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Application Of Peter Chew Theorem For Calculus (Second Order Linear Equations With Constant Coefficients)

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Abstract: Exercising surds to represent figures is a common practice in scientific and Engineering fields, especially in scripts where calculators are banned or unapproachable. Peter Chew Theorem make result becomes simple when dealing with converting Quadratic Surds. The substance of the Peter Chew Theorem lies in enabling the forthcoming generation to simple break problems related to Quadratic Surds more effectively, easing a direct comparison with contemporary results. By employing the Peter Chew Theorem, one can streamline the tutoring and literacy of math, particularly concerning second- order direct equations with constant portions. This theorem's objective aligns with Albert Einstein's famed quotation Everything should be made as simple as possible, but not simpler.

Keywords: calculus; second order linear equations with constant coefficients; peter chew theorem; quadratic surds

1. INTRODUCTION

1.1. *Surd* [1]

Exercising surds to represent figures is a common practice in scientific and Engineering fields, especially in scripts where calculators are banned or unapproachable. Peter Chew Theorem make result becomes simple when dealing with converting Quadratic Surds. The substance of the Peter Chew Theorem lies in enabling the forthcoming generation to simple break problems related to Quadratic Surds more effectively, easing a direct comparison with contemporary results.

1.2. *Surds Explained with Worked Examples by Shefiu S. Zakariyah(PhD)* [1]

In a triangle ABC, AB = BC = $(\sqrt{3} - 1)\,cm$ and $\angle ACB = 30^o$, without using a calculator find the length of AC. Figure 5. (pg 30)

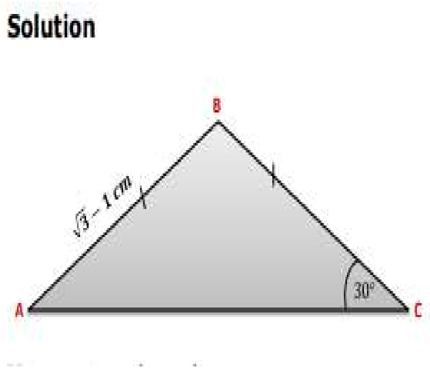

Figure 5.

Where AB = c = $(\sqrt{3} - 1)\,cm$,

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BC = a = $(\sqrt{3} - 1)cm$

AC = b

Also, ∠ BAC = ∠ ACB = 30^o .

AB= BC

∠ B = 180^o - (∠ BAC + ∠ BAC)

= 120^o

$b^2 = (\sqrt{3} - 1)^2 + (\sqrt{3} - 1)^2 - 2(\sqrt{3} - 1)(\sqrt{3} - 1)\cos 120^o$.

= 2 $(\sqrt{3} - 1)^2 - 2(\sqrt{3} - 1)^2(-\cos 60^o)$

$= 2[3 + 1 - 2\sqrt{3}] - 2[3 + 1 - 2\sqrt{3}](-\frac{1}{2})$

$= 2[4 - 2\sqrt{3}] + [4 - 2\sqrt{3}]$

$= 3[4 - 2\sqrt{3}]$

$= 12 - 6\sqrt{3}$

b = $\sqrt{12 - 6\sqrt{3}}$

Current Method:
Let the square root of the $12 - 6\sqrt{3}$ be $\sqrt{x}$ - $\sqrt{y}$ for which x, y ∈ R. Therefore

$12 - 6\sqrt{3} = (\sqrt{x} - \sqrt{y})^2$

= x + y – 2 $\sqrt{xy}$

Comparing the both sides,
We've , x + y = 12
also - 2 $\sqrt{xy} = -6\sqrt{3}$
Divide 2sides by – 2, $\sqrt{xy} = 3\sqrt{3}$
Square 2 sides , xy = 27 ……………..…..(ii)
From (i), y = 12 – x …………………………(iii)
Substitute equation (iii) in equation (ii),

x (12 -x) = 27

12 x - x^2 = 27.

