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Article

Prediction Approach of Crack width for FRP Reinforced Concrete Flexure Members of Pavement and Girder in Highway Structures

Tao Lu ^{1,2}, Fenghui Dong ^{3,4,*}, Yongjie Zhang ⁵, Chaohui Ma ^{1,2}, Mingxing She ^{1,2} and Min Liu ^{1,2}

¹ Poly Changda Engineering Co., LTD, Guang Zhou, Guangdong 510620, China;

² College of Highway, Chang'an University, Xiaan 710064, China

³ College of Civil Engineering, Nanjing Forestry University, Nanjing 210037, Jiangsu, China

⁴ China Railway Bridge & Tunnel Technologies Co., LTD., Nanjing 210061, Jiangsu, China

⁵ Guangdong Transportation Planning, Design and Research Institute Group Co., Ltd, Guangzhou 510507, China;

* Correspondence: nldfh@njfu.edu.cn

Abstract: As the general acceptance of fiber-reinforced polymer (FRP) in the concrete construction industry continues to rise, it is increasingly imperative that the crack width of FRP reinforced concrete members of pavement and girder be predicted with good accuracy due to strengthening of RC pavement, slabs or beams structures using FRP is known to control crack width. However, no specific provisions are provided by most international design codes for predicting crack width in FRP reinforced concrete flexure members. In this paper, five formulas for predicting the crack width for FRP reinforced structures are derived from three common computational theories which including statistical method, comprehensive theory, and bond-slip theory. The formula of statistical method is simplest, but the parameters depend on much sample data. The comprehensive and slippage theory based on semi-empirical statistical crack spacing are obtained through the calculation of non-uniformity coefficient of strain. The comprehensive and slippage theory based on semi-empirical axial force balance are obtained through the calculation of tensile reinforcement factor. The comparative analysis of performance of five formulas is carried out. The results indicate that the formula from comprehensive theory based on semi-empirical statistical crack spacing is recommended for predicting the crack width of FRP reinforced concrete flexural structure.

Keywords: highway structure; concrete pavement; concrete girder; crack width; FRP; cracks spacing; comprehensive theory

1. Introduction

Crack control is an important serviceability criterion in the design of RC highway structures including girder and pavement structure. Cracking of concrete is related to its limited tensile deformation capacity, and is usually expected to develop in RC members under service loads during the lifetime of the structure. However, there should be some control over the width and distribution of structural cracks in order to maintain the aesthetics of the structure, protect the internal steel reinforcement especially when subjected to aggressive environments, and reduce the risk of debonding of the FRP external reinforcement.

The application of externally bonded FRP reinforcements for the flexural strengthening of RC members is becoming popular worldwide. The purpose of FRP strengthening is to increase the ultimate capacity or improve service conditions. Generally, RC members strengthened with FRP laminates have smaller and closely spaced cracks compared with the unstrengthened beams and pavement structure, either considering the same service load or considering the same tension level in the steel. This is due to the additional tension stiffening of the reinforcement that reduces the crack spacing.

The large number of experimental tests on RC elements strengthened by externally bonded FRP laminates and sheets available in the literature focused on studying the flexural behavior at ultimate

conditions [1–13]. Only limited studies dealt with predicting the crack width of FRP-strengthened beams [14–17]. In this paper, formulas for predicting the crack width in flexure RC members from leading international codes and design guidelines are presented. Five formulas for predicting the crack width for FRP reinforced structures are derived from three common computational theories which including statistical method, comprehensive theory, and bond-slip theory. The resulting prediction formulas are evaluated by comparison with the results from the conducted experimental data. In addition, the recommended formula for predicting the crack width for FRP reinforced structures is given.

2. Conventional methods

At present, there are many methods for calculating the width of structural cracks in different codes. But there is rarely special recommended formula for predicting the crack width for FRP reinforced structures. The conventional solution is to make FRP equivalent to steel bar through elastic modulus ratio.

