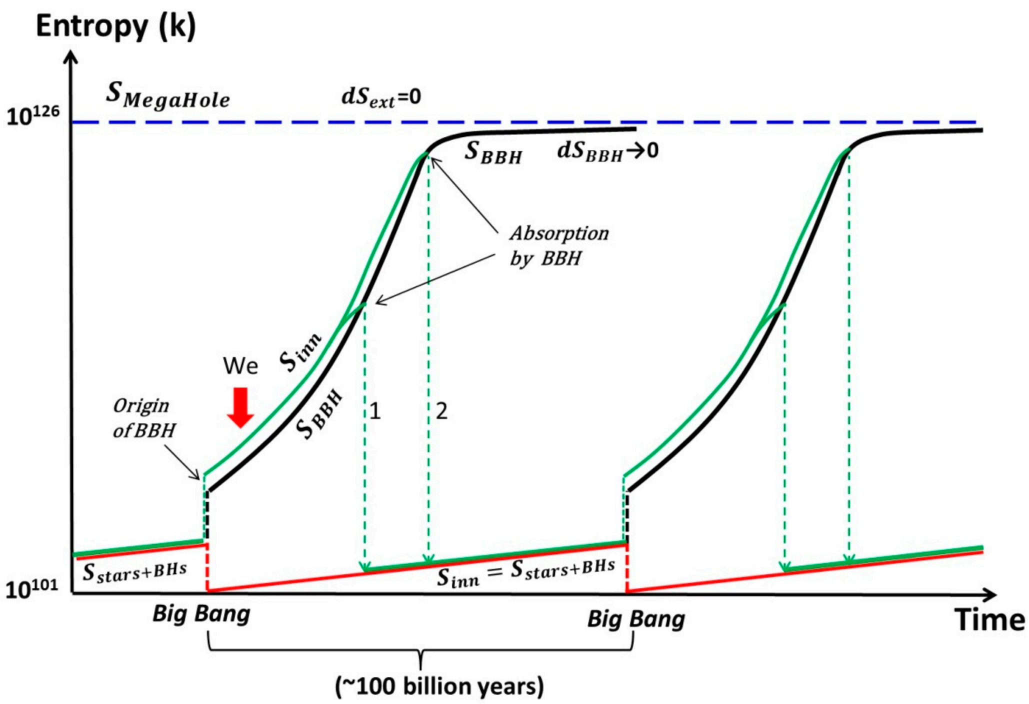

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Figure 2. Evolution of entropy in a cyclic model of the Universe. At the top, a dashed line represents the maximum entropy of the Universe from the perspective of an external observer: S_{ext}. The black line represents the entropy of the Big Black Hole S_{BBH}, which forms at the moment of the Big Bang. Initially, the BBH grows rapidly according to (1), but after consuming almost the entire Universe, its growth sharply slows down due to the nearly zero density of the surrounding medium. The entropy of the visible Universe (excluding the BBH) $S_{stars+BHs}$ is shown in red. It slowly increases between the Big Bangs by a relatively small amount and sharply decreases at the moment of the Big Bang, as a portion of the matter and entropy of the Universe is expended in creating the BBH, whose entropy even at the initial moment exceeds the entropy of the entire rest of the Universe. If we sum $S_{stars+BHs}$ and S_{BBH}, we obtain the overall entropy of the Universe in this cycle, which closely approaches the entropy of the observable part of the Universe S_{inn}, indicated by the green line. Gradually, the Big Black Hole will absorb the Universe, causing a reduction in S_{inn} compared to the sum of $S_{stars+BHs}$ and S_{BBH}. This is why the green curve starts to converge with the black line, and when an observer is absorbed by the growing BBH, the green line discontinues, and the entropy of the observable part of the Universe aligns with $S_{stars+BHs}$. There are many internal observers, some of whom are absorbed by the BBH earlier (dashed green line 1), while others are absorbed later (dashed green line 2).

What happens to the entropy of the Big Black Hole as it expands to its maximum limits and seeks to merge with the MegaHole? MegaHole has the maximum entropy corresponding to its size, and it cannot absorb new portions of entropy without increasing its internal mass and surface area. Therefore, we cannot add the entropy of the Big Black Hole S_{BBH} to the entropy of MegaHole $S_{MegaHole}$, because their surfaces do not have their own mass, so merging the surface of BBH with the surface of MegaHole does not increase the size or entropy of the latter. At the moment of the merger of BBH with MegaHole, the increase in its entropy tends to zero: $dS \to 0$ (see Figure 2), therefore, no matter how much entropy is contained in the surfaces of BBH, the overall condition of entropy invariance from the perspective of an external observer will be maintained.

Both observers (external and internal) believe that the second law of thermodynamics holds, according to which entropy either remains constant (4) or increases (5). For the observable part of the Universe, entropy changes cyclically. The reduction in entropy upon entering the Big Black Hole is not a formality: the expanding Universe reduced its density and increased its entropy. The Universe of the new cycle, after entering the Big Black Hole, starts with the minimum entropy, increases its density, and prepares for a new stage of self-organization.

4. Discussion

Tolman [1] and Penrose [7] note the anomalous behavior of gravitating systems that can self-organize (for example, to form stars) even under conditions of complete isolation. Accounting for the contribution of black holes to entropy removes Tolman's objection to the cyclic Universe and opens up the possibility for constructing cosmological periodic models that do not contradict the second law of thermodynamics.

Zel'dovich and Novikov considered the cyclic universe in two cases: one without entropy growth and the other with entropy increasing from cycle to cycle [33]. They concluded that only a cosmological model with entropy growth is feasible, implying that the Universe must expand in size as cycles accumulate. As demonstrated in this paper, in the case of a Universe pulsating within a black hole, both scenarios are simultaneously realized: for an external observer, the Universe's entropy remains constant, while for an internal observer, it periodically changes during the cycle. Additionally, the size of the Universe is constrained by the radius of the black hole within which it pulsates.

In more realistic models of cosmology, it is necessary to consider the important role of gravitational waves [34–36]. However, we believe that gravitational radiation does not have a significant impact on the question of the evolution of the Universe's entropy.

5. Conclusions

Black holes play a key role in the entropy balance of the Universe. Black holes are entropy devourers. They possess colossal entropy, which they accumulate over a long period and quickly remove from the overall entropy balance of the visible part of the Universe.

The cyclic Universe inside a black hole represents an ideal periodic system in which there are no internal energy losses, and therefore, the entropy increase should be zero (from the perspective of an external observer). However, from the perspective of an internal observer, entropy constantly grows in accordance with the second law of thermodynamics. The excess entropy accumulated in each cycle is carried away by the surface of the Big Black Hole to the periphery of the Universe, allowing each new cycle of the Universe to start with minimum entropy. The entropy of the visible part of the Universe changes strictly periodically and does not tend to accumulate, just like the population of black holes in the pulsating Universe.

Funding: This research received no external funding.

Data Availability Statement: Not applicable.

Conflicts of Interest: The author declares no conflict of interest.

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