x^2 - 12 x + 27 = 0

(x – 9) (x – 3) = 0

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Therefore, either $x - 9 = 0, \quad x - 3 = 0$

$$x = 9, 3$$

We now need to find the values for y, therefore
When $x = 9$, from (iii), $y = 12 - 9 = 3$.
And when $x = 3$, from (iii) , $y = 12 - 3 = 9$
Hence , the square root of the $12 - 6\sqrt{3}$ are

$$\sqrt{x} - \sqrt{y} = \sqrt{9} - \sqrt{3}$$

$$= 3 - \sqrt{3}$$

and $\sqrt{a} + \sqrt{b} = \sqrt{3} - \sqrt{9}$

$$= \sqrt{3} - 3$$

$$\therefore \sqrt{12 - 6\sqrt{3}} = \pm (3 - \sqrt{3}).$$

Since AC cannot be negative, thens AC = ($3 - \sqrt{3}$) cm . [$\sqrt{3} - 3$ is negative].
Peter Chew Theorem^2.

$$\sqrt{12 - 6\sqrt{3}} = \sqrt{12 - 2\sqrt{27}}$$

Cause $x^2 - 12x + 27 = 0$, **then** $x = 9, 3$

$$\therefore \sqrt{12 + 2\sqrt{27}} = \sqrt{9} - \sqrt{3}$$

$$= 3 - \sqrt{3}$$

$\therefore$ **Length AC** $= 3 - \sqrt{3}$

2. Current Method and Peter Chew Theorem

Example: If $\sqrt{12 + 2\sqrt{35}} = \sqrt{x} + \sqrt{y}$, find the value of x and y. and convert $\sqrt{12 + 2\sqrt{35}}$ into the sum of 2 real number in surd form.

Current Method,

i) Solution 1:

$$\sqrt{12 + 2\sqrt{35}} = \sqrt{7 + 5 + 2\sqrt{(7)(5)}}$$

$$= \sqrt{(\sqrt{7})^2 + (\sqrt{5})^2 + 2\sqrt{(7)(5)}}$$

$$= \sqrt{(\sqrt{7} + \sqrt{5})^2}$$

$$= \sqrt{7} + \sqrt{5}$$

ii) Solution 2: If $\sqrt{12 + 2\sqrt{35}}$ be $\sqrt{x} + \sqrt{y}$

$$12 + 2\sqrt{35} = (\sqrt{x} + \sqrt{y})^2$$

$$= x + y + 2\sqrt{xy}$$

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Comparing the both sides,

We've $x + y = 12$

$y = 12 - x$ ….. i)

also $xy = 35$ ……..ii)

From i) and ii), $x (12 - x) = 35$

$x^2 - 12x + 35 = 0$

$(x - 7)(x - 5) = 0$

Therefore, $x = 7, 5$

From i), If $x = 7, y = 12 - 7 = 5$

If $x = 5, y = 12 - 5 = 7$

$\therefore \sqrt{12 + 2\sqrt{35}} = \sqrt{7} + \sqrt{5}$

iii) Peter Chew Theorem,

Cause $x^2 - 12x + 35 = 0$, then $x = 7, 5$

$\therefore \sqrt{12 + 2\sqrt{35}} = \sqrt{7} + \sqrt{5}$

3. Linear second order differential equation with constant coefficient .

Solution to the Differential Equation.Diagram

LINEAR SECOND - ORDER DIFFERENTIAL EQUATIONS WITH CONSTANT COEFFICIENTS. $a\frac{d^2y}{dx^2} + b\frac{dy}{dx} + cy = 0$		
Roots Of $am^2 + bm + c = 0$		**Solution To The Differential Equation**
$b^2 - 4ac = 0$	$m_1 = m_2$	$y = (A + Bx)e^{m_1x}$
$b^2 - 4ac > 0$	$m_1 \neq m_2$	$y = Ae^{m_1x} + Be^{m_2x}$
$b^2 - 4ac < 0$	$m_1 = \alpha + \beta i$ $m_2 = \alpha - \beta i$	$y = e^{\alpha x}(A\cos\beta x + B\sin\beta x)$

Example 1: Find the solution to the differential equation: $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} - 10y = 0.$

Solution: *The auxiliary equation is:* $m^2 - 3m - 10 = 0$.