A total of 98 components from references [17–29] were selected and a comparison of methods form eight specifications is carried out. The details of predicting the crack width for FRP reinforced structures form above references are shown in Table 1. The calculation results of predicting the crack width for FRP reinforced structures from different codes are shown in Figure 1.

Table 1. Collection of different test results.

Ref.	Test number	Crack width	Ref.	Test number	Crack width	Ref.	Test number	Crack width
Huang [17]	1	0.3	Wu [18]	one layer -1	0.07	Feng [20]	BF I-2-1	0.10
	2	0.3		one layer -2	0.18		BF I -3-1	0.14
	4	0.3		three layer -1	0.05		BF I -4-1	0.11
	6	0.3		three layer -2	0.09		BF I -2-2	0.24
	8	0.3	You [19]	CFRP-1-34	0.18		BF I -3-2	0.20
	9	0.3		CFRP-2-48	0.17		BF I -4-2	0.16
	13	0.3		CFRP-3-53	0.14		BF I -2-3	0.40
	15	0.3		GFRP-1-31	0.19		BF I -3-3	0.27
Wu [21]	CL25-2C-8.4	0.06		GFRP-2-37	0.1		BF I -4-3	0.21
	CL25-2G-8.5	0.14		GFRP-3-44	0.1	Hu [24]	BM0	0.09
	CL25-2C-12	0.16	Ahmed [23]	ST-S-1	0.14	Zhang [22]	JL1-30	0.05
	CL25-2G-10.8	0.22		ST-S-2	0.23		JL2-30	0.05
Liew [26]	AB-G1-1m	0.21	Alam [25]	C1	0.16		JL3-30	0.05
	AB-G1-3m	0.2		C2	0.14		YL1-30	0.18
	AB-G1-6m	0.18		C3	0.28		YL2-30	0.10
	AB-G1-1y	0.17	Monica [27]	CFB_01_60	0.1		JL1-40	0.08
	AB-G2-1m	0.13		AFB_01_60	0.1		JL2-40	0.05
	AB-G2-3m	0.09		CFB_02_60	0.06		JL3-40	0.08
	AB-G2-6m	0.07		AFB_02_60	0.1		YL1-40	0.23
	AB-G2-1y	0.1		GFB_02_60	0.1		YL2-40	0.13
	OB-G1-1m	0.25		CFB_01_100	0.4		JL1-50	0.10
	OB-G1-3m	0.17		AFB_01_100	0.4		JL2-50	0.10
	OB-G1-6m	0.1		CFB_02_100	0.15		JL3-50	0.10
	OB-G1-9m	0.15		AFB_02_100	0.2		YL1-50	0.25
	OB-G1-1y	0.11		GFB_02_100	0.2		YL2-50	0.15
	CB-G1-6m	0.18	Abdulaziz [28]	PL-1-0.6	0.24	Hua [29]	CW1	0.35
	CB-G1-1y	0.09		PL-1-0.3	0.35		CW2	0.38

OB-G2-1m	0.18	PL-2-0.6	0.36	CW3	0.25
OB-G2-3m	0.1	S-3-0.2	0.39	DW1	0.43
OB-G2-6m	0.16	PL-1-0.6-1	0.2	DW2	0.41
OB-G2-9m	0.14	PL-1-0.3-1	0.2	DW3	0.3
OB-G2-1y	0.08	PL-2-0.6-1	0.25		
CB-G2-6m	0.08	S-3-0.2-1	0.24	/	
CB-G2-1y	0.06	/			