$m_1 = -2, \quad m_2 = 5$

$\therefore y = C_1e^{-2x} + C_2e^{5x}$

Example 2: Find the general solution to the differential equation: $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 0.$

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Solution: *The auxiliary equation is:* $m^2 - 2m + 1 = 0$.

$m_1 = 1$, $m_2 = 1$

$\therefore y = (A + Bx)e^x$

Example 3: Find the solution to the differential equation: $\frac{d^2y}{dx^2} + 6\frac{dy}{dx} + 13 = 0$.

Solution: *The auxiliary equation is:* $m^2 + 6m + 13 = 0$.

$m_1 = -3 + 2i$, $m_2 = -3 - 2i$, Therefore, $\alpha = -3$, $\beta = 2$

$\therefore y = e^{-3x}(A\cos 2x + B\sin 2x)$

4. Application of Peter Chew Theorem for calculus (Second Order Linear Equations With Constant Coefficients)

Example 1 : Find the solution to the equation: . $y'' + 3y' + \sqrt{2}\,y = 0$

Solution :

The auxiliary equation is: $m^2 + 3m + \sqrt{2} = 0$

$$m_{1,2} = \frac{-3 \pm \sqrt{(3)^2 - 4(1)(\sqrt{2})}}{2(1)}$$

$$= \frac{-3 \pm \sqrt{9 - 2(\sqrt{8})}}{2}$$

Current Method,

If $\sqrt{9 - 2\sqrt{8}} = \sqrt{x} - \sqrt{y}$

Then $9 - 2\sqrt{8} = (\sqrt{x} - \sqrt{y})^2$

$$\therefore \quad 9 - 2\sqrt{8} = x + y - 2\sqrt{xy}$$

Comparing the both sides ,

We've $x + y = 9$

$$y = 9 - x \quad \ldots\ldots \text{ i)}$$

also $xy = 8$ ……..ii)

From i) and ii), $x(9 - x) = 8$

$$x^2 - 9x + 8 = 0$$

$$(x - 8)(x - 1) = 0$$

$$x = 8, 1$$

From i), If $x = 8$, $y = 9 - 8 = 1$

If $x = 1$, $y = 9 - 1 = 8$

$$\therefore \sqrt{9 + 2\sqrt{8}} = \sqrt{8} - \sqrt{1}$$

$$= 2\sqrt{2} - 1$$

iii) Peter Chew Theorem,

Cause $x^2 - 9x + 8 = 0$, then $x = 8, 1$

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$$\therefore \sqrt{9+2\sqrt{8}} = \sqrt{8} - \sqrt{1}$$

$$= 2\sqrt{2} - 1$$

Therefore $m_{1,2} = \frac{-3 \pm \sqrt{9-2(\sqrt{8})}}{2}$

$$= \frac{-3 \pm (2\sqrt{2} - 1)}{2}$$

$$= \frac{2\sqrt{2}-4}{2}, \frac{-2\sqrt{2}-2}{2}$$

$$= \sqrt{2} - 2, -\sqrt{2} - 1$$

For $m_1 \neq m_2$, Solution To The Differential Equation is $y = A\, e^{m_1 x} + B\, e^{m_2 x}$

Therefore $y(x) = A\, e^{(\sqrt{2}-1)x} + B\, e^{(-\sqrt{2}-2)x}$

Example 2 : Find the solution to the equation: . $y'' + 20y' + 10\sqrt{19}\, y = 0$

Solution :