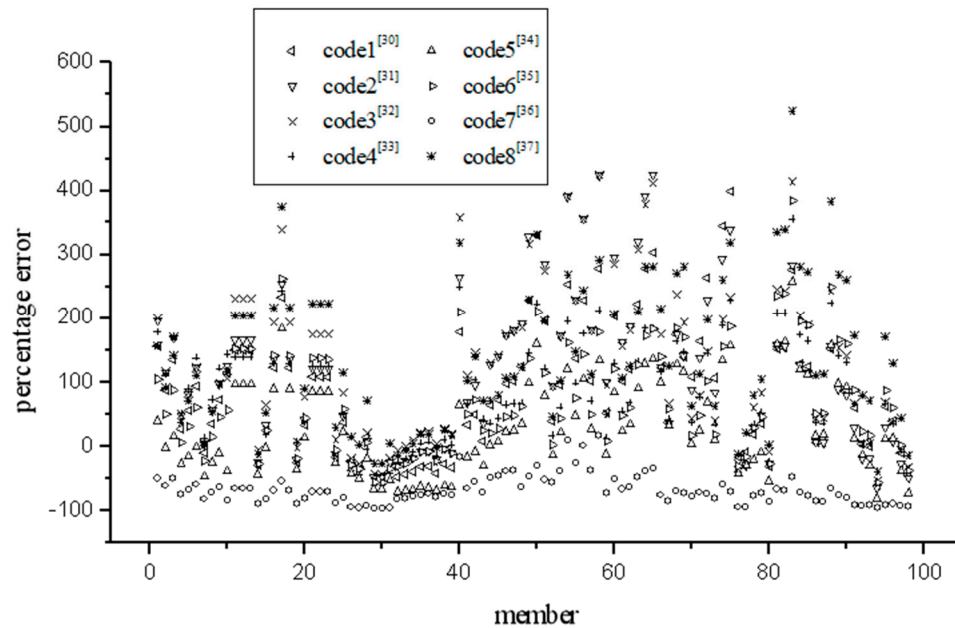


Figure 1. Comparison of different specifications and test results.

The Figure 1 shows that the conventional calculation methods of codes are not very suitable to FRP reinforced structure. The results of predicting the crack width for FRP reinforced structures have a highly discrete feature. The reason is that calculations of crack width are used semi-empirical models and many parameters are derived from the test or statistical results of actual structure without FRP. Therefore, it is necessary to investigate a reasonable method of predict the crack width for FRP reinforcement structure.

3. Statistical methods

Statistical analysis is a common method to deal with crack problems. It is assumed that the total crack width is consisted of two parts which is initial crack width before reinforcement and additional crack width. The total width w can be written as

$$w = w_0 + \Delta w \quad (1)$$

where w_0 is initial crack width and Δw is additional crack width.

The initial crack width is given by

$$w_0 = C_1 C_2 \frac{\sigma_0}{E_s} \left(\frac{30+d}{0.28+10\rho} \right) \quad (2)$$

where σ_0 is stress of steel bar, ρ is steel ration, E_s is elastic modulus of steel bar, d is diameter, C_1 is shape factor of steel bar, C_2 is structural stress characteristic coefficient.

In a similar way, it can be assumed that the value of additional crack width has the same expression as

$$\Delta w = C_1 C_2 \frac{\Delta \sigma}{E_s} \left(\frac{a_1+d}{a_2+a_3\rho_e} \right) \quad (3)$$

where ρ_e is equivalent ration after FRP reinforcement, $\Delta\sigma$ is increased stress after FRP reinforcement, a_1 , a_2 and a_3 are parameters to be determined.

The parameters z_{exp} and z are defined as follows

$$z = \frac{w_0^{exp} \Delta\sigma}{C_1 C_2 E_s \exp} \quad (4)$$

$$z = \frac{a_1 + d}{a_2 + a_3 \rho_e} \quad (5)$$

where w_{exp} is the measured crack width from experiments.

The parameters a_1 , a_2 , and a_3 are the most appropriate values that minimize the formula $\sum(z - z_{exp})^2$. The fitting parameters are determined by the data measured from the references. The final formula is as follows

$$w = C_1 C_2 \frac{\sigma_{s0}}{E_s} \left(\frac{30+d}{0.28+10\rho_e} \right) + C_1 C_2 \frac{\Delta\sigma_s}{E_s} \left(\frac{1.1+d}{0.14-1.98\rho_e} \right) \quad (6)$$

The comparison between the calculation formula and the references results is shown in Figure 2 where

$$\Delta w = w - w_0 \quad \text{and}$$

$\Delta w_{exp} = w_{exp} - w_0$. It can be seen from Figure 2 that the fitting results depend highly on the test sample size. If the sample size increases, the fitting results are more accurate.