The auxiliary equation is: $m^2 + 20m + 10\sqrt{19} = 0$

$$m_{1,2} = \frac{-20 \pm \sqrt{(20)^2 - 4(1)(10\sqrt{19})}}{2(1)}$$

$$= \frac{-20 \pm \sqrt{400 - 40(\sqrt{19})}}{2}$$

$$= -10 \pm \sqrt{100 - 10(\sqrt{19})}$$

$$= -10 \pm \sqrt{100 - 2\sqrt{475}}$$

Current Method,

If $\sqrt{100 - 2\sqrt{475}} = \sqrt{x} - \sqrt{y}$

$Therefore\ 100 - 2\sqrt{475} = (\sqrt{x} - \sqrt{y})^2$

$$= x + y - 2\sqrt{xy}$$

Comparing the both sides,

We've $x + y = 100$

$$y = 100 - x \ ..... \ i)$$

also $x\, y = 475$ ....ii)

From i) and ii), $x(100 - x) = 475$

$$x^2 - 100x + 475 = 0$$

$$(x - 95)(x - 5) = 0$$

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$$x = 95, 5$$

From i), If $x = 95, y = 100 - 95 = 5$

If $x = 5, y = 100 - 5 = 95$

$\therefore \sqrt{100 - 2\sqrt{475}} = \sqrt{95} - \sqrt{5}$

iii) Peter Chew Theorem,

Cause $x^2 - 100x + 475 = 0$, then $x = 95, 5$

$\therefore \sqrt{100 - 2\sqrt{475}} = \sqrt{95} - \sqrt{5}$

From $m_{1,2} = -10 \pm \sqrt{100 - 2\sqrt{475}}$

$= -10 \pm (\sqrt{95} - \sqrt{5})$

$= \sqrt{95} - \sqrt{5} - 10, -\sqrt{95} + \sqrt{5} - 10$

For $m_1 \neq m_2$, Solution To The Differential Equation is $y = A\ e^{m_1 x} + B\ e^{m_2 x}$

Therefore $y(x) = A\ e^{(\sqrt{95} - \sqrt{5} - 10)x} + B\ e^{(-\sqrt{95} + \sqrt{5} - 10)x}$

Example 3 : Find the solution to the equation: . $y'' + 124\,y' + 62\sqrt{123}\ y = 0$

Solution :

The auxiliary equation is: $m^2 +$ **$20\,m + 10\sqrt{19} = 0$**

$$m_{1,2} = \frac{-124 \pm \sqrt{(124)^2 - 4(1)(62\sqrt{123})}}{2(1)}$$

$$= \frac{-124 \pm \sqrt{15376 - 248\sqrt{123}}}{2}$$

$$= -62 \pm \sqrt{3844 - 62\sqrt{123}}$$

$$= -62 \pm \sqrt{3844 - 2\sqrt{118203}}$$

Current Method,

If $\sqrt{3844 - 2\sqrt{118203}} = \sqrt{x} - \sqrt{y}$

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$3844 - 2\sqrt{118\,203} = (\sqrt{x} - \sqrt{y})^2$

$= x + y - 2\sqrt{xy}$

Comparing the both sides,

We've $x + y = 3\,844$

$y = 3\,844 - x$ ….. i)

also $x\,y = 118\,203$ ……..ii)

from i) and ii), $x\,(3\,844 - x) = 118\,203$

$x^2 - 3\,844\,x + 118\,203 = 0$

$(x - 3\,813)\,(x - 31) = 0$

$x = 3\,813, 31$

From i), If $x = 3\,813$, $y = 3\,844 - 3813 = 31$

If $x = 31$, $y = 3\,844 - 31 = 3\,813$

$\therefore \sqrt{3844 - 2\sqrt{118\,203}} = \sqrt{3\,813} - \sqrt{31}$

iii) Peter Chew Theorem,

Cause $x^2 - 100x + 475 = 0$, then $x = 95, 5$

$\therefore x^2 - 3\,844\,x + 118\,203 = 0 = = \sqrt{3\,813} - \sqrt{31}$

From $m_{1,2} = -62 \pm \sqrt{3844 - 2\sqrt{118203}}$

$= -62 \pm (\sqrt{3\,813} - \sqrt{31})$

$= \sqrt{3\,813} - \sqrt{31} - 62, \; - \sqrt{3\,813} + \sqrt{31} - 62$

For $m_1 \neq m_2$, Solution To The Differential Equation is $y = A\,e^{m_1 x} + B\,e^{m_2 x}$

Therefore $y(x) = A\,e^{(\sqrt{3\,813} - \sqrt{31} - 62)x} + B\,e^{(-\sqrt{3\,813} + \sqrt{31} - 62)x}$

Example 4 : Find the solution to the equation: . $2y'' + 2\sqrt{2}\,y' + \sqrt{5}\,y = 0$

Solution :

The auxiliary equation is: $2m^2 + 2\sqrt{2}\,m + \sqrt{5} = 0$

$$m_{1,2} = \frac{-2\sqrt{2} \pm \sqrt{(2\sqrt{2})^2 - 4(2)(\sqrt{5})}}{2(2)}$$

$$= \frac{-2\sqrt{2} \pm \sqrt{8 - 8\sqrt{5}}}{4}$$

$$= \frac{-\sqrt{2} \pm \sqrt{2-2\sqrt{5}}}{2}$$

Current Method,

Let $\sqrt{2-2\sqrt{5}}$ be $\sqrt{x} - \sqrt{y}$

$2-2\sqrt{5} = (\sqrt{x} - \sqrt{y})^2$

$= x + y - 2\sqrt{xy}$

Comparing the both sides,

We've $x + y = 2$

$y = 2 - x$ ….. i)

also $x\,y = 5$ ……..ii)

From i) and ii), $x(2 - x) = 5$

$x^2 - 2x + 5 = 0$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(5)}}{2(1)}$$

$$x = \frac{2 \pm \sqrt{-16}}{2}$$

$$x = \frac{2 \pm 4i}{2}$$

$$x = 1 \pm 2i$$

From i), If $x = 1+2i$, $y = 2 - (1+2i) = 1 - 2i$

If $x = 1-2i$, $y = 2 - (1-2i) = 1 + 2i$

$\therefore \sqrt{2-2\sqrt{5}} = \sqrt{1+2i} - \sqrt{1-2i}$

iii) Peter Chew Theorem,

Cause $x^2 - 2x + 5 = 0$, then $x = 1 + 2i, 1 - 2i$

$\therefore \sqrt{2-2\sqrt{5}} = \sqrt{1+2i} - \sqrt{1-2i}$

From $m_{1,2} = \frac{-\sqrt{2} \pm \sqrt{2-2\sqrt{5}}}{2}$

$$= \frac{-\sqrt{2} \pm (\sqrt{1+2i} - \sqrt{1-2i})}{2}$$

$$= \frac{-\sqrt{2}}{2} + \frac{\sqrt{1+2i} - \sqrt{1-2i}}{2}, \frac{-\sqrt{2}}{2} - \frac{\sqrt{1+2i} - \sqrt{1-2i}}{2}$$

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For **$m_1 = \alpha + \beta i$, $m_2 = \alpha - \beta i$. let $\alpha = \frac{-\sqrt{2}}{2}$, $\beta = \frac{\sqrt{1+2i} - \sqrt{1-2i}}{2}$**

Solution To The Differential Equation is **$y = e^{\alpha x}$ (A cos β x + B sin β x)**

Therefore $y(x) = y = e^{\frac{-\sqrt{2}}{2}x}$ [A cos($\frac{\sqrt{1+2i} - \sqrt{1-2i}}{2}$ x) + B sin) $\frac{\sqrt{1+2i} - \sqrt{1-2i}}{2}$ x)]

Example 5 : Find the solution to the equation: . $2y'' + 2\sqrt{6}\ y' + \sqrt{13}\ y = 0$

Solution :

The auxiliary equation is: $2m^2 + 2\sqrt{6}\ m + \sqrt{13} = 0$

$$m_{1,2} = \frac{-2\sqrt{6} \pm \sqrt{(2\sqrt{6})^2 - 4(2)(\sqrt{13})}}{2(2)}$$

$$= \frac{-2\sqrt{6} \pm \sqrt{24 - 8\sqrt{13}}}{4}$$

$$= \frac{-\sqrt{6} \pm \sqrt{6 - 2\sqrt{13}}}{2}$$

Current Method,

Let $\sqrt{6 - 2\sqrt{13}}$ be $\sqrt{x} - \sqrt{y}$

$6 - 2\sqrt{13} = (\sqrt{x} - \sqrt{y})^2$

$= x + y - 2\sqrt{xy}$

Comparing the both sides,

We've $x + y = 6$

$y = 6 - x$ ….. i)

also $x y = 13$ ……..ii)