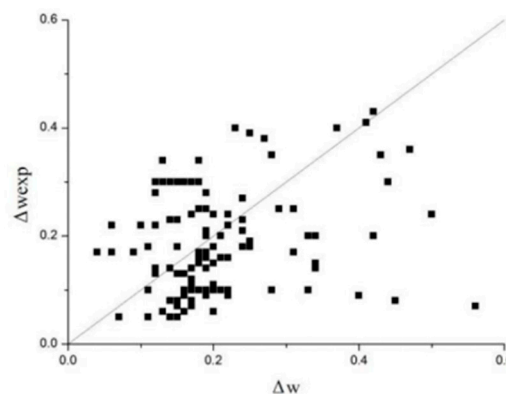


Figure 2. Comparison between statistical method and measured value of experiment.

4. Semi-empirical theory based on statistical crack interval

4.1. Crack interval based on statistical approaches

Above result has shown that there was a positive correlation between crack spacing and cover thickness, and between crack spacing and ration of reinforcement diameter to reinforcement ratio. It is assumed that the crack spacing can be a formula as follows

$$l_m = a_1 c + a_2 \frac{d}{\rho} \quad (7)$$

where c is thickness of concrete cover, a_1 and a_2 are the calculation parameters that can be obtained by least-squares fitting procedure through the experimental data, and a_1 is 1.46, and a_2 is 0.03.

4.2. Non-uniformity coefficient of strain

Concrete is not ideal homogeneous material and the problem will be very complicated if the stress is too much focused at each point. Using average stress is an efficient method. Non-uniformity coefficient of strain of reinforced concrete structure can be written as

$$\psi_s = \frac{\bar{\varepsilon}_s}{\varepsilon_s} \quad (8)$$

where $\bar{\varepsilon}_s$ is average strain of reinforcement, ε_s is steel strain of fracture section.

Similarly, non-uniformity coefficient of strain of FRP concrete was defined by the following expression.

$$\psi_f = \frac{\bar{\varepsilon}_f}{\varepsilon_f} \quad (9)$$

where $\bar{\varepsilon}_f$ is average strain of FRP, ε_f is FRP strain of fracture section.

For reinforced concrete members, the stress of steel bar is very high at the crack, going down to both sides. Because of the characteristic of bonding failure, the bonding stress is zero at the crack. So it is assumed that the stress of steel bar between two cracks can be written as

$$\sigma_s(x) = A_s \cos \frac{2\pi x}{l_m} + B_s \quad (10)$$

where l_m is crack spacing, x is the distance from the crack to any point between two cracks, A_s and B_s are calculation parameters.

Average stress of steel bar between cracks can be written as

$$\bar{\sigma}_s = \int_0^{l_m} (A_s \cos \frac{2\pi x}{l_m} + B_s) dx / \frac{l_m}{2} = B_s \quad (11)$$

And

$$\sigma_s(0) = A_s + B_s \quad (12)$$

$$\sigma_s(\frac{l_m}{2}) = -A_s + B_s \quad (13)$$

So

$$\bar{\sigma}_s = \frac{\sigma_s(0) + \sigma_s(\frac{l_m}{2})}{2} \quad (14)$$

Obtain

$$\psi_s = \frac{\bar{\varepsilon}_s}{\varepsilon_s} = \frac{\bar{\sigma}_s}{\sigma_s} = \frac{\frac{\sigma_s + \sigma_s^c}{2}}{\sigma_s} = \frac{\sigma_s A_s + \sigma_s^c A_s}{2\sigma_s A_s} \quad (15)$$

In the same way, it can be drawn for FRP structure

$$\psi_f = \frac{\sigma_f A_f + \sigma_f^c A_f}{2\sigma_f A_f} \quad (16)$$

where σ_f^c is stress of FRP between two cracks.