From i) and ii), $x (6 - x) = 13$

$6x - x^2 = 13$

$x^2 - 6x + 13 = 0$

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(13)}}{2(1)}$$

$$x = \frac{6 \pm \sqrt{-16}}{2}$$

$$x = \frac{6 \pm 4i}{2}$$

$$x = 3 \pm 2i$$

From i), If $x = 3+2i$, $y = 6 - (3+2i) = 3 - 2i$

If $x = 3-2i$, $y = 6 - (3-2i) = 3 + 2i$

$$\therefore \sqrt{6-2\sqrt{13}} = \sqrt{3+2i} - \sqrt{3-2i}$$

iii) Peter Chew Theorem,

Cause $x^2 - 6x + 13 = 0$, then x = $1 + 2i, 1 - 2i$

$$\therefore \sqrt{6-2\sqrt{13}} = \sqrt{3+2i} - \sqrt{3-2i}$$

From $m_{1,2} = \frac{-\sqrt{6} \pm \sqrt{6-2\sqrt{13}}}{2}$

$$= \frac{-\sqrt{6} \pm (\sqrt{3+2i} - \sqrt{3-2i}\ )}{2}$$

$$= \frac{-\sqrt{6}}{2} + \frac{\sqrt{3+2i} - \sqrt{3-2i}}{2}, \frac{-\sqrt{6}}{2} - \frac{\sqrt{3+2i} - \sqrt{3-2i}}{2}$$

For **$m_1 = \alpha + \beta i$, $m_2 = \alpha - \beta i$. let** $\alpha = \frac{-\sqrt{6}}{2}$, $\beta = \frac{\sqrt{3+2i} - \sqrt{1-2i}}{2}$

Solution To The Differential Equation is **y = $e^{\alpha x}$ (A cos β x + B sin β x)**

Therefore , y(x) = y = $e^{\frac{-\sqrt{6}}{2}x}$ [A cos($\frac{\sqrt{3+2i} - \sqrt{3-2i}}{2}$ x) + B sin) $\frac{\sqrt{3+2i} - \sqrt{3-2i}}{2}$ x)]

5. Conclusion

In scientific and engineering domains, the use of surds as a representation for numbers holds substantial significance, especially in situations where the reliance on calculators is limited or non-existent. Surds become a crucial tool when dealing with complex computations involving irrational values. The application of the Peter Chew Theorem significantly simplifies solutions in the conversion of Quadratic Surds.

The Peter Chew Theorem stands as a transformative asset, aiming to facilitate a more simple and straightforward approach for the upcoming generation in handling and resolving problems associated with Quadratic Surds. Its primary objective revolves around empowering individuals to navigate and solve these intricate problems more efficiently, thereby enabling a direct comparison with the solutions derived through contemporary methodologies.

The integration of the Peter Chew Theorem into mathematical education serves to streamline the teaching and learning processes, particularly within the realm of calculus, specifically concerning second-order linear equations characterized by constant coefficients. This theorem acts as a pedagogical tool, enhancing the understanding and application of mathematical concepts related to Quadratic Surds, contributing to a more comprehensive grasp of calculus principles.

Aligned with the wisdom encapsulated in Albert Einstein's renowned quote — " **Everything should be made as simple as possible, but not simpler** " — the Peter Chew Theorem embodies a philosophy centered on simplification without compromising accuracy or depth. Its pursuit of simplifying solutions to complex problems resonates with the fundamental principles driving mathematical innovation.

This theorem's goal harmonizes with the perpetual quest within mathematics: to distill intricate problems into more manageable forms while preserving the precision required for comprehensive understanding and application.

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