The tensile force between the cracks is shown in Figure 3.

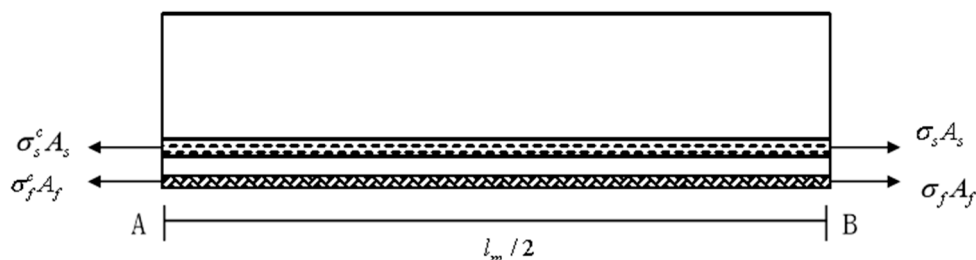


Figure 3. The tensile force between the cracks.

According to the principle of axial force balance, there is a formula as

$$\sigma_f^c A_f + \sigma_s^c A_s = \sigma_f A_f + \sigma_s A_s - F \quad (17)$$

There is an assumption

$$\psi = \psi_s = \psi_f = \frac{(\sigma_s A_s + \sigma_f A_f) + (\sigma_s^c A_s + \sigma_f^c A_f)}{2(\sigma_s A_s + \sigma_f A_f)} \quad (18)$$

Substitute Eq. (17) to Eq. (18), obtain

$$\psi = 1 - \frac{F}{2(\sigma_s A_s + \sigma_f A_f)} \quad (19)$$

There is an equilibrium condition

$$F = 0.5 f_t b (h - x) \quad (20)$$

So

$$\psi = 1 - \frac{0.5 f_t b (h - x)}{2(\sigma_s A_s + \sigma_f A_f)} \quad (21)$$

4.3. Calculation of fracture width by comprehensive theory

There is an assumption that the crack is a triangular shape, and the width reaches maximum on the concrete surface and zero at the neutral axis, which is shown in Figure 4.

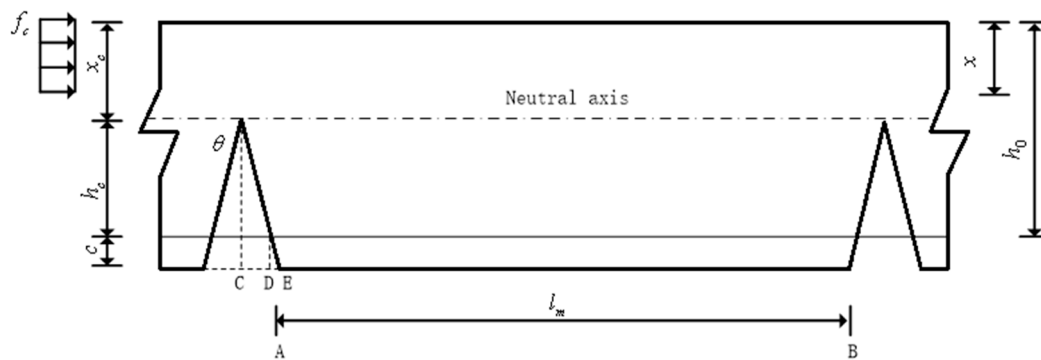


Figure 4. Crack development diagram.

The crack width at the bottom of the beam and pavement structure consists of two parts, CD and DE , where $CD = \frac{l_m}{2} \bar{\epsilon}_s$,

$$DE = \theta c = \frac{l_m \bar{\epsilon}_s}{\frac{2}{h_c}} c. \text{ The average width of crack can be written as}$$

$$w = 2 \left(\frac{l_m}{2} \bar{\epsilon}_s + \frac{l_m \bar{\epsilon}_s}{\frac{2}{h_c}} c \right) = l_m \bar{\epsilon}_s \left(1 + \frac{c}{h_c} \right) = l_m \frac{\bar{\sigma}_s}{E_s} \left(1 + \frac{c}{h_c} \right) \quad (22)$$

where x_c is concrete compression height, x is relative height of compression zone.

There is a relationship between compression height and relative height of compression zone for C50 concrete in normal structure where $x=0.8 x_c$. The formula can be written as

$$h_c = h_0 - x_c = h_0 - \frac{2(h_0 - 0.87h_0)}{0.8} = 0.675h_0 \quad (23)$$

The average width of crack can be written as

$$\bar{w} = l_m \frac{\bar{\sigma}_s}{E_s} \left(1 + \frac{c}{0.675h_0} \right) \quad (24)$$

The maximum width of the crack can be written as

$$\begin{aligned}
 w &= \alpha_s \bar{w} &= \alpha_s \psi \frac{\sigma_s}{E_s} l_m \left(1 + \frac{c}{h_c}\right) &= 1.07 \psi \frac{\sigma_s}{E_s} \left(1.46c + \right. \\
 &0.03 \frac{d_e}{\rho_e} \left. \right) \left(1 + \frac{c}{h_c}\right) && (25)
 \end{aligned}$$

where α_s is expanded coefficient that is determined by least square method.

The comparison between the calculation formula and the references results is shown in the Figure 5. It can be seen form Figure 5 that the fitting results depend highly on the test sample size. If the sample size increases, the fitting results are more accurate.

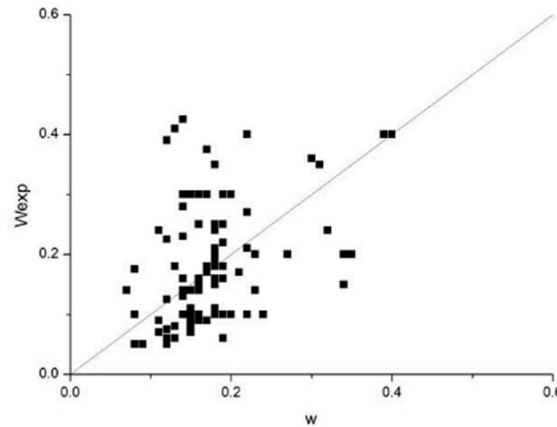


Figure 5. Comparison between calculated and measured value of experiment.

4.4. Calculation of fracture width by slippage theory

Similar to reinforced concrete structure .It is assumed that the development of cracks is due to relative slippage between FRP and concrete on the bottom of the beam and the pavement structure.

The average width of crack can be written as

$$\bar{w} = \bar{\epsilon}_f l_m \quad (26)$$

The maximum width of the crack can be written as

$$w = \alpha_f \bar{w} = \alpha_f \psi \frac{\sigma_f}{E_f} l_m = 1.04 \psi \frac{\sigma_f}{E_f} \left(1.46c + 0.03 \frac{d_e}{\rho_e}\right) \quad (27)$$

where

σ_f is expanded coefficient that is determined by least square

method, the value of

σ_f is assumed to be 1.04.

5. Semi-empirical theory based on bond stress theory

5.1. Crack spacing based on axial force balance

According to axial force balance theory in crack prediction, there is an equation that can be written as

$$f_t b h_{c,eff} = \frac{l_m}{2} u_s \tau_s + \frac{l_m}{2} b_f \tau_f \quad (28)$$

where

f_t is compressive strength of concrete, b is width of section,

$h_{c,eff}$ is effective tension height, τ_s and

τ_f is bond stress of steel bar and FRP, u_s

and

u_f is perimeter of steel bar and FRP. And

$$l_m = \frac{2f_t h_{c,eff} b}{\tau_s u_s + \tau_f b_f} \quad (29)$$

$$\tau_s \approx 1.8f_t \quad (30)$$

$$\tau_f \approx 1.25f_f \quad (31)$$

Crack spacing can be expressed as

$$l_m = \frac{2bh_{c,eff}}{1.8u_s + 1.25b_f} \quad (32)$$

5.2. Tensile reinforcement factor ζ

When the tension of the reinforced concrete member beyond the specified limitation, the concrete is cracking. The cracking section of the concrete is out of work, but the concrete between cracks can still bear tension. Tensile reinforcement factor is as follows

$$\zeta = 1 - \beta \frac{M_{cr}}{M} \quad (33)$$

where M is bending moment caused by external load,
 M_{cr} is crack load, β is shape coefficient.

5.3. Calculation of fracture width by comprehensive theory

In the same way as described in section 3.3, the maximum width of the crack can be written as

$$w_{\max} = \alpha_s \zeta \frac{\sigma_s}{E_s} l_m \left(1 + \frac{c}{h_c}\right) = 1.47 \zeta \frac{\sigma_s}{E_s} \left(\frac{2bh_{c,eff}}{1.8u_s + 1.25b_f}\right) \left(1 + \frac{c}{h_c}\right) \quad (34)$$

where α_s is expanded coefficient that is determined by least square method, the value of α_s is assumed to be 1.47.

The comparison between the calculation formula and the references results is shown in Figure 6. It can be seen form Figure 6 that the calculation formula agrees with the experimental data well, which can be used in prediction of crack width in FRP reinforcement structure.

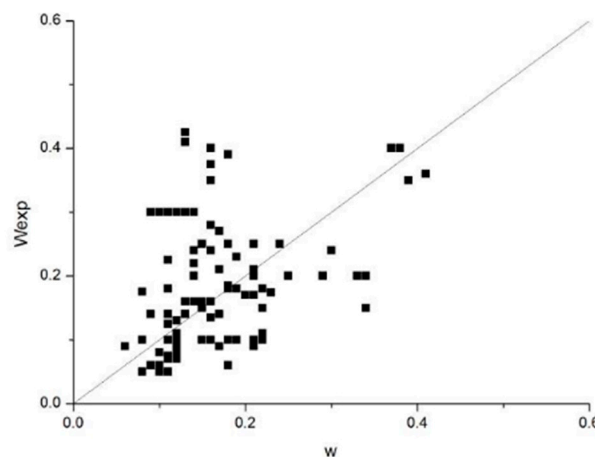


Figure 6. Comparison between calculated and measured value of experiment.

5.4. Calculation of fracture width by slippage theory

In the same way as described in section 3.3, the maximum width of the crack can be written as

$$w_{\max} = \alpha_f \zeta \frac{\sigma_f}{E_f} l_m = 1.41 \zeta \frac{\sigma_f}{E_f} \left(\frac{2bh_{c,eff}}{1.8u_s + 1.25b_f} \right) \quad (35)$$

where α_f is expanded coefficient that is determined by least square method, the value of α_f is assumed to be 1.41.

The comparison between the calculation formula and the references results is shown in Figure 7. It can be seen from Figure 7 that the calculation formula agrees with the experimental data well, which can be used in prediction of crack width in FRP structure.

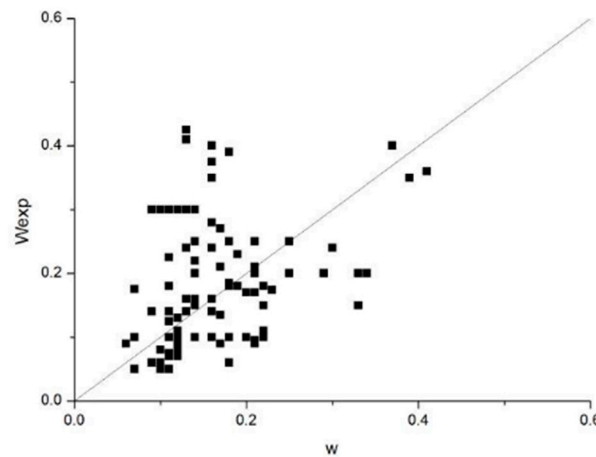


Figure 7. Comparison between calculated and measured value of experiment.

6. Comparison of different formulas

There are three computational theories of crack width in FRP structure, based on which five formulas are derived. These formulas are applied to the above model tests, the comparison and analysis of calculation results are shown in Figure 8. We can see from Figure 8 that (1) the results of different methods are very close except statistical calculating method; (2) distribution of errors is very similar, mostly concentrated at $\pm 50\%$. Although the statistical calculating method is the closest to the measured value, the calculation parameters will depend on the sample size, which not very accurate in theory, but just as a simple way to deal with the problem.

In general, the other four formulas are all available for predicting of crack width for FPR structure. Using trigonometric function to simulate the stress unevenness coefficient is more meaningful than axial force balance in point view of theory. Meanwhile, comprehensive theory covers more factors than slippage theory. Therefore, the equal (25) of prediction of crack width by comprehensive theory based on statistical crack interval is recommended as the most suitable calculation method.

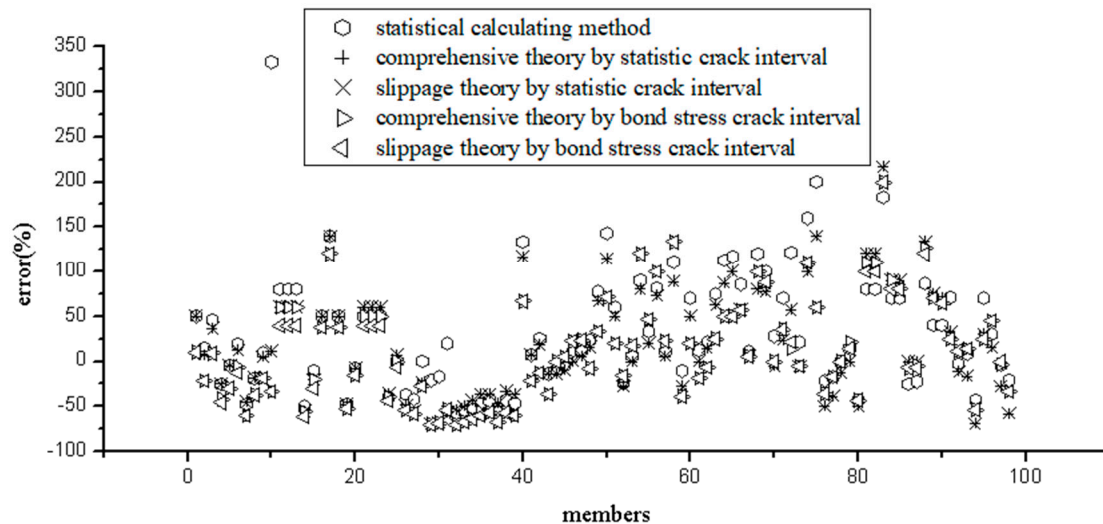


Figure 8. Comparative analysis of different formulas.

7. Conclusions

This paper presents five different formulas based on three common theories which including statistical method, comprehensive theory, and bond-slip theory. The calculating parameters in the formulas come from the test results of different resences. The applicability performance of five formulas is compared. The following conclusions can be obtained.

(1) The statistical formula is the simplest, the advantage of which has less parameters and is easily applied to predict crack width in FRP reinforced concrete flexure members. However, different test results have great influence on its own formula parameters and the tests on FRP cracks are very limited. This method can only be used as a simple method to solve problems of cracking in FRP reinforced concrete flexure members.

(2) Analysis of crack spacing based on axial force balance is not very accurate in point view of theory because of ignoring the effect of force distance, so the formula based on which is also not very reasonable.

(3) Comprehensive and slippage theory are both widely used in codes or specifications all over the world. There are more calculation factors involved in comprehensive theory. Therefore, the proposed formula for predict crack width in FRP reinforced concrete flexure members is recommended. In brief, the method of comprehensive theory using statistical crack spacing is recommended in predicting crack width in FRP reinforced concrete flexure members.

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